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USE OF CHAOTIC AND RANDOM VIBRATIONS TO GENERATE HIGH FREQUENCY TEST INPUTS - PART II, CHAOTIC VIBRATIONS*

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Biographies

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Abstract

This paper and a companion paper show that the traditional limits on amplitude and frequency that can be generated in a laboratory test on a vibration exciter can be substantially extended. This is accomplished by attaching a device to the shaker that permits controlled metal to metal impacts that generate a high acceleration, high frequency environment on a test surface. A companion paper (Reference 1) derives some of the mechanical relations for the system. This paper shows that a sinusoidal shaker input can be used to excite deterministic chaotic dynamics of the system yielding a random vibration environment on the test surface, or a random motion of the shaker can be used to generate a random vibration environment on the test surface. Numerical examples are presented to show the kind of environments that can be generated in this system.

Introduction

Traditional methods for laboratory vibration testing on shakers can be used to produce environments with limits that are associated with the shaker size and capacity. Generally, electrodynamic shakers can generate environments with amplitudes up to hundreds of g's (on the shaker head) and with frequency content up to 2000, 3000, or perhaps 5000 Hz.

A companion paper (Reference 1) has described a mechanical system that can be used to extend the limits of environments that can be generated on shakers. A schematic of the system is shown in Figure 1. The system is composed of two horizontal, large, flat metal plates, equal in area, that are separated from one another by relatively small space on the order of one inch. The plates are held together along all edges with elements that keep the space between the plates (at least along the edges) constant. The bottom plate is attached to the shaker head. A relatively soft elastic material is attached to the top of the lower plate and the space between the top of the soft elastic material and the bottom of the upper plate is divided into compartments. The size of each compartment is approximately one half inch on a side. A steel sphere is placed in each compartment. The size of the steel sphere must be compatible with the compartment size and the space between the elastic material above the lower plate and the bottom of the top plate.

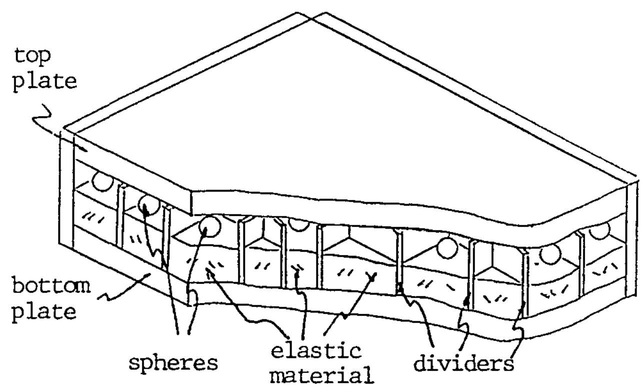

Figure 1. Schematic of experimental dynamic testing system.

When a vertical motion is induced in this system by movement of the shaker head, motions on the top surface of the top plate are also induced. The top surface of the top plate is the test surface. The motion of the test surface is composed of three component parts. The first component of motion is related to the dilatational shock waves that travel through the plate when a sphere strikes the underside of the top plate. The second component of motion is the part induced by flexural response of the top plate responding to impacts that occur when the metal spheres hit the underside of the top plate. The third component of motion is induced by the motion at the perimeter of the top plate. The dynamics of all three of these types of motion were investigated in Reference 1 Specifically, Reference 1 established the impulse response functions and frequency response functions of the three components of motion.

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In this paper we consider (1) the dynamics of the elastic sphere moving between the soft elastic material attached to the top of the lower plate and the top plate, and (2) the spectral density of motion on the top surface of the top plate related to the three components of motion. We consider two types of motion excited by the vibration exciter. First, we consider sinusoidal motion, and then we consider random motion. We show that when the vibration exciter is used to excite a sinusoidal input, the response of the sphere can be made chaotic. When the response of the sphere is chaotic it strikes the upper plate at times whose intervals are unpredictable. This creates a motion in the upper plate, two of whose components are quasi-random.

The second input generated with the vibration exciter is a stationary random process. A stationary random process causes the motion of the sphere to also be a stationary random process. When the motion of the sphere is a stationary random process it impacts the upper plate at random times and the forces generated when the sphere impacts the upper plate form a shot noise type random process.

Numerical examples are presented which demonstrate the typical behavior of the experimental dynamic testing system. It is shown that when the vibration exciter is used to generate very nominal environments, the component of response due to shockwave generation is very substantial. Specifically, low displacement, very high acceleration and high frequency environments are generated. For example, environments with frequency content well beyond 10,000 Hz and with rms acceleration beyond 100,000 g's are generated.

Description of the System

One aspect of the experimental dynamic testing system to be analyzed in this paper is the motion of a sphere between the top plate and the elastic material attached to the bottom plate. The dynamic model for this system is shown in Figure 2. In that system the sphere has mass m and is assumed to move vertically between a massless base plate, atop a spring with stiffness k and damper with damping value c, and the upper plate. The distance between the top of the sphere and the upper plate when the system is in static equilibrium is denoted d. The distance between the upper plate and lower plate is assumed to be held fixed. This is clearly an approximation since both the upper and lower plates are flexible, but it is a good approximation because the motion of the upper and lower plates is small relative to the motion of the sphere. The motion of the sphere is denoted by z(t), where z(t) is the absolute displacement of the sphere. The motion of the plate system is denoted by x(t).

The equation of motion of the sphere can always be given by

$$\ddot{y} = -\frac{1}{m} R(y,\dot{y}) + \ddot{x}, \qquad (1)$$

Where y=z-x, and dots denote differentiation with respect to time. $R(y,\dot{y})$ is the restoring force for the system. Two forms for the restoring force and two forms of excitation will be considered in the following. Other forms of the restoring force will also be discussed.

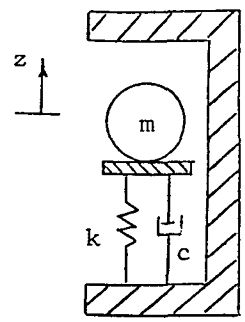

Figure 2. Model of the system to be analyzed.

Chaotic Vibrations of a Fundamental System

The first system to be be considered has the displacement restoring force shown schematically in Figure 3 and sinusoidal excitation. The equation of motion for the system is given by

$$\ddot{y} + \frac{c}{m}\dot{y} + \frac{k}{m} y = -X_0 \sin(\omega_f t), \quad y \le 0 \qquad (2)$$

where X_0 is the amplitude of the acceleration input and ω_f is its frequency (in units of rad/sec).

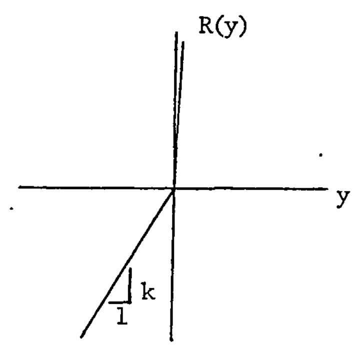

Figure 3. Restoring force of the first system.

In this system the space between the top of the sphere and the bottom of the upper plate in static equilibrium is set to zero. It is assumed that the spring and the damper, below the sphere behave linearly, and that the collisions between the sphere and the upper plate are elastic.

References 2 through 6 consider the nonlinear vibrations of this system in great detail Reference 2 recommends that the vibrations of this system be considered in a normalized form. It is recommended that Equation 2 be nondimensionalized in the following way. First, define the period of the system as

$$T = \pi\sqrt{(m/k)}, \qquad (3)$$

This corresponds to a system natural frequency defined

$$\omega = 2\pi/T = \sqrt{(K/m)}, \qquad (4)$$

Where K is the equivalent system stiffness and is given by

$$K = 4k, \qquad (5)$$

A critical damping for the system can be defined

$$c_c = 2m\omega, \qquad (6)$$

and the system damping factor is

$$\zeta = c/c_c = c/2m\omega, \qquad (7)$$

Now, a ratio of input forcing frequency to natural frequency for the system can be defined as

$$\eta = \omega_f/\omega, \qquad (8)$$

The displacement response can be nondimensionalized by dividing through by the quantity mX_0/K. This yields

$$Y = \frac{y}{(mX_0/K)}, \qquad (9)$$

Further, the time base of the system can be nondimensionalized by defining

$$\tau = \omega_f t, \qquad (10)$$

With these changes the equation of motion becomes

$$Y'' + 2\frac{\zeta}{\eta}Y' + \frac{1}{\eta^2}Y = \frac{1}{\eta^2}\sin\tau, \quad Y \le 0, \qquad (11)$$

where a prime denotes differentiation with respect to the variable τ.

It is shown References 2 through 6 that chaotic vibrations occur in this system at about $\eta = 4.5$. This implies that when the forcing frequency is given by the value

$$\omega_f = 4.5\,\omega = 9\sqrt{(k/m)}, \qquad (12)$$

chaotic vibrations occur in the experimental system. Some chaotic responses of the normalized system were calculated. This was accomplished using a Runge-Kutta fourth order approach, with variable step size. (See Reference 7.) The results of one calculation that show the nondimensionalized relative displacement response of the system over a period of nondimensionalized time are shown in Figure 4.

Several things are apparent from this short time segment of response. First, the response appears random because the displacement does not follow any set pattern, however, the input is not random and none of the system parameters are random. Therefore, the response cannot be considered to be truly random. This is a characteristic of chaotic response. This type of behavior is sometimes called quasi-random. Second, the response appears to be limited in amplitude and, indeed, being a chaotic response, it is. Third, the impacts occur at unpredictable times with unpredictable velocities.

References 2 through 6 characterize the behavior of this system using standard measures applied in chaotic vibration analysis. For example, they present measures of the system gain for various values of η (forcing frequency to system equivalent natural frequency ratio). They present time histories of the response, and they present Poincare maps. However, they do not present two aspects of the system response which are very important in the present application, namely, the characteristics of impact time intervals and impact velocities.

We used our own numerical analyses to investigate these quantities. We found that the average impact time interval for the nondimensionalized response, $Y(\tau)$, is 20 (time units). This implies that the average impact rate for the actual response, $y(t)$, is

$$\lambda = 0.05\,\omega_f, \qquad (13)$$

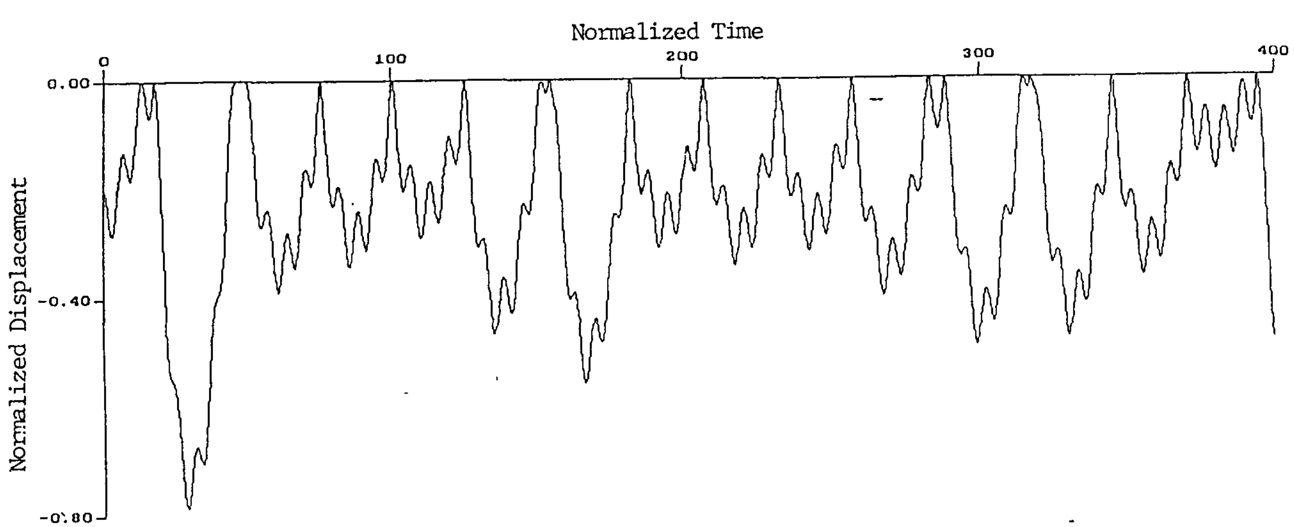

Figure 4. Segment of chaotic response of the system shown in Figure 2. Relative displacement and time are nondimensionalized. $\eta=4.5$, $\zeta=0.1$

This is the average frequency of impacts of the chaotically vibrating system. Further, our investigations showed that the mean and standard deviation of the nondimensionalized impact velocity are 0.018 and 5×10^{-4} (displacement unit)/(time unit), respectively. This implies that the mean and variance of the actual impact velocity for the system in chaotic vibration are

$$\mu_V = 0.365\ X_0/\omega_f, \quad \sigma_V = 0.453\ X_0/\omega_f, \tag{14}$$

The V subscript denotes impact velocity. The units on both these quantities depend on the units of X_0 and might be, for example, in/sec.

The impact time intervals of the response were plotted on exponential probability paper to establish whether or not they might have an exponential probability distribution. The results indicate that the interarrival times of the impacts do not follow an exponential distribution.

In summary, the results presented here imply that a wide range of chaotic impact behaviors can be established for this system. Equation 12 shows that when the system mass and stiffness are chosen, the frequency of the input that causes chaos is established. Equation 13 shows that the frequency of impacts is about one twentith the input frequency. Equation 14 shows that the mean and the variance of the impact velocity can be set simply by varying the amplitude of the input motion X_0.

Random Vibration of the Experimental Dynamic Testing System

When the experimental system is excited with a random input, the motion of the sphere is a random process. We excited the nondimensionalized system with a stationary white noise random process whose root-mean-square (rms) value is equal to the rms of the sinusoidal input in the previous section. Specifically, we replaced the term, sin τ, in Equation 11 with a term $r(\tau)$, where $r(\tau)$ is a white noise random process with with rms value $1/\sqrt{2}$. The system response was again computed using the Runga-Kutta, variable step size algorithm, and a realization of the response random process is shown in Figure 5.

The character of this response is somewhat different from that shown in Figure 4, the chaotic response. In theory, the random response is unbounded, though the response of actual systems is obviously always bounded. As in the chaotic response, the impact times are unpredictable, and in this case, they are actually random. The impact velocities are also random. The mean rate of impacts in the randomly driven system is

$$\lambda = 0.035\ \omega_f, \tag{15}$$

That is, the rate of impact is lower than in the chaotically driven system. The mean and standard deviation of impact velocity are

$$\mu_V = 0.541\ X_0/\omega_f, \quad \sigma_V = 1.07\ X_0/\omega_f, \tag{16}$$

On the average, the impacts of the randomly driven system are more severe and have greater variability. The correlation between impact time intervals and impact velocities was calculated in both cases and shown to be very small, essentially zero. Further, the correlation between consecutive impact times was calculated in both cases and shown to be approximately zero.

The impact time intervals of the response were plotted on exponential probability paper to establish whether or not they might have an exponential probability distribution. The results indicate that the interarrival times of the impacts are exponentially distributed in this case. This indicates that the impact force random process is a shot noise random process. (See Reference 8.)

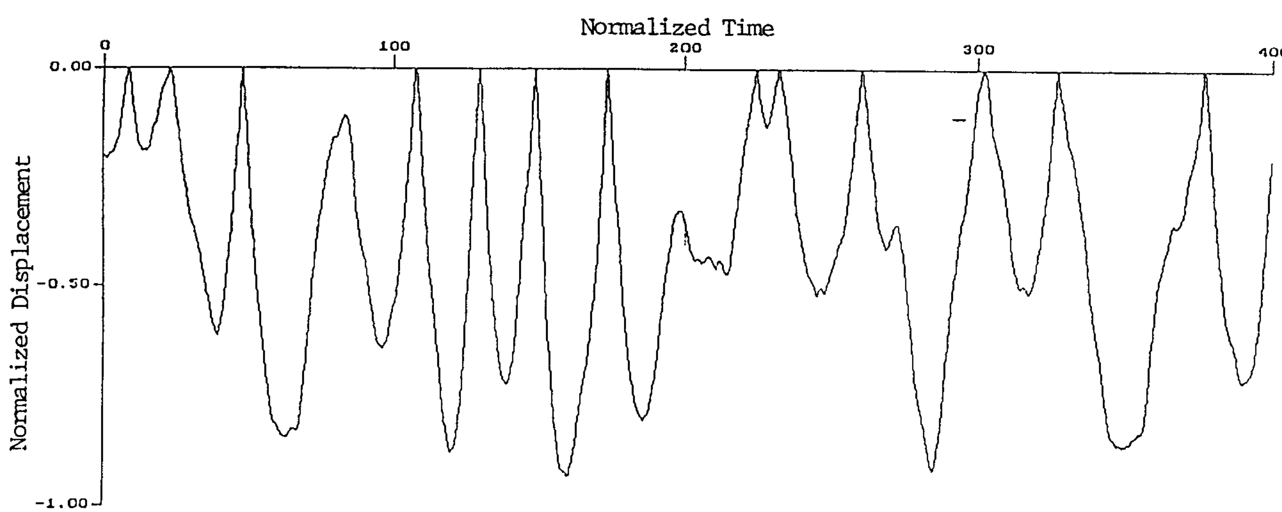

Figure 5 Segment of random vibration response of the system shown in Figure 2. Relative displacement and time are nondimensionalized. $\eta=4.5$, $\zeta=0.1$

Chaotic and Random Vibrations of Other Systems

The behavior of some other systems were evaluated for comparison to those shown in the previous two sections. The first system considered was one in which the gap between the top of the sphere and the bottom of the upper plate in static equilibrium was set at some small value. It was shown when the excitation provided by the vibration exciter was sinusoidal, chaotic modes of vibration do occur in the system.

The random vibration response of the same system was also computed. Again, the random input was taken to be a stationary random process whose spectral density is white. Response calculation showed that the random vibration response is similar to the chaotic response.

Other systems with nonlinear base spring restoring forces were investigated, and it is believed that chaotic type responses occur with this type of system, also.

Spectral Density of Dynamic Test System Motion

It was shown in Reference 1 that the motion at the surface of the test system (atop a location where a sphere impacts) is composed of three components. These three components are (1) the shock wave component related to dilatational motion excited by impact of the sphere against the bottom of the top plate, (2) the flexural motion at a point caused by impacts of spheres at multiple points on the bottom of the top plate, and (3) flexural related motion at a point on the surface of the top plate excited by motion of the vibration exciter at the base of the system. The displacement response at the surface of the top plate can be expressed

$$u(y,z,t) = \int_{-\infty}^{t} h_1(t-\tau)\, f_1(\tau)\, d\tau + \sum_{j_1}\sum_{j_2} \int_{-\infty}^{t} h_2(y,z,\xi_{j_1},\eta_{j_2},t-\tau)\, f_2(j_1,j_2,\tau)\, d\tau + \int_{-\infty}^{t} h_3(y,z,t-\tau)\, r(\tau)\, d\tau,$$

$$0 \le y \le a, \quad 0 \le z \le b, \quad -\infty < t < \infty \qquad (17)$$

where h_1, h_2, and h_3 are the displacement impulse response functions for the three components of motion developed in Reference 1. The function h_2 depends on the location where the response is analyzed, (y,z), and the location where the input is generated, (ζ_{j1},η_{j2}). The function h_3 depends only on (y,z). The functions $f_1(t)$ and $f_2(j_1,j_2,t)$ refer to forces on the bottom of the upper plate. The j_1 and j_2 refer to the location of the input force. The function $r(t)$ refers to the acceleration induced through the boundary. The quantities y, z, ξ, and η are coordinates on the surface of the plate. The plate has length a and width b.

To establish the spectral density of the structural response it is necessary to finite Fourier transform the displacement response. The finite Fourier transform of the displacement response is given by

$$U(y,z,\omega) = H_1(\omega)\, F_1(\omega) + \sum_{j_1}\sum_{j_2} H_2(y,z,\xi_{j_1},\eta_{j_2},\omega)\, F_2(j_1,j_2,\omega) + H_3(y,z,\omega)\, R(\omega),$$

$$0 \le y \le a, \quad 0 \le z \le b, \quad -\infty < \omega < \infty \qquad (18)$$

where H_1, H_2, and H_3 are system frequency response functions and are the Fourier transforms of the impulse response functions, h_1, h_2, and h_3. F_1, F_2, and R are the Fourier transforms of the inputs f_1, f_2, and r.

The next step in computing the spectral density of the response is to establish the modulus squared of the finite Fourier transform. This is established by multiplying each side of the above equation by its complex conjugate. The result is

$$|U(y,z,\omega)|^2 = |H_1(\omega)|^2\, |F_1(\omega)|^2 + \left| \sum_{j_1}\sum_{j_2} H_2(y,z,\zeta_{j_1},\eta_{j_2},\omega)\, F_2(j_1,j_2,\omega) \right|^2 + |H_3(y,z,\omega)|^2\, |R(w)|^2 + (\text{cross-terms}),$$

$$0 \le y \le a, \quad 0 \le z \le b, \quad -\infty < \omega < \infty \qquad (19)$$

The cross-terms in the above relation are not written out because they vanish in the final expression.

In order to establish the spectral density of the response at a point on the top of the plate, we take the expected value on both sides in Equation 19. Next, we multiply both sides by π/T_f, where T_f is the duration of the finite Fourier transform, and then we take the limit as $T_f \to \infty$. In our analysis, we assume that the various forces appearing in the above equations are uncorrelated. This causes the expected value of the cross terms in the above expression to vanish. This includes not only the cross terms listed explicitly, but also the cross terms that are generated when the modulus squared of the second term is computed. When the experimental system is responding either in chaotic vibrations or in random vibrations this is probably a reasonable assumption.

The spectral density of the response at a point on the top plate over a sphere is

$$S_{UU}(y,z,\omega) = |H_1(\omega)|^2\, S_{F_1F_1}(\omega) + \sum_{j_1}\sum_{j_2} |H_2(y,z,\xi_{j_1},\eta_{j_2},\omega)|^2\, S_{F_2F_2}(j_1,j_2,\omega) + |H_3(y,z,\omega)|^2\, S_{RR}(\omega),$$

$$0 \le y \le a, \quad 0 \le z \le b, \quad -\infty < \omega < \infty \qquad (20)$$

The spectral densities appearing in the first and second components of the above equation are identical because they are simply the spectral densities of the forcing function random process related to impact of a sphere on the bottom of the upper plate. (Recall that component 1 is the dilatational response to the impact of one sphere, and component 2 is the flexural response to the impacts of all spheres on the bottom of the top plate.) The spectral density appearing in the third component of the above equation is simply the spectral density of the base motion, and it is assumed that this is known because it is controlled in any experiment. In the following, we pursue the derivation of the impact force random process spectral density.

Reference 9 shows that when an elastic sphere impacts a massive elastic object the force time history is approximately

$$f_{1c}(t) = \begin{cases} A\sin(\omega_0 t), & 0 \le t \le \pi/\omega_0 \\ 0, & \text{elsewhere} \end{cases} \qquad (21)$$

where $A = 1.33\, R_0^2 \left(\frac{\rho^3}{\delta^2} \right)^{0.2} V^{1.2}$,

$$\omega_0 = \frac{0.297}{R_0(\rho\delta)^{0.4}} V^{0.2}$$

where R_0 is the radius of the sphere, ρ is the mass density of the materials, V is the impact velocity, and $\delta=(1-\nu^2)/E\pi$ is a parameter for a material with Poisson's ratio ν and modulus of elasticity E. Therefore, the impact force random process can be represented

$$f_1(t) = \sum_{j=1}^{\infty} f_{1cj}(t-T_j), \quad 0 \le t < \infty \qquad (22)$$

where the j subscript on $f_{1cj}(t-T_j)$ indicates the fact that the amplitude, A, in Equation 21 is a random variable, A_j, j=1,2,3,..., in the impact force random process, and the T_j, j=1,2,3,..., are the random variables denoting impact times for the sphere on the upper plate. The finite Fourier transform of this random process is

$$F_1(\omega) = \sum_{j=1}^{N_{T_f}} \exp(-i\omega T_j)\, F_{1cj}(\omega), \quad -\infty<\omega<\infty \qquad (23)$$

Where $F_{1cj}(\omega)$ is the finite Fourier transform of Equation 21, and the exponential is included to show that the Fourier transforms of the individual pulses simply equal the Fourier transform of Equation 21 times a phase shift. The limit on the above sum is denoted N_{Tf}. N_{Tf} is a random variable equal to the number of impacts in the time interval $(0,T_f)$.

The spectral density of the impact force random process is the limit as $T_f \to \infty$, of π/T_f times the expected value of the modulus squared of $F_1(\omega)$. This is

$$S_{F_1F_1}(\omega) = \lim_{T_f\to\infty} E[N_{T_f}]\, E[|F_{1cj}(\omega)|^2], \quad -\infty<\omega<\infty \qquad (24)$$

To establish this expression it is assumed that (1) the number of impacts in the time interval $(0,T_f)$ is independent the random impact amplitudes, and (2) the interarrival times of the impacts are independent random variables.

To evaluate the second expected value in Equation 24 it is necessary have an expression for the modulus squared of $F_{1cj}(\omega)$. This is

$$|F_{1cj}(\omega)|^2 = \frac{A_j^2}{(2\pi)^2} \left(\frac{\sin\frac{\pi}{2\omega_0}(\omega-\omega_0)}{(\omega-\omega_0)} - \frac{\sin\frac{\pi}{2\omega_0}(\omega+\omega_0)}{(\omega+\omega_0)} \right)^2, \quad -\infty<\omega<\infty \qquad (25)$$

The above expression is a random variable because it depends on the impact velocity V, which is a random variable. The form of the dependence is given in the expressions following Equation 21, and it is apparent that the dependence is quite complex.

There are several approaches that might be taken to estimate the mean value of this complicated function of the random variable V, but a direct (and usually quite accurate) method is to write a Taylor series expansion for the function in the variable V, truncate this expression following the quadratic terms, then take the expected value of the result. The result is

$$E[|F_{1cj}(\omega)|^2] = g(\omega,\mu_V) + \frac{\sigma_V^2}{2} g''(\omega,\mu_V), \quad -\infty<\omega<\infty \qquad (26)$$

where $g(\omega,V)$ is the function on the right-hand side in Equation 25, $g''(\omega,V)$ is its second derivative with respect to V, and the moments of the impact velocity, V, are given in Equations 14 and 16.

To evaluate Equation 24 it is now only necessary to establish the expected value of N_{Tf}, the number of impacts in the time interval $(0,T_f)$. The quantity λ, given in Equations 13 and 15 is the average number of impacts in the system per unit time. The average of N_{Tf} is simply

$$E[N_{T_f}] = \lambda\, T_f, \qquad (27)$$

Based on this result, the spectral density of the impact force random process is

$$S_{F_1F_1}(w) = \pi\lambda \left[g(\omega,\mu_V) + \frac{\sigma_V^2}{2} g''(\omega,\mu_V) \right], \quad -\infty<\omega<\infty \qquad (28)$$

Use of this expression in Equation 20 completely establishes the spectral density of the displacement response on the surface of the test system.

To determine the spectral density of the acceleration response on the surface of the test system we merely multiply the above expression by the quantity ω^4. This is

$$S_{\ddot{U}\ddot{U}}(\omega) = \omega^4 S_{UU}(\omega), \quad -\infty<\omega<\infty \tag{29}$$

It will be shown in the example to follow that the spectral density related to the first component of response (i.e., that due to passage of dilatational waves through the top plate) is very substantial.

Numerical Example

A numerical example corresponding to the one in Reference 1 is presented here. The impulse response and frequency response functions for the dynamic test system were derived in Reference 1, and plots of these were shown for a specific case. The system considered uses steel plates and steel spheres. The material parameters are

$E = 30\times10^6$ psi
$\rho = 7.36\times10^{-4}$ lb-sec^2/in^4
$\nu = 0.3$

The dimensions of the steel plates are

a = 15 inches
b = 9 inches
thickness = 0.5 inch

The spheres have a diameter of 0.25 inch, and there are 15 of them placed on a cartesian grid with y locations 1.5, 4.5, 7.5, 10.5, and 13.5 inches, and z locations 1.5, 4.5, and 7.5 inches. The elastic material attached to the lower plate is assumed to have stiffness k = 1 lb/in.

For the case of the chaotic vibrations, the input is given the amplitude X_0 =52g=20000 in/sec^2. With all the other parameters, this indicates that the parameters of the chaotic vibration are

$\omega_f = 3665$ rad/sec (583 Hz)
$\lambda = 183$ impacts/sec
$\mu_V = 1.99$ in/sec
$\sigma_V = 2.47$ in/sec

These system and input parameters combine to yield an impact force random process whose one-sided spectral density is that shown in Figure 6, over the frequency interval (0,10000)Hz. The first two components of the response excited by the force random process (Equation 20) are shown in Figures 7 and 8. The rms value of the first component (including only that mean square power in the frequency interval (0,10000)Hz) is 1.12×10^5g. This seems extraordinarily high, but when it is noted from Equation 21 and Reference 1 that the peak acceleration excited by the average impact is about 17000g, and the rate of occurrence is about 183 impacts/sec, the response is understandable. Further, this is the rms response of the free surface. When a test item is attached to the plate surface the severity of motion will decrease as a function of the test item impedance.

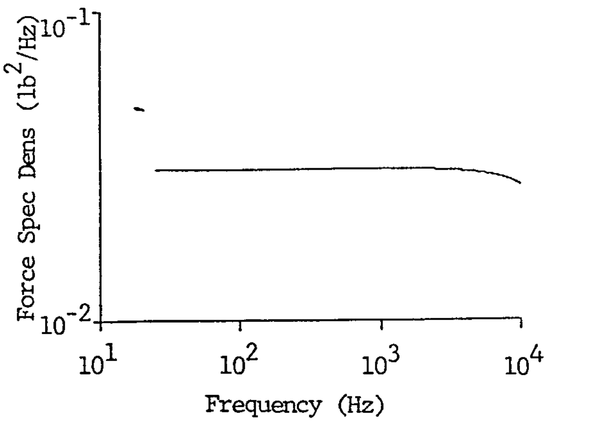

Figure 6. Spectral density of the force random process for the example.

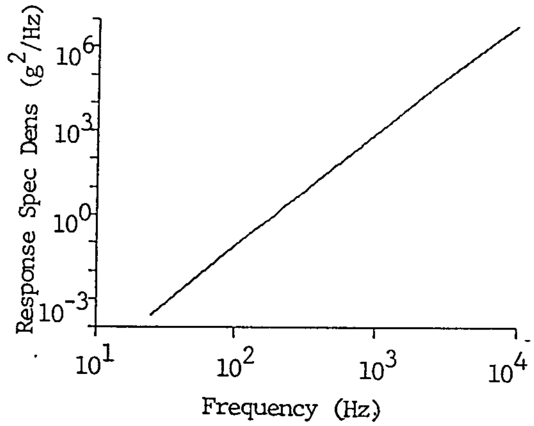

Figure 7. Acceleration spectral density of component 1 of the response random process for the example.

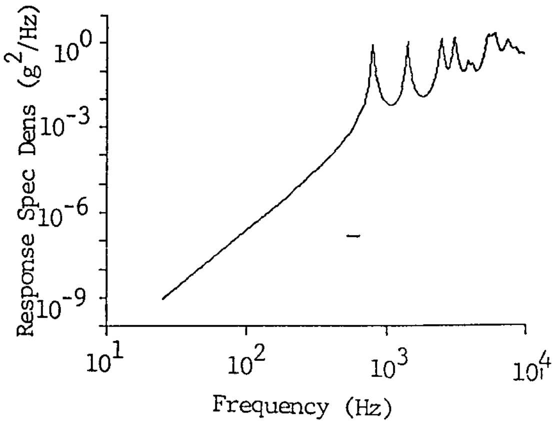

Figure 8. Acceleration spectral density of component 2 of the response random process for the example.

The second component of motion has an rms value of 72g. While this is high by many standards, it is completely overwhelmed by the first component. The component of motion caused by the base input is a 583 Hz sinusoidal component with amplitude of 70g. This component contributes 50g to the rms response, and is also completely overwhelmed by the first component of response.

It is important to keep in mind that the first component of the plate surface motion (the predominant component) is not a continuous vibration signal, but rather a random sequence of pulses like a shot noise random process. A time history for this type of random process is shown in Figure 9. This is similar to the sort of time history that component 1 in this example might execute, in the sense that acceleration pulses with amplitudes of 10000 to 20000g are realized. However, it is different from the sort of time history that might be realized in connection with component 1, in the sense that the average frequency of impacts in Figure 9 is 5000 impacts/sec. (This change was made in the generation of Figure 9 so that some character of the accelaration pulses could be shown, and so that several pulses could be shown in one time history.)

Random vibration of the dynamic test system was also considered. The only substantial difference between this and the chaotic vibrations case is that the impact statistics are slightly changed. Specifically, the impact rate and velocity statistics are

$\lambda = 127$ impacts/sec
$\mu_V = 2.95$ in/sec
$\sigma_V = 5.83$ in/sec

These parameters combine to yield an impact force random process whose spectral density is practically identical to that shown in Figure 6, therefore, the primary component of response is essentially the same as in the chaotic vibrations case.

Conclusions

Reference 1 and the present paper propose a mechanical system for the generation of test environments that have high frequency, low displacement, and high acceleretion. It is shown that shot noise type acceleration environments with frequency content well beyond 10000 Hz and rms acceleration beyond 10^5g are possible. The possibility of generating these types of environments in the vibration laboratory establishes the potential for testing small components to extreme mechanical environments.

This approach does have limitations, though. First, it does not appear possible to shape the test spectrum in an arbitrary manner because the displacement environment is primarily a shot noise type environment, and this simply has a white spectral density. (The spectral density is band-limited because the displacement pulse has finite duration, and the bandwidth can be controlled, to some extent, through choice of the parameters of the plates and the spheres.) Second, it only appears feasible, at present, to test relatively small components because large components will substantially reduce the test environment.

Several aspects of this and similar test systems bear further investigation. First, the effects on the test environments caused by system and input parameter variations should be investigated. Second, the effects on the test environment caused by the presence of various test items must be investigated. Third, the possibility of generating environments using similar test devices should be considered. Finally, a vibration test system should be built and evaluated in the laboratory.

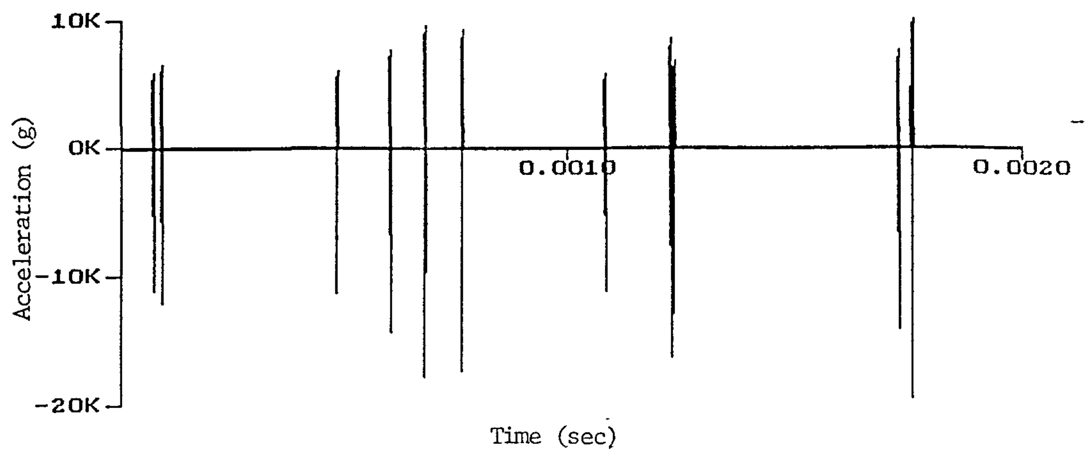

Figure 9. Realization of a shot noise random process.

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