A Method for Comparing Impacts with Real Targets to Impacts onto the IAEA Unyielding Target*

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INTRODUCTION

The severity of the IAEA accident conditions test requirement (IAEA 1990) of an impact onto an essentially unyielding target from a drop height of 9 meters encompasses a large fraction of all real world impacts. This is true, in part, because of the unyielding nature of the impact target. Impacts onto the unyielding target have severities equivalent to higher velocity impacts onto real targets which are not unyielding. The severity of impacts with yielding targets is decreased by the amount of the impact energy absorbed in damaging the target. In demonstrating the severity of the regulatory impact event it is advantageous to be able to relate this impact onto an essentially unyielding target to impacts with yielding targets.

BACKGROUND

There are several reasons for wanting to relate the severity of impacts with yielding targets to that of impacts with an unyielding target. The motivation for making the comparison will somewhat dictate the way the comparison is made. In the Final Environmental Statement on the Transportation of Radioactive Material by Air and Other Modes (US NRC 1977), which is a risk assessment for the shipment of all types of radioactive material, the properties of the packaging were not known. This forces the relationship between impact velocities for yielding and unyielding surfaces to be independent of package stiffness. For this reason a method was developed that compared the penetration of a rigid sphere into different surfaces, with steel considered to be the unyielding target. Velocities resulting in equal penetration depth were considered to be equivalent. This led to the following relationship for determining equivalent impact velocities:

$$\frac{V_{yielding}}{V_{steel}} = \left[\frac{1-\nu_y^2}{1-\nu_s^2}\right]\left[\frac{E_s}{E_y}\right]^{1/3} \qquad \text{(EQ 1)}$$

where $V_{yielding}$ is the velocity for impact onto a yielding surface, V_{steel} is the velocity for impact onto an unyielding surface, ν_y and E_y are Poisson's ratio and Young's modulus for the yielding surface material, and ν_s and E_s are Poison's ratio and Young's modulus for steel. This method was only applied to aircraft accident scenarios and the distribution of target hardness was determined by the ground surface composition along airline flight paths.

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In the Modal Study (Fischer et al. 1987), a risk assessment for the transport of spent fuel, the properties of the package were known. This allows the relationship between yielding and unyielding targets to depend on package characteristics. To determine equivalent impact velocities an equivalent damage technique was used. This technique resulted in a relationship for velocities of:

$$\frac{V_{yielding}}{V_{unyielding}} = \sqrt{1 + \frac{d_s}{d_c}} \qquad \text{(EQ 2)}$$

where $V_{unyielding}$ is the impact velocity for impacts onto an unyielding surface, d_s is the deformation of the yielding target caused by an impact of a rigid package at a velocity such that the impact force is the same as for the impact of the package on an unyielding target, and d_c is the deformation of the package caused by impact on an unyielding target.

METHOD

The method discussed in this paper for relating impacts with yielding targets to an impact with an unyielding target will apply the principle of conservation of energy. Immediately before the impact the energy of the package and target is equal to the kinetic energy of the package. At the point of maximum deformation of the package and the target the velocity is zero, so all of the energy in the system is strain energy. For impacts onto a rigid target the strain energy of the system is all in the package. During an impact with a real target the strain energy of the system is in both the package and the target. For casks, the strain energy in the package is typically divided into strain energy in the impact limiter and strain energy in the cask body, with the strain energy in the impact limiter typically being orders of magnitude larger than the strain energy in the cask body. If inertial effects are ignored the force acting on the cask body is the same as the force acting on the impact limiter and target for any time during the impact event. This condition can be viewed as a spring-mass system with a set of three massless nonlinear springs acting in series. Figure 1 shows this simplification of the impact event. Notice in this figure that the impact limiter target springs are treated as massless. For the impact limiter this assumption is generally quite accurate because its mass is usually much less than the mass of the cask. Neglecting the mass of the target in most cases does not introduce a large error in the analysis because the velocity, and therefore kinetic energy, of this mass is usually very small.

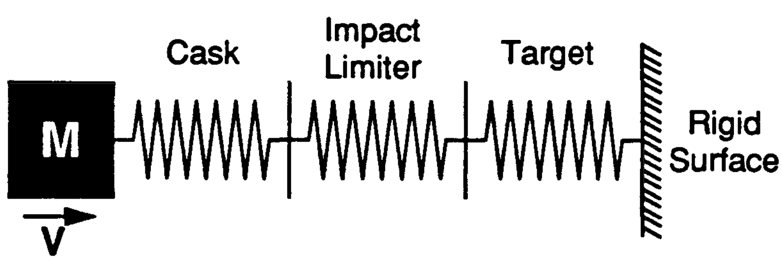

Figure 1 - Simplified spring model for impacts.

The strain energy in each of the springs for a given displacement is equal to the area under the force-deflection curve up to that displacement. For a linear spring this results in the familiar equation $E = 1/2K\delta^2$, where E is the strain energy in the spring, K is the linear spring constant, and δ is the displacement of the spring. For a non-linear spring with a force-deflection relationship defined by F(x), equation 3 shows the mathematical expression for the strain energy:

$$E = \int_0^{\delta} F(x)\,dx \qquad \text{(EQ 3)}$$

Where:

- E = The strain energy in the spring.
- F(x) = The force in the spring as a function of displacement.
- x = The displacement of the spring.
- δ = The displacement of the spring at the force level of interest.

In the system depicted in Figure 1, the total strain energy of the three springs must be equal to the kinetic energy of the mass at impact, and the force in the three springs is equal. These two conditions are the constraints on the problem and may be expressed mathematically as:

$$\frac{1}{2}MV_{yielding}^{2} = E_c + E_i + E_t \qquad \text{(EQ 4)}$$

and

$$F_c = F_i = F_t \qquad \text{(EQ 5)}$$

where M is the mass of the cask and impact limiter, $V_{yielding}$ is the impact velocity onto a yielding target, E_c, E_i, and E_t are the strain energies in the springs representing the cask body, impact limiter, and target, and F_c, F_i, and F_t are the instantaneous forces in these springs.

For impacts onto an unyielding target the entire kinetic energy of the mass must be converted into strain energy of the cask and impact limiter. This implies that the strain energy in the springs representing the cask and impact limiter is equal to the kinetic energy of the mass for an impact onto an unyielding target. Expressing this mathematically:

$$E_c + E_i = \frac{1}{2}MV_{unyielding}^{2} \qquad \text{(EQ 6)}$$

where $V_{unyielding}$ is the impact velocity onto an unyielding target. Equations 4 and 6 can be combined to provide a relationship for velocities of:

$$\frac{V_{yielding}}{V_{unyielidng}} = \sqrt{1 + \frac{E_t}{E_c + E_i}} \qquad \text{(EQ 7)}$$

EXAMPLE PROBLEM

The method described above will be demonstrated with the following example problem. A 90,700 kg (100 ton) rail cask impacts a hard soil with a velocity of 26.8 m/s (60 MPH). The impact limiter for this cask is designed using simplified relationships to limit the deceleration from the regulatory drop to 40 g with a crush of 0.23 m, which is below the lock-up deflection of the impact limiting material. This impact limiter is within the normal range used for this type of package, but it is softer than most. In the regulatory 9 meter drop the cask has an actual acceleration of 43.5 g and there is 0.236 m of crush in the impact limiter. The force deflection curve for the impact limiter is shown in Figure 2, along with force deflection curves for the cask body and the hard soil target. For this case the force displacement relationship for the cask body is:

$$F_c = A(1 - e^{-Bx_c} + Cx_c) \qquad \text{(EQ 8)}$$

the force displacement relationship for the impact limiter is given by:

$$F_i = D[1 - e^{-Ex_i} + F(e^{G(x_i - H)} - e^{-GH})] \qquad \text{(EQ 9)}$$

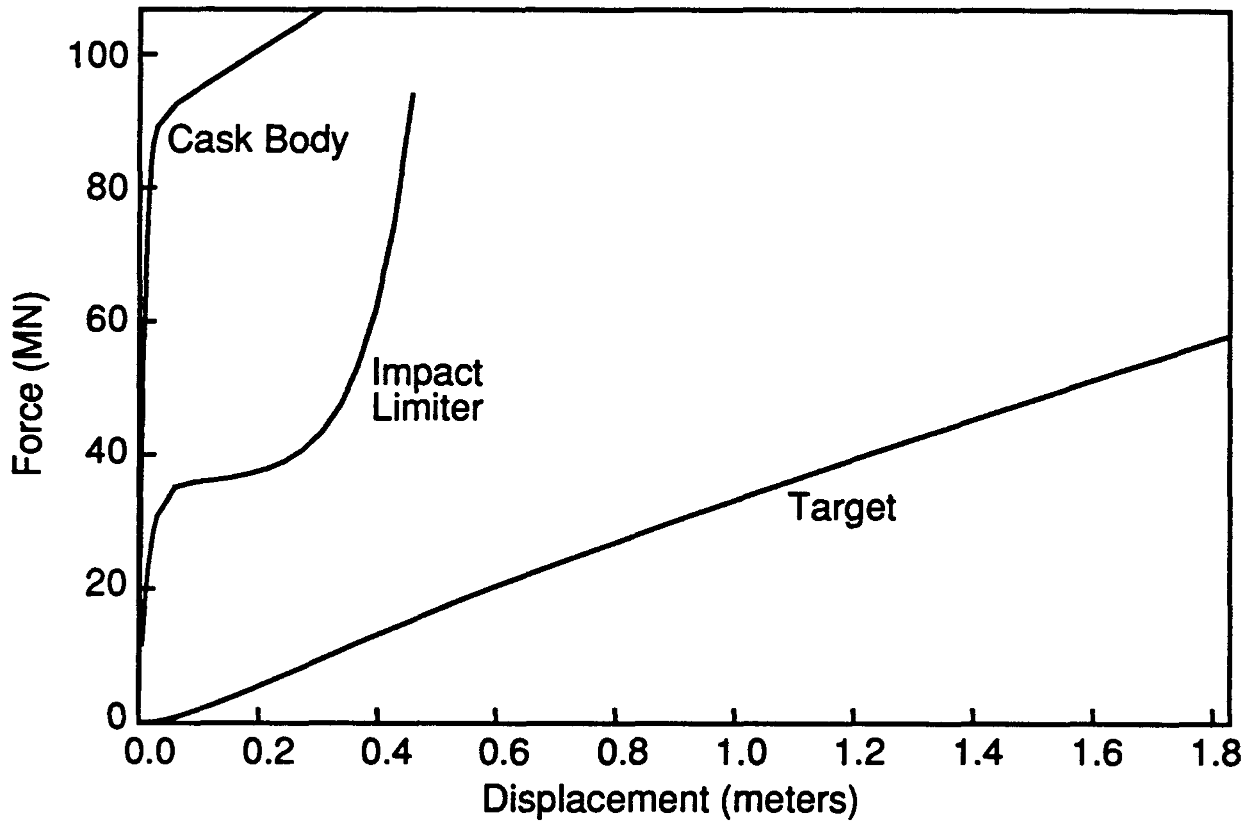

Figure 2 - Force-displacement curves for a rail cask body, its impact limiter, and a hard soil target.

and the force displacement curve for the hard soil target is given by:

$$F_t = K[JLx_t^{L-1} - P(e^{-Px_t} - e^{-Nx_t})] \qquad \text{(EQ 10)}$$

In these equations A-P are constants that define the curves with the values listed below, x_c, x_i, and x_t are expressed in meters and the forces F_c, F_i, and F_t are expressed in Newtons:

A = 89.0×10^6 N
B = 131 m^{-1}
C = 0.656 m^{-1}
D = 35.6×10^6 N
E = 98.4 m^{-1}
F = 0.1
G = 13.12 m^{-1}
H = 0.244 m
J = 9.81 $m^{-1.922}$
K = 1.76×10^6 N.m
L = 1.922
N = 9.84 m^{-1}
P = 4.92 m^{-1}

These equations were developed by fitting experimental (Bonzon and Schamaun 1976, Gonzales 1987, and Waddoups 1975) and analytical data. It would also be possible to use experimental data directly and express the relationships betwe:n force and displacement in tabular form. This method will require numerical integration of the force-displacement curves to calculate the strain energy associated with each spring. For the equations above it is possible to integrate explicitly, resulting in the expressions below for strain energy.

$$E_c = \int_0^{\delta_c} F_c dx_c = A\left[\delta_c + \frac{1}{B}e^{-B\delta_c} - \frac{1}{B} + \frac{C}{2}\delta_c^2\right] \qquad \text{(EQ 11)}$$

$$E_i = \int_0^{\delta_i} F_i dx_1$$

$$= D\left[\delta_i + \frac{1}{E}(e^{-E\delta_i} - 1) + \frac{F}{G}e^{-GH}(e^{G\delta_i} - 1 - G\delta_i)\right] \qquad \text{(EQ 12)}$$

$$E_t = \int_0^{\delta_t} F_t dx_t = K\left[J\delta_t^L + e^{-P\delta_t} - 1 - \frac{P}{N}(e^{-N\delta_t} - 1)\right] \qquad \text{(EQ 13)}$$

The sum of the strain energies for the three springs must be equal to the kinetic energy at impact, which is equal to $1/2MV_1^2$, where V_1 is equal to 26.8 m/s and M is equal to 90,700 kg. This gives a value for the kinetic energy of 32.6 x 10^6 N·m. To determine how this energy is distributed between the cask body, the impact limiter, and the target a complex system of non-linear equations must be solved. Generally for problems of this nature it is easier to solve them numerically with the aid of a computer, but it is possible to use a trial and error method for the solution. Solving this system of equations for this problem yields the following results. The strain energy in the cask body is 0.09 x 10^6 N·m, the strain energy in the impact limiter is 8.69 x 10^6 N·m, and the strain energy in the target is 23.82 x 10^6 N·m. The force acting on the three springs is 39.4 x 10^6 N (equivalent to 44.3 g acceleration). The elastic displacement of the cask body spring is 4.4 mm, the displacement of the impact limiter spring is 0.252 m, and the displacement of the target spring is 1.20 m. The sum of the energy in the cask and impact limiter springs is 8.78 x 10^6 N·m, which is the kinetic energy for a 13.9 m/s impact onto an unyielding target, using Eq. 6.

If we consider a 26.8 m/s impact of this cask onto the yielding target without its impact limiter the force in the cask and target springs is 45.5 x 10^6 N (equivalent to 51.1 g acceleration), the strain energy in the cask body spring is 0.14 x 10^6 N·m, and the strain energy in the target spring is 32.5 x 10^6 N·m. The elastic displacement of the cask body spring is 5.4 mm and the displacement of the target spring is 1.41 m. The equivalent velocity for an impact onto an unyielding target is 1.74 m/s (3.9 MPH). In the two cases the damage to the cask body is likely to be very small or non-existent. This is indicated by the lack of inelastic deformation in the cask body springs. (Note from Figure 2 that a force of 45.5 x 10^6 N is still well within the linear portion of the force-displacement curve for the cask body spring.)

This example demonstrates an important fact concerning target hardness. A target that is hard for one package may be soft for another package. The package system with an impact limiter is not as stiff as the package without the impact limiter. In the case of the package with the impact limiter a significant amount of the impact energy is absorbed by the impact limiter, which is only slightly stiffer than the target for this level of loading. For the package without an impact limiter almost all of the impact energy is absorbed by the target because the cask body is much stiffer than the target.

EFFECT OF PACKAGE AND TARGET STIFFNESS

The effect of package and target stiffness on the relative damage, as measured by deformation, caused by impacts onto yielding targets can be demonstrated by varying the impact limiter and target stiffnesses. For this exercise the energy absorbed by the package itself is ignored because it is insignificant compared to the amount absorbed by the impact limiter and target. The target is considered to be a linear spring with variable stiffness and the impact limiter is considered to be a bi-linear spring with nearly constant crush force. The crush force for the impact limiter depends on the g level desired for the impact. For each impact limiter and target stiffness the impact velocity required to produce the same amount of damage as that from a 9 m free fall (13.4 m/s impact velocity) onto an unyielding target is calculated. Two packages are considered, a 90,700 kg rail cask and a 23,000 kg truck cask. For the rail cask three different impact limiters are used, one resulting in approximately 40 g acceleration, one with approximately 60 g acceleration, and one with approximately 80 g acceleration. For the truck cask four impact limiters are considered, with approximate accelerations of 40, 60, 80, and 100 g. Figure 3 shows the resulting equivalent impact velocities required for the three rail casks and Figure 4 shows the equivalent velocities for the four truck casks. The linear stiffness that approximates the force deflection curve for the hard soil target in the preceding example is 3.3×10^7 N/m. From these two figures it can be seen that targets with stiffness greater than about 1×10^9 N/m can be treated as essentially unyielding and targets with stiffness less than about 1×10^6 N/m cause very little damage to the package. This result is very package specific and should not be thought of as globally applicable. For smaller, less stiff packages targets with stiffnesses in the range of 1×10^6 N/m may appear to be essentially unyielding. For these smaller packages it is less likely to have targets with these high stiffness levels because the contact area between the package and the target is also smaller.

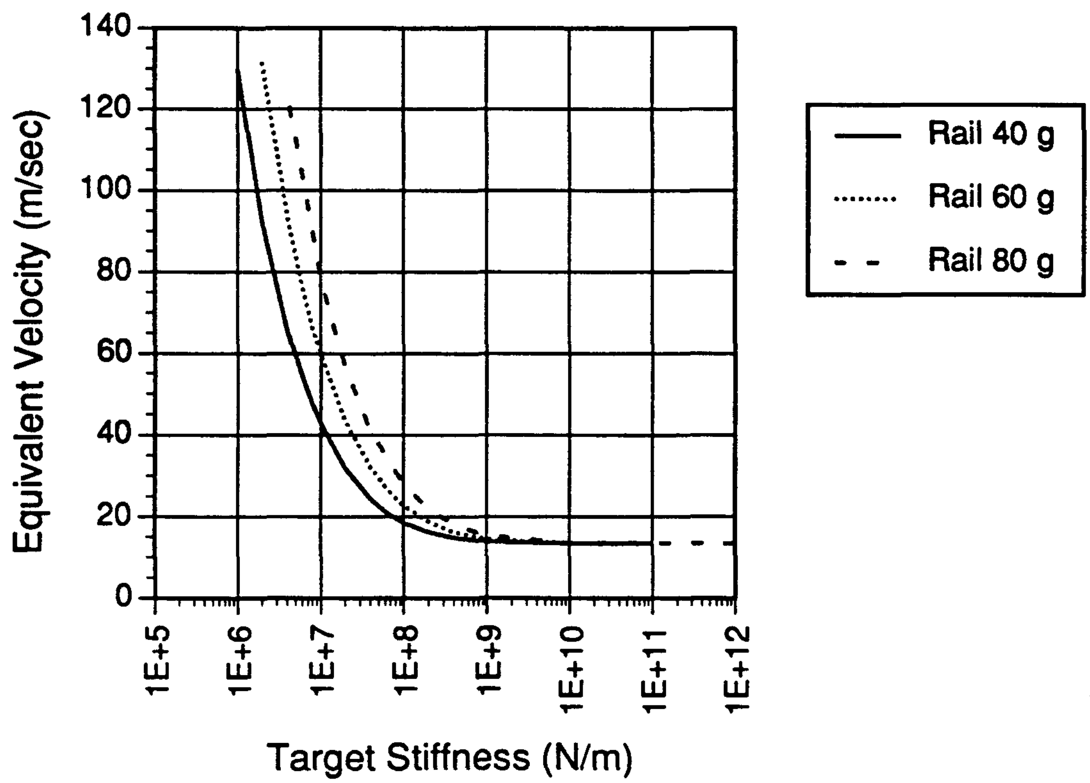

Figure 3 - Impact velocity onto a yielding target that causes the same damage as a 9 m impact onto an unyielding target for a 90,700 kg rail cask.

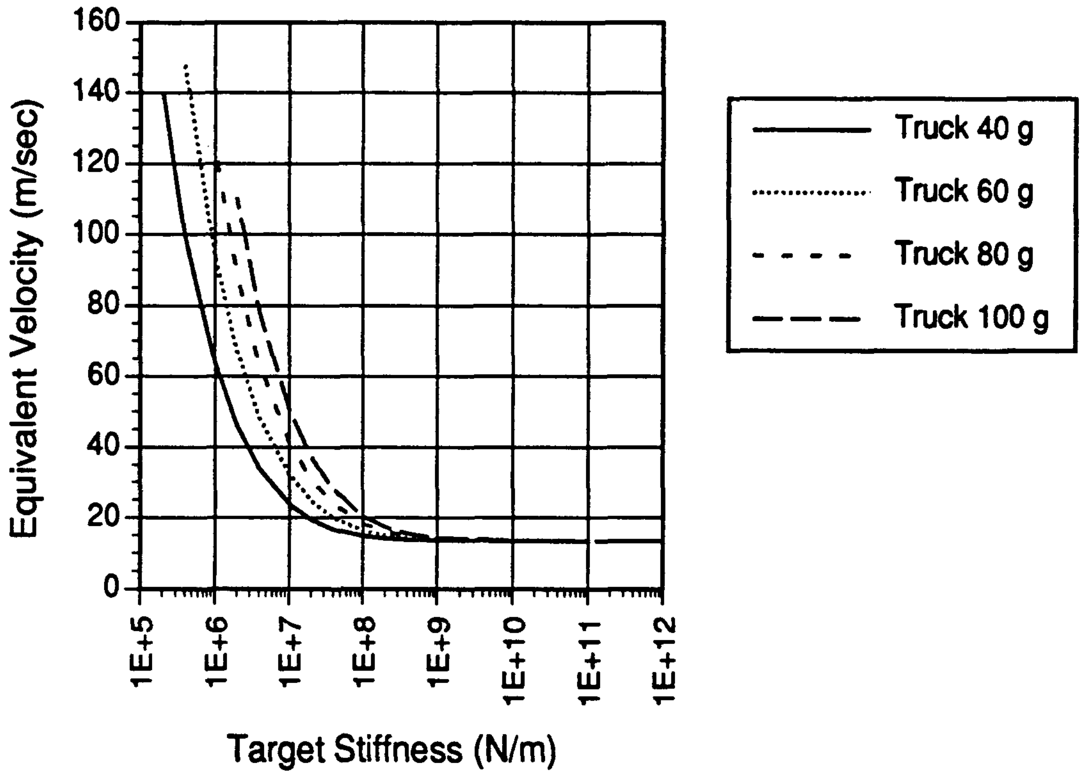

Figure 4 - Impact velocity onto a yielding target that causes the same damage as a 9 m impact onto an unyielding target for a 23,000 kg truck cask.

LIMITATIONS

To apply the method described in this paper for relating impacts with yielding targets to impacts with an unyielding target the user must know the load-displacement properties of the target as well as the cask body and impact limiter. For most radioactive material shipping packages the cask body is much more rigid than the impact limiter, and a close approximation to the solution can be obtained by assuming the cask is rigid. This reduces the spring system to two springs: one representing the impact limiter and one representing the target. For many targets, such as vehicles and posts, the amount of energy they can absorb before failing is finite. In these cases, if the impact energy is greater than the energy absorbed by the cask body, impact limiter, and target at the time the target fails, the package will not be stopped by the impact and will have a residual kinetic energy.

Modelling the cask body, the impact limiter, and the target as massless springs implies that the impact event is one-dimensional and quasistatic. That is, there is no load transmitted normal to the direction of motion, the forces are applied as distributed loads, and there are no inertial or strain rate effects. For packages such as the one in the example, where the cask body is much stiffer than the impact limiter, loads at this interface that are normal to the direction of motion have little significance and point loads are unlikely so the one dimensional crush is an accurate approximation. At the interface between the impact limiter and the target it is quite likely that loads in the transverse direction will cause crushing of either the impact limiter or the target, which will result in some energy absorption. This fact will tend to reduce the severity of the impact on the yielding target compared to the impact modelled as one dimensional crush. Severe impact tests on small packages (Bonzon and Schamaun 1976) showed this result by differences in failure mode. Impacts onto soil targets that had deformations of the cask body similar to lower velocity impacts onto an unyielding target did not result in gross failure of the containment boundary, while the impacts on the unyielding target did. This result could also be caused by higher strain rates for the impacts onto the unyielding target. The change in failure mode caused by transverse forces or strain rate effects is impossible to model as an impact onto an unyielding target at a lower velocity. The method of this paper considers the impact onto the yielding target to be more severe than it actually is. For the purpose of risk assessments or hazard communications this result is conservative.

CONCLUSIONS

A mathematically rigorous method is developed for relating impacts with yielding targets to lower velocity impacts with unyielding targets. The method correctly models the mechanics of the impact and the conversion of kinetic energy to strain energy. An important result shown by the example problem is that apparent target hardness depends on the stiffness of the impacting package. For a cask with impact limiters a 26.8 m/s impact onto hard soil results in equivalent forces as a 13.9 m/s impact onto an unyielding target. For the same cask without the impact limiters a 26.8 m/s impact onto hard soil is equivalent to a 1.74 m/s impact onto an unyielding target. This is one reason why non-technical members of the public often have difficulty realizing the severity of the regulatory impact. For most people, objects such as trucks and bridge columns appear to be very hard, but to many radioactive material shipping packages these objects are relatively soft.

The method discussed in this paper for relating impacts with yielding targets to lower velocity impacts with unyielding targets helps to explain how the regulatory impact accident provides a high degree of safety to the public. This methodology is relatively simple to use, and can be applied to the "What if" scenarios brought up by interveners.

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