

INCOMMENSURATE LATTICE-INSTABILITY IN  $K_2SeO_4$ 

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**ABSTRACT**

Successive phase transformations in  $K_2SeO_4$  at  $T_i = 130K$  and  $T_c = 93K$  have been studied by the triple-axis neutron scattering technique. Above  $T_i$ , a  $\Sigma_2$  optic phonon branch along  $(\xi, 0, 0)$  shows striking softening and  $\omega(q)$  for  $q \sim (1/3, 0, 0)$  tends to zero at  $T_i$ . This softening results from a temperature dependent decrease of the interlayer forces with range  $a/2$  and  $a$  ( $a$  is one unit cell length along the  $a$  axis) in the presence of strong and persisting forces with a range  $3a/2$ . The critical scattering above  $T_i$  has a maximum at an incommensurate position:  $q_\delta = (1-\delta)a^*/3$  with  $\delta \sim 0.08$  and peaks at  $E=0$  near  $T_i$ . At  $T_i$ , superlattice reflections appear at incommensurate positions,  $q_\delta$ . The deviation  $\delta$  decreases with decreasing temperature with an apparently discontinuous jump to  $\delta=0$  at  $T_c$ . Below this temperature the crystal remains commensurate and is ferroelectric. The incommensurate transition, the simultaneous lock-in of the commensurate phase and the ferroelectricity are discussed using a Landau type expansion of the free energy.

**INTRODUCTION**

Certain materials exhibit an instability against atomic displacements which are characterized by incommensurate wavevectors.<sup>1</sup> In those materials a phase transition to an incommensurate phase occurs first and in many cases a "lock-in" transformation to a commensurate phase follows. The phase transitions in potassium selenate ( $K_2SeO_4$ ) are interesting not only because they represent another example of this kind of lattice instability but also because the primary incommensurate atomic

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displacements induce a polarization wave which becomes the macroscopic polarization in the commensurate phase.<sup>2</sup> This is thus a new type of improper ferroelectric—the incommensurate-commensurate-type improper ferroelectrics.

Potassium selenate undergoes two phase transformations at  $T_i = 128\text{K}$  and  $T_c = 93\text{K}$ . Above  $T_i$  (P-phase) the crystal structure is

orthorhombic with a space group of Pnam.<sup>3</sup> Between  $T_i$  and  $T_c$  (I-phase) x-ray studies<sup>4</sup> revealed superlattice reflections which are characterized by a wavevector of  $\vec{a}^*/3$ . The detailed neutron scattering studies<sup>2</sup>

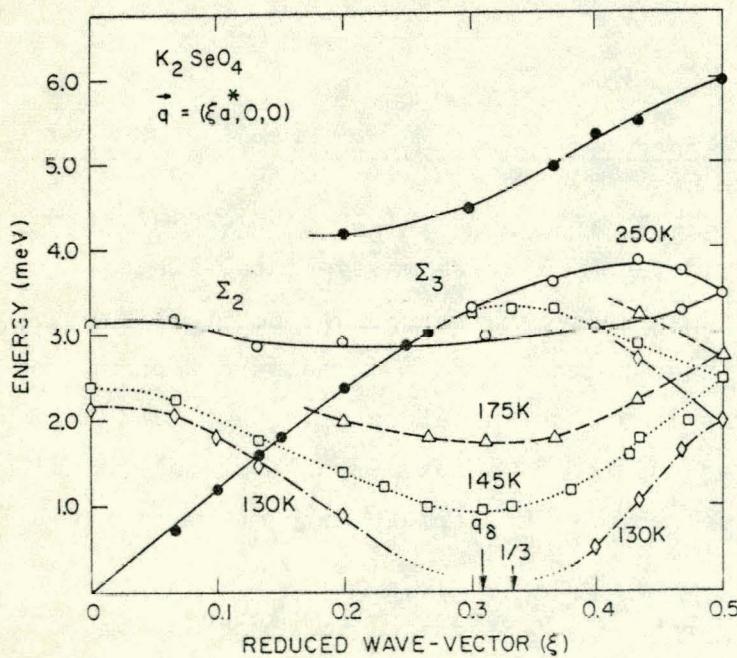


Fig. 2. Dispersion relations of transverse modes propagating along [100].

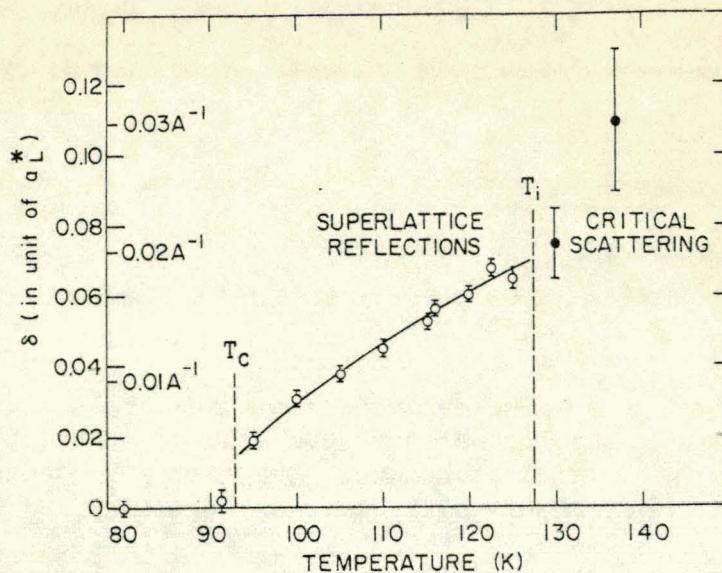


Fig. 1. Temperature change of the deviation  $\delta$

disclosed, however, that the superlattice reflections do not appear at exactly  $\vec{a}^*/3$  but they are shifted slightly from the commensurate points. The wavevector characterizing the superlattice structure is  $\vec{q}_\delta = (1-\delta)\vec{a}^*/a$  with  $\delta$  changing with temperature from 0.07 just below  $T_i$  to zero at  $T_c$  as shown in Fig. 1. Below  $T_c$  (F-phase) the superlattice structure

remains commensurate and shows weak ferroelectricity.<sup>5</sup>

In this paper results of neutron scattering studies on the lattice instability in the P-phase are presented and discussed. All the measurements were made by using triple-axis spectrometers at the Brookhaven High Flux Beam Reactor.

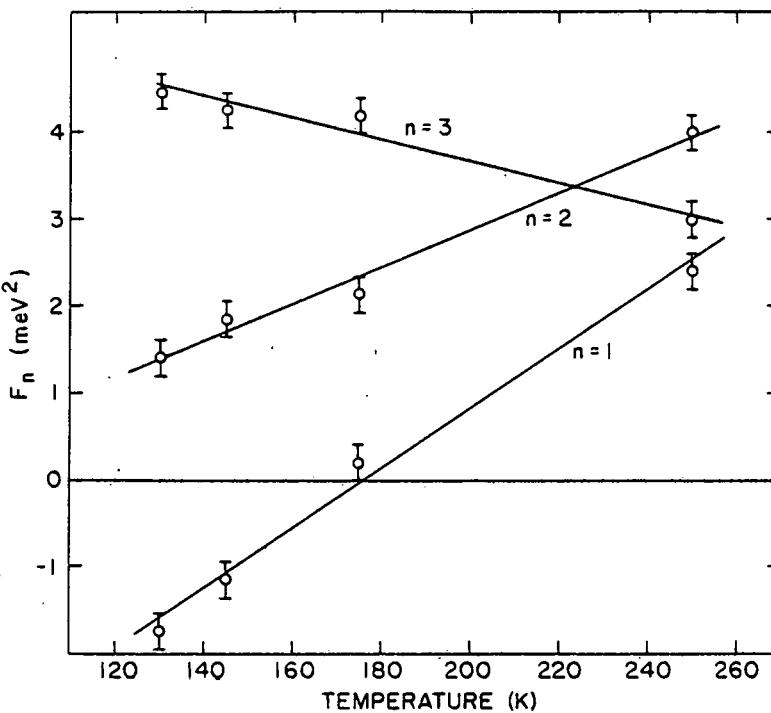


Fig. 3. Temperature dependence of the interlayer force constants.

#### SOFT MODE DISPERSION RELATION

The dispersion relations for the low-lying transverse modes propagating in the [100] direction were measured at several temperatures and are shown in Fig. 2. The lowest  $\Sigma_2$  optic branch shows a striking softening towards  $T_f$ . The softening is conspicuous around  $\xi=1/3$  and extends to a wide range along [100]. Measurements of the dispersion surface in the perpendicular [001] direction shows that the softening is confined to the vicinity of  $a^*$  axis in the  $\vec{q}$ -space.

The  $\Sigma_2$  mode is degenerate at the zone boundary with the  $\Sigma_3$  acoustic mode. We may, therefore, regard the two branches to be a single branch in the extended zone which is doubled along the  $a^*$ -axis and in which the instability takes place at  $\xi \approx 2/3$ . In order to characterize the dispersion relation we Fourier analyze it using the relation

$$[\hbar\omega(\xi)]^2 = \sum_n F_n (1 - \cos n\pi\xi) \quad (1)$$

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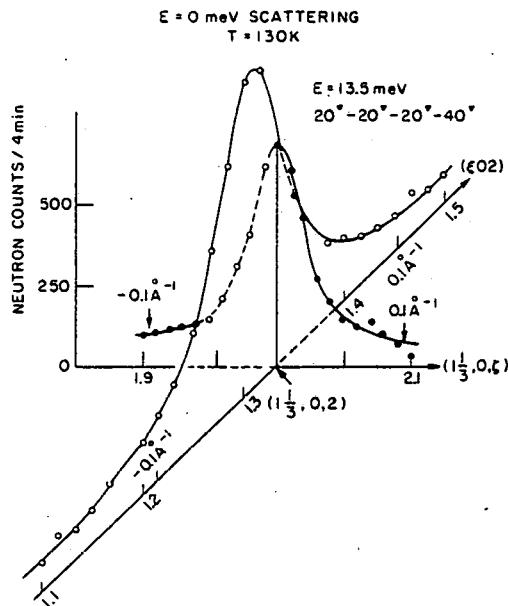


Fig.4.  $E=0$  scattering distribution around  $(1/3, 0, 2)$  at 130K.

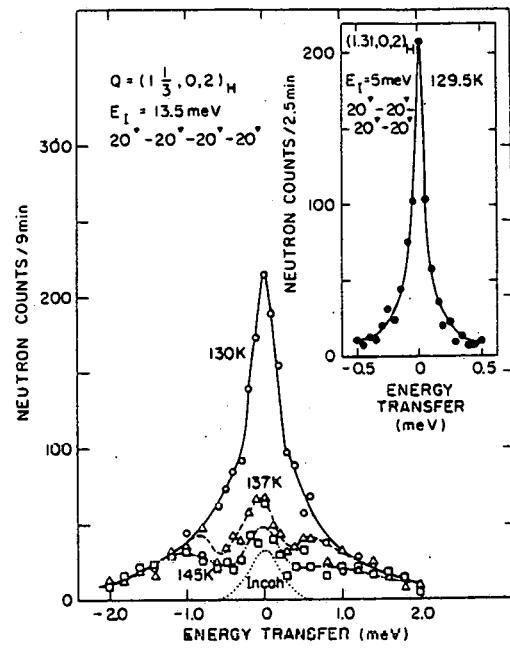


Fig.5. Energy spectrum of the critical scattering at several temperatures. Inset shows a result obtained by a better energy resolution.

in which the coefficients  $F_n$  correspond to fictitious effective force constants coupling planes in the crystal separated by a length equal to  $na/2$ . Sets of force constants obtained are shown in Fig. 3 as a function of temperature. The force constants beyond  $n=3$  are insignificant and are not shown in the figure. The force constant between nearest neighbor "planes"  $F_1$  decreases linearly with decreasing temperature and changes sign at about 175K.  $F_2$  is larger than  $F_1$  and also decreases as the temperature is lowered. In contrast, the force constant between the third neighbor "planes"  $F_3$  is strong and increases slightly with decreasing temperature. At temperatures just above  $T_i$   $F_3$  becomes a predominant component which characterizes the softening of the phonon energy around  $\xi=2/3$ . Notice that  $F_3$  alone produces a phonon instability at  $\xi=2/3$ . The presence of  $F_1$  and  $F_2$  displaces the minimum from the commensurate point. This simple picture explains how the incommensurate instability develops in  $K_2SeO_4$ .

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## CRITICAL SCATTERING

As shown in Fig. 4, as the transformation temperature is approached from above, the well-defined phonon sidebands disappear and very near  $T_i$  the critical scattering is peaked at  $\Delta E=0$ . If the incoherent scattering is subtracted, there is no central peak at  $T>145K$ , but there is weak evidence for a three-peak structure at 137K, and the unusual width of the wings of the scattering closer to  $T_i$  also suggest the presence of more than one relaxation time. A high resolution scan of the critical scattering (see inset of Fig. 3) revealed no detectable energy width. Fig. 5 shows the distribution of the  $E=0$  critical scattering in reciprocal space, and clearly shows the incommensurate nature of the instability.

## INDUCTION OF SPONTANEOUS POLARIZATION

Below  $T_i$  the  $\Sigma_2$ -type lattice distortion  $Q(\vec{q}_\delta)$  characterized by the wavevector  $q_\delta$  develops. Due to an interaction term<sup>2</sup>

$$F_{int} = A Q^3(\vec{q}_\delta) P_z(\vec{q}_{3\delta}) \quad (2)$$

in the free energy expansion, the secondary lattice distortion with amplitude  $P_z(\vec{q}_{3\delta})$  representing a long wavelength displacement wave with an associated polarization and characterized by a wavevector  $\vec{q}_{3\delta} = 3\delta(\vec{a}^*/3)$  is induced. As  $q_{3\delta}$  becomes zero together with  $\delta$  at  $T_c$ , the polarization wave transforms to the spontaneous polarization. The interaction term (2) also provides a driving force for the incommensurate-commensurate phase transition. A more detailed account of the phase transformations and the excitations of the low temperature phase will be published elsewhere.

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