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OCEAN THERMAL ENERGY CONVERSION PROGRAM

Extended NONSAP Program For OTEC Structural Systems

by
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ABSTRACT

A constitutive relation and failure criterion for concrete material under general three-dimensional stress states has been developed using the work-hardening theory of plasticity. The formulation has all the required properties of concrete and gives a close estimate to experimental stresses for complete general stress states.

In order that the results of research be readily usable in the analysis of suboceanic structures such as the large shells proposed for adoption in the Ocean Thermal Energy Conversion program (OTEC), corresponding computer codes must be developed to reflect this material.

This report describes the development of the corresponding computer code in the form of a subroutine for the general purpose nonlinear finite element analysis program called NONSAP which was originally developed by the University of California at Berkeley in 1974.

INTRODUCTION

For the most part, the development of stress-strain relations of concrete have, in the past, been limited to the formulations for concrete subjected to uniaxial and biaxial stress states. With the present state of rapid development in finite element analysis and the increasing use of triaxially-loaded massive concrete structures, e.g. prestressed concrete reactor vessels and containments, dams, offshore drilling platforms, offshore storage tanks, large concrete barges, etc., the formulation of a constitutive relationship for concrete under general three-dimensional stress states is of increasing importance. In this connection, a constitutive model capable of representing the nonlinear behavior of concrete under monotonic loading has been proposed by research workers at Lehigh University [1, 2]. In their model, the concrete is assumed to be a continuous, isotropic, and linearly elastic-plastic strain-hardening-fracture material. An initial discontinuous surface, equivalent to the initial yield surface of metal, was proposed to have two different functions: one is for the compressive stress state and the other is for the tension-compression stress state. Similar to the concept in plasticity, the subsequent loading surfaces and fracture surface were obtained from the uniform expansion of the initial discontinuous surface following a strain hardening function. Further, the incremental stress-strain relations for both two-dimensional and three-dimensional stress states were developed in matrix form using classical theory of plasticity.

In this report, computer algorithm for the constitutive relations of concrete to be used in the finite element analysis is given.

According to this algorithm, computer subroutines for concrete structures subjected to either two-dimensional or three-dimensional stress state were developed. These subroutines can be readily adapted by any nonlinear finite element computer program in which the solution method is based on the incremental approach with a tangent modulus formulation for solving elastic-plastic problems [3]. The subroutines have been implemented in a general purpose nonlinear finite element program, NONSAP-A [4], which is a modified version of the NONSAP program developed by Bathe, Iding and Wilson [5]. In addition, two numerical examples are presented in the report to demonstrate the utility of the concrete constitutive subroutines in finite element analysis. In addition, three examples are given to show the analysis capability of the NONSAP-A program other than concrete structures.

CONSTITUTIVE RELATIONS

The nonlinear stress-strain response of concrete, as proposed in references [1,2], is represented by a plasticity model with work-hardening. In this model the behavior of concrete can be described by two relationships: a loading function (including the initial yield surface and subsequent loading surface) and an incremental stress-strain law (or associated flow law). Detailed development of these relationships can be found in [1] and only a brief outline of the theory is given as follows.

1. Loading Function

According to reference [1], the failure surface of concrete is assumed to be a function of I_1, the first invarient of the stress tensor σ_{ij}; and J_2, the second invariant of the deviatoric stress tensor S_{ij}. Then the loading function has the form

$$f(\sigma_{ij}) = \frac{\frac{\kappa^2}{3} J_2 - \frac{\kappa^2}{36} I_1^2 \pm \frac{1}{12} I_1^2 + \frac{\beta}{3} I_1}{1 - \frac{\alpha}{3} I_1} = \tau^2 \qquad (1)$$

where the positive-negative sign in the third term represents the loading function in the compression region and the tension-compression region, respectively; the stress invariants I_1 and J_2 are defined by

$$\begin{aligned} I_1 &= \sigma_{mm} \\ J_2 &= \frac{1}{2} S_{ij} S_{ij} \\ &= \frac{1}{2} \sigma_{ij} \sigma_{ij} - \frac{1}{6} \sigma_{mm}^2 \end{aligned} \qquad (2)$$

The constants κ, α, and β in Eq. 1 are given by

$$\kappa^2 = 3$$

$$\alpha = \frac{A_u - A_o}{\tau_u^2 - \tau_o^2} \qquad (3)$$

$$\beta = \frac{A_o \tau_u^2 - A_u \tau_o^2}{\tau_u^2 - \tau_o^2}$$

in which, A_o, A_u, τ_o, and τ_u are the material constants and they are functions of f'_c, f_c, f'_t, f_t, f'_{bc}, and f_{bc}. Herein, f'_c, f'_t and f'_{bc} denote the ultimate strength of concrete under uniaxial compression, uniaxial tension, and equal biaxial compression, respectively; while f_c, f_t, f_{bc} denote the initial yield strength of concrete under the corresponding loading. The constants A_o, A_u, τ_o and τ_u assume different values in the compression and tension-compression regions.

For the compression region ($I_1 < 0$ and $\sqrt{J_2} + I_1/\sqrt{3} < 0$)

$$\frac{A_o}{f'_c} = \frac{\bar{\bar{f}}_{bc}^{\,2} - \bar{\bar{f}}_c^{\,2}}{2\bar{f}_{bc} - \bar{\bar{f}}_c} \; ; \quad \frac{A_u}{f'_c} = \frac{\bar{f}'^{\,2}_{bc} - 1}{2\bar{f}'_{bc} - 1} \qquad (4)$$

$$\left(\frac{\tau_o}{f'_c}\right)^2 = \frac{\bar{f}_c \; \bar{f}_{bc} \; (2\bar{f}_c - \bar{f}_{bc})}{3(2\,\bar{f}_{bc} - \bar{f}_c)} \; ; \quad \left(\frac{\tau_u}{f'_c}\right)^2 = \frac{\bar{f}'_{bc} \; (2 - \bar{f}'_{bc})}{3(2\bar{f}'_{bc} - 1)}$$

For the tension-compression region: (Either $I_1 \geq 0$ or $\sqrt{J_2} + I_1/\sqrt{3} \geq 0$)

$$\frac{A_o}{f'_c} = \frac{\bar{f}_c - \bar{f}_t}{2} \quad ; \quad \frac{A_u}{f'_c} = \frac{1 - \bar{f}'_t}{2}$$

$$\left(\frac{\tau_o}{f'_c}\right)^2 = \frac{\bar{f}_c \cdot \bar{f}_t}{6} \quad ; \quad \left(\frac{\tau_u}{f'_c}\right)^2 = \bar{f}'_t/6 \tag{5}$$

where $(\bar{\ })$ denotes the nondimensionalized quantity of the corresponding term with respect to f'_c.

In Eq. 1, if $\tau = \tau_o$, the loading function reduces to the initial yield surface of concrete. On the other hand, if $\tau = \tau_u$, Eq. 1 becomes the failure surface of concrete. For the stress state between τ_o and τ_u, τ is a function of the plastic strain which can be characterized by a simple compression test, i.e.

$$\tau = \tau \ (\varepsilon^P) \tag{6}$$

where ε^P represents the equivalent plastic strain,

$$\varepsilon^P = \int d\varepsilon^P = \int (d\varepsilon^P_{ij} \ d\varepsilon^P_{ij})^{\frac{1}{2}} \tag{7}$$

2. Incremental Stress-Stain Law

The incremental stress-strain relations in matrix form for the cases of three-dimensional, axisymmetric, plane strain and plane stress problems have been derived in reference [1]. These relationships are summarized as the following:

a) Three-Dimensional State

Referring to a rectangular Cartesian coordinate system (x,y,z), the incremental stress-strain law is given by

$$\begin{Bmatrix} d\sigma_x \\ d\sigma_y \\ d\sigma_z \\ d\sigma_{xy} \\ d\sigma_{yz} \\ d\sigma_{zx} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu-\omega\Phi_{11} & \nu-\omega\Phi_{12} & \nu-\omega\Phi_{13} & -\omega\Phi_{14} & -\omega\Phi_{15} & -\omega\Phi_{16} \\ & 1-\nu-\omega\Phi_{22} & \nu-\omega\Phi_{23} & -\omega\Phi_{24} & -\omega\Phi_{25} & -\omega\Phi_{26} \\ & & 1-\nu-\omega\Phi_{33} & -\omega\Phi_{34} & -\omega\Phi_{35} & -\omega\Phi_{36} \\ \text{SYMMETRIC} & & & \frac{1-2\nu}{2}-\omega\Phi_{44} & -\omega\Phi_{45} & -\omega\Phi_{46} \\ & & & & \frac{1-2\nu}{2}-\omega\Phi_{55} & -\omega\Phi_{56} \\ & & & & & \frac{1-2}{2}-\omega\Phi_{66} \end{bmatrix} \begin{Bmatrix} d\varepsilon_x \\ d\varepsilon_y \\ d\varepsilon_z \\ d\gamma_{xy} \\ d\gamma_{yz} \\ d\gamma_{zx} \end{Bmatrix} \quad (8)$$

or symbolically, the above equations can be written as

$$\{ d\sigma_i \} = [C_{ij}]\{ d\varepsilon_j \} \quad (8\text{-a})$$

where C represents the elastic-plastic matrix and

$$\frac{1}{\omega} = \{(1-2\nu)(2\eta^2 J_2+3\rho^2) + 9\nu\rho^2\} + \frac{mH(1+\nu)(1-2\nu)}{E} \sqrt{(2\eta^2 J_2 + 3\rho^2)}$$

$$\Phi_{11} = \{(1-2\nu)(\eta S_{xx} +\rho) + 3\nu\rho\}^2$$

$$\Phi_{12} = \{(1-2\nu)(\eta S_{xx} +\rho) + 3\nu\rho\} \ \{(1-2\nu)(\eta S_{yy} +\rho) + 3\nu\rho\}$$

$$\Phi_{13} = \{(1-2\nu)(\eta S_{xx} +\rho) + 3\nu\rho\} \ \{(1-2\nu)(\eta S_{zz} +\rho) + 3\nu\rho\}$$

$$\Phi_{14} = \{(1-2\nu)(\eta S_{xx} +\rho) + 3\nu\rho\} \ \{(1-2\nu)\eta\tau_{xy}\}$$

$$\Phi_{15} = \{(1-2\upsilon)(\eta S_{xx} + \rho) + 3\upsilon\rho\} \{(1-2\upsilon)\eta\tau_{yz}\}$$

$$\Phi_{16} = \{(1-2\upsilon)(\eta S_{xx} + \rho) + 3\upsilon\rho\} \{(1-2\upsilon)\eta\tau_{zx}\}$$

$$\Phi_{22} = \{(1-2\upsilon)(\eta S_{yy} + \rho) + 3\upsilon\rho\}^2$$

$$\Phi_{23} = \{(1-2\upsilon)(\eta S_{yy} + \rho) + 3\upsilon\rho\} \{(1-2\upsilon)(\eta S_{zz} + \rho) + 3\upsilon\rho\}$$

$$\Phi_{24} = \{(1-2\upsilon)(\eta S_{yy} + \rho) + 3\upsilon\rho\} \{(1-2\upsilon)\eta\tau_{xy}\}$$

$$\Phi_{25} = \{(1-2\upsilon)(\eta S_{yy} + \rho) + 3\upsilon\rho\} \{(1-2\upsilon)\eta\tau_{yz}\}$$

$$\Phi_{26} = \{(1-2\upsilon)(\eta S_{yy} + \rho) + 3\upsilon\rho\} \{(1-2\upsilon)\eta\tau_{zx}\}$$

$$\Phi_{33} = \{(1-2\upsilon)(\eta S_{zz} + \rho) + 3\upsilon\rho\}^2$$

$$\Phi_{34} = \{(1-2\upsilon)(\eta S_{zz} + \rho) + 3\upsilon\rho\} \{(1-2\upsilon)\eta\tau_{xy}\}$$

$$\Phi_{35} = \{(1-2\upsilon)(\eta S_{zz} + \rho) + 3\upsilon\rho\} \{(1-2\upsilon)\eta\tau_{yz}\}$$

$$\Phi_{36} = \{(1-2\upsilon)(\eta S_{zz} + \rho) + 3\upsilon\rho\} \{(1-2\upsilon)\eta\tau_{zx}\}$$

$$\Phi_{44} = \{(1-2\upsilon)\eta\tau_{xy}\}^2$$

$$\Phi_{45} = \{(1-2\upsilon)\eta\tau_{xy}\} \{(1-2\upsilon)\eta\tau_{yz}\}$$

$$\Phi_{46} = \{(1-2\upsilon)\eta\tau_{xy}\} \{(1-2\upsilon)\eta\tau_{zx}\}$$

$$\Phi_{55} = \{(1-2\upsilon)\eta\tau_{yz}\}^2$$

$$\Phi_{56} = \{(1-2\upsilon)\eta\tau_{yz}\} \{(1-2\upsilon)\eta\tau_{zx}\}$$

$$\Phi_{66} = \{(1-2\upsilon)\eta\tau_{zx}\}^2 \qquad (9)$$

and $m = 1 - \frac{\alpha}{3} I_1$

$\eta = 1$

$\rho = n I_1 + \frac{\beta + \alpha\tau^2}{3}$

$n = 0$ in the compression region

$n = -\frac{1}{3}$ in the tension-compression region (10)

(b) Axisymmetric State

Herein, the stress components with respect to a polar coordinate system (r, z, θ) are denoted by σ_r, σ_z, τ_{rz}, and σ_θ , and the corresponding strain components are ε_r, ε_z, γ_{rz}, and ε_z. Then the incremental stress-strain relations are

$$\begin{Bmatrix} d\sigma_r \\ d\sigma_z \\ d\tau_{rz} \\ d\sigma_\theta \end{Bmatrix} = \frac{E}{(1+\upsilon)(1-2\upsilon)} \begin{bmatrix} 1-\upsilon-\omega\Phi_{11} & \upsilon-\omega\Phi_{12} & -\omega\Phi_{13} & \upsilon-\omega\Phi_{14} \\ & 1-\upsilon-\omega\Phi_{22} & -\omega\Phi_{23} & \upsilon-\omega\Phi_{24} \\ & & \frac{1-2\upsilon}{2}-\omega\Phi_{33} & -\omega\Phi_{34} \\ \text{SYMMETRIC} & & & 1-\upsilon-\omega\Phi_{44} \end{bmatrix} \begin{Bmatrix} d\varepsilon_r \\ d\varepsilon_z \\ d\gamma_{rz} \\ d\varepsilon_\theta \end{Bmatrix} \quad (11)$$

or, $\{d\sigma_i\} = [C_{ij}] \{d\varepsilon_j\}$ (11-a)

where

$$\frac{1}{\omega} = \{(1-2\upsilon)(2\eta^2 J_2 + 3\rho^2) + 9\upsilon\rho^2\} + \frac{mH (1+\upsilon)(1-2\upsilon)}{E} \sqrt{(2\eta^2 \quad J \quad + 3\rho^2)}$$

$$\Phi_{11} = \{(1-2\upsilon)(\eta S_r + \rho) + 3\,\upsilon\rho\}^2$$

$$\Phi_{12} = \{(1-2\upsilon)\ (\eta S_r + \rho) + 3\upsilon\rho\}\ \ \{(1-2\upsilon)(\eta S_z + \rho) + 3\upsilon\rho\}$$

$$\Phi_{13} = \{(1-2\upsilon)\ (\eta S_r + \rho) + 3\upsilon\rho\}\ \ \{(1-2\upsilon)\eta\tau_{rz}\}$$

$$\Phi_{14} = \{(1-2\upsilon)\ (\eta S_r + \rho) + 3\upsilon\rho\}\ \ \{(1-2\upsilon)(\eta S_\theta + \rho) + 3\upsilon\rho\}$$

$$\Phi_{22} = \{(1-2\upsilon)\ (\eta S_z + \rho) + 3\upsilon\rho\}^2$$

$$\Phi_{23} = \{(1-2\upsilon)\ (\eta S_z + \rho) + 3\upsilon\rho\}\ \ \{(1-2\nu)\eta\tau_{rz}\}$$

$$\Phi_{24} = \{(1-2\upsilon)\ (\eta S_z + \rho) + 3\upsilon\rho\}\ \ \{(1-2\upsilon)(\eta S_\theta + \rho) + 3\upsilon\rho\}$$

$$\Phi_{33} = \{(1-2\upsilon)\eta\tau_{rz}\}^2$$

$$\Phi_{34} = \{(1-2\upsilon)\eta\ \tau_{rz}\}\ \ \{(1-2\upsilon)(\eta S_\theta + \rho) + 3\upsilon\rho\}$$

$$\Phi_{44} = \{(1-2\upsilon)(\eta S_\theta + \rho) + 3\upsilon\rho\}^2 \qquad (12)$$

(c) Plane Strain State

In the case of plane strain, the non-vanishing stress components are σ_x, σ_y, τ_{xy}, and σ_z, and the corresponding strain components are ε_x, ε_y, γ_{xy}, and $\varepsilon_z = 0$. Equation (11-a)holds except that the stress and strain vectors are redefined as

$$\{d\sigma_i\} = \begin{Bmatrix} d\sigma_x \\ d\sigma_y \\ d\tau_{xy} \\ d\sigma_z \end{Bmatrix} \quad , \quad \{d\varepsilon_i\} = \begin{Bmatrix} d\varepsilon_x \\ d\varepsilon_y \\ d\tau_{xy} \\ 0 \end{Bmatrix} \tag{13}$$

(d) Plane Stress State

In the case of plane stress, the non-vanishing stress components are σ_x, σ_y, and τ_{xy}, and the strain components are ε_x, ε_y, τ_{xy}, and ε_z. The incremental stress-strain law is given by

$$\begin{Bmatrix} d\sigma_x \\ d\sigma_y \\ d\tau_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{C}_{11} & \overline{C}_{12} & \overline{C}_{13} \\ & \overline{C}_{22} & \overline{C}_{23} \\ \text{SYMMETRIC} & & \overline{C}_{33} \end{bmatrix} \begin{Bmatrix} d\varepsilon_x \\ d\varepsilon_y \\ d\gamma_{xy} \end{Bmatrix} \tag{14}$$

and $d\varepsilon_z = -(C_{41} \cdot d\varepsilon_x + C_{42}\, d\varepsilon_y + C_{43}\, d\gamma_{xy})/C_{44}$ (15)

where $\overline{C}_{ij} = C_{ij} - \dfrac{C_{14}\, C_{4j}}{C_{44}}$, for i, j = 1, 2, and 3 (16)

and C_{ij} are defined in Eqs. (11) and (11-a).

COMPUTER ALGORITHM

It is anticipated that the nonlinear analysis of a concrete structure will be carried out by use of the finite element method with an incremental solution approach. In this approach the total load is divided into several small increments and the solution corresponding to each load increment is obtained from a set of linearized equations.

Within the context of incremental solution, two different methods have been employed to solve nonlinear structrual problems: one is the initial strain method and the other is the tangent modulus method [3]. The computer algorithm presented in this report is suitable only for the latter method, i.e. the tangent modulus method.

Consider a typical load increment, e.g. the n-th load increment. The constitutive relations of concrete are used for two-different stages: at first the assembladge stage during which the element stiffness and in turn, the structural stiffness are formed, and then the stress calculation stage during which stresses and strains in each element are computed. An outline on the numerical algorithm of the constitutive relations to be used in a finite element analysis is given as follows.

1. Let (N-1) and N be the previous and the current load increment respectively; the corresponding strain components are denoted by ε_{ij}^{N-1} and ε_{ij}^{N}. At the beginning of N-th load increment, the incremental strains are calculated from

$$d\varepsilon_{ij} = \varepsilon_{ij}^{N} - \varepsilon_{ij}^{N-1} \tag{17}$$

2. Assuming elastic behavior, the trial stress increments are calculated from the elastic stress-strain law

$$\{d\sigma\} = [C^E] \{d\varepsilon\} \tag{18}$$

where C^E is the elastic matrix involving the Young's modulus and Poisson's ratio only.

3. Calculate the total stresses

$$\sigma_{ij} = \sigma_{ij}^N + d\sigma_{ij} \tag{19}$$

where σ_{ij}^N are the total stresses for the N-th load increment.

4. Check the present load increment to determine whether the element is under elastic or plastic loading:

a) Based on the yield criterion, Eq. (1), if $f(\sigma_{ij}) - \tau^2 \leq 0$, the element is under elastic loading. In this case, the elastic material matrix is formed during the assemblage stage or the incremental stresses are calculated from the elastic stress-strain law during the stress calculation stage.

b) On the other hand, if $f(\sigma_{ij}) - \tau^2 > 0$, the element is under plastic loading. In this case, the elastic portion of the stress increment, $d\sigma_{ij}$, is scaled as the following: let $\bar{\sigma}_{ij} = \sigma_{ij}^N$, we require

$$f(\bar{\sigma}_{ij} + R\, d\sigma_{ij}) - \tau^2 = 0 \tag{20}$$

and the scaling factor R is found to be

$$R = \frac{-B \pm \sqrt{B^2 - AC}}{A} \tag{21}$$

where

$$A = \frac{\kappa^2}{3}\left(\frac{1}{2}\, d\sigma_{ij}\; d\sigma_{ij} - \frac{1}{6}\, d\sigma_{mm}^2\right) + n\, d\sigma_{mm}^2$$

$$B = \frac{\kappa^2}{3}\left(\frac{1}{2}\,\bar{\sigma}_{ij}\; d\sigma_{ij} - \frac{1}{6}\,\bar{\sigma}_{mm}\; d\sigma_{nn}\right) + n\;\bar{\sigma}_{mm}\; d\sigma_{nn} + \frac{d\sigma_{mm}}{6}\,(\beta + \alpha\tau^2)$$

$$C = \frac{\kappa^2}{3}\left(\frac{1}{2}\,\bar{\sigma}_{ij}\;\bar{\sigma}_{ij} - \frac{1}{6}\,\bar{\sigma}^2_{nn}\right) + n\,\bar{\sigma}^2_{mm} + \frac{\bar{\sigma}_{mm}}{3}\,(\beta + \alpha\tau^2) - \tau^2 \tag{22}$$

Thus, the remaining portion of the incremental strains is

$$d\bar{\varepsilon}_{ij} = (1 - R)\; d\varepsilon_{ij} \tag{23}$$

To achieve better numerical accuracy, the strain components $d\bar{\sigma}_{ij}$ are further divided into small increments and the elastic-plastic stress-strain matrix is formed according to either Eq. (8) or Eq. (1). Subsequently, the incremental stresses $d\sigma_{ij}$ are calculated.

5. At the end of the stress calculation stage, the stresses and strains in an element are updated, i.e.,

$$\sigma_{ij}^{N+1} = \sigma_{ij}^{N} + d\sigma_{ij}$$

$$\varepsilon_{ij}^{N+1} = \varepsilon_{ij}^{N+1} + d\varepsilon_{ij} \tag{24}$$

6. Proceed with the same calculation for the next loading increment.

A flow diagram for the above computation procedures is shown in Fig. 1. A listing of constitutive subroutines used in NONSAP-A for two-dimensional and three-dimensional structural problems is given in the Appendix.

DESCRIPTION OF NONSAP-A PROGRAM

The computer subroutines for the constitutive relations of concrete have been incorporated in a nonlinear structural analysis program called NONSAP-A [4], which is a modified version of the NONSAP program originally developed by Bathe, Iding and Wilson of the University of California at Berkeley [5]. The program can be used for conducting linear and nonlinear, static, and dynamic analysis for a range of complex structures. Nonlinearities considered in the program include nonlinear materials, large displacement and large strain. The element library consists of truss, beam, two-dimensional and three-dimensional isoparametric elements. The material models include linear elastic, von-Mises plasticity, Drucker-Prager plasticity, concrete plasticity (Lehigh) and Mooney-Rivlin material for rubber. A list of element types and material models is given in Table 1. The program was organized on a modular basis so that any new structural element and material model can be implemented with only minor programming changes.

To demonstrate the utility and analysis capability of the NONSAP-A program, five numerical examples are presented in the following:

1. Indirect Tension of a Concrete Cylinder

A concrete cylinder subjected to two equal and opposite forces through rigid metal blocks, as shown in Fig. 2, was analyzed by the NONSAP-A program. The metal blocks were assumed to be of steel for which the Young's modulus and Poisson's ratio are respectively: $E_s = 30 \times 10^6$ psi, $\nu_s = 0.3$. The material constants used for concrete are shown in Table 2, same as those of reference [1]. The cylinder was assumed to be fairly long compared with its diameter and therefore a plane strain model can be used for the analysis. Due to symmetry in loading as well as geometry, only one quarter of the cylinder was considered. The stress-strain relations for the steel blocks were assumed to be linearly elastic. This assumption is justified since the stress level in the steel blocks is well below its yield limit. For concrete, three different constitutive relations were used for the purpose of comparison; namely, Lehigh's model, Drucker-Prager model, and von Mises plasticity for metals. The load-deflection curves obtained from the three different models were plotted in Fig. 3. It is noted, however, that the friction angle for the Drucker-Prager model calculated directly from the yield stresses of concrete in tension and compression was too high, i.e. $\theta = 56.6°$. Then, a lower value, $\theta = 30°$, was adopted in the analysis. As seen in Fig. 3, the von Mises model represents higher deformability once the material becomes plastic, That is, considerable strains are developed in the cylinder when the load is increased near the fracture of the cylinder. On the contrary, the Drucker-Prager model gives much stiffer response and the stress-strain law is very sensitive to the hydrostatic stress in concrete. Fig. 4 shows the spreading of yielded and fractured zones in the cylinder based on the Lehigh's model for the applied load P= 6.53 kips. A large portion of the cylinder has become fractured at this load level.

2. Punch Identation of a Concrete Cylinder

A short concrete cylinder subjected to vertical punches from the top and bottom surfaces, as shown in Fig. 5, was analyzed. From symmetric, only one quarter of the cylinder was considered for the analysis. Axisymmetric finite element model consisting of 144 quadrilateral elements and 176 nodes is also shown in Fig. 5. The material properties of concrete are the same as those of problem 1. The load-deflection curve obtained from NONSAP-A as seen in Fig. 6 is somewhat higher than that obtained by Chen [1]. This discrepancy is due to the fact that Chen's solution considered the reduced strength of concrete resulting from cracking developed in the cylinder whereas the NONSAP-A does not at this time include such an effect. Therefore, the material response of the present analysis is stiffer than that Chen obtained from his analysis.

3. Plastic Deformation of A Thick-Walled Cylinder

A thick-walled cylinder with inner radius 1.25" and outer radius 2.5" subjected to internal pressure was analyzed by the NONSAP-A program using twenty 6-node axisymmetric elements as shown in Fig. 7. The cylinder was made of aluminum 17SO with considerable strain hardening and its elastic-plastic stress strain curve is given in reference [6]. For the elastic-plastic analysis, von Mises yield criterion with isotropic hardening was assumed. Shown in Fig. 7 are the plots of the internal pressure vs. the hoop strains in the inner and outer surfaces of the cylinder. The strain response of the outer surface obtained from NONSAP-A program is identical to the results published by Hartzman [7], however, it is slightly below the analytical solution obtained by MacGregor [6].

4. Large Displacement Analysis of a Spherical Shell

An elastic spherical shell subjected to a concentrated apex load shown in Fig. 8 was analyzed by NONSAP-A. The geometry of the shell and elastic constants used are

Radius: $R = 4.76$ in.

Height: $h = 0.01576$ in.

Apex height: $H = 0.0859$ in.

Angle of half span $\theta = 10.9°$

Young's Modulus: $E = 10 \times 10^6$ psi

Poisson's ratio: $\nu = 0.3$

The finite element consists of ten 8 node elements with axisymmetric deformation. A total of 18 load steps was applied and equilibrium iterations were performed for each load increment. Shown in Fig. 8 is the load-deflection curve at the apex of the shell. At the beginning part, the shell exhibits softening behavior as the load is approaching to the snap through value. Near the snap through stage, very small load steps have to be used in order to obtain convergent solution. After the shell has become snapped through, it exhibits considerable stiffening behavior as shown in the figure. The same problem has been analyzed previously by NONSAP [5], Stricklin [8] and Mescall [9] and it can be seen that fairly good agreement was obtained among the three different solution methods.

5. Large Deformation Analysis of an Arch

A clamped circular arch shown in Fig. 9 was analyzed by NONSAP-A. Isotropic and linearly elastic material property was assumed and one-half of the arch span was represented by six 8-node plane stress elements. The arch geometry and material constants are given by

$$R = 100 \text{ in}$$

$$h = 2 \text{ in}$$

$$t = 1 \text{ in (width)}$$

$$\beta = 0.245 \text{ rad.}$$

$$E = 10 \times 10^6 \text{ psi}$$

$$\nu = 0.3$$

A non-dimensionalized load parameter is defined by

$$\bar{p} = \frac{12 \beta R^2}{\pi^2 E h^3} P$$

where P is the applied load at the apex.

Again, this is a large displacement, snap through problem and the load-deflection curve exhibit similar behavior as the spherical shell considered in the previous example. An analytical solution for this problem was obtained by Schreyer and Masur [10]. As seen in Fig. 8, the comparison between NONSAP-A solution and the analytical solution is quite good.

Conclusions

The computer subroutines for the constitutive relations of plain concrete have been prepared and implemented into a general purpose finite element program, NONSAP-A, for conducting nonlinear analysis of concrete structures. From the sample problems shown, the NONSAP-A program can be applied not only to concrete structures, but also to the elastic-plastic analysis of metal structures under small or large deformations.

Future effort will be devoted to extending the present concrete plasticity model to include the effect of cracking in concrete and steel reinforcement. In this way, a more realistic analysis can be carried out for reinforced concrete structures.

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Table 1 Element Types and Material Models in NONSAP-A

	Truss	Beam	2-D Element	3-D Element
Linear Elastic	Yes	Yes	Yes	Yes
von Mises Plasticity	Yes		Yes	Yes
Drucker-Prager Plasticity			Yes	
Concrete Plasticity			Yes	Yes
Mooney-Rivlin Material			Yes	
Large Displacement and Strain	Yes		Yes	Yes

Table 2 Material Constants for Concrete

	τ_o (ksi)	τ_u (ksi)	α (ksi^{-1})	β (ksi)
Compression Zone	1.324	2.208	0.149	0.437
Tension Zone	0.327	0.545	4.260	0.759

Young's Modulus $E = 3791$ ksi

Poisson's Ratio $\nu = 0.188$

$f'_{bc} = 1.160 f'_c$

$f'_t = 0.090 f'_c$

$f_t = 0.054 f'_c$

$f_c = 0.600 f'_c$

$f'_c = 9.450$ ksi

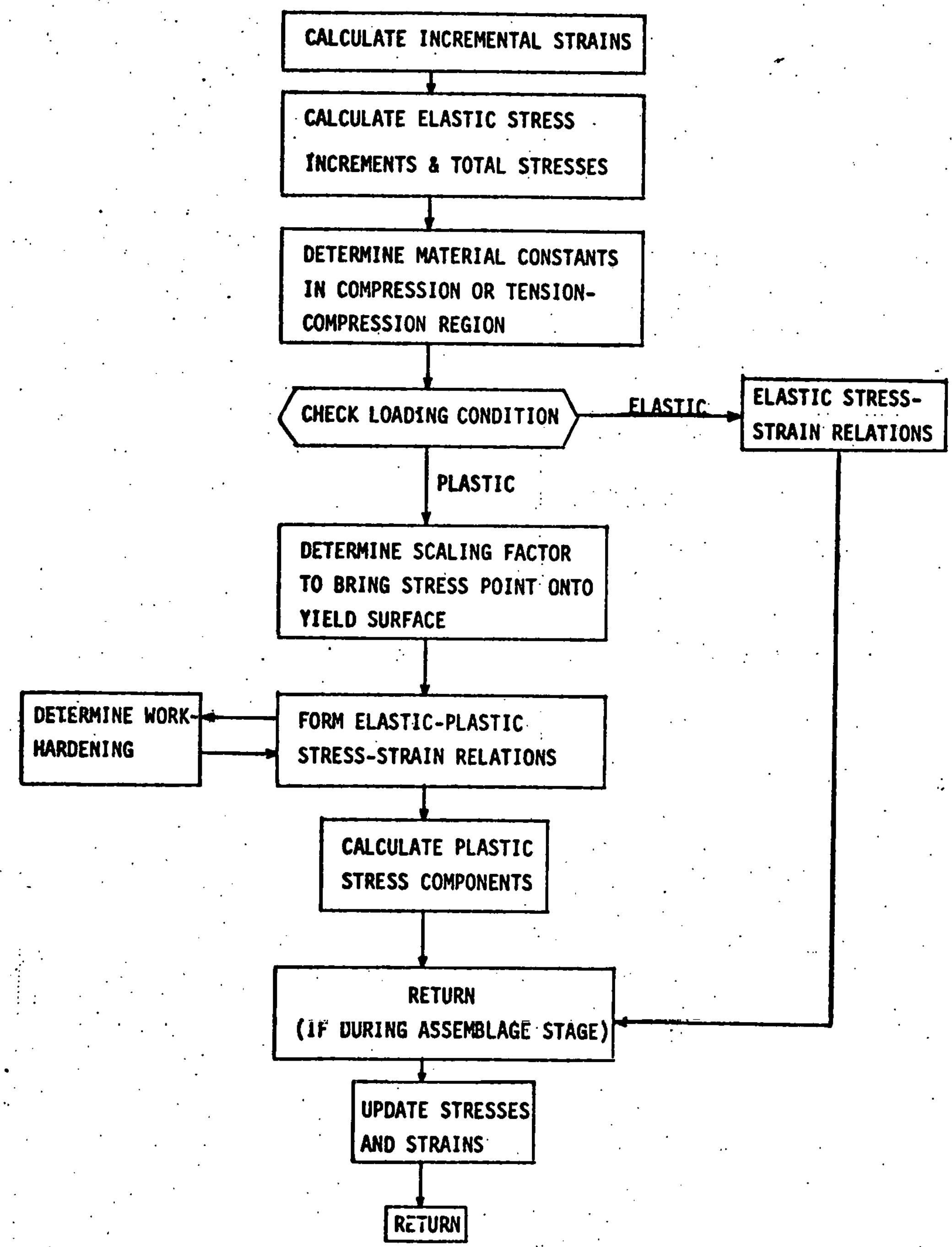

Fig.1 Flow diagram for the computer algorithm of concrete plasticity.

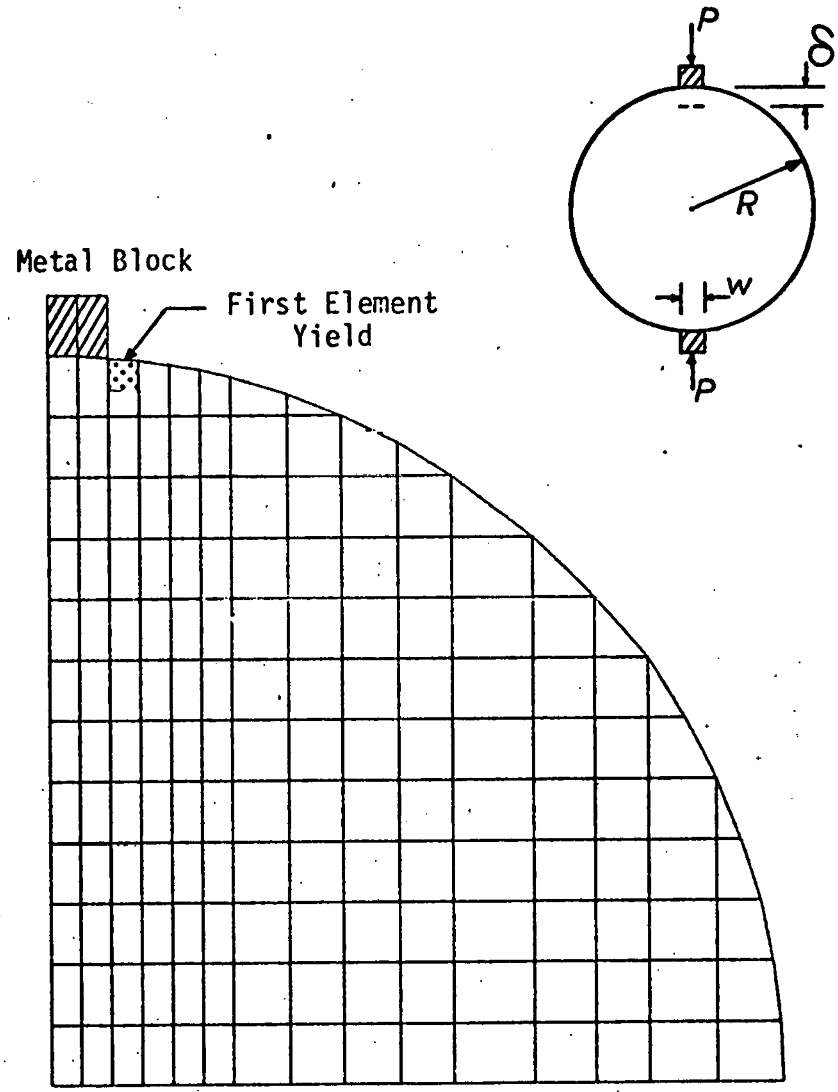

Fig. 2 Finite element model for indirect tension of a concrete cylinder.

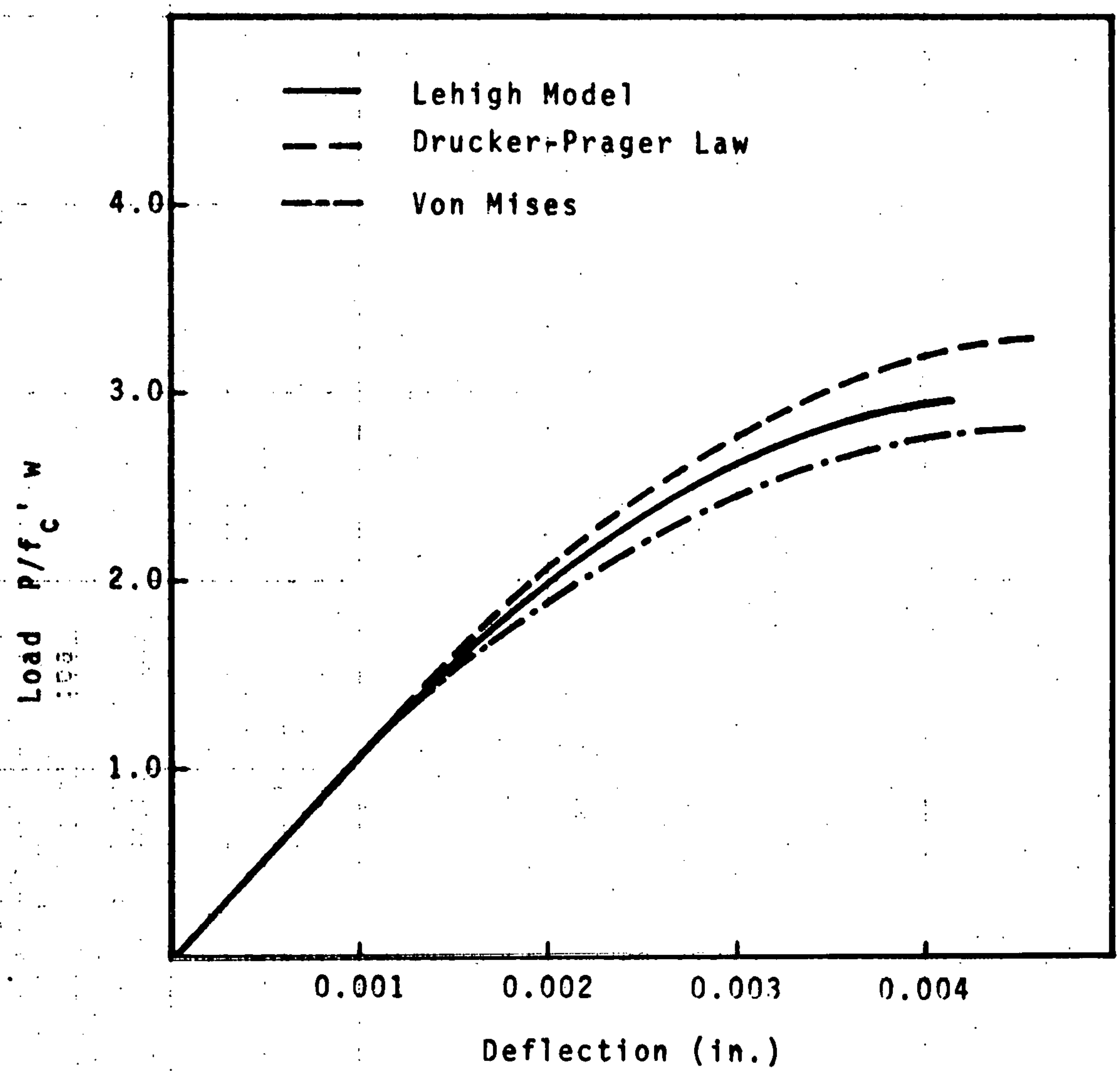

Fig. 3 Load-deflection response of a concrete cylinder.

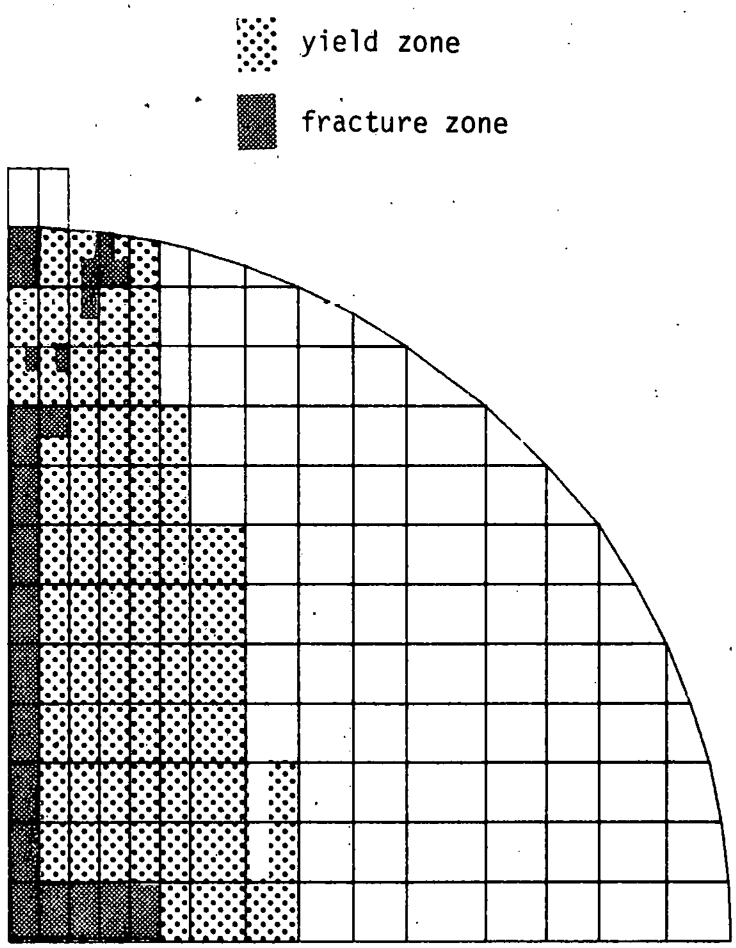

Fig. 4 Spread of yielded and fractured zones in the cylinder for load P = 6.53 K.

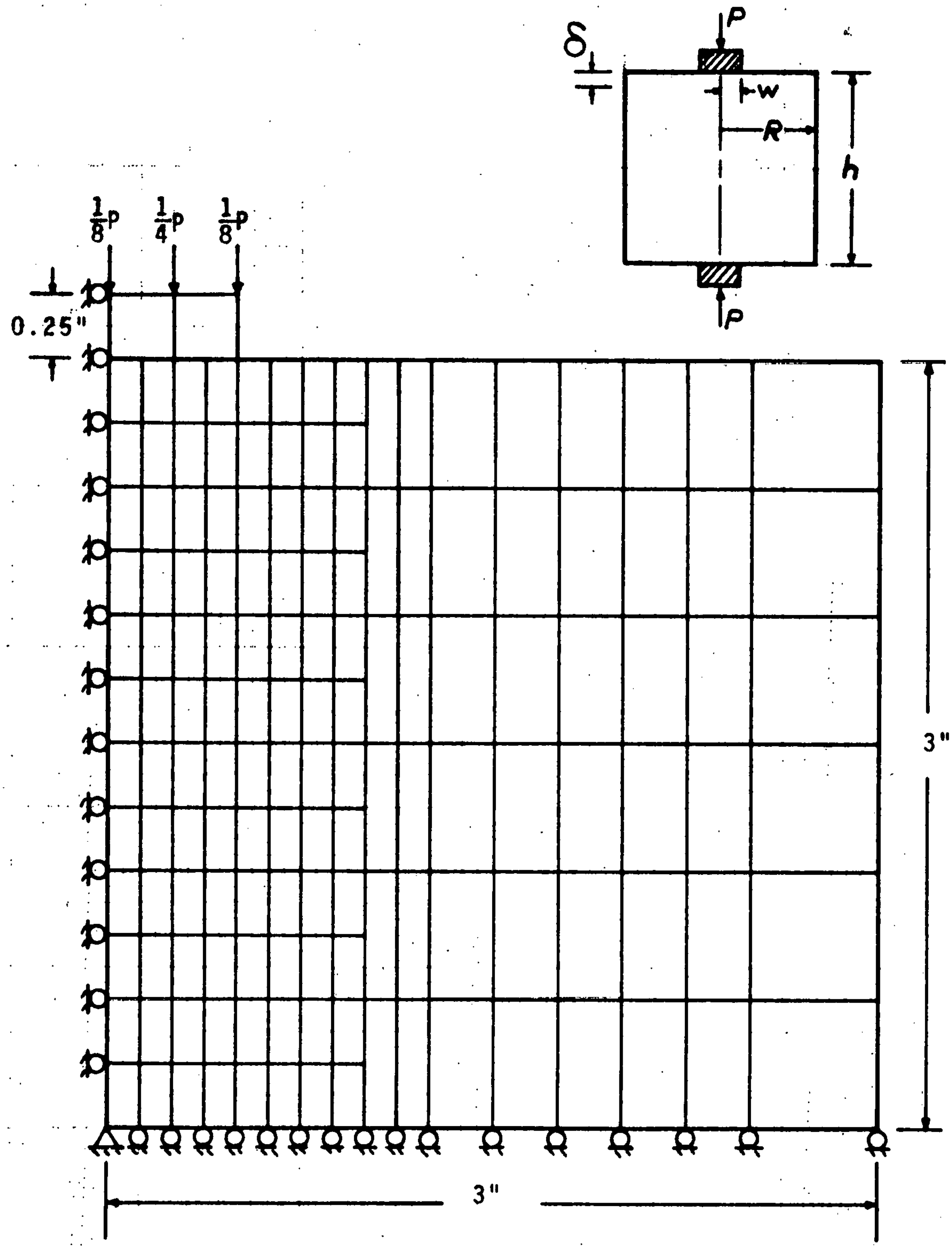

Fig. 5 Finite element model for punch indentation of concrete cylinder.

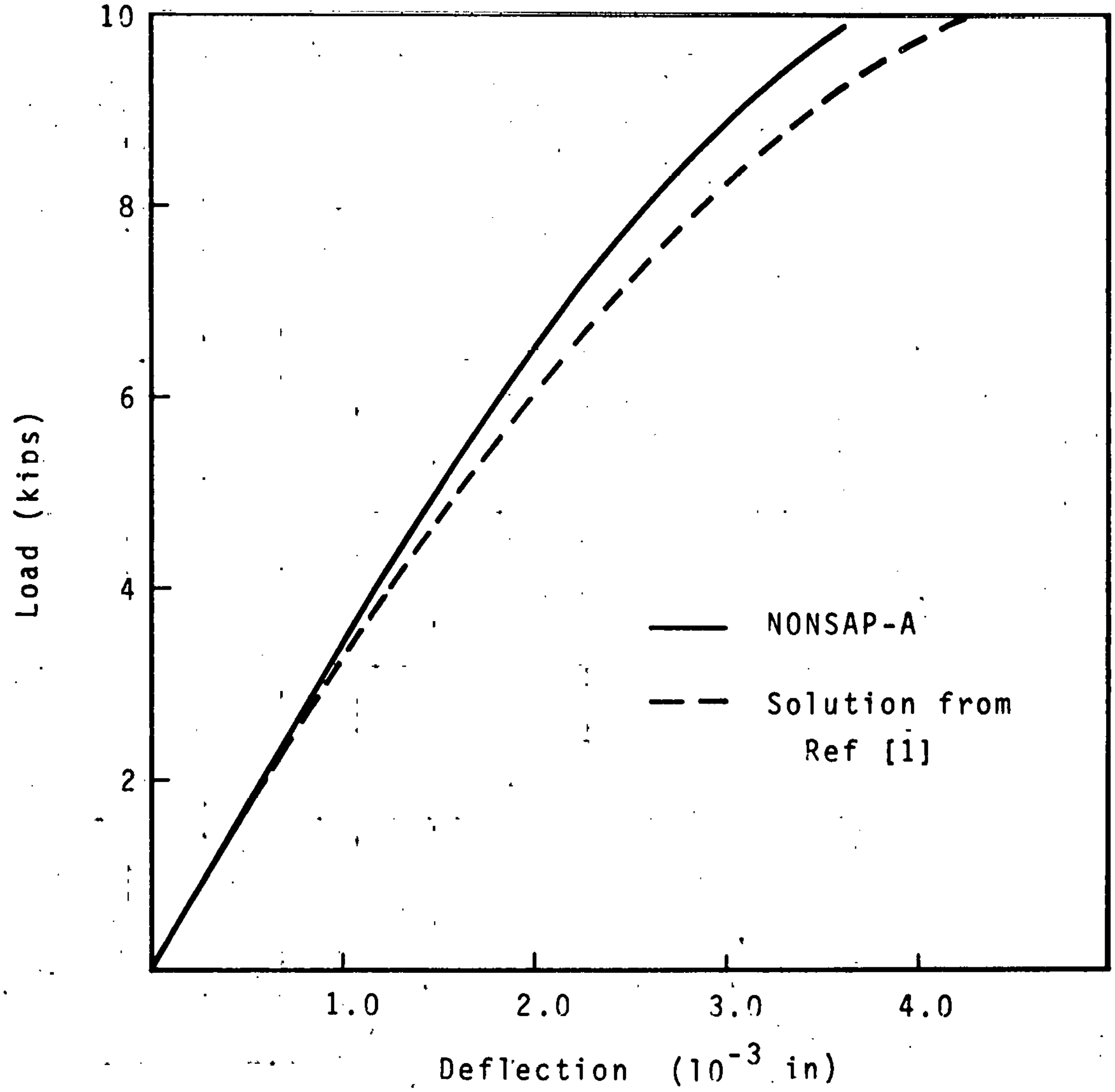

Fig. 6 Load deflection curve of a concrete cylinder subjected to rigid punch.

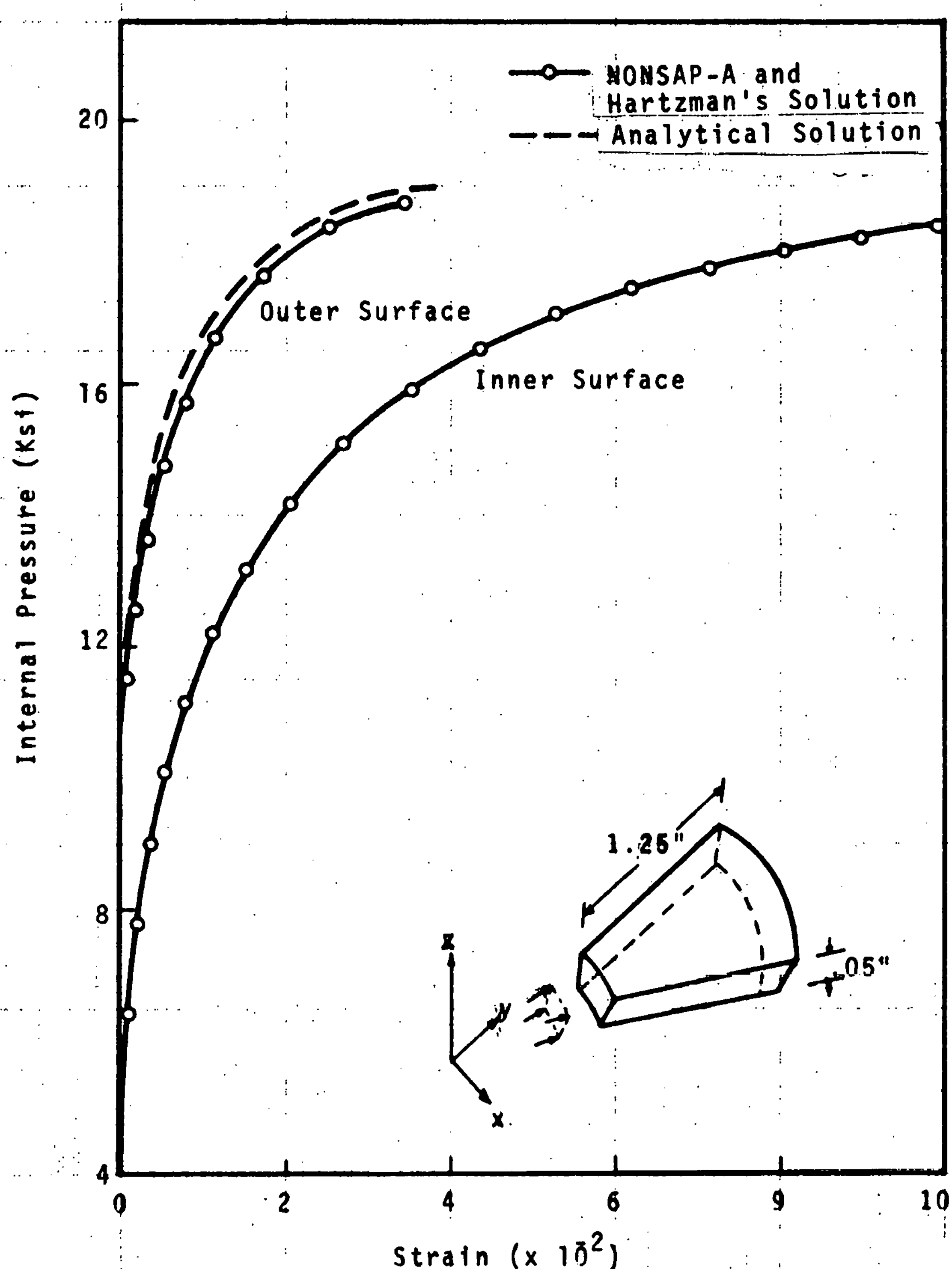

Fig. 7 Internal pressure vs. hoop strain at the inner and outer surface

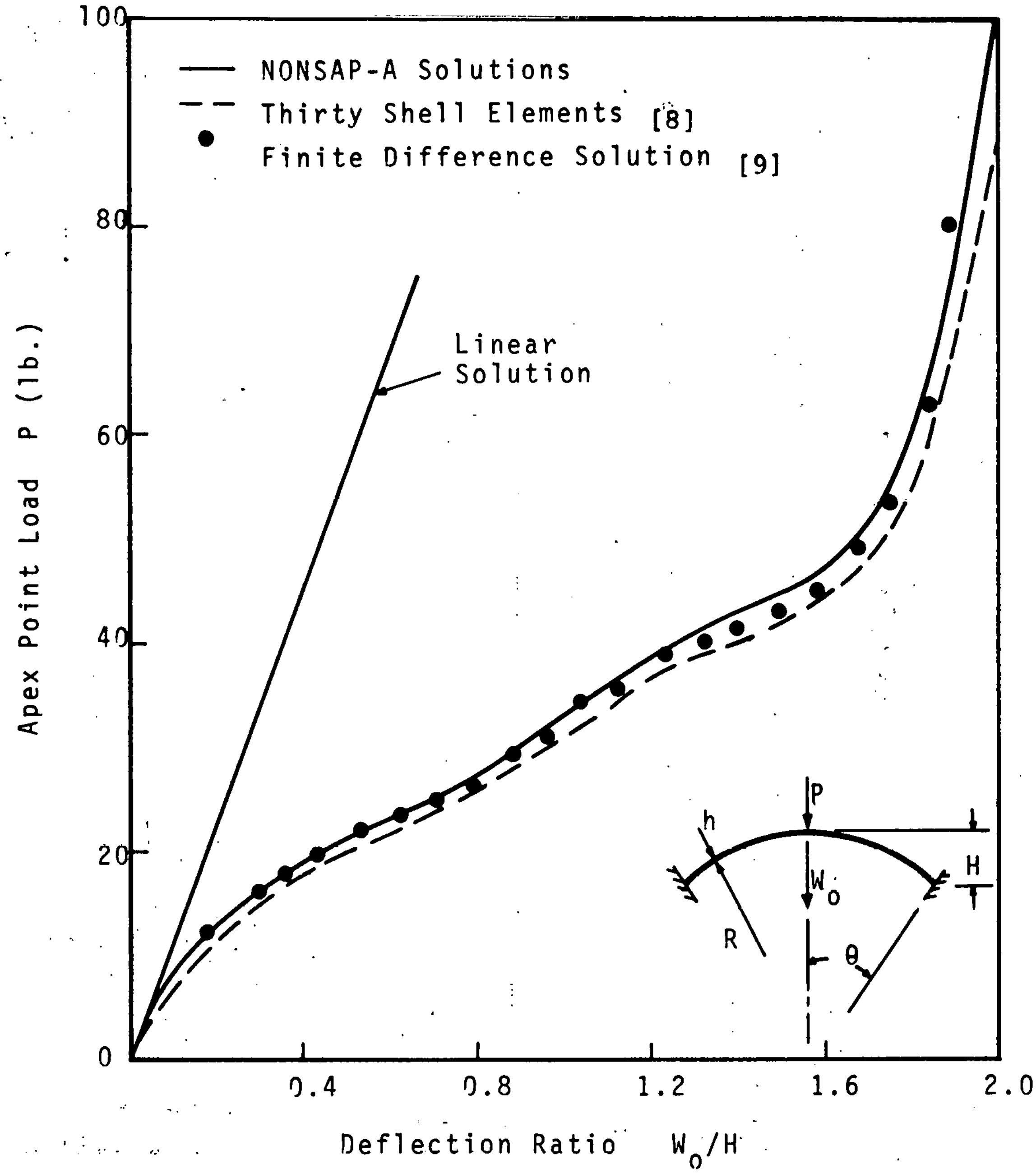

Fig. 8 Load-deflection of a Spherical Shell.

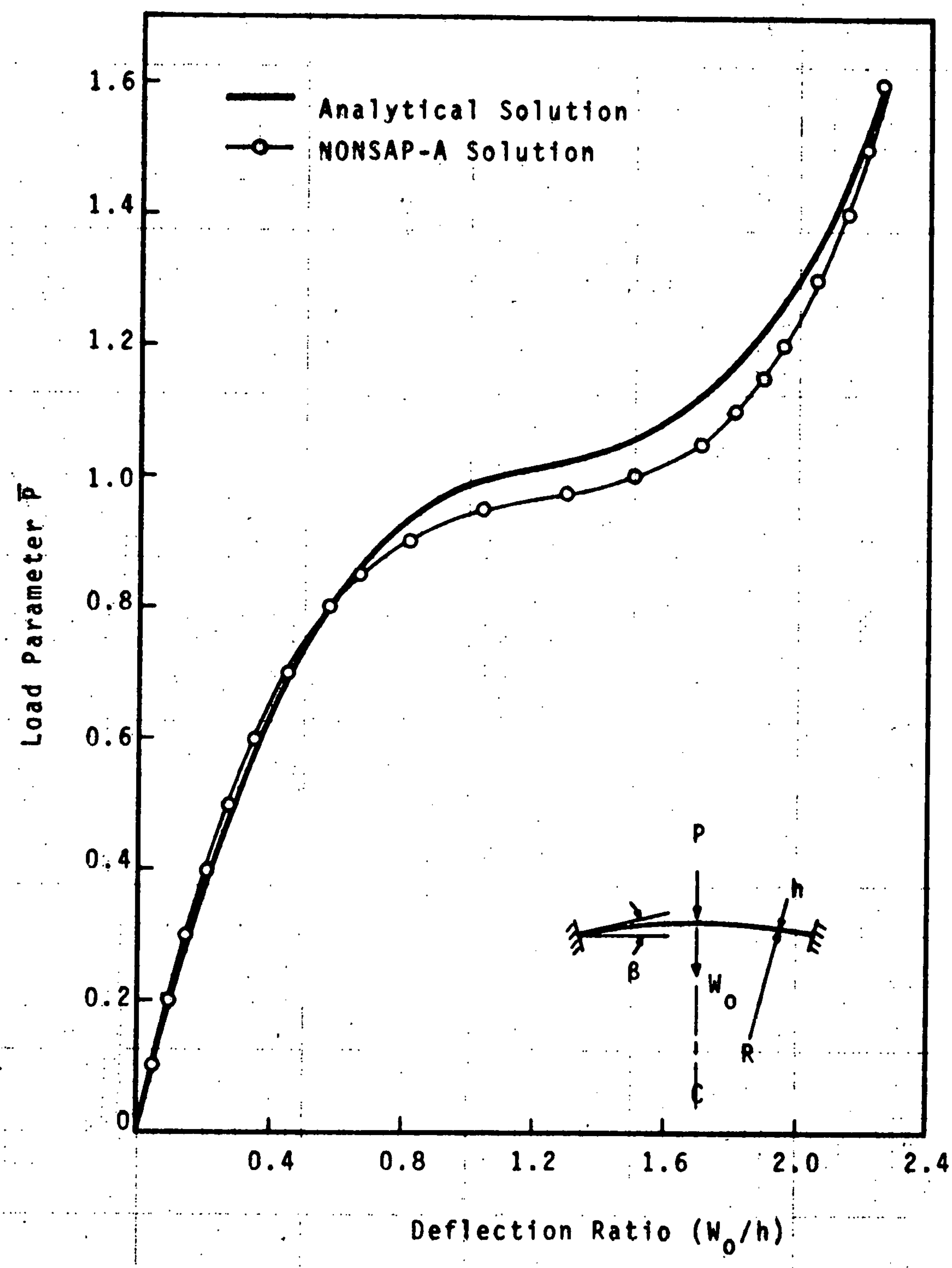

Fig. 9 Load-deflection of a shallow arch.

APPENDIX - COMPUTER SUBROUTINES

CONC2D - Check the yield condition and calculate the elastic-plastic stresses for two-dimensional stress state.

MIDEP2 - Form the elasto-plastic material matrix for two-dimensional stress state.

EPSLOP - Determine the strain-hardening function "H" from the effective stress - effective plastic strain curve.

CONC3D - Check the yield condition and calculate the elastic-plastic stresses for three-dimensional stress state.

MIDEP3 - Form the elasto-plastic material matrix for three-dimensional stress state.

```
      SUBROUTINE CONC2D(PROP,SIG,EPS,YIELD,IPEL)
C
C     CONSTITUTIVE RELATIONS OF CONCRETE UNDER 2-D STRESSES
C     PLANE STRESS, PLANE STRAIN, OR AXISYMMETRIC DEFORMATION STATE
C
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
C .                                                                   .
C .      IST      NUMBER OF  STRESS COMPONENTS                        .
C .      ISR      NUMBER OF  STRAIN COMPONENTS                        .
C .      SIG      STRESSES AT THE END OF THE PREVIOUS UPDATE          .
C .      EPS      STRAINS  AT THE END OF THE PREVIOUS UPDATE          .
C .      RATIO    PART OF STRAIN INCREMENT TAKEN ELASTICALLY          .
C .      DELEPS   INCREMENT IN STRAINS                                .
C .      DELSIG   INCREMENT IN STRESSES, ASSUMING ELASTIC BEHAVIOR    .
C .                                                                   .
C .      PROP(1) YOUNG S  MODULUS                                     .
C .      PROP(2) POISSON S  RATIO                                     .
C .      PROP(3) - PROP(6) MATERIAL CONSTANTS OF CONCRETE FOR COMP ZONE
C .      PROP(7) -PROP(10) MATERIAL CONSTANTS OF CONCRETE FOR TENS ZONE
C .                                                                   .
C .      IPEL     = 1, MATERIAL ELASTIC                               .
C .               = 2, MATERIAL PLASTIC                               .
C .      ITYP2D   = 0, AXISYMMETRIC DEFORMATION
C .               = 1, PLANE STRAIN
C .               = 2, PLANE STRESS
C .                                                                   .
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
C
C
      COMMON /EL/ IND,ICOUNT,NPAR(20),NUMEG,NEGL,NEGNL,IMASS,IDAMP,ISTAT
     1            ,NDOF,KLIN,IEIG,IMASSN,IDAMPN
      COMMON /VAR    / NG,KPRI,MODEX,KSTEP,ITE,ITEMAX,IREF,IEQREF,INOCMD
      COMMON /ELPMID/ YM,PV,YLD,XN,XA,XB,XK,RTC
     1                , A1,A2,B1,B2,C1,C2,D1,BM,YLDC,YLDT,ISR,IST
      COMMON /MATMOD/ STRESS(4),STRAIN(4),C(4,4),IPT,NEL
      COMMON /DISDER/ DISD(5)
C
      DIMENSION PROP(1),SIG(1),EPS(1)
      DIMENSION TAU(4),DELSIG(4),DELEPS(4),DEPS(4),STATE(2)
      DIMENSION LPR(4),LPC(4)
C
      EQUIVALENCE (NPAR(3),INDNL),(NPAR(5),ITYP2D),(DELEPS(1),DEPS(1))
C
      DATA NGLAST/1000/, STATE/1HE,1HP/
C
C
C
C
      IF (IPT.NE.1) GO TO 110
      IST=4
```

```
      IF (ITYP2D.EQ.2) IST=3
      ISR=3
      IF (ITYP2D.EQ.0) ISR=4
      YM=PROP(1)
      PV=PROP(2)
      XK = 1.732
      D1=PV/(PV - 1.)
      A2=YM/(1.+PV)
      B2=(1.-PV)/(1.-2.*PV)
      C2=PV/(1.-2.*PV)
      C1=A2/2.
      BM=YM/(1. - 2.*PV)/3.
      YLDC = PROP(4)
      YLDT = PROP(8)
C
      IF (ITYP2D.EQ.2) GO TO 105
C
C     PLANE STRAIN  /  AXISYMMETRIC
      B1=A2*C2
      A1=B1+A2
      GO TO 110
C
C     PLANE STRESS
  105 A1=YM/(1.-PV*PV)
      B1=A1*PV
C
  110 YLD = YIELD
C
C
C        1. CALCULATE INCREMENTAL STRAINS
C
      DO 120 I=1,ISR
  120 DELEPS(I) = STRAIN(I) - EPS(I)
C
C
C        2. CALCULATE THE STRESS INCREMENT,
C           ASSUMING ELASTIC BEHAVIOR
C
      DELSIG(1) = A1*DELEPS(1) + B1*DELEPS(2)
      DELSIG(2) = B1*DELEPS(1) + A1*DELEPS(2)
      DELSIG(3) = C1*DELEPS(3)
      DELSIG(4) = 0.
      IF (ITYP2D.EQ.2) GO TO 150
      DELSIG(4) = B1 * (DELEPS(1)+DELEPS(2))
      IF (ITYP2D.EQ.1) GO TO 150
      DELSIG(1) = DELSIG(1) + B1*DELEPS(4)
      DELSIG(2) = DELSIG(2) + B1*DELEPS(4)
      DELSIG(4) = DELSIG(4) + A1*DELEPS(4)
C
C
```

```
C          3. CALCULATE TOTAL STRESSES,
C             ASSUMING ELASTIC BEHAVIOR
C
  150 TAU(4) = 0.
      DO 160 I=1,IST
  160 TAU(I) = SIG(I) + DELSIG(I)
C
      SM = TAU(1)+TAU(2)+TAU(4)
      SX = TAU(1)
      SY = TAU(2)
      SS = TAU(3)
      SZ = TAU(4)
      XJ2 = 0.5*(SX*SX +SY*SY+SZ*SZ+2.*SS*SS)-SM*SM/6.0
      RTC=1.0
      IF (SM .GT. 0.0) RTC=-1.0
      XX = SQRT(XJ2) + SM/1.732
      IF (XX .GT. 0.0) RTC=-1.0
      IF (RTC .LT. 0.) GO TO 166
C
C     CONSTANTS FOR COMPRESSION REGION
C
      AO = PROP(3)
      TO = PROP(4)
      AU = PROP(5)
      TU = PROP(6)
      GO TO 167
  166 CONTINUE
C
C     CONSTANTS FOR TENSION - COMPRESSION REGION
C
      AO = PROP(7)
      TO = PROP(8)
      AU = PROP(9)
      TU = PROP(10)
  167 CONTINUE
      XA = (AU-AO)/(TU*TU-TO*TO)
      XB = (AO*TU*TU-AU*TO*TO)/(TU*TU-TO*TO)
      XN = (3.*RTC-XK*XK)/36.
      IF (YLD .LT. 0.) YLD = TO
C
C        4. CHECK WHETHER *TAU* STATE OF STRESS FALLS
C           OUTSIDE THE LOADING SURFACE
C
C
      F1 = ((XK*XK/3.)*(0.5*(SX*SX+SY*SY+SZ*SZ+2.*SS*SS)-SM*SM/6.)
     1       +XN*SM*SM+XB*SM/3.)/(1.-XA*SM/3.)
      FT = F1-YLD*YLD
C
      IF (FT) 170,170,300
C
```

```
C     STATE OF STRESS WITHIN LOADING SURFACE  -  ELASTIC BEHAVIOR
C
  170 IPEL=1
      YLD = -1.0
      STRESS(4) = 0.
      DO 180 I=1,IST
  180 STRESS(I) = TAU(I)
      IF (ITYP2D.EQ.2) STRAIN(4)=EPS(4) + D1*(DELEPS(1) + DELEPS(2))
      GO TO 400
C
C
C     STATE OF STRESS OUTSIDE LOADING SURFACE  -  PLASTIC BEHAVIOR
C
C
C     DETERMINE PART OF STRAIN TAKEN ELASTICLY
C
  300 IPEL=2
C
      SM = SIG(1)+SIG(2)+SIG(4)
      SX = SIG(1)
      SY = SIG(2)
      SS = SIG(3)
      SZ = SIG(4)
C
      DM = DELSIG(1)+DELSIG(2)+DELSIG(4)
      DX = DELSIG(1)
      DY = DELSIG(2)
      DS = DELSIG(3)
      DZ = DELSIG(4)
C
      X1 = XB + XA*YLD*YLD
      A = (XK*XK/3.)*(0.5*(DX*DX+DY*DY+DZ*DZ+2.*DS*DS)-DM*DM/6.)
     1  + XN*DM*DM
      B = (XK*XK/3.)*(0.5*(SX*DX+SY*DY+SZ*DZ+2.*SS*DS)-SM*DM/6.)
     1     +XN*SM*DM+X1*DM/6.
      E = (XK*XK/3.)*(0.5*(SX*SX+SY*SY+SZ*SZ+2.*SS*SS)-SM*SM/6.)
     1     +XN*SM*SM+X1*SM/3.-YLD*YLD
C
      RR = B*B - A*E
      RATIO=(-B + SQRT(RR))/A
C
      DO 350 I=1,IST
  350 TAU (I) = SIG(I) + RATIO*DELSIG(I)
      IF (ITYP2D.EQ.2) STRAIN(4)=EPS(4) + RATIO*D1*(DELEPS(1)
     1                  + DELEPS(2))
C
C     *TAU* NOW CONTAINS (PREVIOUS STRESSES +
C     STRESSES DUE TO ELASTIC STRAIN INCREMENTS)
C
C
```

```
C          5. CALCULATE PLASTIC  STRESSES
C
C       DETERMINE  INCREMENT INTERVAL
  370 M=10.*SQRT(FT)/YLD + 1
      IF (M.GT.30) M=30
      XM = (1. - RATIO)/M
C
C
      DO 380 I=1,4
C
C     ELASTIC-PLASTIC MATERIAL MATRIX
C
  380 DEPS(I) = XM*DELEPS(I)
C
C
C     .....  CALCULATION OF ELASTOPLASTIC STRESSES  .....  (START)
C
      DO 600 IM=1,M
C
      CALL MIDEP2(TAU,DEPS,C)
C
      DO 560 I=1,IST
      DO 560 J=1,ISR
  560 TAU(I) = TAU(I) + C(I,J) * DEPS(J)
C
C
C     UPDATE YLD
C
      SM = TAU(1)+TAU(2)+TAU(4)
      SX = TAU(1)
      SY = TAU(2)
      SS = TAU(3)
      SZ = TAU(4)
      F1 = ((XK*XK/3.)*(0.5*(SX*SX+SY*SY+SZ*SZ+2.*SS*SS)-SM*SM/6.)
     1        +XN*SM*SM+XB*SM/3.)/(1.-XA*SM/3.)
      YLD = SQRT (F1)
C
  600 CONTINUE
C
C     .....  CALCULATION OF ELASTOPLASTIC STRESSES  .....  ( END )
C
      STRESS(4) = 0.
      DO 390  I = 1,IST
  390 STRESS(I) = TAU(I)
C
C
C         6. UPDATING STRESSES, STRAINS, YIELD, NS
C
  400 DO 410 I=1,IST
  410 SIG(I)  = STRESS(I)
```

```
      DO 420 I=1,ISR
  420 EPS(I)   = STRAIN(I)
      YIELD    = YLD
      IF (ITYP2D.EQ.2) EPS(4)=STRAIN(4)
      IF (KPRI.EQ.0) GO TO 700
C
C
      IF (ICOUNT.EQ.3) RETURN
C
C        7. FORM THE MATERIAL LAW
C
      IF (IPEL.EQ.1) GO TO 450
C
C     ELASTIC-PLASTIC MATERIAL MATRIX
C
      CALL MIDEP2(TAU,DEPS,C)
      RETURN
C
C     ELASTIC
  450 DO 460 I=1,ISR
      DO 460 J=1,ISR
  460 C(I,J)=0.
      C(1,1)=A1
      C(2,1)=B1
      C(1,2)=B1
      C(2,2)=A1
      C(3,3)=C1
      IF (ITYP2D .GT.0) RETURN
      C(1,4)=B1
      C(2,4)=B1
      C(4,1)=B1
      C(4,2)=B1
      C(4,4)=A1
C
      RETURN
C
C
C     P R I N T I N G   O F   S T R E S S E S
C
  700 CONTINUE
C
      SM = STRESS(1)+STRESS(2)+STRESS(4)
      SX = STRESS(1)
      SY = STRESS(2)
      SS = STRESS(3)
      SZ = STRESS(4)
C
C
  800 IF (NG.NE.NGLAST) GO TO 802
      IF (NEL.GT.NELAST) GO TO 806
```

```
      IF (IPT-1) 810,808,810
C
  802 NGLAST = NG
  808 IF (ITYP2D) 803,805,803
  803 WRITE (6,2002)
      GO TO 806
  805 WRITE (6,2003)
C
  806 NELAST=NEL
      WRITE (6,2004) NEL
  810 CONTINUE
      F1 = ((XK*XK/3.)*(0.5*(SX*SX+SY*SY+SZ*SZ+2.*SS*SS)-SM*SM/6.)
     1         +XN*SM*SM+XB*SM/3.)/(1.-XA*SM/3.)
      EFSG = 0.
      IF (F1 .GT. 0.) EFSG = SQRT(F1)
      IF (ITYP2D) 813,815,813
C
  813 WRITE (6,2005) IPT,STATE(IPEL),
     1                 (STRESS(I),I=1,3),(STRAIN(I),I=1,3),EFSG
      RETURN
C
  815 WRITE (6,2007) IPT,STATE(IPEL),STRESS(4),
     1                 (STRESS(I),I=1,3),STRAIN(4),(STRAIN(I),I=1,3),
     2                  EFSG  , RTC
      RETURN
C
C
 2002 FORMAT(101H ELEMENT  STRESS  STRESS-YY  STRESS-ZZ  STRESS-YZ    ST
     1RAIN-YY    STRAIN-ZZ    STRAIN-YZ    EFFECTIVE , /
     2          16H NUM/IPT   STATE,79X,6HSTRESS,/)
 2003 FORMAT(120H ELEMENT  STRESS  STRESS-XX  STRESS-YY  STRESS-ZZ  STRE
     1SS-YZ   STRAIN-XX   STRAIN-YY   STRAIN-ZZ   STRAIN-YZ   EFFECTIVE,
     2   /,    16H NUM/IPT   STATE,98X,6HSTRESS,/)
 2004 FORMAT (I4/)
 2005 FORMAT(5X,I2,2X,A1,6HLASTIC,3(2X,F9.1),3(3X,E10.3),4X,F9.1)
 2007 FORMAT(5X,I2,2X,A1,6HLASTIC,4(2X,F9.1),4(2X,E10.3),2X,F9.1,F5.1)
 4000 FORMAT(1H ,'ELEMENT NO.',I5,5X,'INT. PT.',I5,5X,'PLASTIC',3E12.5)
C
      END
```

```
      SUBROUTINE MIDEP2(TAU,DEPS,DP)
C
C        FORMS THE ELASTO-PLASTIC MATERIAL MATRIX
C
      COMMON /EL/ IND,ICOUNT,NPAR(20),NUMEG,NEGL,NEGNL,IMASS,IDAMP,ISTAT
     1             ,NDOF,KLIN,IEIG,IMASSN,IDAMPN
      COMMON /MATMOD/ STRESS(4),STRAIN(4),C(4,4),IPT,NEL
      COMMON /ELPMID/ YM,PV,YLD,XN,XA,XB,XK,RTC
     1                , A1,A2,B1,B2,C1,C2,D1,BM,YLDC,YLDT,ISR,IST
      DIMENSION TAU(1),DEPS(1),DP(1)
C
      EQUIVALENCE (NPAR(5),ITYP2D)
C
C
      SM = (TAU(1)+TAU(2)+TAU(4))/3.
      SX = TAU(1) - SM
      SY = TAU(2) - SM
      SS = TAU(3)
      SZ = TAU(4) - SM
C
C     CALCULATE STRAIN-HARDENING FUNCTION 'H'
C
      STR = YLD
      IF (RTC.LT.0.) STR = (1.0 +(YLD-YLDT)/YLDT)*YLDC
C
      CALL EPSLOP (STR,HP)
      H = 2.*YLD*HP
      XM = 1.-XA*SM
      XJ2 = 0.5*(SX*SX +SY*SY+SZ*SZ+2.*SS*SS)
      ET = XK*XK/3.
      RO = XN*SM*6.+(XB+XA*YLD*YLD)/3.
      Y1 = 1+PV
      Y2 = 1-2.*PV
      Y3 = 3.*PV*RO
      WINV = Y2*(2.*ET*ET*XJ2+3.*RO*RO)+9.*PV*RO*RO+XM*H*Y1*Y2/YM
     1        *SQRT(2.*ET*ET*XJ2+3.*RO*RO)
      W   = 1./WINV
C
      F1  = Y2*(ET*SX+RO) + Y3
      F11 = F1*F1
      F12 = F1*(Y2*(ET*SZ+RO)+Y3)
      F13 = F1*(Y2*(ET*SY+RO)+Y3)
      F14 = F1*(Y2*ET*SS)
      F2  = Y2*(ET*SZ+RO)+Y3
      F22 = F2*F2
      F23 = F2*(Y2*(ET*SY+RO)+Y3)
      F24 = F2*(Y2*ET*SS)
      F3  = Y2*(ET*SY+RO)+Y3
      F33 = F3*F3
      F34 = F3*(Y2*ET*SS)
```

```
      F44 = (Y2*ET*SS)*(Y2*ET*SS)
C
C
      A1 = YM/(Y1*Y2)
      B1 = 1. - PV
      C1 = PV
C
      DP(1) = A1*(B1-W*F11)
      DP(2) = A1*(C1-W*F13)
      DP(3) = A1*( -W*F14)
      DP(4) = A1*(C1-W*F12)
      DP(5) = DP(2)
      DP(6) = A1*(B1-W*F33)
      DP(7) = A1*(   -W*F34)
      DP(8) = A1*(C1-W*F23)
      DP(9) = DP(3)
      DP(10) = DP(7)
      DP(11) = A1*(.5*Y2 - W*F44)
      DP(12) = A1*( -W*F24)
C
      IF (ITYP2D.EQ.1) RETURN
C
      DP(13) = DP(4)
      DP(14) = DP(8)
      DP(15) = DP(12)
      DP(16) = A1*(B1-W*F24)
C
      IF (ITYP2D.EQ.0) RETURN
C
C     PLANE STRESS / MODIFY DP MATRIX
C
      DEPS(4) = -(DP(4)*DEPS(1)+DP(8)*DEPS(2)+DP(12)*DEPS(3))/DP(16)
      STRAIN(4)=STRAIN(4) + DEPS(4)
      DO 120 I=1,3
      A=C(I,4)/C(4,4)
      DO 120 J=I,3
      C(I,J)=C(I,J) - C(4,J)*A
  120 C(J,I) = C(I,J)
C
C
      RETURN
      END
```

```
      SUBROUTINE EPSLOP (STR,EST)
      COMMON /EL/ IND,ICOUNT,NPAR(20),NUMEG,NEGL,NEGNL,IMASS,IDAMP,ISTAT
     1            ,NDOF,KLIN,IEIG,IMASSN,IDAMPN
      COMMON /WHF/ STSS(30),STRN(30)
      COMMON /ELPMID/ YM,PV,YLD,XN,XA,XB,XK,RTC
      EQUIVALENCE (NPAR(17),NCON)
C
      MAX = (NCON-10)/2
      SIGE = STR
      IF (SIGE .LE. STSS(1) ) GO TO 40
      DO 10 K = 2, MAX
      N = K
      IF (SIGE .GE. STSS(K-1) .AND. SIGE .LT. STSS(K)) GO TO 20
   10 CONTINUE
      IF (SIGE .GE. STSS(MAX)) GO TO 30
      WRITE(6,200)
      STOP
C
   20 CONTINUE
      RANGE = STSS(N)-STSS(N-1)
      RATIO = (STSS(N)-SIGE) / RANGE
      IF (RATIO .LE. 0.1) N = N+1
      IF (N .GE. MAX) GO TO 30
      EST    = (STSS(N)-STSS(N-1))/(STRN(N)-STRN(N-1))
      GO TO 100
C
   30 CONTINUE
      EST    = 0.0
      GO TO 100
   40 EST    = (STSS(2)-STSS(1)) / (STRN(2)-STRN(1))
  100 CONTINUE
      RETURN
  200 FORMAT(1H0, 37HERROR - EFFECTIVE STRESS OUT OF RANGE )
      END
```

```
      SUBROUTINE CONC3D(PROP,SIG,EPS,YIELD,IPEL)
C
C     CONSTITUTIVE RELATIONS OF CONCRETE UNDER 3-D STRESSES
C
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
C .                                                                   .
C .      SIG     STRESSES AT THE END OF THE PREVIOUS UPDATE           .
C .      EPS     STRAINS  AT THE END OF THE PREVIOUS UPDATE           .
C .      RATIO   PART OF STRAIN INCREMENT TAKEN ELASTICALLY           .
C .      DELEPS  INCREMENT IN STRAINS                                 .
C .      DELSIG  INCREMENT IN STRESSES, ASSUMING ELASTIC BEHAVIOR     .
C .                                                                   .
C .      PROP(1) YOUNG S  MODULUS                                     .
C .      PROP(2) POISSON S  RATIO                                     .
C .      PROP(3) - PROP(6) MATERIAL CONSTANTS OF CONCRETE FOR COMP  ZONE
C .      PROP(7) -PROP(10) MATERIAL CONSTANTS OF CONCRETE FOR TENS  ZONE
C .                                                                   .
C .      IPEL    = 1, MATERIAL ELASTIC                                .
C .              = 2, MATERIAL PLASTIC                                .
C .                                                                   .
C . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
C
C
      COMMON /EL/ IND,ICOUNT,NPAR(20),NUMEG,NEGL,NEGNL,IMASS,IDAMP,ISTAT
     1           ,NDOF,KLIN,IEIG,IMASSN,IDAMPN
      COMMON /VAR   / NG,KPRI,MODEX,KSTEP,ITE,ITEMAX,IREF,IEQREF,INOCMD
      COMMON /ELPMID/ YM,PV,YLD,XN,XA,XB,XK,RTC,YLDC,YLDT,A1,B1
      COMMON /MATMOD/ STRESS(6),STRAIN(6),C(6,6),IPT,NEL
      COMMON /DISDER/ DISD(5)
C
      DIMENSION PROP(1),SIG(1),EPS(1)
      DIMENSION TAU(6),DELSIG(6),DELEPS(6),DEPS(6),STATE(2)
      DIMENSION LPR(4),LPC(4)
C
      EQUIVALENCE (NPAR(3),INDNL),(NPAR(5),ITYP2D),(DELEPS(1),DEPS(1))
C
      DATA NGLAST/1000/, STATE/1HE,1HP/
C
C
C
C
      KK = NEL + IPT
      IF (KK.GT.2) GO TO 110
      YM=PROP(1)
      PV=PROP(2)
      XK=1.732
      A1=YM/((1.+PV)*(1.-2.*PV))
      B1=A1*PV
      A1=A1*(1.-PV)
      YLDC = PROP(4)
```

```
      YLDT = PROP(8)
C
      DO 40 I = 1,6
      DO 40 J = 1,6
   40 C(I,J) = 0.
C
      C(1,1) = A1
      C(1,2) = B1
      C(1,3) = B1
      C(2,1) = C(1,2)
      C(2,2) = A1
      C(2,3) = B1
      C(3,1) = C(1,3)
      C(3,2) = C(2,3)
      C(3,3) = A1
      C(4,4) = YM/(1.+PV)/2.
      C(5,5) = C(4,4)
      C(6,6) = C(4,4)
C
  110 YLD = YIELD
C
C
C        1. CALCULATE INCREMENTAL STRAINS
C
      DO 120 I=1,6
  120 DELEPS(I) = STRAIN(I) - EPS(I)
C
C
C        2. CALCULATE THE STRESS INCREMENT,
C           ASSUMING ELASTIC BEHAVIOR
C
      DO 130 I=1,6
      DELSIG(I) = 0.
      DO 130 J=1,6
  130 DELSIG(I) = DELSIG(I) + C(I,J)*DELEPS(J)
C
C
C        3. CALCULATE TOTAL STRESSES,
C           ASSUMING ELASTIC BEHAVIOR
C
      DO 160 I=1,6
  160 TAU(I) = SIG(I) + DELSIG(I)
C
      SM = TAU(1)+TAU(2)+TAU(3)
      SX = TAU(1)
      SY = TAU(2)
      SZ = TAU(3)
      XY = TAU(4)
      YZ = TAU(5)
      ZX = TAU(6)
```

```
      XJ2 = 0.5*(SX*SX+SY*SY+SZ*SZ) - SM*SM/6. + XY*XY+YZ*YZ+ZX*ZX
      RTC=1.0
      IF (SM .GT. 0.0) RTC=-1.0
      XX = SQRT(XJ2) + SM/1.732
      IF (XX .GT. 0.0) RTC=-1.0
      IF (RTC .LT. 0.) GO TO 166
C
C     CONSTANTS FOR COMPRESSION REGION
C
      AO = PROP(3)
      TO = PROP(4)
      AU = PROP(5)
      TU = PROP(6)
      GO TO 167
  166 CONTINUE
C
C     CONSTANTS FOR TENSION - COMPRESSION REGION
C
      AO = PROP(7)
      TO = PROP(8)
      AU = PROP(9)
      TU = PROP(10)
  167 CONTINUE
      XA = (AU-AO)/(TU*TU-TO*TO)
      XB = (AO*TU*TU-AU*TO*TO)/(TU*TU-TO*TO)
      XN = (3.*RTC-XK*XK)/36.
      IF (YLD .LT. 0.) YLD = TO
C
C        4. CHECK WHETHER *TAU* STATE OF STRESS FALLS
C           OUTSIDE THE LOADING SURFACE
C
C
      F1 = (XK*XK/3.)*(XJ2+XN*SM*SM+XB*SM/3.)/(1.-XA*SM/3.)
      FT = F1-YLD*YLD
C
      IF (FT) 170,170,300
C
C     STATE OF STRESS WITHIN LOADING SURFACE  -  ELASTIC BEHAVIOR
C
  170 IPEL=1
      YLD = -1.0
      DO 180 I=1,6
  180 STRESS(I) = TAU(I)
      GO TO 400
C
C
C     STATE OF STRESS OUTSIDE LOADING SURFACE  -  PLASTIC BEHAVIOR
C
C
C     DETERMINE PART OF STRAIN TAKEN ELASTICLY
```

```
C
  300 IPEL=2
C
      SM = SIG(1)+SIG(2)+SIG(3)
      SX = SIG(1)
      SY = SIG(2)
      SZ = SIG(3)
      XY = TAU(4)
      YZ = TAU(5)
      ZX = TAU(6)
C
      DM = DELSIG(1)+DELSIG(2)+DELSIG(3)
      DX = DELSIG(1)
      DY = DELSIG(2)
      DZ = DELSIG(3)
      DXY = DELSIG(4)
      DYZ = DELSIG(5)
      DZX = DELSIG(6)
C
      X1 = XB + XA*YLD*YLD
      A = (XK*XK/3.)*(0.5*(DX*DX+DY*DY+DZ*DZ+2.*(DXY*DXY+DYZ*DYZ
     1     +DZX*DZX))-DM*DM/6.+XN*DM*DM
      B = (XK*XK/3.)*(0.5*(SX*DX+SY*DY+SZ*DZ+2.*(XY*DXY+YZ*DYZ+ZX*DZX))
     1     -SM*DM/6.)+XN*SM*DM+X1*DM/6.
      E = (XK*XK/3.)*(0.5*(SX*SX+SY*SY+SZ*SZ+2.*(XY*XY+YZ*YZ+ZX*ZX))
     1     -SM*SM/6.)+XN*SM*SM+X1*SM/3.-YLD*YLD
C
      RR = B*B - A*E
      RATIO=(-B + SQRT(RR))/A
C
      DO 350 I=1,6
  350 TAU (I) = SIG(I) + RATIO*DELSIG(I)
C
C     *TAU* NOW CONTAINS (PREVIOUS STRESSES +
C     STRESSES DUE TO ELASTIC STRAIN INCREMENTS)
C
C
C        5. CALCULATE PLASTIC STRESSES
C
C     DETERMINE INCREMENT INTERVAL
  370 M=10.*SQRT(FT)/YLD + 1
      IF (M.GT.30) M=30
      XM = (1. - RATIO)/M
C
C
      DO 380 I=1,6
C
C     ELASTIC-PLASTIC MATERIAL MATRIX
C
  380 DEPS(I) = XM*DELEPS(I)
```

```
C
C
C     .....  CALCULATION OF ELASTOPLASTIC STRESSES  .....  (START)
C
      DO 600 IM=1,M
C
      CALL MIDEP3(TAU,DEPS,C)
C
      DO 560 I=1,6
      DO 560 J
  560 TAU(I) = TAU(I) + C(I,J) * DEPS(J)
C
C
C     UPDATE YLD
C
      SM = TAU(1)+TAU(2)+TAU(3)
      SX = TAU(1)
      SY = TAU(2)
      SZ = TAU(3)
      XY = TAU(4)
      YZ = TAU(5)
      ZX = TAU(6)
      XJ2 = 0.5*(SX*SX+SY*SY+SZ*SZ) - SM*SM/6. + XY*XY+YZ*YZ+ZX*ZX
      F1 = (XK*XK/3.)*(XJ2+XN*SM*SM+XB*SM/3.)/(1.-XA*SM/3.)
      YLD = SQRT (F1)
C
  600 CONTINUE
C
C     .....  CALCULATION OF ELASTOPLASTIC STRESSES  .....  ( END )
C
      DO 390 I=1,6
  390 STRESS(I) = TAU(I)
C
C
C        6. UPDATING STRESSES, STRAINS, YIELD, NS
C
  400 DO 410 I=1,6
  410 SIG(I)   = STRESS(I)
      DO 420 I=1,6
  420 EPS(I)   = STRAIN(I)
      YIELD    = YLD
      IF (KPRI.EQ.0) GO TO 700
C
C
      IF (ICOUNT.EQ.3) RETURN
C
C        7. FORM THE MATERIAL LAW
C
      IF (IPEL.EQ.1) GO TO 450
C
```

```
C      ELASTIC-PLASTIC MATERIAL MATRIX
C
       CALL MIDEP3(TAU,DEPS,C)
       RETURN
C
C
C      P R I N T I N G   O F   S T R E S S E S
C
  700 CONTINUE
C
       SM = STRESS(1)+STRESS(2)+STRESS(3)
       SX = STRESS(1)
       SY = STRESS(2)
       SZ = STRESS(3)
       XY = STRESS(4)
       YZ = STRESS(5)
       ZX = STRESS(6)
C
C
  800 IF (NG.NE.NGLAST) GO TO 802
       IF (NEL.GT.NELAST) GO TO 806
       IF (IPT-1) 810,808,810
C
  802 NGLAST = NG
  805 WRITE (6,2003)
C
  806 NELAST=NEL
       WRITE (6,2004) NEL
  810 CONTINUE
       XJ2 = 0.5*(SX*SX+SY*SY+SZ*SZ) - SM*SM/6. + XY*XY+YZ*YZ+ZX*ZX
       F1 = (XK*XK/3.)*(XJ2+XN*SM*SM+XB*SM/3.)/(1.-XA*SM/3.)
       EFSG = 0.
       IF (F1 .GT. 0.) EFSG = SQRT(F1)
C
       WRITE (6,2005) IPT,STATE(IPEL),
     1                 (STRESS(I),I=1,3),(STRAIN(I),I=1,3),EFSG
       RETURN
C
 2003 FORMAT(104H ELEMENT  STRESS  STRESS-XX  STRESS-YY  STRESS-ZZ    ST
     1RAIN-XX    STRAIN-YY    STRAIN-ZZ     EFFECTIVE,  /,
     2 16H NUM/IPT   STATE,80X,6HSTRESS,/)
 2004 FORMAT (I4/)
 2005 FORMAT(5X,I2,2X,A1,6HLASTIC,3(2X,F9.1),3(3X,E10.3),4X,F9.1)
C
       END
```

```
      SUBROUTINE MIDEP3(TAU,DEPS,C)
C
C        FORMS THE ELASTO-PLASTIC MATERIAL MATRIX
C
      COMMON /EL/ IND,ICOUNT,NPAR(20),NUMEG,NEGL,NEGNL,IMASS,IDAMP,ISTAT
     1            ,NDOF,KLIN,IEIG,IMASSN,IDAMPN
      COMMON /MATMOD/ STRESS(6),STRAIN(6),C(6,6),IPT,NEL
      COMMON /ELPMID/ YM,PV,YLD,XN,XA,XB,XK,RTC,YLDC,YLDT,A1,B1
      DIMENSION TAU(1),DEPS(1),C(1,1),FE(6,6),F(6)
C
      EQUIVALENCE (NPAR(5),ITYP2D)
C
C
      SM = (TAU(1)+TAU(2)+TAU(3))/3.
      SX = TAU(1) - SM
      SY = TAU(2) - SM
      SZ = TAU(3) - SM
      XY = TAU(4)
      YZ = TAU(5)
      ZX = TAU(6)
C
C     CALCULATE STRAIN-HARDENING FUNCTION 'H'
C
      STR = YLD
      IF (RTC.LT.0.) STR = (1.0 +(YLD-YLDT)/YLDT)*YLDC
C
      CALL EPSLOP (STR,HP)
C
      H = 2.*YLD*HP
      XM = 1.-XA*SM
      XJ2 = 0.5*(SX*SX+SY*SY+SZ*SZ) - SM*SM/6. + XY*XY+YZ*YZ+ZX*ZX
      ET = XK*XK/3.
      RO = XN*SM*6.+(XB+XA*YLD*YLD)/3.
      Y1 = 1+PV
      Y2 = 1-2.*PV
      Y3 = 3.*PV*RO
      WINV = Y2*(2.*ET*ET*XJ2+3.*RO*RO)+9.*PV*RO*RO+XM*H*Y1*Y2/YM
     1       *SQRT(2.*ET*ET*XJ2+3.*RO*RO)
      W    = 1./WINV
      F(1) = Y2*(ET*SX+RO) + Y3
      F(2) = Y2*(ET*SY+RO) + Y3
      F(3) = Y2*(ET*SZ+RO) + Y3
      F(4) = Y2*ET*XY
      F(5) = Y2*ET*YZ
      F(6) = Y2*ET*ZX
      DO 50 I=1,6
      DO 50 J=1,6
   50 FE(I,J) = F(I)*F(J)
C
C
```

```
      D1 = W*YM/(Y1*Y2)
      DO 120 I=1,6
      DO 120 J=I,6
      C(I,J) = C(I,J)-D1*F(I,J)
  120 C(J,I) = C(I,J)
C
C
      RETURN
      END
```