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The CYGRO-4 Fuel Rod Analysis Computer Program (LWBR/AWBA Development Program)

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NOTICE

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Abstract

The CYGRO-4 computer program is an advanced version of the CYGRO program for the analysis of axisymmetric deformations of the cross-section of an oxide fueled rod-type fuel element. CYGRO-4, which replaces the CYGRO-3 program, incorporates improved numerical methods, improved time step control, a new fuel-clad and rod-support interaction model, a new thermal conductivity model for the fuel-clad gap, and a new fuel cracking, crack deformation and crack healing model.

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I. Introduction

This memorandum documents development of the CYGRO-4 finite element computer program for the analysis of axisymmetric deformations of the cross-section of an oxide fueled rod-type fuel element. CYGRO-4 is a production program developed from the CYGRO-1, CYGRO-2, and CYGRO-3 series of fuel rod analysis programs.* It was derived from CYGRO-3 by the inclusion of improved numerical methods, improved programming procedures and revised physical models aimed at improving accuracy and reliability for the analysis of Light Water Breeder Reactor (LWBR) Project and Advanced Water Breeder Application (AWBA) Project fuel elements. Significant developments include time step control based on accuracy considerations, a new fuel-clad and rod-support interaction model, a new thermal conductivity model for the fuel-clad gap, and a new fuel cracking, crack deformation and crack healing model.

Input to CYGRO-4 is divided into two parts: rod data and operating history. The rod data comprise all quantities needed to

* References (1), (2), (3), and (4).

characterize the geometry and physical properties of the rod, including such information as clad diameter and thickness, fuel diameter, and fuel and clad thermal conductivities, thermal expansion properties, creep behavior and response to the reactor environment. The operating history consists of the measured, or predicted, values of coolant pressure, coolant temperature, reactor power level, and the relevant relationships between power level, fission rate and neutron flux. The program computes the corresponding history of rod temperature, dimensional changes, and stresses.

Temperature computations assume thermal equilibrium. The temperature distribution is thus a function of current reactor power level, gamma heating and nuclear self shielding, and the thermal conductivities of fuel, clad, fuel-clad gap and rod-coolant interface. Fuel conductivity is calculated from the input data as a function of the current, local temperature, porosity and cracking. Gap conductivity depends on gap size, thermal radiation, conduction at points of fuel-clad contact, and the conductivity of the fluid in the gap. This conductivity depends on composition, on temperature, and - where appropriate - on pressure.

Deformation of the fuel and clad, the stresses in them, and the forces of interaction between fuel and clad and rod and support are computed from a Large Deflection Finite Element model incorporating representations for thermo-elasticity, creep resulting from thermally activated mechanisms or mechanisms associated with bombardment by high energy particles, fuel swelling and densification, clad growth

resulting from neutron bombardment, and fuel cracking and crack healing. In keeping with the Finite Element representation, the program accounts for the effects of spatial - as well as temporal - variation of temperature, stress, strain, cracking, and densification. This is an important feature of CYGRO-4.

Another important feature of CYGRO-4 is the comprehensive model for fuel-clad and rod-support interaction. In addition to interacting by radial stresses, the fuel and clad can interact through axial forces. Fuel and clad axial strain rates may be the same or different depending on fuel-clad gap size, the direction and magnitude of the axial interaction force, and the effects of pellet "hourglassing," eccentric pellets, lodged chips, and clad collapse, where the effects are relevant. Similarly, the rod axial strain rate may be the same as, or different from, the rate of support extension. Relative motion between rod and support depends on the magnitude of the axial interaction force and the force needed for slippage. The support extension includes the effects of flexibility and thermal expansion, as well as user input extension as might arise, perhaps, from the representation of support extension associated with neutron flux induced growth. The comprehensive form of the fuel-clad and rod-support interaction model is particularly important in the correct representation of rod behavior in response to reactor power changes such as those encountered in "peak load following" operation.

Throughout the work on CYGRO-4 the fundamental objective has been the development of a reliable, accurate production program. For this reason the changes have been of an evolutionary character, and comparatively little effort has been expended in developing new mathematical models. In fact, changes in mathematical modeling were undertaken only when the results of analyses demonstrated that the changes were needed. The cumulative effect of the changes discussed in this memorandum has been to increase the reliability of the program and to decrease its sensitivity to small changes in input data while providing a substantially improved representation of physical processes.

II. Temperature Calculations

The major phenomena of fuel and cladding mechanical behavior are strong functions of temperature. For this reason reliable fuel element modeling depends on an accurate knowledge of the temperature distribution throughout the element. In CYGRO-1, CYGRO-2 and CYGRO-3 (References 1, 3, and 4) the thermal conductivity of the fuel and of the fuel-clad gap were expressed in a simple, piecewise-linear form, and the coefficients in these expressions were adjusted to achieve temperatures that approximated those computed by the FIGRO computer program (References 5, 6 and 7). Unfortunately, close approximation to FIGRO was difficult to achieve for any given time in life, and the heat conduction and heat transfer models in the earlier versions of CYGRO were too restrictive to allow satisfying agreement with FIGRO at more than a few widely spaced times. Moreover, the procedure of using CYGRO and FIGRO in parallel proved to be time consuming and cumbersome.

For these reasons, it seemed profitable to modify CYGRO-3 to incorporate the thermal conductivity model from FIGRO so that CYGRO-4 would embody the best available models. This would allow accurate analyses to be performed with CYGRO-4 without having to rely on FIGRO to provide the temperature modeling. In the course of this change other changes were made as well. These have resulted in the accurate and adaptable procedure which is the subject of this section. The material in this section progresses from an analysis of the temperature distribution in an annulus through a discussion of the thermal conductivities of fuel and cladding to a discussion of the thermal conductance of the fuel-clad gap and the effect of oxidation on the conductivity of the cladding

Consider an annulus occupying the region $r_1 \leq r \leq r_n$. Writing Q_v for the volume averaged heat generation rate, the total rate of heat generation is $q = \pi Q_v(r_n^2 - r_1^2)$ per unit length. When there is heat generation q_o per unit length in the cylindrical region $0 \leq r \leq r_1$, the total heat generation per unit length in the region $0 \leq r \leq r_n$ is $\bar{q} = q_o + q = q_o + \pi Q_v(r_n^2 - r_1^2)$.

Calculation of the temperature distribution requires knowledge of the power distribution. When it is known that the relative power distribution, $p(r)$, satisfies the relations

$$\underset{r \to r_i;\ r < r_i}{\text{Limit}} \quad p(r) = p_i^{(-)}$$

and

$$\underset{r \to r_i;\ r > r_i}{\text{Limit}} \quad p(r) = p_i^{(+)}$$

the volumetric heating rate in the annulus $r_k \leq r \leq r_{k+1}$ can be approximated by

$$Q(r) \approx \frac{p_k^{(+)} (r_{k+1} - r) + p_{k+1}^{(-)} (r - r_k)}{(r_{k+1} - r_k)} RQ_v \qquad \text{(II.1)}$$

where R is a factor of proportionality to be obtained from (II.7) of this analysis.

The heat generated in the annulus $r_k \leq r \leq r_{k+1}$; $k = 1,\ldots,(n-1)$ is given by

$$q_k = \int_0^{2\pi} \int_{r_k}^{r_{k+1}} Q(r) r \, dr \, d\theta$$

or

$$q_k \approx 2\pi RQ_v \int_{r_k}^{r_{k+1}} \frac{p_k^{(+)} (r_{k+1} - r) + p_{k+1}^{(-)} (r - r_k)}{(r_{k+1} - r_k)} r \, dr \quad . \qquad \text{(II.2)}$$

This is equivalent to

$$q_k \approx \pi RQ_v \left[B_k p_k^{(+)} + A_{k+1} p_{k+1}^{(-)}\right] \qquad \text{(II.3)}$$

where A_ℓ and B_ℓ are given by

$$A_\ell = \frac{1}{3}(r_\ell - r_{\ell-1})(r_{\ell-1} + 2r_\ell) \qquad \text{(II.4)}$$

$$B_\ell = \frac{1}{3}(r_{\ell+1} - r_\ell)(2r_\ell + r_{\ell+1}) \quad , \qquad \text{(II.5)}$$

with the understanding that $A_1 = B_n = 0$. In this notation, the total heat generated in the annulus $r_1 \le r \le r_n$ is

$$q = \sum_{k=1}^{n-1} q_k = \pi R Q_v \sum_{k=1}^{n} [A_k p_k^{(-)} + B_k p_k^{(+)}] \qquad \text{(II.6)}$$

per unit length. This expression determines the value of R in terms of the r_k, $p_k^{(+)}$ and $p_k^{(-)}$. For the specified volumetric average heat generation rate, Q_v, is attained when

$$R = \frac{r_n^2 - r_1^2}{\sum_{k=1}^{n} [A_k p_k^{(-)} + B_k p_k^{(+)}]} \quad . \qquad \text{(II.7)}$$

The temperature satisfies the differential equation

$$-C(T)\frac{\partial T}{\partial r} = \frac{q(r)}{2\pi r} \quad , \qquad \text{(II.8)}$$

where C(T) is the temperature dependent thermal conductivity and q(r) is the heat energy per unit length flowing through the surface r = constant. When T(r) is approximated as a piecewise linear function of r by

$$T \approx \frac{T_k(r_{k+1} - r) + T_{k+1}(r - r_k)}{(r_{k+1} - r_k)} \; ; \; r_k \le r \le r_{k+1} \qquad \text{(II.9)}$$

Equation (II.8) can be reduced to the approximate form

$$C\left(\frac{T_k + T_{k+1}}{2}\right) \frac{(T_{k+1} - T_k)}{(r_{k+1} - r_k)} = \frac{-q(\frac{1}{2}[r_k + r_{k+1}])}{\pi(r_k + r_{k+1})} \qquad \text{(II.10)}$$

in which the partial derivative of T at $r = \frac{1}{2}(r_k + r_{k+1})$ is approximated by the derivative of (II.9) and the value of T is approximated by $\frac{1}{2}(T_k + T_{k+1})$.

The most efficient computational procedure results when (II.10) is rearranged to the form

$$T_k = T_{k+1} + D_k \frac{q(\frac{1}{2}[r_k + r_{k+1}])}{\pi C(\frac{1}{2}[T_k + T_{k+1}])} \quad , \qquad \text{(II.11)}$$

in which

$$D_k = \frac{(r_{k+1} - r_k)}{(r_{k+1} + r_k)} \qquad \text{(II.12)}$$

In this form, T_k is determined from T_{k+1} and the known data once T_{k+1} has been determined from T_{k+2} or the boundary conditions. Consequently, the nonlinear differential equation (II.8) is solved as a sequence of nonlinear equations (II.11) involving a single unknown quantity.

In evaluating the right hand side of (II.11), $q(\frac{1}{2}[r_k + r_{k+1}])$ can be expressed as

$$q(\tfrac{1}{2}[r_k + r_{k+1}]) = q_o + \int_0^{2\pi} \int_0^{\frac{1}{2}(r_k + r_{k+1})} Q(r)\, r\, dr\, d\theta$$

Substitution into this from Equation (II.1) gives

$$q(\tfrac{1}{2}[r_k + r_{k+1}]) \approx \pi Q_v R \{ \sum_{j=1}^{k-1} (A_j p_j^{(-)} + B_j p_j^{(+)}) + A_k p_k^{(-)} + \frac{r_{k+1} - r_k}{12} [p_k^{(+)} (7r_k + 2r_{k+1}) + p_{k+1}^{(-)} (2r_k + r_{k+1})]\} + q_o \quad \text{(II.13)}$$

Returning to (II.11), definition of

$$F_k = \frac{1}{12} (7r_k + 2r_{k+1}) (r_{k+1} - r_k)^2/(r_{k+1} + r_k) \quad \text{(II.14)}$$

and

$$G_k = \frac{1}{12} (2r_k + r_{k+1}) (r_{k+1} - r_k)^2/(r_{k+1} + r_k) \quad \text{(II.15)}$$

and use of (II.13) produces the expression

$$T_k = T_{k+1} + \frac{D_k}{C(\frac{1}{2}[T_k + T_{k+1}])} [q_o + \sum_{j=1}^{k-1} (A_j P_j^{(-)} + B_j P_j^{(+)})Q_v + A_k P_k^{(-)} Q_v] + \frac{Q_v}{C(\frac{1}{2}[T_k + T_{k+1}])} (F_k P_k^{(+)} + G_k P_{k+1}^{(-)}) \quad \text{(II.16)}$$

where $P_k^{(+)} = Rp_k^{(+)}$ and $P_k^{(-)} = Rp_k^{(-)}$. In this computationally efficient form, A_ℓ, B_ℓ, D_k, F_k and G_k are known from the initial values of the r_k, the $P_k^{(+)}$ and $P_k^{(-)}$ are known from the power distribution, and the values of Q_v and q_o are determined by the reactor operating state. Thus, the A_ℓ, B_ℓ, D_k, F_k and G_k can be computed from the input data and stored for use at all times.

The $p_k^{(+)}$ and $p_k^{(-)}$ are specified for certain times $t = t_1, t_2, \ldots, t_m$. Thus, the input consists of a set of quantities

$$\bar{p}_{k\ell}^{(+)},\ \bar{p}_{k\ell}^{(-)} : \quad k = 1, \ldots, n;\ \ell = 1, \ldots, m \quad .$$

These are normalized by use of the relation $\bar{P}_{k\ell}^{(a)} = R_\ell \bar{p}_{k\ell}^{(a)}$ to obtain a set of quantities

$$\bar{P}_{k\ell}^{(+)} = \bar{p}_{k\ell}^{(+)} R_\ell \qquad \text{(II.17a)}$$

and

$$\bar{P}_{k\ell}^{(-)} = \bar{p}_{k\ell}^{(-)} R_\ell \qquad \text{(II.17b)}$$

with

$$R_\ell = \frac{r_n^2 - r_1^2}{\sum_{k=1}^{n} \left[A_k \bar{p}_{k\ell}^{(-)} + B_k \bar{p}_{k\ell}^{(+)}\right]} \qquad \text{(II.18)}$$

in correspondence with (II.7). Temperature calculations then proceed using (II.16) where $P_k^{(a)}$ is understood to be the value

$$P_k^{(a)} = \frac{\bar{P}_{k\ell}^{(a)}(t_{\ell+1} - t) + \bar{P}_{k(\ell+1)}^{(a)}(t - t_\ell)}{(t_{\ell+1} - t_\ell)} \qquad \text{(II.19)}$$

and (a) is (+) or (-) as appropriate. In practice, $p_k^{(+)} = p_k^{(-)}$ for all k except at a possible single discontinuity at the interface between fuel regions.

The fuel thermal conductivity $C_f(T)$ defined by the relation (II.8) is the same as that used in FIGRO (Reference 6):

$$C_f(T) = \frac{(1 - p)/(1 + \beta p)}{A + BT + (C+DF)/T} \quad . \qquad \text{(II.20)}$$

In this expression, p is the porosity, T is the absolute temperature, F is the net depletion in fissions per cubic centimeter and the quantities β, A, B, C, and D are input quantities chosen to represent the experimentally determined conductivity.

The clad thermal conductivity $C_c(T)$ defined by (II.8) is expressed as the piecewise linear function

$$C_c(T) = \frac{(T - T_k)C(T_{k+1}) + (T_{k+1} - T)C(T_k)}{T_{k+1} - T_k} \; ; \; T_k < T \leq T_{k+1} \quad . \text{(II.21)}$$

In this expression the $C(T_i)$ are the experimentally determined conductivities at the tabulation temperatures T_i.

As in FIGRO (Reference 6) the temperature drop across the fuel-clad gap or interface is given by

$$T_f - T_c = \frac{q}{\pi h_g (r_f + r_c)} \quad . \qquad \text{(II.22)}$$

In this expression, T_f is the temperature at the outer surface of the fuel, T_c is the temperature at the inner surface of the clad, r_f is the outer radius of the fuel, r_c is the inner radius of the clad, and q is the heat generation rate. The gap heat transfer coefficient is given by

$$h_g = \frac{1}{(h_s + h_r + h_f)^{-1} + \mu_{oxide}} \quad . \qquad \text{(II.23)}$$

The conduction through points of contact between the solids is given by

$$h_s = \frac{2C_fC_c}{C_f + C_c} \frac{P_c}{a_o H[\frac{1}{2}(R_f^2 + R_c^2)]^{\frac{1}{2}}} \quad . \qquad \text{(II.24)}$$

Here C_f is the fuel thermal conductivity, C_c is the clad thermal conductivity, P_c is the contact pressure, a_o is an empirical constant which determines the mean radius of contact spots, R_f and R_c are the arithmetic mean roughness heights of the fuel and clad surfaces, and H is the Meyer hardness of the softer solid.

In (II.23) heat transfer by radiation is represented by

$$h_r = \frac{\mathcal{F}_{1,2}\, \sigma(T_f^4 - T_c^4)}{T_f - T_c} \quad . \qquad \text{(II.25)}$$

In this expression, $\mathcal{F}_{1,2}$ is given by

$$\mathcal{F}_{1,2} = \frac{1}{1/F_{1,2} + (\frac{1}{\epsilon_f} - 1) + \frac{r_f}{r_c}(1/\epsilon_c - 1)} \quad , \qquad \text{(II.26)}$$

while σ is the Stefan-Boltzman constant, ϵ_f and ϵ_c are the fuel and clad emissivities, and $F_{1,2}$ is a geometric factor whose value is 1.0 for infinite parallel plates.

In (II.23) the conduction coefficient for the gas layer, h_f, accounts for the effects of initial fill gases, fission gases and volatile gases that accumulate in the fuel-clad gap. This coefficient is given by

$$h_f = \frac{K_{mix}}{Q[R_1 + R_2] + g + r_c - r_f} \qquad \text{(II.27)}$$

where $Q[R_1 + R_2]$ is the effective gap thickness caused by surface roughness, g is the "temperature jump distance" for the gas in the interface, K_{mix} is the thermal conductivity of the gas mixture in the gap at the average interface temperature, and $(r_c - r_f)$ is the nominal gap size. The dimensionless coefficient Q is assigned the value $Q = 1.0$ when the radial gap is greater than 0.0005 inch and $Q = 2.75 - 1.758 \times 10^{-4} P_c$ for gaps smaller than 0.0005 inch.

In (II.27) the thermal conductivities of the component gases are taken to be of the form

$$k_i = a_i T^{n_i} \qquad \text{(II.28)}$$

with a_i and n_i chosen to represent the relevant experimental data. The conductivity of a mixture of gases is then given in terms of the k_i,

the relative mole fractions, x_i, and the molecular weights, M_i, by

$$K_{mix} = \frac{\Sigma_i \, x_i \, M_i^{1/3} k_i}{\Sigma_i \, x_i \, M_i^{1/3}} \quad . \tag{II.29}$$

Upon distinguishing between the initial fill gas and the gases resulting from released volatiles and fission, this expression can be written in the form $K_{mix} = \frac{a + c}{b + d}$. This leads to a reduction of the jump distance expression for (II.27) from the form

$$g = \frac{1}{K_{fg} - K_i} \left[(g_{fg} - g_i) K_{mix} + (K_{fg} g_i - K_i g_{fg}) \right] \frac{P_r}{P} \frac{T}{T_r}$$

to the more tractible form

$$g = \frac{b g_{fg} + d g_i}{b + d} \frac{P_r}{P} \frac{T}{T_r} \quad . \tag{II.30}$$

Here, $b = \left\{ \sum_i x_i \, M_i^{1/3} \right\}_{\text{fission gases}}$ and $d = \left\{ \sum_i x_i \, M_i^{1/3} \right\}_{\text{initial fill}}$ while g_i and g_{fg} are the "jump distances" for initial fill gas and fission gases, respectively. The reference temperature is T_r and the reference pressure is P_r. The pressure is given by the ideal gas law

$$P = nRT/V \tag{II.31}$$

in which n is the total number of moles of gas, R is the universal gas constant, and V is the volume composed of the plenum volume plus the volume of the fuel-clad gap. The total number of moles of gas represents

the sum of the number of moles of fill gas, the number of moles of helium resulting from ternary fissions, the number of moles of other fission gases, and the number of moles of volatile compounds released from the fuel. The helium resulting from ternary fissions is determined as the product of the net depletion (the number of fissions), the number of atoms of helium per fission event, and the fraction of the helium released from the fuel matrix. The population of other fission gases depends on the net depletion, the fraction of fission gas released, and the observed composition of fission gases. The volatile gas released from the fuel is determined by outgassing tests.

The volume of the plenum plus the volume of the fuel-clad gap is calculated as

$$V = V_{plen} + \pi\ell(r_f + r_c)[r_c - r_f + 1.2(R_f + R_c)] \quad . \qquad (II.32)$$

This is used in (II.31) to relate the temperature and pressure of the gas.

When there is a perforation in the cladding of the fuel element, the fill gas and any fission gas will vent into the coolant and water will enter the rod. As a result, the fuel-clad gap will be filled by water when the reactor is at very low power and the rod is at low temperature and by steam when the reactor is operating at any but the lowest power level. For this reason, adequate analysis of defected rods requires a representation for the conductivity of steam to replace the gas conductivity representation (II.29) and the "jump distance" representation (II.30). Since the fluid inside a defected rod will be at essentially the pressure of the reactor coolant, the rod will be filled with

to Reference (8), this representation is accurate within ± 6 percent over the range of from boiling to 700°C and pressures from atmospheric to 2500 psi (175 bar). No accuracy indications are given for temperatures above 700°C. However, since steam approaches an ideal gas at these temperatures, satisfactory results are to be expected. Unfortunately, there is very little data for temperatures above 700°C to serve as a standard for comparison.

The density of steam, ρ_ℓ, expressed in lbm/ft^3, is given in terms of temperature, T_k, in Kelvin and pressure, P, in atmospheres by the expression (Reference 9)

$$1/\rho_\ell = 0.0160185 \left(\frac{4.55504\ T_k}{P} + B \right) \quad , \qquad \text{(II.34)}$$

where B is given by

$$B = B_o \left[1 + \frac{B_o P}{T_k^2} \left\{ B_2 - B_3 + \frac{B_o^2 P^2}{T_k^2} (B_4 - B_5) \right\} \right] \quad . \qquad \text{(II.35)}$$

In this, the B_i are given by

$$B_o = 1.89 - B_1 \qquad \text{(II.36a)}$$

$$B_1 = \frac{2641.62}{T_k}\ 10^{80870/T_k^2} \qquad \text{(II.36b)}$$

$$B_2 = 82.546 \qquad \text{(II.36c)}$$

$$B_3 = 162460/T_k \qquad \text{(II.36d)}$$

$$B_4 = 0.21828\ T_k \qquad \text{(II.36e)}$$

and

$$B_5 = 126970/T_k \qquad \text{(II.36f)}$$

According to the discussion in Reference (9) this representation is accurate within 0.05 percent or less at all temperatures above 315°C (600°F).

Consideration of the accuracy limits given in References (8) and (9) indicates that the representation of conductivity by (II.33) using the density given by (II.34), (II.35) and (II.36) is sufficiently accurate for practical fuel rod analyses. The principal limit on accuracy is at lower temperatures and high pressures. Fortunately, this is not crucial to the accuracy of practical analyses, for inaccuracies at relatively low temperatures correspond to inaccuracies at zero power or at very low power levels. Good accuracy in computing the conductivity is important only at substantial power levels.

As a result of its computational structure, CYGRO-4 must have a representation for the conductivity even when there is no power generation. Fortunately, the only requirement in this case is that the conductivity be nonzero. This makes it possible to assign the conductivity at temperatures in the range from 282°C (540°F) to 315°C (600°F) as

$$k^*(T,P) = k(315,P) + (T - 315)\left.\frac{\partial k(T,P)}{\partial T}\right|_{T=315} .$$

When T_c is below 282°C (540°F), the conductivity of the steam is taken to be infinite. If the program input indicates that the reactor is generating appreciable power when this condition prevails, the program will stop and indicate an error condition.

In keeping with the assumptions in FIGRO (Reference 6), when the cladding is defected the temperature jump distance, g, appearing in (II.27) is computed by use of

$$g = g_r \frac{T}{T_r} \frac{P_r}{P_{system}} . \qquad (II.37)$$

Here g_r is an experimentally determined constant and P_{system} is the system coolant pressure.

In (II.23) the thermal resistance of the oxide layer at the inner surface of the clad, ρ_{oxide}, is determined by the thickness of the layer and the experimentally determined conductivity of the oxide. A similar layer is assumed to affect heat transfer at the rod-to-coolant interface. This corresponds to the representation in FIGRO (Reference 7).

III. Fuel Cracking

Examination of irradiated fuel pellets* makes it clear that they have cracked, and that, in many cases, the cracks have "healed." Since cracked fuel will obviously respond to loads and conduct heat

* See, for example, Reference (10).

differently than uncracked fuel, adequate representation of fuel behavior must include models for cracking, crack healing, and the effect of cracks on heat conduction.

A fuel cracking model was incorporated in CYGRO-3 (Reference 4). However, this model gives its best results using a cracking stress more than one hundred times smaller than the expected value. Furthermore, analyses using this model occasionally display unacceptably large discrepancies between the strains calculated from changes in geometry and the strains calculated from the history of stress, temperature and fission rate. Finally, results obtained using this model often include tensile stresses across cracks that are substantially larger than the stress at which cracking was said to occur. These facts suggested the need for a new cracking and crack healing model that would give results in keeping with basic expectations of crack behavior.

Up to the present, the CYGRO model has neglected the effect of fuel cracks on heat conduction. Thus, the model presented here provides an opportunity for more complete representation of fuel behavior.

The CYGRO finite element formulation* is based on a flexibility model in which the strains and, thus, the deformations of each ring element are expressed as functions of the stresses in that element.

* See Reference (3).

The basic incremental stress-strain expression for an element takes the form

$$\begin{bmatrix} \frac{\partial\epsilon_1}{\partial\sigma_1} & \frac{\partial\epsilon_1}{\partial\sigma_2} & \frac{\partial\epsilon_1}{\partial\sigma_3} \\ \frac{\partial\epsilon_2}{\partial\sigma_1} & \frac{\partial\epsilon_2}{\partial\sigma_2} & \frac{\partial\epsilon_2}{\partial\sigma_3} \\ \frac{\partial\epsilon_3}{\partial\sigma_1} & \frac{\partial\epsilon_3}{\partial\sigma_2} & \frac{\partial\epsilon_3}{\partial\sigma_3} \end{bmatrix} \begin{bmatrix} \delta\sigma_1 + \dot{\sigma}_1\Delta t \\ \delta\sigma_2 + \dot{\sigma}_2\Delta t \\ \delta\sigma_3 + \dot{\sigma}_3\Delta t \end{bmatrix} + \begin{bmatrix} \delta\epsilon_1 + \dot{\epsilon}_1\Delta t \\ \delta\epsilon_2 + \dot{\epsilon}_2\Delta t \\ \delta\epsilon_3 + \dot{\epsilon}_3\Delta t \end{bmatrix} = \begin{bmatrix} \Delta\epsilon_1 \\ \Delta\epsilon_2 \\ \Delta\epsilon_3 \end{bmatrix} \quad \text{(III.1)}$$

Here*, the coordinates x_i; i = 1, 2 or 3, are cylindrical polar coordinates $x_1 = r$, $x_2 = \theta$, $x_3 = z$ in a coordinate system whose z-axis coincides with the axis of the rod. The increment of strain is given by $\Delta\epsilon_i$; i = 1, 2 or 3. The change in stress, $\Delta\sigma_i$, is said to occur as a "jump" $\delta\sigma_i$ plus a rate $\dot{\sigma}_i$, and the processes of growth, swelling, and thermal expansion contribute to both "jumps" $\delta\epsilon_i$ and rates of change $\dot{\epsilon}_i$. The formulation of a model for any deformation process consists of establishing the effect that process has on $\partial\epsilon_i/\partial\sigma_j$, $\delta\epsilon_i$, and $\dot{\epsilon}_i$.

Because of the axial symmetry of the CYGRO model and because the CYGRO cross-section analysis neglects axial variations of stresses and displacements, the stresses σ_i are principal stresses and the strains ϵ_i are principal strains. Thus, adopting a maximum tensile stress criterion for cracking, we reason that cracks will coincide with the

* In the notation of Reference (3), (III.1) is written as $\dot{D}_s = A_{st} + A_{ss}\dot{F}$, where $\dot{D}_s$ is the strain rate, A_{ss} is the matrix $\partial\epsilon_i/\partial\sigma_j$, $\dot{F}_\sigma$ is the stress rate, and A_{st} corresponds to $\delta\epsilon_i + \dot{\epsilon}_i\Delta t$. (See Equation H1 of Reference 3).

x_i coordinate surfaces. That is, cracks occur only on surfaces of constant $x_1 = r$, surfaces of constant $x_2 = \theta$, or surfaces of constant $x_3 = z$. In keeping with the assumption of axial symmetry, a crack coinciding with a surface of constant $x_1 = r$ extends over every point on that surface. Moreover, since the CYGRO model assumes the stress state and strain state are uniform throughout each finite element, all cracks must be assumed to be distributed uniformly and continuously throughout the ring element in which they exist. Of course, a continuous, uniform distribution of cracks is not physically realistic. However, this concept provides the basis for a phenomenological finite element representation of fuel cracking that gives analytical results in close agreement with experimental data.

During the interval $t_{k+1} - t_k = \Delta t_k$, cracks on surfaces of constant x_i can affect the strain increment $\Delta\epsilon_i$ in one of four ways: the element can crack on surfaces of constant x_i, the element can remain cracked on these surfaces, the cracks on surfaces of constant x_i can close, or - in the trivial case - the element can remain uncracked throughout the interval. Clearly a cracking model needs to be concerned only with the first three of these, except for contributions to $\Delta\epsilon_i$ from cracks on surfaces perpendicular to the surface of constant x_i.

To provide the basis for a methematical model, consider the phenomena encountered in the deformation and cracking of a brittle elastic tensile specimen with free lateral boundaries subjected to a

uniform strain $\epsilon_3 = e_o$ for which the stress is $\sigma_3 = Ee_o$ is equal to the cracking stress σ^*:

1. Let the strain increase to $\epsilon_3 = e_o + e_1 = \frac{\sigma^*}{E} + e_1$ during the interval $\Delta t_1 = t_2 - t_1$. This causes the specimen to crack. The stress σ_3 falls to zero. The strain associated with the crack is $\epsilon_3^c = \sigma^*/E + e_1$, while the transverse strains, ϵ_1 and ϵ_2, change from $-\nu\sigma^*/E$ to zero.

2. Let the strain increase to $\epsilon_3 = \frac{\sigma^*}{E} + e_1 + e_2$ during the interval $\Delta t_2 = t_3 - t_2$. This causes the crack to open further: $\epsilon_3^c = \frac{\sigma^*}{E} + e_1 + e_2$. The stress σ_3 and the transverse strains, ϵ_1 and ϵ_2, remain zero.

3. During the interval $\Delta t_3 = t_4 - t_3$, let the lateral boundaries of the specimen be subjected to normal stress $\sigma_1 = \sigma_2 = \Sigma_\ell < 0$. For $|\Sigma_\ell| \leq \frac{1}{2\nu}[\sigma^* + E(e_1 + e_2)]$ the crack remains open and σ_3 remains zero. The crack strain is changed to $\epsilon_3^c = \frac{\sigma^*}{E} + e_1 + e_2 + \frac{2\nu}{E}\Sigma_\ell$, and the transverse strains become $\epsilon_1 = \epsilon_2 = \Sigma_\ell(1 - \nu)/E$.

4. During the interval $\Delta t_4 = t_5 - t_4$, let the strain ϵ_3 be reduced to zero. Since the lateral stresses, $\sigma_1 = \sigma_2 = \Sigma_\ell$ ($\Sigma_\ell < 0$), are compressive, the crack closes, and the transverse

strains are $\epsilon_1 = \epsilon_2 = \frac{(1 - 2\nu)(1 + \nu)}{E} \Sigma_\ell$ while the stress σ_3 becomes $\sigma_3 = 2\nu \Sigma_\ell$.

In the CYGRO analysis the stress flexibility matrix $[\partial\epsilon_i/\partial\sigma_j]$ appearing in (III.1) is defined for the whole of the current interval. Consequently, the flexibility matrix will assume one of two characteristic forms. One is the form for an uncracked ring element (or an element where the cracks are held closed). The other is the form for an element with cracks on surfaces of constant x_1, x_2 or x_3, or on combinations of these surfaces. The stress flexibility matrix for an element in which there are no cracks (or the cracks are held closed) is the flexibility matrix calculated from the elasticity, creep and swelling models.

The stress flexibility matrix for an element with cracks can be inferred from the phenomena of the tensile test:

1. During the interval Δt_2 the increase in strain did not affect the stress σ_3. This implies that the crack is infinitely flexible. Unfortunately, this would require an infinite term in the stress flexibility matrix. Thus, practical calculations require assignment of a large, but finite, flexibility to a crack. For a material with elastic modulus E, the crack is assigned a compliance S/E, where S is a number larger than 1 that will be assigned the value providing the maximum correspondence between analysis and experiment.

2. During the interval Δt_3 the lateral stresses, σ_1 and σ_2, did not affect the strain ϵ_3 or the stress σ_3. This implies that the off-diagonal entries in the i-th row of the flexibility matrix are zero when there are open cracks on surfaces of constant x_i. However, this leads to difficulties in formulating a model that is consistent with the principle of conservation of energy. For this reason, the off-diagonal terms of the matrix are assumed to be unaffected by cracking. For sufficiently large values of S the off-diagonal terms are negligibly small compared to the diagonal term.

According to this reasoning, an elastic element with cracks on surfaces of constant $x_1 = r$ has the stress flexibility matrix

$$\left[\frac{\partial \epsilon_i}{\partial \sigma_j}\right] = \begin{bmatrix} \frac{S+1}{E} & \frac{-\nu}{E} & \frac{-\nu}{E} \\ \frac{-\nu}{E} & \frac{1}{E} & \frac{-\nu}{E} \\ \frac{-\nu}{E} & \frac{-\nu}{E} & \frac{1}{E} \end{bmatrix} \qquad \text{(III.2)}$$

Similarly an element with cracks on surfaces of constant $x_1 = r$ and $x_2 = \theta$ has the stress flexibility matrix

$$\left[\frac{\partial \epsilon_i}{\partial \sigma_j}\right] = \begin{bmatrix} \frac{S+1}{E} & \frac{-\nu}{E} & \frac{-\nu}{E} \\ \frac{-\nu}{E} & \frac{S+1}{E} & \frac{-\nu}{E} \\ \frac{-\nu}{E} & \frac{-\nu}{E} & \frac{1}{E} \end{bmatrix} \qquad \text{(III.3)}$$

The stress flexibility matrix for an element with cracks on all three sets of coordinate surfaces has three diagonal entries $(S+1)/E$ and off-diagonal entries $\frac{-\nu}{E}$. Matrices corresponding to other configurations of cracks can be obtained from (III.2) and (III.3) by suitable exchange of variables.

Since the stress flexibility matrix is used to calculate an improved estimate for the stresses at the end of the time interval, it must represent the state at the end of that interval. Thus, whether an element cracks during an interval or remains cracked throughout that interval, the appropriate flexibility matrix will be determined by the configuration of cracks at the end of the interval. Likewise, when the element has no open cracks at the end of an interval, the flexibility matrix will be the matrix for an uncracked element, regardless of whether the element was uncracked throughout the interval, or the element was cracked at the beginning of the interval and the cracks closed during the interval.

As a consequence of this reasoning, both cracking and crack closing result in a change of stress flexibility matrix. In keeping with the CYGRO model both processes are said to occur at constant stress. This results in "jump strains" $\delta\epsilon_i$. To calculate the $\delta\epsilon_i$, consider the sum of the elastic and crack strains at stress state $\underset{\sim}{\sigma}^\circ = [\sigma_i^\circ]$ when the flexibility matrix is $\underset{\sim}{F}^\circ = [\delta\epsilon_i/\delta\sigma_j]$:

$$\underset{\sim}{\epsilon}^\circ = \underset{\sim}{F}^\circ \cdot \underset{\sim}{\sigma}^\circ \quad .$$

For a different flexibility matrix $\underset{\sim}{F}^1$ corresponding to a different crack configuration, the sum of elastic and crack strains is

$$\underset{\sim}{\varepsilon}^1 = \underset{\sim}{F}^1 \cdot \underset{\sim}{\sigma}^\circ \quad .$$

The transition from a crack state having flexibility matrix $\underset{\sim}{F}^\circ$ to a state with flexibility matrix $\underset{\sim}{F}^1$ can thus be seen to induce a jump in strain

$$\delta\underset{\sim}{\varepsilon} = [\delta\varepsilon_i] = \underset{\sim}{\varepsilon}^1 - \underset{\sim}{\varepsilon}^\circ = (\underset{\sim}{F}^1 - \underset{\sim}{F}^\circ) \cdot \underset{\sim}{\sigma}^\circ \quad . \qquad \text{(III.4)}$$

Since cracking on surfaces x_i = constant results in addition of the crack flexibility S/E to the component $\delta\varepsilon_i/\delta\sigma_i$ of the flexibility matrix for the uncracked element, the jump in strain at cracking is $\delta\varepsilon_i = S\sigma_i/E$ and the jump in strain at crack closing is $\delta\varepsilon_i = -S\sigma_i/E$.

It is instructive to apply this model to the analysis of the uniaxial tensile specimen:

1. During interval Δt_1 the specimen cracks at stress $\sigma_3 = \sigma^*$. The process leads to $\delta\varepsilon_3 = S\sigma^*/E$. Straining to $\varepsilon_3 = \frac{\sigma^*}{E} + e_1$ then results in $\Delta\sigma_3 = -\frac{S\sigma^*}{S+1} + \frac{E\varepsilon_1}{S+1}$, so the stress at $t = t_2$ is $\sigma_3\big|_{t=t_1} = \frac{\sigma^*}{S+1} + \frac{Ee_1}{S+1}$ and the transverse strains at $t = t_2$ are $\varepsilon_1 = \varepsilon_2 = \frac{-\nu}{S+1}\left(\frac{\sigma^*}{E} + e_1\right)$.

2. During the interval Δt_2 the increase in ϵ_3 leads to the stress $\sigma_3|_{t=t_2} = \frac{\sigma^*}{S+1} + \frac{E(e_1+e_2)}{S+1}$. The transverse strains are then $\epsilon_1 = \epsilon_2 = \frac{-\nu}{S+1}\left(\frac{\sigma^*}{E} + e_1 + e_2\right)$.

3. During the interval Δt_3 imposition of the lateral stresses $\sigma_1 = \sigma_2 = \Sigma_\ell < 0$ causes the axial stress to decrease to $\sigma_3|_{t=t_3} = \frac{\sigma^*}{S+1} + \frac{E(e_1+e_2)}{S+1} + \frac{2\nu}{S+1}\Sigma_\ell$ and the lateral strains become $\epsilon_1 = \epsilon_2 = \frac{-\nu}{S+1}\left(\frac{\sigma^*}{E} + e_1+e_2\right) + \frac{\Sigma_\ell}{E}\left(1 - \nu - \frac{2\nu^2}{S+1}\right)$.

4. The crack closes during the interval Δt_4. However, as a result of its computational structure, the CYGRO program performs its computations for the interval based of the assumption that the crack remains open, and then, at the beginning of the next interval, performs the computations needed to correct the resulting minor discrepancies. The computations made using the assumption that the crack remains open lead to the result that the stress is $\sigma_3|_{t=t_4} = \frac{2\nu\Sigma_\ell}{S+1}$. Since Σ_ℓ is negative, σ_3 is negative - a result that indicates the crack has "overclosed." This necessitates a computation to adjust the results to the state where the crack strain is zero. The transition from the flexibility matrix for an element with cracks on planes of constant $x_3 = z$ to the matrix for an element without cracks causes strain jump $\delta\epsilon_3 = \frac{-2\nu S}{S+1}\frac{\Sigma_\ell}{E}$. Using $\epsilon_3 = 0$ and $\Delta\epsilon_3 = 0$ then gives $\Delta\sigma_3 = \frac{2\nu S}{S+1}\Sigma_\ell$. Thus, the resulting stress is $\sigma_3|_{t=t_4^+} = 2\nu\,\Sigma_\ell$,

and the transverse strains are $\epsilon_2 = \epsilon_3 = \frac{(1+\nu)(1-2\nu)\Sigma_\ell}{E}$.

The results of this analysis indicate the fundamental correctness of the cracking model described here. For the results of the analysis approach the exact results rapidly as S becomes large. Values of S in the range of 100 to 1000 provide results of suitable accuracy for most purposes.

Generalization of this model to problems involving creep, thermal strains, and growth and swelling is immediate - the flexibility of the cracks and the strain jumps associated with opening and closing of cracks act to augment the flexibility and the strains of the uncracked material. Thus, if the stress flexibility matrix for the uncracked material is $\underset{\sim}{F}^\circ = [\delta\epsilon_i/\delta\sigma_j]$ as a result of its elastic and creep properties, cracking on surfaces of constant $x_1 = r$ results in addition of S/E to the entry on the first row and the first column of $\underset{\sim}{F}^\circ$. Likewise, if the uncracked material displays a strain jump $\delta\epsilon_1^\circ$, cracking at $\sigma_1 = \sigma^*$ augments the strain jump to $\delta\epsilon_1 = \delta\epsilon_1^\circ + S\sigma^*/E$.

In the original CYGRO cracking model the cracking stress was independent of temperature, and complete strength recovery occurred when the crack closes. Available experimental evidence suggests that neither of these representations is correct. The cracking strength of uranium oxide varies by more than fifty percent over temperatures ranging from 600°C to 1200°C (1112°F to 2192°F) (Reference 11). Moreover, the rate

of healing for small cracks in uranium oxide increases by a factor of eight or more over the temperature range from 1600°C to 2000°C (2912°F to 3632°F) (Reference 12). At temperatures near the upper end of this range, strength recovery from microcracking to the original strength requires about four hours of annealing. The substantial temperature dependence and the significant time required for strength recovery suggest that healing may be very slow, or even nonexistent, at the temperatures encountered in the outer region of the fuel especially in low power regions of the rod.

A suitable representation of the temperature dependence of the cracking stress, $\sigma^*(T)$, can be obtained by use of the piecewise linear expression*

$$\sigma^*(T) = [A_k + B_k T]\sigma^*_{nominal} \qquad \text{(III.5)}$$

In this expression T is the temperature and A_k and B_k are parameters applicable for $T_{k-1} < T \leq T_k$.

Experimental observations on uranium oxide reported by Reynolds, Burton and Speight [Reference 13] suggest that the growth of pores at the intersection of grain boundaries controls the fracture process at high strain rates ($\sim 2 \times 10^{-3}$/second). At lower strain rates ($\sim 10^{-6}$/second) fracture appears to result from the growth of numerous cavities on the grain boundaries.

* See the data presented in Reference (11).

In both cases, fairly regular arrays of pores are formed at intervals comparable to the linear dimensions of a grain. Thus, writing the nominal grain size as ℓ and assuming cylindrical pores of uniform diameter, a, regularly arrayed at a center-to-center spacing of $\beta\ell$, the results of Paris and Sih (Reference 14) provide an estimated cracking stress

$$\sigma^*(T) = \frac{K_{IC}(T)}{[2\beta\ell \tan (\frac{\pi a}{2\beta\ell})]^{1/2}} \quad . \qquad \text{(III.6)}$$

Here, $K_{IC}(T)$ is the critical stress intensity factor for the material under consideration. It is appropriate for the CYGRO analysis to assume that there are no stress concentrations. Thus, writing K_{IC} in the form suggested by (III.5) and (III.6):

$$\sigma^*(T) = \frac{[A_k + B_k T]K_{nominal}}{[2\beta\ell \tan (\frac{\pi a}{2\beta\ell})]^{1/2}} \quad , \qquad \text{(III.7)}$$

the initial strength data can be represented in terms of β, $a/\beta\ell = a_o/\beta\ell$ and the magnitude of the coefficients A_k and B_k in (III.5). Unfortunately, there is no prescribed way to assign these quantities. The most appealing approach is to choose $a_o/\beta\ell$ such that the porosity of the material is accurately represented. That is, $(a_o/\beta\ell)^2$ equals the initial porosity. Then the strength data can be used to define β and the magnitude of the coefficients A_k, B_k and $K_{nominal}$ in terms of the average grain size. High strain rate fracture would correspond to values of β near 1.0, while low strain rate fracture would require $\beta \approx 0.3$. The low strain rate values would be more appropriate to normal reactor operating conditions.

According to the model discussed earlier in this section, the material will fracture when the tensile stress reaches $\sigma^*(T)$ given by (III.7). When the resulting crack closes due to mechanical action, strength recovery begins. The analysis of Dutton (Reference 15) proposes that crack-like pores heal at a rate

$$\frac{da}{dt} = -a^*[1 - \exp(-Q_a/RT)]\exp(-Q_b/RT) \quad . \qquad \text{(III.8)}$$

Here, a is the major radius of the crack, a* is an experimentally determined diffusion rate constant, Q_a and Q_b are activation energies, and R is Boltzmann's constant.

At crack closing $a = \beta\ell$, and pore closure can be assumed to cease when $a = a_o$. Over each time interval the value of a will be taken as the value at the beginning of the interval. The value of a at the end of the interval will be computed by use of (III.8). The cracking stress for the interval will be the value of σ^* given by (III.7) for the appropriate values of a and T. Limitation of a to $a \geq a_o$ ensures that the material does not become stronger than its initial value.

This model neglects all aspects of creep rupture. No account is taken of the growth of pores at grain boundaries as discussed by Reynolds, Burton and Speight (Reference 13). Likewise, strength recovery is taken to be independent of the stress state and the fission rate. One might expect that these parameters and others will prove to be important to

cracking and strength recovery following crack closing. However, there is relatively little fundamental data on these matters, and the program could easily suffer from the inclusion of an overly sophisticated model. Thus the present model provides improved modeling capability without overreaching the available data.

In the CYGRO-4 heat conduction calculations discussed in Section II the temperature difference between the inside and the outside of a ring element is proportional to the product of the heat flux and the radial thickness of the element and inversely proportional to the thermal conductivity and the average radius of the element. Thus, the effective thermal resistance is the ratio of the radial thickness divided by the conductivity. When there are cracks on surfaces of constant r the thermal resistance is increased above that of uncracked fuel. The cracks create small gaps within the fuel that behave in much the same way as the gap between the fuel and the clad. Thus heat flows across the cracks by radiative transfer and by convection in the mixture of fission gas and initial fill gas existing throughout the interior of the rod.

To represent this effect the thermal resistance of a fuel ring element can be taken as the sum of the resistance of the uncracked fuel and the resistance across the cracks. Since the exact distribution of the cracks cannot be known and the gap conductivity model is not amenable to a continuum representation, it is convenient to calculate the thermal resistance of the cracks as if there existed a single crack at the mid-radius of the ring element with a gap given by the product of the initial radial dimension of the element and the crack strain in the radial

direction. The gas in the crack gap is at the same pressure as the gas in the fuel-clad gap and has the same composition as the gas in the gap - whether fill gas plus fuel volatiles and fission gases or steam. Its temperature is the temperature at the mid-radius of the fuel ring.

Application of the gap conductivity model of Section II is immediate. The conduction through the gas results in conductivity h_f. In the notation of (II.27)

$$h_f = \frac{K_{mix}}{2CR_s + g + t} \qquad \text{(III.9)}$$

where K_{mix} is the thermal conductivity of the gas, C is a dimensionless form coefficient, R_s is the roughness of the surface of the crack, g can be calculated as in Section II, and t is the thickness of the crack gap - the ring radial dimension times the radial component of crack strain. The only new quantity introduced here is R_s which is to be determined from the input to the program. In the absence of relevant data on the roughness of the crack surface it would seem appropriate to assume the roughness at least as large as the roughness of the outer surface of the fuel, a quantity already supplied for use in the gap conductivity calculations.

Radiative heat transfer across the crack results in an effective conductivity

$$h_r = 4\,\epsilon_f\,\sigma\,T_{Av}^3/(2 - \epsilon_f) \qquad \text{(III.10)}$$

Here, σ is the Stefan-Boltzman constant, T_{Av} is the fuel ring average temperature, and ϵ_f is the emissivity of the fuel. This expression assumes that the crack gap is so small compared to the radius that the surfaces of the crack are effectively infinite parallel plates. The net crack gap conductivity, h_c, is the sum of h_f and h_r. Thus, the gap resistance is

$$1/h_c = 1/(h_f + h_r) \quad .$$

Transformation of this to the form adapted to the CYGRO temperature computations gives

$$\frac{1}{(r_o - r_i)h_c} = \frac{1}{(r_o - r_i)(h_f + h_r)} \quad . \qquad \text{(III.11)}$$

Hence, the thermal resistance of a cracked fuel ring is given by

$$\frac{1}{C_f^*} = \frac{1}{C_f} + \frac{\alpha}{(h_f + h_r)(r_o - r_i)} \quad . \qquad \text{(III.12)}$$

In this expression, α is an input constant whose value can be adjusted to give the best agreement with experimental data. Assignment of a value of zero to α reduces the resistance to the value for uncracked fuel. It is not likely that a value of α much larger than 1.0 would prove useful or desirable.

IV. Fuel-Clad and Rod-Support Interaction

Mechanical interaction between the fuel and the cladding and between the fuel rod and the supporting structure has a strong effect on the deformations of the cladding. For this reason, accurate fuel rod analysis requires a reliable and realistic model for the various interaction processes.

A fuel-clad interaction model has existed in the CYGRO programs since CYGRO-2 (Reference 3) and a rod-support interaction model was incorporated in CYGRO-3 (Reference 4). However, these models occasionally would lead to anomalous situations in which the results of computations violated the assumptions made in setting up the governing equations. For example, the fuel might have an axial strain rate larger than that of the cladding even though the governing equations were valid only for situations in which the fuel strain rate was smaller than that of the cladding. This suggested the need for a revised fuel-clad and rod-support interaction model that would be more reliable and would be more consistent with the revised time step controls discussed in Section VI of this memorandum. At time $t = t_{k+1} = t_k + \Delta t_k$ the fuel radius and length, r_f and ℓ_f, are given in terms of the radial fuel-clad interaction force, F_1, and the axial fuel-clad interaction force, F_2, by the linearized equations

$$\begin{bmatrix} r_{f(k+1)} \\ \ell_{f(k+1)} \end{bmatrix} = \begin{bmatrix} r_{f(k)} \\ \ell_{f(k)} \end{bmatrix} + \begin{bmatrix} \delta r_{f(k)} + \dot{r}_{f(k)} \Delta t_k \\ \delta \ell_{f(k)} + \dot{\ell}_{f(k)} \Delta t_k \end{bmatrix} - \begin{bmatrix} A^f_{ij} \end{bmatrix} \begin{bmatrix} \delta F_{1(k)} + \dot{F}_{1(k)} \Delta t_k \\ \delta F_{2(k)} + \dot{F}_{2(k)} \Delta t_k \end{bmatrix} . \tag{IV.1}$$

Here, the A^f_{ij} are the components of the two-by-two, symmetric fuel flexibility matrix. The form of this equation reflects the fact that F_1 acts to decrease the fuel radius and F_2 acts to decrease the fuel length.

Similarly, the clad inner radius and length, r_c and ℓ_c, are given in terms of F_1, F_2 and the rod-support axial interaction force, F_3, by the linearized equations

$$\begin{bmatrix} r_{c(k+1)} \\ \ell_{c(k+1)} \end{bmatrix} = \begin{bmatrix} r_{c(k)} \\ \ell_{c(k)} \end{bmatrix} + \begin{bmatrix} \delta r_{c(k)} + \dot{r}_{c(k)} \Delta t_k \\ \delta \ell_{c(k)} + \dot{\ell}_{c(k)} \Delta t_k \end{bmatrix} + \begin{bmatrix} A^c_{ij} \end{bmatrix} \begin{bmatrix} \delta F_{1(k)} + \dot{F}_{1(k)} \Delta t_k \\ \delta F_{2(k)} + \dot{F}_{2(k)} \Delta t_k \\ \delta F_{3(k)} + \dot{F}_{3(k)} \Delta t_k \end{bmatrix} . \tag{IV.2}$$

In this expression the A^c_{ij} are the components of the two-by-three clad flexibility matrix. The flexibility matrix has entries A^c_{ij} that satisfy the equations $A^c_{12} = A^c_{21} = -A^c_{13}$ and $A^c_{23} = -A^c_{22}$.

Finally, the length of the rod support structure ℓ_s, is given in terms of F_3 by

$$\ell_{s(k+1)} = \ell_{s(k)} + \delta \ell_{s(k)} + \dot{\ell}_{s(k)} \Delta t_{(k)} + A^s \left(\delta F_{3(k)} + \dot{F}_{3(k)} \Delta t_k \right) . \tag{IV.3}$$

The fuel-clad interaction and the rod-support interaction are characterized by the interaction forces, F_i; i = 1,2,3, and the relative

dimensions, V_i; i = 1,2,3, which correspond to the radial gap between fuel and clad, the difference between clad axial extension and fuel extension, and the difference between support extension and rod extension. At $t = t_{k+1}$ the V_i are related to the F_i by the matrix equations

$$[V_{i(k+1)}] = [V_{i(k)}] + [\delta V_{i(k)} + \dot{V}_{i(k)}\Delta t_k] + [A^I_{ij}][\delta F_{j(k)} + \dot{F}_{j(k)}\Delta t_k] \quad . \tag{IV.4}$$

The terms in these equations can be obtained from the relations $V_1 = r_c - r_f$, $V_2 = \ell_c - \ell_f$, and $V_3 = \ell_s - \ell_c$. In particular, the terms of the flexibility matrix $[A^I_{ij}]$ are

$$\begin{aligned}
A^I_{11} &= A^c_{11} + A^F_{11} \\
A^I_{12} &= A^I_{21} = (A^c_{12} + A^F_{12}) = (A^c_{21} + A^F_{21}) \\
A^I_{22} &= (A^c_{22} + A^F_{22}) \\
A^I_{13} &= A^I_{31} = A^c_{13} = -A^c_{12} \\
A^I_{23} &= A^I_{32} = -A^c_{22} \\
A^I_{33} &= (A^s - A^c_{23}) = (A^s + A^c_{22}) \quad .
\end{aligned} \tag{IV.5}$$

The matrix equation (IV.4) relates the three interaction forces F_i, i = 1,2,3 to the three relative dimensions V_i; i = 1,2,3. Six other equations are needed to obtain a unique solution for the $\delta F_{i(k)}$, $\delta V_{i(k)}$, $\dot{F}_{i(k)}$ and $\dot{V}_{i(k)}$. These equations can be obtained from the assumed state of fuel-clad and rod-support interaction. The values of $F_{i(k+1)}$

and $V_{i(k+1)}$ resulting from solution of the six pairs of equations must then be checked to show that they are consistent with the assumptions made. If the results are inconsistent with the assumptions, new assumptions must be made and the process must be repeated until a set of assumptions produces results that agree with the assumptions.

Fuel-clad radial interaction results when the fuel-clad gap, V_1, is zero. In that case, the contact force, F_1, must be positive. When there is a gap, V_1 is positive and F_1 must be zero. Negative values of F_1 and V_1 are not valid. Thus, a fourth equation relating the $\delta V_{i(k)} + V_{i(k)}\Delta t_k$ and $\delta F_{i(k)} + F_{i(k)}\Delta t_k$ can be obtained by setting

$$V_{1(k+1)} = 0 \text{ provided } F_{1(k+1)} \geq 0$$

or (IV.6)

$$F_{1(k+1)} = 0 \text{ provided } V_{1(k+1)} \geq 0 \quad .$$

There are three possible states of fuel-clad axial interaction: $(V_{2(k+1)} - V_{2(k)}) = 0$, $(V_{2(k+1)} - V_{2(k)}) > 0$, and $(V_{2(k+1)} - V_{2(k)}) < 0$. In the first case, the magnitude of $F_{2(k+1)}$ is limited. In the second and third cases, the magnitude of $F_{2(k+1)}$ is known, and the sign is specified so that the sign of the product $F_{2(k+1)}[V_{2(k+1)} - V_{2(k)}]$ is negative. Thus, a fifth equation relating the $\delta V_{i(k)} + \dot{V}_{i(k)}\Delta t_k$ and $\delta F_{i(k)} + \dot{F}_{i(k)}\Delta t_k$ can be obtained by setting

$$V_{2(k+1)} = V_{2(k)} \text{ provided } -M^{\ell}_{2(k+1)} \leq F_{2(k+1)} \leq M^{u}_{2(k+1)}$$

$$F_{2(k+1)} = -M^{\ell}_{2(k+1)} \text{ provided } V_{2(k+1)} > V_{2(k)} \qquad \text{(IV.7)}$$

or

$$F_{2(k+1)} = M^{u}_{2(k+1)} \text{ provided } V_{2(k+1)} < V_{2(k)} .$$

Finally, there are three possible states of rod-support interaction: $(V_{3(k+1)} - V_{3(k)}) = 0$, $(V_{3(k+1)} - V_{3(k)}) > 0$ and $(V_{3(k+1)} - V_{3(k)}) < 0$. As in the case of fuel-clad axial interaction, the first case implies a limit on the magnitude of the interaction force. In the second and third cases, the magnitude of the interaction force is known and the sign of the interaction force is such that the force acts to impede the relative motion of rod and support. Thus, the sixth equation relating the $\delta V_{i(k)} + \dot{V}_{i(k)}\Delta t_k$ and $\delta F_{i(k)} + \dot{F}_{i(k)}\Delta t_k$ can be obtained by setting

$$V_{3(k+1)} = V_{3(k)} \text{ provided } -M^{\ell}_{3(k+1)} \leq F_{3(k+1)} \leq M^{u}_{3(k+1)}$$

$$F_{3(k+1)} = -M^{\ell}_{3(k+1)} \text{ provided } V_{3(k+1)} > V_{3(k)} \qquad \text{(IV.8)}$$

or

$$F_{3(k+1)} = M^{u}_{3(k+1)} \text{ provided } V_{3(k+1)} < V_{3(k)} .$$

The analogy between (IV.7) and (IV.8) is complete.

As a result of the form of Equations (IV.6),(IV.7), and (IV.8) determination of the six variables $\delta V_{i(k)} + \dot{V}_{i(k)}\Delta t_k$ and $\delta F_{i(k)} + \dot{F}_{i(k)}\Delta t_k$

can be reduced to the solution of three simultaneous equations in the three variables $(\delta F_{i(k)} + \dot{F}_{i(k)}\Delta t_k)$ followed by determination of the $(\delta V_{i(k)} + \dot{V}_{i(k)}\Delta t_k)$ from these. The solution must then satisfy the assumptions stated in Equations (IV.6), (IV.7), and (IV.8). These assumptions are equivalent to limitations on the size of Δt_k. In some instances the limits are on the maximum value of Δt_k, while in others the minimum value of Δt_k is limited. The interaction assumptions are consistent when the maximum acceptable value of Δt_k is larger than, or equal to, the minimum acceptable value of Δt_k. Whenever the assumptions are not consistent, they must be changed systematically until a consistent set of assumptions is found.

A circumstance requiring consideration is the situation where the upper bound on Δt_k limits Δt_k excessively. In this circumstance the program can be reduced to a sequence of trivially short time steps. To avoid this, the limits expressed in Equations (IV.6), (IV.7), and (IV.8) are modified so that the calculated time step will be large enough to produce an "overshoot" of amount d of displacement or f of interaction force.

Returning now to Equations (IV.6), (IV.7), and (IV.8), consider the limits imposed on Δt_k by the assumptions listed there. When, $V_{l(k+l)} = 0$, (IV.6) requires $F_{l(k+l)} \geq -f$. For $\dot{F}_{l(k)} > 0$, this implies

$$\Delta t_k > \frac{-[f + (F_{l(k)} + \delta F_{l(k)})]}{\dot{F}_{lk}} \qquad \text{(IV.9a)}$$

while for $\dot{F}_{l(k)} < 0$

$$\Delta t_k \leq \frac{-[f + (F_{l(k)} + \delta F_{l(k)})]}{\dot{F}_{lk}} \qquad \text{(IV.9b)}$$

When $F_{1(k+1)} = 0$, $V_{1(k+1)} \geq -d$.

For $\dot{V}_{1(k)} \geq 0$ this implies

$$\Delta t_k \geq \frac{-[d + V_{1(k)} + \delta V_{1(k)}]}{\dot{V}_{1(k)}} \qquad \text{(IV.10a)}$$

while for $\dot{V}_{1(k)} \leq 0$, this implies

$$\Delta t_k \leq \frac{-[d + V_{1(k)} + \delta V_{1(k)}]}{\dot{V}_{1(k)}} \qquad \text{(IV.10b)}$$

When $V_{2(k+1)} = V_{2(k)}$, (IV.7) requires $-[M^{\ell}_{2(k+1)} + f] \leq F_{2(k+1)} \leq [M^{u}_{2(k+1)} + f]$. Writing $M^{\ell}_{2(k+1)} = M^{\ell}_{2(k)} + \delta M^{\ell}_{2(k)} + \dot{M}^{\ell}_{2(k)} \Delta t_k$, the limits become

$$[\dot{F}_{2(k)} + \dot{M}^{\ell}_{2(k)}]\Delta t_k \geq -[M^{\ell}_{2(k)} + F_{2(k)} + \delta M^{\ell}_{2(k)} + \delta F_{2(k)} + f] \qquad \text{(IV.11a)}$$

and

$$[\dot{F}_{2(k)} - \dot{M}^{u}_{2(k)}]\Delta t_k \leq [M^{u}_{2(k)} - F_{2(k)} + \delta M^{u}_{2(k)} - \delta F_{2(k)} + f] \qquad \text{(IV.11b)}$$

For $[\dot{F}_{2(k)} + \dot{M}^{\ell}_{2(k)}] > 0$, (IV.11a) requires

$$\Delta t_k \geq -\frac{[M^{\ell}_{2(k)} + F_{2(k)} + \delta M^{\ell}_{2(k)} + \delta F_{2(k)}] + f}{[\dot{F}_{2(k)} + \dot{M}^{\ell}_{2(k)}]} \qquad \text{(IV.12a)}$$

while for $[\dot{F}_{2(k)} + \dot{M}^{\ell}_{2(k)}] < 0$,

$$\Delta t_k \leq - \frac{[M^{\ell}_{2(k)} + F_{2(k)} + \delta M^{\ell}_{2(k)} + \delta F_{2(k)}] + f}{[\dot{F}_{2(k)} + \dot{M}^{\ell}_{2(k)}]} \quad . \qquad \text{(IV.12b)}$$

Similarly, for $[\dot{F}_{2(k)} - \dot{M}^{u}_{2(k)}] > 0$, (IV.11b) requires

$$\Delta t_k \leq \frac{[M^{u}_{2(k)} - F_{2(k)} + \delta M^{u}_{2(k)} - \delta F_{2(k)}] + f}{[\dot{F}_{2(k)} - \dot{M}^{u}_{2(k)}]} \qquad \text{(IV.13a)}$$

while for $[\dot{F}_{2(k)} - \dot{M}^{u}_{2(k)}] < 0$

$$\Delta t_k \geq \frac{[M^{u}_{2(k)} - F_{2(k)} + \delta M^{u}_{2(k)} - \delta F_{2(k)}] + f}{[\dot{F}_{2(k)} - \dot{M}^{u}_{2(k)}]} \quad . \qquad \text{(IV.13b)}$$

When $F_{2(k+1)} = -M^{\ell}_{2(k+1)}$, $V_{2(k+1)} \geq V_{2(k)} - d$. This implies $\dot{V}_{2(k)} \Delta t_k + \delta V_{2(k)} \geq - d$

$$\Delta t_k \geq \frac{-[d + \delta V_{2(k)}]}{\dot{V}_{2(k)}} \quad . \qquad \text{(IV.14a)}$$

Similarly, when $F_{2(k+1)} = M^{u}_{2(k+1)}$, $V_{2(k+1)} \leq V_{2(k)} + d$. This implies $\dot{V}_{2(k)} < 0$ and

$$\Delta t_k \geq \frac{d - \delta V_{2(k)}}{\dot{V}_{2(k)}} \quad . \qquad \text{(IV.14b)}$$

Further limits on the time step are imposed by the requirement that $F_{2(k+1)}$ remain negative when $V_{2(k+1)} - V_{2(k)} > 0$ and positive when $V_{2(k+1)} - V_{2(k)} < 0$. Thus, for $\dot{V}_{2(k)} > 0$ and $\dot{F}_{2(k)} < 0$

$$\Delta t_k \geq \frac{f - [F_{2(k)} + \delta F_{2(k)}]}{\dot{F}_{2(k)}} \qquad \text{(IV.15a)}$$

and for $\dot{V}_{2(k)} > 0$ and $\dot{F}_{2(k)} > 0$

$$\Delta t_k \leq \frac{f - [F_{2(k)} + \delta F_{2(k)}]}{\dot{F}_{2(k)}} \qquad \text{(IV.15b)}$$

while for $\dot{V}_{2(k)} < 0$ and $\dot{F}_{2(k)} > 0$

$$\Delta t_k \geq - \frac{f + [F_{2(k)} + \delta F_{2(k)}]}{\dot{F}_{2(k)}} \quad . \qquad \text{(IV.16a)}$$

and for $\dot{V}_{2(k)} < 0$ and $\dot{F}_{2(k)} < 0$

$$\Delta t_k \leq - \frac{f + [F_{2(k)} + \delta F_{2(k)}]}{\dot{F}_{2(k)}} \quad . \qquad \text{(IV.16b)}$$

The cases $V_{3(k+1)} = V_{3(k)}$, $F_{3(k+1)} = M^u_{3(k+1)}$ and $F_{3(k+1)} = -M^\ell_{3(k+\ell)}$ lead to limits which can be derived from the limits given by (IV.12) through (IV.16). The limits for these cases are identical to the form obtained by exchanging 3 for 2 every place 2 appears in (IV.12) through (IV.16). Accordingly, they will not be given here.

The functions M^u_2, M^ℓ_2, M^u_3 and M^ℓ_3 which limit the magnitude of interaction forces F_2 and F_3 have been assigned a representation

$$M^x_{i(k+1)} = M^x_{i(k)} + \delta M^x_{i(k)} + \dot{M}^x_{i(k)} \Delta t_k \quad .$$

for i = 2, $\delta M^x_{2(k)}$ and $\dot{M}^x_{2(k)}$ depend on reactor pressure, temperature and the fuel-clad gap. For i = 3, $\delta M^x_{3(k)} = 0$, and $\dot{M}^x_{3(k)}$ is defined from the input data.

The functional form of M_2^{ℓ} is

$$M_2^{\ell} = \mu_c \, r_{f(o)} \, p_x + \mu_e \, h_g^{\ell} + \mu_I \, F_1 \quad , \tag{IV.17}$$

while the functional form of M_2^{u} is

$$M_2^{u} = \mu_c \, r_{f(o)} \, p_x + \mu_e \, h_g^{u} + \mu_T \, r_{f(o)} \, T_x + \mu_I \, F_1 \quad . \tag{IV.18}$$

In these expressions $r_{f(o)}$ is the initial fuel radius and μ_c, μ_e, μ_I and μ_T are user specified friction coefficients. The other new variables are defined by

$$p_x = \text{Maximum of} \begin{cases} 0 \\ P_{coolant} - P_{gap} - P_{collapse} \end{cases} \tag{IV.19}$$

$$T_x = \text{Maximum of} \begin{cases} 0 \\ \left[\frac{1}{2}(T_c + T_s) - T_A\right] \end{cases} \tag{IV.20}$$

$$h_g^{\ell} = \text{Maximum of} \begin{cases} 0 \\ C_{end}\left[\alpha_e(T_c - T_s)r_{f(o)} - V_1\right] \end{cases} \tag{IV.21}$$

and

$$h_g^{u} = \text{Maximum of} \begin{cases} 0 \\ C_{end}\left[\alpha_e(T_c - T_s)r_{f(o)} + \epsilon_e - V_1\right] \end{cases} \quad . \tag{IV.22}$$

In these, α_e is a user input quantity representing the pellet "hour-glassing" coefficient, ϵ_e is the fuel eccentricity or chip effect measure, T_A is a user input temperature effect parameter and T_c and T_s are the fuel centerline - and surface temperatures. The quantity C_{end} is a measure of the interaction force for a given interference.

Examination of Equations (IV.17) through (IV.22) shows that p_x, T_x, h_g^{ℓ}, h_g^{u} and F_1 all vary with time. Thus, writing

$$M^{\ell}_{2(k+1)} = M^{\ell}_{2(k)} + \delta M^{\ell}_{2(k)} + \dot{M}^{\ell}_{2(k)} \Delta t_k \qquad \text{(IV.23)}$$

and

$$M^{u}_{2(k+1)} = M^{u}_{2(k)} + \delta M^{u}_{2(k)} + \dot{M}^{u}_{2(k)} \Delta t_k \qquad \text{(IV.24)}$$

the explicit values of $M^{\ell}_{2(k+1)}$ and $M^{u}_{2(k+1)}$ can be deduced. Specifically, those values are

$$M^{\ell}_{2(k)} + \delta M^{\ell}_{2(k)} = \mu_c \, r_{f(o)} \, p_{x(k)} + \mu_e h^{\ell}_{g(k)} + \mu_I [F_{1(k)} + \delta F_{1(k)}] \qquad \text{(IV.25a)}$$

$$\dot{M}^{\ell}_{2(k)} = \mu_c \, r_{f(o)} \, \dot{p}_{x(k)} + \mu_e \, \dot{h}^{\ell}_{g(k)} + \mu_I \, \dot{F}_{1(k)} \qquad \text{(IV.25b)}$$

$$M^{u}_{2(k)} + \delta M^{u}_{2(k)} = \mu_c \, r_{f(o)} \, p_{x(k)} + \mu_e \, h^{u}_{g(k)} + \mu_T \, r_{f(o)} \, T_{x(k)} + \mu_I [F_{1(k)} + \delta F_{1(k)}] \qquad \text{(IV.26a)}$$

and

$$\dot{M}^{u}_{2(k)} = \mu_c \, r_{f(o)} \, \dot{p}_{x(k)} + \mu_e \, \dot{h}^{u}_{g(k)} + \mu_T r_{f(o)} \dot{T}_{x(k)} + \mu_I \, \dot{F}_{1(k)} \qquad \text{(IV.26b)}$$

In these expressions the derivatives $\dot{p}_{x(k)}$, $\dot{h}^{u}_{g(k)}$, et cetera are understood to be zero whenever the corresponding quantity, $p_{x(k)}$, $h^{u}_{g(k)}$, et cetera, is zero. When these quantities are non-zero, the derivatives can be determined from the expressions

$$\dot{h}^{\ell}_{g(k)} = C_{end}[\alpha_e(\dot{T}_{c(k)} - \dot{T}_{s(k)})r_{f(o)} - \dot{V}_{1(k)}] \qquad \text{(IV.27)}$$

$$\dot{h}^{u}_{g(k)} = C_{end}[\alpha_e(\dot{T}_{c(k)} - \dot{T}_{s(k)})r_{f(o)} - \dot{V}_{1(k)}] \qquad \text{(IV.28)}$$

and

$$\dot{T}_{x(k)} = \tfrac{1}{2}[\dot{T}_{c(k)} - \dot{T}_{s(k)}] \quad . \qquad \text{(IV.29)}$$

Evaluation of $h^{\ell}_{g(k)}$ and $h^{u}_{g(k)}$ is based on the value $V_{l(k)} + \delta V_{l(k)}$, with the approximation that $h^{\ell}_{g(k)}$ and $h^{u}_{g(k)}$ are zero throughout the interval if they are zero at $t = t_k$, and non-zero throughout the interval if they are non-zero at $t = t_k$. This results in limits on the size of the time interval that act in addition to the limits imposed by fuel-clad interaction and rod-support interaction.

The central aspect of the fuel-clad and rod-support interaction computations is the procedure used to determine the states of interaction. Since the states are discrete (contact or non-contact, slipping or non-slipping) it is impossible to use a method such as the Newton-Raphson method or the method of bisection that requires continuous variation. Two alternatives appear: solution of the equations for all possible interaction states, or use of an iteration procedure such as that in Reference (16) in which the trial states for a given iteration are determined from the results of the previous iteration by changing only those assumptions which are inconsistent with the results of the computations. As indicated above the second alternative was chosen. The remainder of this section is devoted to a description of its implementation.

Since the inequalities stated in (IV.6),(IV.7), and (IV.8) lead to minimum and maximum limits on the time step, let the values of the three minimum limits be indicated by $\Delta t_{min}^{(i)}; i = 1,2,3$, and let the values of the three maximum limits be $\Delta t_{max}^{(i)}$; $i = 1,2,3$. Thus, when it is assumed that $V_{2(k+1)} = V_{2(k)}$, the values $\Delta t_{min}^{(2)}$ and $\Delta t_{max}^{(2)}$ are defined by (IV.12) and (IV.13). In particular, for $\dot{F}_{2(k)} > 0$ and $\dot{M}^{u}_{2(k)} = \dot{M}^{\ell}_{2(k)} = 0$, $\Delta t_{min}^{(2)}$ corresponds to the right hand side of (IV.12a) while $\Delta t_{max}^{(2)}$ corresponds to the right hand side of (IV.13a). Conversely, for $\dot{F}_{2(k)} < 0$, $\Delta t_{min}^{(2)}$ corresponds to the right hand side of (IV.13b) while $\Delta t_{max}^{(2)}$ corresponds to the right hand side of (IV.12b).

In this way, each of the aspects of fuel-clad and rod-support interaction leads to limits on Δt_k: $\Delta t_{min}^{(i)} \leq \Delta t_k$ and $\Delta t_k \leq \Delta t_{max}^{(i)}$. Obviously, any assumption that leads to $\Delta t_{min}^{(i)} > \Delta t_{max}^{(i)}$ is invalid and must be changed. In the example just given, the assumption $V_{2(k+1)} = V_{2(k)}$ is invalid when $\Delta t_{min}^{(2)} > \Delta t_{max}^{(2)}$. The most appropriate assumption for the next iteration is then to assume that the fuel slips in the cladding. The direction of slipping is determined by the sign of $F_{2(k+1)}$ - slipping occurs so that the sign of $[V_{2(k+1)} - V_{2(k)}]F_{2(k+1)}$ is negative.

The procedure begins by assuming that the state of fuel-clad and rod-support interaction for $t = t_{k+1}$ will be the same as for $t = t_k$. This allows use of (IV.6), (IV.7), and (IV.8) with (IV.4) to determine the $\delta V_{i(k)}$, $\delta F_{i(k)}$, $\dot{V}_{i(k)}$ and $\dot{F}_{i(k)}$ that result from the assumed interaction

state. These quantities are used in expressions obtained from (IV.9) through (IV.16) to determine the $\Delta t_{max}^{(i)}$ and $\Delta t_{min}^{(i)}$. If the largest of the $\Delta t_{min}^{(i)}$ is consistent with the smallest of the $\Delta t_{max}^{(i)}$, the assumed interaction state is valid and the program proceeds to other computations. Whenever one of the $\Delta t_{min}^{(i)}$ exceeds the corresponding $\Delta t_{max}^{(i)}$, the assumed interaction state is changed to agree with the state indicated by the results of the computations based on the previous assumptions. Consistent assumptions are retained, and inconsistent assumptions are changed. Thus, if assuming no-slipping leads to a contradiction, slipping will be assumed, or if assuming a gap leads to a contradiction, fuel-clad radial contact will be assumed. The assumed sign of slipping is chosen as indicated in the example just given.

This procedure converges to a consistent solution rapidly in the great majority of cases. However, in certain unusual circumstances it is not effective, and the new assumptions must be chosen according to different criteria. This is done solely to arrive at a set of consistent assumptions. The consequences of the assumptions still must meet the requirements imposed by (IV.6), (IV.7), and (IV.8), and expressed in (IV.9) through (IV.16).

The usual procedure for arriving at improved assumptions proves to be ineffective in a number of special cases that are usually associated with the initiation or termination of slipping between the fuel and cladding. For example, when the fuel and cladding were in intimate contact at $t = t_k$ and there has been no axial slippage between them, a decrease in reactor power at $t = t_{k+1}$ can create a situation where it is incorrect to assume termination of contact and initiation of slipping

or to assume that contact continues with no slippage between fuel and cladding. Unfortunately, the first pair of assumptions will sometimes lead to results which suggest the second set, and vice versa. The usual procedure will not arrive at the assumption of contact with slipping or non-contact without slipping. In this circumstance, rapid convergence occurs if the non-slip assumption is continued whenever the contact assumption proves invalid - even though the non-slip assumption was invalid during that iteration. This allows assumption of non-contact with no slipping which may then lead to the assumption of contact and slipping. One of these latter assumptions will prove to be consistent. Obviously this special procedure should only be used after several iterations with the usual procedure because the usual procedure is successful in most cases, and the end of contact often coincides with initiation of slipping.

When the fuel-clad gap is closing and the fuel and cladding are coming into contact other special cases are encountered. As the gap closes the values of $M^{\ell}_{2(k)} + \delta M^{\ell}_{2(k)}$ and $M^{u}_{2(k)} + \delta M^{u}_{2(k)}$ rise abruptly. Thus, if there was a gap at $t = t_k$ and the results of the previous iteration indicate that the gap is closed and there will be slipping between the fuel and clad, it can prove fruitful to assume no slipping. Another phenomenon associated with closing of the fuel-clad gap is the suppression of the clad extension that could occur if there was a fuel-clad gap. In this instance it is often profitable to assume that there is no rod-support slippage even though the results of the previous iteration indicate there should be such slippage.

When the support is exceedingly stiff compared to the rod axial stiffness the value of $\dot{V}_{3(k)}$ can prove to be the best indicator of the rod-support slip direction. In other cases, rod-support slipping may not be tried by the usual iteration procedure. This is most likely when the fuel-clad gap is closed and fuel-clad slipping is starting. Thus, when the usual procedure does not converge in this circumstance, it is profitable to force an iteration in which rod-support slipping is assumed.

Other special cases involve situations where the sign of $\dot{F}_{2(k)}$ is the opposite of the sign of $F_{2(k)} + \delta F_{2(k)}$ or the sign of $\dot{F}_{3(k)}$ is the opposite of the sign of $F_{3(k)} + \delta F_{3(k)}$. In these cases, the usual procedure (where the calculated value of $F_{i(k+1)}$ is used to determine the direction of slipping) is likely to perform poorly. An alternative procedure based on the sign of $F_{i(k)} + \delta F_{i(k)}$ leads to rapid convergence and is used except when $\dot{F}_{i(k)}$ is exceedingly large - so large that $F_{i(k)}$ times the maximum of the $\Delta t_{max}^{(i)}$ is larger than twice the maximum of $M_{i(k)}^{\ell}$ and $M_{i(k)}^{u}$.

This fuel-clad and rod-support interaction model has proved to be quite reliable and has contributed to a reduction in the number of anomalous discrepancies between the results of closely related analyses. During the initial phases of its implementation special cases like those discussed here often prevented convergence to a valid answer. The difficulties resulting from these cases were circumvented by developing ad hoc procedures which use the features of the special case to determine assumptions that are more profitable than those that would result from the usual procedure.

While the iteration procedure discussed here provides an efficient means of finding the correct state of fuel-clad and rod-support interaction, it is only a part of the revised fuel-clad and rod-support interaction model. Another vital aspect of the revised model is the requirement is that the results of the interaction computations agree with the assumptions upon which the computations are based. This feature provides increased confidence in the physical reality of the results, for the iteration procedure will converge only for an interaction state that gives consistent results. The failures to converge that occurred in the development of the model resulted in program stops to prevent the generation of results that might be invalid.

V. Fuel Densification

The phenomenon of fuel densification is represented in CYGRO-4 by the use of the "hot pressing" model of CYGRO-1 (Reference 2). In this representation, there are one-to-five categories of pores. The process of densification involves the shrinking of these pores in response to the applied external pressure and the surface tension of the fuel.

For a category of pores with volume fraction ϵ_h consisting of N_s pores per unit solid volume, the nominal pore radius is

$$r_p = \left[\frac{3\epsilon_h}{4\pi N_s (1 - \epsilon_h)} \right]^{1/3} \qquad (V.1)$$

Thus, when the surface tension is γ and the "pressure" is $p = -(\sigma_1 + \sigma_2 + \sigma_3)/3$, the densification rate is

$$\dot{\epsilon}_h = -\frac{3\epsilon_h}{2\eta}\left[\frac{\gamma}{2r_p} + \frac{P}{2}\right] + \frac{3\sqrt{2}}{4}\frac{S_y}{\eta}\epsilon_h \ln\left(\frac{1}{\epsilon_h}\right) \quad . \qquad (V.2)$$

The remaining variables are the "local yield stress" around pores, S_y, and the effective viscosity, η. The value of S_y is an input variable, while η is determined from the "steady state" creep rate.

Returning to the basic definition of η—$1/\eta = \partial\dot{\epsilon}/\partial\sigma$—$\eta$ can be obtained from

$$\frac{1}{\eta} = \frac{1}{R_{sc}}\left\{10^{C-3G}\,|P|^{G-1} + A\varphi\right\} \quad ,$$

where R_{sc} is an input constant and A is the coefficient of fission rate dependent creep. Thus, η is

$$\eta = \frac{R_{sc}}{10^{C-3G}\,|P|^{G-1} + A\varphi} \quad , \qquad (V.3)$$

or using (A.I.12) and (A.I.13) from Appendix I

$$\eta = \frac{R_{sc}}{\left[\frac{Q_d^{R_r/(R_r-R_d)}}{Q_r^{R_d/(R_r-R_d)}}\right]|P|^{[R_r(W_d-1)+R_r]/(R_r-R_d)} + A\varphi} \quad . \qquad (V.4)$$

In order to establish limits on the size of the time step as needed in the procedure described in Section VI, consider the derivative of $\dot{\epsilon}_h$ with respect to ϵ_h. This is

$$d\dot{\epsilon}_h/d\epsilon_h = \frac{1}{\eta}\left\{(2-\epsilon_h)\left[\frac{4\pi N_s}{3\epsilon_h(1-\epsilon_h)^2}\right]^{1/3} - \frac{3P}{4} + \frac{3\sqrt{2}}{4}S_y(1+\epsilon_h)\right\} \quad .$$

If densification occurs at the rate given by (V.2), a linearized Taylor series expansion of (V.2) indicates that the rate of expansion will change from $\dot{\epsilon}_h$ to $\dot{\epsilon}_h \left(1 + \frac{d\dot{\epsilon}_h}{d\epsilon_h} \Delta t\right)$ in time Δt. Thus, if the change in densification rate is to be limited to the fraction D_d of the densification rate existing at the beginning of the time step, Δt must be limited to

$$\Delta t_k \leq \frac{\eta D_d}{(2-\epsilon)\left[\frac{4\pi N_s}{3\epsilon_{h1}(1-\epsilon_{h1})^2}\right]^{1/3} - \frac{3P}{4} + \frac{3\sqrt{2}}{4} S_y(1+\epsilon_h)} \quad . \quad (V.5)$$

This limit is considered in determining the size of the time step as discussed in the following section.

VI. Time Step Limits

The computations in CYGRO-4 use the rates of change at $t = t_k$ to calculate increments for the interval $t_k < t \leq t_{k+1}$. This means that the time step $\Delta t_k = t_{k+1} - t_k$ must be limited to values for which the rates of change are accurate. Table VI.1 provides a list of limits to the size of the time step that have been incorporated in CYGRO-4. This section discusses a number of these limits in sufficient detail to provide an understanding of the basic reasoning involved in developing time step limits. This illustrates the fact that there are time step limits associated with all the physical processes affecting fuel elements and that mathematical models for these physical processes must include procedures for calculating relevant time step limits.

Table VI.1 Limits to Time Step in CYGRO-4

1. Discontinuity in rate of change of reactor data: power level, coolant temperature, coolant pressure, et cetera.

2. Change in state of fuel-clad radial interaction.

3. Change in state of fuel-clad axial interaction.

4. Change in state of rod-support interaction.

5. Fuel cracking or closing of cracks in fuel.

6. Change of strain rate resulting from hardening or change of stress.

7. Change of compliance from value estimated using assumed time step.

8. Change of geometry resulting from large increment of strain.

9. Effect of temperature on deformation rates.

10. Changes in conditions such as clad collapse or initiation of "hourglassing" interaction between fuel and clad that require a change in the equations describing the behavior of the element.

The first time step limit in Table VI.1 arises from the input data. Input to CYGRO-4 includes data on the reactor conditions at a series of times $t = \tau_n$; $n = 1,\ldots$ referred to as History Card Times. For values of t in the range $\tau_n < t \leq \tau_{n+1}$ the reactor conditions are calculated by interpolating linearly between the data at times $t = \tau_n$ and $t = \tau_{n+1}$. Thus, the rate of change of the reactor data is usually discontinuous at each of the times $t = \tau_m$. This fact forms the basis for the fundamental time step limit. That is, for $\tau_{n-1} \leq t_k < \tau_n$, the value of t_{k+1} is limited by $t_{k+1} \leq \tau_n$. In general this limit is not as restrictive as the other limits discussed in the remainder of this section.

The second, third, and fourth limits in Table VI.1 are provided by the values $\Delta t_{max}^{(i)}$; $i = 1,2,3$ introduced in Section IV. As discussed there, the fuel-clad and rod-support model leads to limits on the minimum and the maximum size of the time step. These generally prove to be more restrictive than the limit imposed by the History Card Time. They are implemented by requiring that the program time step Δt_k is smaller than the minimum of the $\Delta t_{max}^{(i)}$; $i = 1,2,3$ calculated according to the procedures discussed in Section IV.

The fifth time step limit is associated with fuel cracking. As explained in Section III, the fuel in a finite element ring is said to crack whenever one of the stresses σ_i; $i = 1,2,3$ there reaches the cracking stress σ^*. At that point, the compliance matrix for the finite element must change. This means that if the stress σ_i in an uncracked finite element has value $\sigma_{i(k)} < \sigma^*$ at $t = t_k$ while the stress jump

is $\delta\sigma_{i(k)}$ and the rate of change is $\dot{\sigma}_{i(k)}$, the time step Δt_k must be consistent with

$$\sigma_{i(k)} + \delta\sigma_{i(k)} + \dot{\sigma}_{i(k)}\,\Delta t_k \leq \sigma^* .$$

As a result, Δt_k must assume a trivially small value if $\sigma_{i(k)} + \delta\sigma_{i(k)} \geq \sigma^*$, while for $\sigma_{i(k)} + \delta\sigma_{i(k)} < \sigma^*$ and $\dot{\sigma}_{i(k)} > 0$,

$$\Delta t_k \leq \frac{\sigma^* - \sigma_{i(k)} - \delta\sigma_{i(k)}}{\dot{\sigma}_{i(k)}} . \qquad \text{(VI.1)}$$

Cracks in a finite element of the fuel are said to close when the strain associated with the crack becomes zero or negative. This means that if the strain ϵ_i in a cracked finite element has value $\epsilon_{i(k)} > 0$ at $t = t_k$, while the strain jump is $\delta\epsilon_{i(k)}$, the strain rate is $\dot{\epsilon}_{i(k)}$, the strain associated with elastic, thermal, creep and growth deformation is $\epsilon^t_{i(k)}$, the jump of the elastic and thermal strains is $\delta\epsilon^e_{i(k)}$, and the rate of change of elastic, thermal, creep and swelling strains is $\dot{\epsilon}^t_{i(k)}$, the time step Δt_k must be consistent with

$$\epsilon_{i(k)} + \delta\epsilon_{i(k)} + \dot{\epsilon}_{i(k)}\,\Delta t_k > \epsilon^t_{i(k)} + \delta\epsilon^e_{i(k)} + \dot{\epsilon}^t_{i(k)}\,\Delta t_k .$$

As a result, Δt_k must assume a trivially small value if $[\epsilon_{i(k)} - \epsilon^t_{i(k)} + \delta\epsilon_{i(k)} - \delta\epsilon^e_{i(k)}] \leq 0$, while for $[\epsilon_{i(k)} - \epsilon^t_{i(k)} + \delta\epsilon_{i(k)} - \delta\epsilon^e_{i(k)}] > 0$ and $[\dot{\epsilon}^t_{i(k)} - \dot{\epsilon}_{i(k)}] > 0$

$$\Delta t_k \leq \frac{\epsilon_{i(k)} - \epsilon^t_{i(k)} + \delta\epsilon_{i(k)} - \delta\epsilon^e_{i(k)}}{\dot{\epsilon}^t_{i(k)} - \dot{\epsilon}_{i(k)}} . \qquad \text{(VI.2)}$$

The calculation of creep strains in CYGRO-4 is based on the fundamental inelastic strain rate expression

$$\dot{\varepsilon}_i^p = \frac{\partial \sigma_g}{\partial \sigma_i} \dot{\varepsilon}_g^p(\sigma_g) \qquad \text{(VI.3)}$$

for i = 1,2,3 corresponding to radial, circumferential and axial directions respectively. The value of $\partial\sigma_g/\partial\sigma_i$ is computed in terms of the values of the σ_i at the beginning of the time step and the value of $\dot{\varepsilon}_g(\sigma_g)$ is computed in terms of these quantities and the strain hardening at the beginning of the time step, but a correction for the effect of changes of stress is included. The change of strain hardening and the change of stress during the time step lead to limits on Δt.

The functional form of σ_g is

$$\sigma_g = [A(\sigma_1-\sigma_2)^2 + B(\sigma_2-\sigma_3)^2 + C(\sigma_3-\sigma_1)^2]^{1/2} . \qquad \text{(VI.4)}$$

Thus, the partial derivatives $\partial\sigma_g/\partial\sigma_i$ are

$$\frac{\partial \sigma_g}{\partial \sigma_1} = \frac{A(\sigma_1-\sigma_2) + C(\sigma_1-\sigma_3)}{\sigma_g} ,$$

$$\frac{\partial \sigma_g}{\partial \sigma_2} = \frac{B(\sigma_2-\sigma_3) + A(\sigma_2-\sigma_1)}{\sigma_g} \qquad \text{(VI.5)}$$

and

$$\frac{\partial \sigma_g}{\partial \sigma_3} = \frac{C(\sigma_3-\sigma_1) + B(\sigma_3-\sigma_2)}{\sigma_g} ,$$

while the second partial derivatives $\partial^2\sigma_g/\partial\sigma_i\partial\sigma_j$ are

$$\frac{\partial^2\sigma_g}{\partial\sigma_1^2} = (AB+BC+CA)\frac{(\sigma_2-\sigma_3)^2}{\sigma_g^3}$$

$$\frac{\partial^2\sigma_g}{\partial\sigma_1\partial\sigma_2} = \frac{\partial^2\sigma_g}{\partial\sigma_2\partial\sigma_1} = (AB+BC+CA)\frac{(\sigma_2-\sigma_3)(\sigma_3-\sigma_1)}{\sigma_g^3} \qquad \text{(VI.6)}$$

$$\frac{\partial^2\sigma_g}{\partial\sigma_1\partial\sigma_3} = \frac{\partial^2\sigma_g}{\partial\sigma_3\partial\sigma_1} = (AB+BC+CA)\frac{(\sigma_1-\sigma_2)(\sigma_2-\sigma_3)}{\sigma_g^3}$$

$$\frac{\partial^2\sigma_g}{\partial\sigma_2^2} = (AB+BC+CA)\frac{(\sigma_3-\sigma_1)^2}{\sigma_g}$$

$$\frac{\partial^2\sigma_g}{\partial\sigma_2\partial\sigma_3} = \frac{\partial^2\sigma_g}{\partial\sigma_3\partial\sigma_2} = (AB+BC+CA)\frac{(\sigma_3-\sigma_1)(\sigma_1-\sigma_2)}{\sigma_g^3}$$

$$\frac{\partial^2\sigma_g}{\partial\sigma_3^2} = (AB+BC+CA)\frac{(\sigma_1-\sigma_2)^2}{\sigma_g^2}$$

Consider now the effect of changes $\Delta\sigma_{i(k)} = \delta\sigma_{i(k)} + \dot{\sigma}_{i(k)}\,\Delta t_k$ in the σ_i during the interval $t_k < t \le t_{k+1}$. These induce corresponding changes in the values of σ_g, $\dot{\epsilon}_g^p$ and $\partial\sigma_g/\partial\sigma_i$ and, thus, in the value of $\dot{\epsilon}_i^p$. To estimate this change, let $\Delta\epsilon_{i(k)}^p = \dot{\epsilon}_i^p\,\Delta t_k$ be calculated by use of a linearized Taylor series about the stress state $\sigma_i = \sigma_{i(k)}$

$$\Delta\epsilon_{i(k)}^p = \frac{\partial\sigma_g}{\partial\sigma_i}\,\dot{\epsilon}_g^p\Big|_{\sigma=\sigma_{i(k)}}\Delta t_k + \sum_{j=1}^{3}\frac{\partial}{\partial\sigma_j}\left[\frac{\partial\sigma_g}{\partial\sigma_i}\,\dot{\epsilon}_g^p\,\frac{dt_k}{2}\right]\Delta\sigma_{j(k)} \qquad \text{(VI.7)}$$

In this expression the dt_k corresponds to a preliminary estimate of the value of Δt_k. This arises because of the computational structure of CYGRO - the effective compliance for a time step represents the sum of the elastic compliance and the "creep compliance." Thus, the creep

computations include an estimated time step. The validity of this estimate is the subject of an analysis presented later in this section.

To compute the terms in the summation appearing in (VI.7), consider its explicit form

$$\sum_{j=1}^{3} \frac{\partial}{\partial \sigma_j}\left[\frac{\partial \sigma_g}{\partial \sigma_i}\,\dot{\epsilon}_g^p\,\frac{dt_k}{2}\right]\Delta\sigma_{j(k)} = \frac{\partial \sigma_g}{\partial \sigma_i}\sum_{j=1}^{3}\left[\frac{\partial}{\partial \sigma_j}\,\dot{\sigma}_g^p\,\frac{dt_k}{2}\right]\Delta\sigma_{j(k)} + \dot{\epsilon}_g^p\,\frac{dt_k}{2}\sum_{j=1}^{3}\frac{\partial^2 \sigma_g}{\partial \sigma_i \partial \sigma_j}\,\Delta\sigma_{j(k)} \tag{VI.8}$$

The first summation on the right hand side of (VI.8) can be written as

$$\frac{\partial \sigma_g}{\partial \sigma_i}\sum_{j=1}^{3}\left[\frac{\partial}{\partial \sigma_j}\,\dot{\epsilon}_g^p\,\frac{dt_k}{2}\right]\Delta\sigma_{j(k)} = \frac{\partial \sigma_g}{\partial \sigma_i}\,\frac{\partial \dot{\epsilon}_g^p}{\partial \sigma_g}\,\frac{dt_k}{2}\sum_{j=1}^{3}\frac{\partial \sigma_g}{\partial \sigma_j}\,\Delta\sigma_{j(k)} \quad . \tag{VI.9}$$

The second summation depends on $\partial^2\sigma_g/\partial\sigma_i\partial\sigma_j$. To simplify its presentation, let

$$F = AB + BC + CA$$

while

$$\begin{aligned} Q_1 &= \sigma_{2(k)} - \sigma_{3(k)} \\ Q_2 &= \sigma_{3(k)} - \sigma_{1(k)} \\ Q_3 &= \sigma_{1(k)} - \sigma_{2(k)} \end{aligned} \tag{VI.10}$$

This notation transforms the second summation on the right hand side of (VI.8) to the form

$$\dot{\epsilon}_g^p \frac{dt_k}{2} \sum_{j=1}^{3} \frac{\partial^2 \sigma_g}{\partial \sigma_i \partial \sigma_j} \Delta\sigma_{j(k)} = \frac{F}{\sigma_g^3} \dot{\epsilon}_g^p \frac{dt_k}{2} \sum_{j=1}^{3} Q_i \, Q_j \, \Delta\sigma_{j(k)} \quad . \qquad \text{(VI.11)}$$

Returning now to (VI.7), the results of (VI.9) and (VI.11) provide the expression

$$\Delta\epsilon_{i(k)}^p = \frac{\partial \sigma_g}{\partial \sigma_i} \dot{\epsilon}_g^p \Big|_{\sigma=\sigma_{i(k)}} \Delta t_k + \sum_{j=1}^{3} P_{ij} \, \Delta\sigma_{j(k)} \, dt_k \qquad \text{(VI.12)}$$

in which the P_{ij} take the form

$$P_{ij} = \tfrac{1}{2} \left[\frac{\partial \dot{\epsilon}_g^p}{\partial \sigma_g} \frac{\partial \sigma_g}{\partial \sigma_i} \frac{\partial \sigma_g}{\partial \sigma_j} + \frac{F}{\sigma_g^3} \dot{\epsilon}_g^p \, Q_i \, Q_j \right] \quad . \qquad \text{(VI.13)}$$

These are the terms of the "creep compliance."

Accuracy of computations requires that the second term in (VI.12) be small compared to the first term. Thus, if the second term is to be smaller than a fraction V_m of the first term dt_k, and thus Δt_k, must satisfy

$$\Delta t_k \leq V_m \left| \frac{\frac{\partial \sigma_g}{\partial \sigma_i} \dot{\epsilon}_g^p}{\sum_{j=1}^{3} P_{ij} \, \dot{\sigma}_{j(k)}} \right| \quad . \qquad \text{(VI.14)}$$

The coefficient V_m is provided with the program input.

In the CYGRO-4 creep representation the creep resulting from "thermally activated" creep mechanisms is*

$$\dot{\epsilon}_g^t(\sigma_g,h) = Q_d h^{R_d} \sigma_g^{W_d} \qquad \text{(VI.15)}$$

with h determined by solution of the differential equation

$$\dot{h}(\sigma_g,h) = Q_d h^{R_d} \sigma_g^{W_d} - Q_r h^{R_r} \quad . \qquad \text{(VI.16)}$$

To estimate the effect of strain hardening on the creep rate $\dot{\epsilon}_g^t$, (VI.16) can be approximated by the linearized difference equation

$$\frac{\Delta h}{\Delta t} \approx Q_d h^{R_d} \sigma_g^{W_d} \left[1 + R_d \frac{\Delta h}{h}\right] - Q_r h^{R_r} \left[1 + R_r \frac{\Delta h}{h}\right] \quad .$$

This has the solution

$$\Delta h \approx \frac{(Q_d h^{R_d} \sigma_g^{W_d} - Q_r h^{R_r})\Delta t}{1 - \frac{\Delta h}{h}(R_d Q_d h^{R_d} \sigma_g^{W_d} - R_r Q_r h^{R_r})} \quad . \qquad \text{(VI.17)}$$

Application of the same procedure to $\Delta\epsilon_g^t/\Delta t$ gives

$$\frac{\Delta \epsilon_g^t}{\Delta t} = Q_d h^{R_d} \sigma_g^{W_d} \left(1 + R_d \frac{\Delta h}{h}\right)$$

or, upon use of (VI.17)

$$\frac{\Delta \epsilon_g^t}{\Delta t} = Q_d h^{R_d} \sigma_g^{W_d} \left[1 + \frac{R_d(Q_d h^{R_d} \sigma_g^{W_d} - Q_r h^{R_r})\Delta t}{h - \Delta t(R_d Q_d h^{R_d} \sigma_g^{W_d} - R_r Q_r h^{R_r})}\right] .$$

* This representation is the same as that used in CYGRO-3 (See Section II.L of Reference 4) but the notation has been changed to a less cumbersome form.

This provides the basis for a time step limit based on strain hardening for it can be expressed in the form

$$\frac{\Delta\varepsilon_g^t}{\Delta t}\left[Q_d\, h^{R_d}\, \sigma_g^{W_d - 1}\right] = 1 + \delta \quad .$$

Restriction of Δt_k to values for which $|\delta| \leq \delta^*$ is equivalent to restricting Δt_k to

$$\Delta t_k \leq \frac{h\delta^*}{R_d\left|\left(Q_d\, h^{R_d}\, \sigma_g^{W_d} - Q_r\, h^{R_r}\right)\right| + \delta^*\left(R_d\, Q_d\, h^{R_d}\, \sigma_g^{W_d} - R_r\, Q_r\, h^{R_r}\right)} \quad . \tag{VI.18}$$

The coefficient δ^* is provided with the program input.

The terms of the inelastic compliance matrix are of the form $E_{ij} + P^*_{ij}\, dt_k$, where the E_{ij} are the terms of the elastic compliance matrix and the P^*_{ij} account for the effects of creep and densification. Thus, for a different time increment, ρdt_k, the inelastic compliance matrix is $E_{ij} + \rho\, P^*_{ij}\, dt_k$. In the course of computations the estimated time step, dt_k, may be found to be too long or too short. If the acceptable time step, $\rho\, dt_k$, is sufficiently close to the estimated time step, the inelastic compliance matrices for the actual - and the estimated - time steps will be essentially the same. When this is not the case, the size of the step must be limited or the computations for the step must be repeated. To establish criteria for determining when the time

step must be limited or the computation must be repeated, consider the effect of the size of the time step on the largest diagonal component of the inelastic compliance matrix. If the change in the size of the time step results in a sufficiently small change in this component, no recomputation is necessary. To be precise, let the acceptable change in the component be R times the value computed for time step dt_k, where R is a user specified ratio. Thus, the time step $\rho\ dt_k$ can use the inelastic flexibility matrix computed for the time step dt_k if ρ satisfies the inequalities

$$(1 - R)(E_{ii} + P^*_{ii}\ dt_k) \leq E_{ii} + \rho\ P^*_{ii}\ dt_k \leq (1 + R)(E_{ii} + P^*_{ii}\ dt_k)$$

for all the diagonal entries of the inelastic compliance matrix. These inequalities can be rearranged to

$$1 - R\,\frac{E_{ii} + P^*_{ii}\ dt_k}{P^*_{ii}\ dt_k} \leq \rho \leq 1 + R\,\frac{E_{ii} + P^*_{ii}\ dt_k}{P^*_{ii}\ dt_k} \qquad \text{(VI.19)}$$

whenever $P^*_{ii}\ dt_k > 0$. For $P^*_{ii} = 0$ the original inequalities are satisfied for all values of ρ.

The inequalities (VI.19) mean that the change in the compliance matrix limits the time step to

$$\Delta t_k \leq \rho_{max}\ dt_k = \left[1 + R\,\frac{E_{ii} + P^*_{ii}\ dt_k}{P^*_{ii}\ dt_k}\right] dt_k, \qquad \text{(VI.20)}$$

while the computations for the time step are of suitable accuracy for

$$\Delta t_k \geq \rho_{min}\, dt_k = \left[1 - R\,\frac{E_{ii} + P^*_{ii}\, dt_k}{P^*_{ii}\, dt_k}\right] dt_k . \qquad \text{(VI.21)}$$

At temperatures encountered in nuclear power reactors, deformation rates in crystalline materials are governed by the process of "self diffusion" in which an atom of the material migrates through the crystal lattice. The temperature dependence of self diffusion is essentially

$$D = D_o \exp(-Q/RT) \quad , \qquad \text{(VI.22)}$$

where T is the temperature (in degrees Kelvin), R = 1.984 cal/°K mole is the universal gas constant, Q is the "activation energy" for the material, and D_o is the rate factor.

To estimate the effect of changes of temperature, consider the derivative

$$\frac{dD}{dT} = \frac{D_o Q}{RT^2} \exp\left(\frac{-Q}{RT}\right) .$$

By use of this expression, one can see that a change ΔT in temperature results in a corresponding change in D approximated by

$$\Delta D = \frac{D_o\, Q\, \Delta T}{RT^2} \exp\left(\frac{-Q}{RT}\right) . \qquad \text{(VI.23)}$$

When this is expressed as a fraction of the rate $D(T_R)$ at a reference temperature $T = T_R$

$$\frac{\Delta D}{D(T_R)} = \rho = \frac{Q\Delta T}{RT^2} \exp\left(\frac{-Q[T_R - T]}{RTT_R}\right) .$$

Thus, to produce a given ratio $\rho = \Delta D/D(T_R)$, the increment of temperature is approximately

$$\Delta T^*(T) = \frac{\rho RT^2}{Q} \exp\left(\frac{Q[T_R - T]}{RTT_R}\right) . \qquad \text{(VI.24)}$$

This expression indicates that, to a first approximation the change in the diffusion rate is less than $\rho D(T_R)$ for $\Delta T < \Delta T^*(T)$ given by (VI.24). This provides a convenient limit on the size of a temperature increment in a numerical analysis once the ratio ρ and the reference temperature T_R are defined.

For uranium dioxide (UO_2) Q is given in Reference (17) as

$$Q_{UO_2} = 95{,}000 \text{ cal/mole}$$

while for α-zirconium (α-Zr), the largest - and thus most restrictive - estimate of Q given in Reference (18) is

$$Q_{\alpha\text{-}Zr} = 52{,}000 \text{ cal/mole}$$

Due to the form of (VI.24) the values of D_o are immaterial in the present context.

Assignment of a value for the reference temperature, T_R, is a matter of judgment. A reasonable value can be based on the observation that creep in Zircaloy becomes rapid at temperatures above one half the α to β phase transition temperature. This transition temperature is given in Reference (19) as 863 ± 3°C. Even this kind of reasoning does not suggest a reference temperature for uranium dioxide. Accordingly, it would appear productive to use a temperature at, or above, the highest temperatures for which in-pile data exist - about 2500°C (4532°F).

For temperatures above $T = T_R$ the limit imposed by (VI.24) is very restrictive. For this reason, at such temperatures it is more practical to use a limit derived from (VI.22) and (VI.23). Thus, consider the ratio $\Delta D/D(T)$ given by

$$\frac{\Delta D}{D(T)} = \frac{\Delta T Q}{RT^2} \quad .$$

To produce a given ratio $\rho = \Delta D/D(T)$ the increment of temperature is

$$\Delta T^{**}(T) = \frac{\rho R T^2}{Q} \quad . \qquad \text{(VI.25)}$$

A limit on the size of the time step can now be derived from (VI.24) and (VI.25). When the temperature is T_K and the rate of change

of temperature is $\dot{T}_k$, the effect of temperature change on deformation rates will be suitably bounded for

$$\Delta t_k \leq \Delta t_{thermal} = \begin{cases} \rho R T_k^2 / Q|\dot{T}_k|; & T \geq T_R \\ \dfrac{\rho R T_k^2}{Q|\dot{T}_k|} \exp\left(\dfrac{Q[T_R - T_k]}{R T_R T_k}\right); & T < T_R \end{cases} \quad \text{(VI.27)}$$

As discussed in this section, considerations of accuracy and the limits of the physical processes involved in the deformation of the fuel element provide limits on the size of the time step taken by CYGRO-4. These illustrate the principle that there are time step limits associated with each physical process and each mathematical model. While the assignment of numerical values to these limits requires judgment, the derivation of the mathematical form of the limits is relatively straightforward.

In summary, the actual CYGRO-4 time step is limited by the end of the History Interval, by the limits derived as part of the fuel-clad and rod-support interaction model, by fuel cracking and crack closing, by the effect of changes of stress or of strain hardening, by restrictions on the tolerable inaccuracy in the creep compliance, and by the effects of temperature change. The explicit limits are given by definitions (IV.9) through (IV.16), (VI.1), (VI.2), (VI.14), (VI.18), (VI.20), and (VI.27). Computations for the time step must be repeated when the limit expressed in (VI.21) is not met.

VII. Input

The CYGRO-4 program accepts input data in a form derived from the form used in CYGRO-3 (Reference 4). In fact, only small changes are required to transform a CYGRO-3 input deck to CYGRO-4 form.

The input data include data needed to control the program or describe the properties and geometry of the fuel element, and data describing the reactor operating state. This section has two subsections: the first devoted to the input of the control, property and geometry data, and the second devoted to the input of the reactor operating data.

The input data are supplied in "Bettis Input Package Format" (see Reference 20) in which the "logical cards" can consist of one or more physical cards or card images. The first physical card of a "logical card" has a card number followed by a comma. The data fields on the card are separated by commas, and "logical cards" can be extended onto several physical cards by making the first character on the continuation cards be the symbol "+". The data can be Integer (denoted here by I) - in which the allowable characters are "+", "-", or the ten numerical characters 0 through 9 - Alphanumeric (denoted here by A) - in which the allowable characters are any in the FORTRAN character set so long as the string begins with a letter or the characters are enclosed in parentheses - or Floating Point (denoted here by F) - in which the allowable characters are "+" or "-" followed by numerals with-or without a decimal point then by an exponential part comprising "+" or "-"

and the numerals of the exponent of 10. In Floating Point format the decimal is assumed to precede the leading set of numerals whenever the decimal is not given explicitly.

The "logical cards" can appear in any order and two or more physical cards can have the same "logical card" number. In that case only the last appearance of the logical card will be processed. Supplying a physical card with a card number followed by blanks is thus equivalent to removing the preceding "logical card" of the same number.

When it is desired to supply a single entry on a "logical card" this can be done by preceding the desired entry with suitable "filler data" - usually Floating Point zeroes (0.0) or Integer zeroes (0), depending on the required format on the "logical card." Alphanumeric "filler data" can be provided by entering a left and right parenthesis.

The following subsection has a format intended to simplify the preparation of input. The significant card numbers appear in numerical order in the left hand column. The entries on the card are then indicated by data type and name in the following columns and a brief explanation of the significance of the variable is included at the right hand column.

Since the CYGRO-4 input format represents an evolution from the CYGRO-3 format a number of CYGRO-3 input data have been suppressed or deleted. The CYGRO-4 input routine ignores non-zero entries in these

locations so that CYGRO-4 will execute a CYGRO-3 input deck with minor revision. When preparing a new CYGRO-4 deck it is best to supply zero values in these locations. Thus, for example card 904 might take the form

904,0.0,0.0,0.0,1.0 .

Note that values for the first three entries must be given even though the program ignores their values.

A. Data for Program Control, Fuel Rod Geometry and Material Properties

Card	Entry	Type	Name	Significance
2 (Optional)	1 to 8	I	KPRINT(i); i=1,...,8	Controls for output. Edits are suppressed if corresponding entry is zero and are provided if entry is 3. The entries control the following sets of data. 1. Input summary 2. Creep properties of material 3. Reactor state 4. Rod cross-section data 5. Temperature and densification data 6. Stress and strain data 7. Porosity data-"debug" use only 8. Short summary-to replace output groups 4 through 7
30	1	I	ISTART	Optional except when a new Input Package Case after the first is to be the start of a new analysis. To start new analysis, set value to 1.

Card	Entry	Type	Name	Significance
900	1 to 3	F	F(i);i=1,2,3	Three values of integrated fast flux (fluence) used in tabulation of parameters on cards 26JK. F(1) < F(2) < F(3)
	4 to 6	F	FDOT(i); i=1,2,3	Three fission rates used in tabulation of parameters on cards 16JK. FDOT(1) < FDOT(2) < FDOT(3)
	7	F	SURFTM	Supplants input values of coolant temperature (cards $792+10 \cdot I_{cycle}$) whenever input value is above 72°F. Useful for special design evaluation.
	8	F	SURFILM	Supplants value HWATER on 904 when non-zero value is supplied. Provides for expedient change of input for special analyses.
904	4	F	HWATER	Heat transfer coefficient at rod-to-coolant interface.
905	1	F	DENSRATIO	Corresponds to value D_d in Equation V.5. Controls size of time step based on densification rate.
	8	F	TCENTER-MAX	Maximum centerline temperature allowed. Program stops automatically and prints MAXIMUM TEMPERATURE if center temperature exceeds TCENTER-MAX.
906	5	F	FPLEN	Initial force in plenum spring.
	6	F	KPLEN	Stiffness of plenum spring multiplied by fuel stack height.
	7	F	ALFAEND	"Hourglassing" coefficient. See α_e in Equations (IV.21) and (IV.22).

Card	Entry	Type	Name	Significance
906 (Cont'd)	8		ENDMU	Hourglassing coefficient of friction. See μ_e in Equations (IV.17) and (IV.18).
907	1	F	SIZERATIO	Tolerable change in strain in a single time step. Serves in limit on time step.
	7	F	MU	Coefficient of friction between fuel and cladding when fuel-clad gap is zero. See μ_T in Equations (IV.17) and (IV.18).
	8	F	PCOLL	Pressure at which clad collapses. See $P_{collapse}$ in Equation (IV.19). Not needed if cards 3399+I are supplied.
908	1	F	FRCOL	Friction factor μ_c in Equation (IV.17) relating the fuel-clad axial slip force to the excess of pressure above $P_{collapse}$.
	2	F	DTMIN	Minimum time step - the size a "trivial" time step - to be taken when conditions prevent a sizable time step.
	3	F	SIGMA	Fuel cracking stress-see σ^* in Section III.
	4	F	SUPER	Excess compliance associated with a crack. See S in Equation (III.2) et cetera.
	7	F	ECCEN	Pellet eccentricity or lodged chip dimension. See ε_e in Equation (IV.22).
	8	F	EMISS	Fuel emissivity-replaces EMISSF on card 4002 if EMISSF not supplied. See Equation (II.26).

Card	Entry	Type	Name	Significance
909	1	F	RFACTT	Tensile anisotropy parameter R. See Equation (II.1) of Reference (4).
	2	F	PFACTT	Tensile anisotropy parameter P.
	3	F	RFACTC	Compressive anisotropy parameter R.
	4	F	PFACTC	Compressive anisotropy parameter P.
	5	F	XNPRP	Control variable-set to 1.0 for alternative input of material property parameters on cards 164J and 264J instead of cards 16JK and 26JK.
	6	F	KEEPGOING	Control variable for "debug" use.
	7	F	COMPRATIO	Corresponds to R in Equations (VI.19) and (VI.20). Limits change in effective compliance due to change in estimated time step.
	8	F	MINHARDNS	Minimum value of h in "thermal creep" representation (A.I.5).
910	1	F	FMSUP	Value of slipping force between rod and support. See M_3^u and M_2^l in (IV.0). Not needed if cards 3299+I are supplied.
	2	F	FSUP	Compressive force acting on rod. Not needed if cards $595+10 \cdot I_{cycle}$ are supplied.
	3	F	KSUP	Product of support stiffness times rod length.
	6	F	ASUP	Ratio of support thermal expansion to clad thermal expansion.

Card	Entry	Type	Name	Significance
910 (Cont'd)	7	F	TAXREF	Reference temperature in "excess temperature" representation of fuel-clad axial interaction. Corresponds to T_A in Equation (IV.20).
	8	F	MUAX	Force coefficient in "excess temperature" representation of fuel-clad axial interaction. Corresponds to μ_T in Equation (IV.18).
911	1	F	GAPOVR	Tolerance for overclosing gap. Corresponds to d in (IV.10).
	2	F	SLIPOVR	Tolerance for reverse slipping. Corresponds to d in (IV.14).
	3	F	FGAPOVR	Tolerance for tensile fuel-clad radial interaction force.
	4	F	FZOVR	Tolerance for axial interaction force above slip limit. Corresponds to f in (IV.15).
	5	F	FZGMIN	Minimum value of fuel-clad axial slip force.
	6	F	FZSMIN	Minimum value of rod-support slip force.
	7	F	GAPOSC	Value of change in gap size corresponding to "jump" term (δV_1) above which the change in gap size is "softened" to reduce anomalous temperature oscillations.
	8	F	GAPLIM	Value of change point in representation of Q in Equation (II.27). Nominally 0.48×10^{-3} inch.

Card	Entry	Type	Name	Significance
912	1	F	DTEMPMAX	Maximum change of temperature in a time step. Establishes limit on Δt_k supplementing (VI.27).
	2	F	RATEFRAC	Coefficient in Equation (VI.27). Establishes tolerable fraction of nominal rate in controlling temperature changes.
	3	F	TREF-F	Reference temperature for fuel. See (IV.27).
	4	F	Q-F	"Activation energy" for fuel. See (VI.27).
	5	F	TREF-C	Reference temperature for cladding. See (VI.27).
	6	F	Q-C	"Activation energy" for cladding. (See VI.27).
	7	F	SCJUMP	Used in limit on time step based on "jump" in creep rate.
	8	F	CRJUMP	Used in limit on time step based on "jump" in stress.
913	1	F	CRAKOVR	Tolerance on cracking stress used in cracking time step limit. Expressed as fraction of σ^*. See (VI.1).
	2	F	CLOSOVR	Tolerance on crack closing used in crack closing time step limit. See (VI.2).
	3	F	EDOT-ROBOT	Time step limit for "debug" use.

Card	Entry	Type	Name	Significance
913 (Cont'd)	4	F	EDOT-CYGRO	Corresponds to δ^* in (VI.18). Used in time step limit based on change in strain hardening.
	5	F	NOM-CREEP	Nominal value of creep rate serving to soften time step limit (VI.14).
	6	F	NOM-STRESS	Nominal value of equivalent stress serving to soften time step limit based on proportional change in equivalent stress.
	7	F	JUMP STRN	Fraction of SIZERATIO (Entry 1 on Card 907) which can result from creep strain associated with "jump" in stress. Limits size of time step.
	8	F	TAU-ROBOT	Time step limit for "debug" use.
914	1	F	ROD SPAN	Component of "debug" time step limit.
	2	F	BOW ERROR	Component of "debug" time step limit.
I000				I=1 for fuel and I=2 for cladding.
	1	F	NR(I)	Number of radii bounding finite elements in region I. Note NR(1)+NR(2) $\leq$ 32 and NR(I) $\geq$ 2.
	2	F	NG	Number of gas pore categories (fuel only). Value $0 \leq NG \leq 5$. If NG = 0 the 120K cards are not read.

Card	Entry	Type	Name	Significance
I000 (Cont'd)	6	F	QGMAX(I)	Heat generation at full power expressed in terms of original volume.
	8	F	S.TEN.	Surface tension of gas pores (fuel only).
1001	1	F	R3C	Corresponds to R_{oo} in Equations (V.3) and (V.4). Provides scaling of estimated "viscosity."
	2	F	RSP	Corresponds to S_y in Equation (V.2). Accounts for effect of material "yield stress."
I100 through I099+NR	1	F	R(K_r)	NR cards with K_r = 1 to NR(I). Radius of surfaces bounding the finite elements in region I.
1200 through 1199+NG	4	F	VH(K_g)	NG cards with K_g = 1 to NG. Total volume of pores in pore category K_g.
	5	F	V(K_g)	Total volume of region in pore category K_g.
	8	F	NHT(K_g)	Number of pores per unit volume in pore category K_g.
I300	1 to 3	F	RALPHA(R,C,Z)	Radial, circumferential and axial factors for thermal expansion.
	4 to 6	F	RV(R,C,Z)	Radial, circumferential and axial factors of swelling (for region 1) or growth (for region 2).

Card	Entry	Type	Name	Significance
1400	1 to 3	F	P(i);i=1,2,3	Three pressures used in tabulation of RVP(I, J) on cards 16J0. P(1) < P(2) < P(3).
I610 I620 I630 I640	1	F	TEMPT(J)	Data for tabulation of temperature dependence in Region I. J=1,2,3,4. Tabulation temperature.
	2	F	ALPHA(J)	Thermal expansion coefficient at TEMPT(J).
	3	F	ET(J)	Modulus of elasticity at TEMPT(J).
	4	F	NUT(J)	Poisson's ratio at TEMPT(J).
	5	F	CT(J)	Thermal conductivity at TEMPT(J)- for clad region only (I=2).
	6,7,8	F	RVP(i,J); i=1,2,3	Pressure dependence of fuel swelling at temperature TEMPT(J).
I61K, I62K, I63K, I64K				K=1 to 3 for tabulation of fuel properties with FDOT from card 900 for I=1 and for tabulation of cladding properties with F from card 900 for I=2. Not used if XNTRT on card 909 is set to 1.0.
	2	F	CFT(K,J)	Exponent in creep coefficient for thermal creep. See Equations (A.I.1) and (A.I.3).
	3	F	SFT(K,J)	Exponent of stress in "steady state" creep. See Equation (A.I.3).
	4	F	PFT(K,J)	Exponent in coefficient of strain hardening parameter for thermal creep. See Equation (A.I.1).

Card	Entry	Type	Name	Significance
I6JK (Cont'd)	5	F	QFT(K,J)	Component in exponent of strain hardening parameter for thermal creep. See Equation (A.I.1).
	6	F	WCS(K,J)	Exponent of stress in thermal creep. See Equation (A.I.1).
I641	1 to 12	F	CFT(K,J)	Used only if XNPRP on Card 909 is set to 1.0. Twelve values of C(K,J)-Exponent in creep coefficient for thermal creep. See Equations (A.I.1) and (A.I.3)- The values are given so that the first three values correspond to the value of TEMPT(1) given on card I610, the second three correspond to TEMPT(2) from card I620, et cetera.
I642	1 to 12	F	GFT(K,J)	Used only if XNPRP on card 909 is set to 1.0. Exponent of stress in "steady state" creep. See Equation (A.I.3). See card I641 for format of card.
I643	1 to 12	F	PFT(K,J)	Used only if XNPRP on card 909 is set to 1.0. Exponent in coefficient of strain hardening paremeter for thermal creep. See Equation (A.I.1). See card I641 for format of card.
I644	1 to 12	F	QFT(K,J)	Used only if XNPRP on card 909 is set to 1.0. Component in exponent of strain hardening parameter for thermal creep. See Equation (A.I.1). See card I641 for format of card.

Card	Entry	Type	Name	Significance
I645	1 to 12	F	WCS(K,J)	Used only if XNPRP on card 909 is set to 1.0. Exponent of stress in thermal creep. See Equation (A.I.1). See card I641 for format of card.
I700	1	F	CLIN	Coefficient in fission rate-(for fuel) or flux-(for clad) dependent creep rate expression. See Equation IIH(4) of Reference 4.
	2	F	CPOW	Coefficient in non-linear flux dependent creep expression for clad. See Equation IIH(3) in Reference 4.
	3	F	XPOW	Exponent in nonlinear flux dependent creep expression for clad.
	4	F	EINT	Value of ϵ_{int} used in intrinsic strain rate expression for clad. See Equation IIH(1) of Reference 4.
	6	F	ATRAN	Coefficient in transient flux dependent creep of cladding.
	7	F	BTRAN	Fluence (integrated flux) coefficient in transient flux dependent creep of cladding.
	8	F	FREVSM	Scale factor for fuel swelling.
2800 through 2824	1,3,5,7	F	TIME	Values of time in tabulation of fast flux at full power. Up to 100 values of time can be given. 4 values per logical card.

Card	Entry	Type	Name	Significance
28IJ (Cont'd)	2,4,6,8	F	FLUX	Nominal value of fast flux at full power corresponding to preceding time. The flux at a given time is obtained by linear interpolation of the tabulated values. Note that this differs from CYGRO-3.
3001	Up to NR_f	F	RADIUS	Radii at which relative power distribution is to be specified in fuel. Distribution is uniform if this card is not supplied.
3003	Up to NR_c	F	RADIUS	Radii at which relative power distribution is to be specified in cladding. Distribution is uniform if card is not supplied.
3011 to 3020	1	F	TIME	Reactor time at which following power distribution applies. Up to 10 values of time may be given. Distribution unchanged after last time.
	2,3,...	F	P(r)	Relative power level at each value of radius given on cards 3001 and 3003. The p_k in Equation (II.1) are the values obtained by linear interpolation of P(r) to the radii given on cards I100 through I099+N_r. Note that P(r) is relative power. The program normalizes the values so that the average power level is as specified by the average power input.

Card	Entry	Type	Name	Significance
3300 through 3324	1,3,5,7	F	$TSUPMX_i$	Values of time in tabulation of support slip force.
	2,4,6,8	F	$FSUPMX_i$	Values of support slip force corresponding to preceding time. Supplant entry 1 on card 910 if given.
3400 through 3424	1,3,5,7	F	$TCOLAP_i$	Values of time in tabulation of clad collapse pressure.
	2,4,6,8	F	$COLAPP_i$	Values of clad collapse pressure corresponding to preceding time. Supplant entry 8 on card 907 if given.
4000	1	F	ENG FACT	Design factor multiplying conductivity of fuel given by Equation (II.20).
	2	F	PORE. CORR	Porosity correction coefficient β in Equation (II.20).
	3	F	WEIGHT PC	Weight fraction of UO_2. Used in representation of thermal conductivity based on input from card 4003.
	4	F	DENSITY	Ratio of observed fuel density to theoretical density.
4001	1	F	A_o	The term A in Equation (II.20) has the form $A = A_o+A_1U+A_2U^2$ where U is the weight per cent of UO_2 from card 4000.
	2	F	A_1	
	3	F	A_2	
	4	F	B_o	The term B in Equation (II.20) has the form $B=B_o+B_1U+B_2U_2$.

Card	Entry	Type	Name	Significance
4001 (Cont'd)	5	F	B_1	
	6	F	B_2	
	7	F	C	The term C in Equation (II.20).
	8	F	D	The term D in Equation (II.20).
4002	1	F	R1	Surface roughness of fuel. Corresponds to R_f in Equation (II.24).
	2	F	R2	Surface roughness of cladding. Corresponds to R_c in Equation (II.24).
	3	F	HARD	Meyer hardness of harder surface. Corresponds to H in Equation (II.24).
	4	F	EMISSF	Emissivity of fuel. Corresponds to $1/\epsilon_f$ in Equation (II.26).
	5	F	EMISSC	Emissivity of cladding. Corresponds to $1/\epsilon_c$ in Equation (II.26).
	6	F	PCONIN	Correlation constant allowing for pressure acting at points of solid contact even when fuel and clad are not in contact.
	7	F	CCRACK	Coefficient of crack effect on heat conduction in fuel. Corresponds to α in (III.12).
	8	F	GASCON	Design coefficient multiplying value of K_{mix} given by (II.29).

Card	Entry	Type	Name	Significance
4010	1	F	GFG	Jump distance for fission gas at reference temperature and pressure. Corresponds to g_{fg} in Equation (II.30).
	2	F	GI	Jump distance for initial fill gas at reference temperature and pressure. Corresponds to g_i in Equation (II.30).
	3	F	R	Ratio of initial gap volume to the total volume of initial gap plus initial plenum size.
	4	F	NATM	Initial pressure of fill gas.
	5	F	ALPHAFG	Atoms of fission gas produced per fission.
	6	F	RATIO	Ratio of rod average fissions/cc to local fissions/cc. If this is given as 0.0 the values are supplied on the 4020 card series. This value is used in calculating the gas released into the plenum and fuel-clad gap.
	7	F	ALPHAHE	Number of helium atoms produced per fission. This is used to calculate ternary fission helium release.
	8	F	DELTAHE	Fraction of helium released. This is used in the calculation of ternary fission helium release.
4020 through 4029	1 to 8	F	$f_{average}$ f_{local}	Tabulation of average depletions (odd numbered entries) to local depletion (even numbered entries). Up to 40 pairs can be given. This is used in the calculation of gas release.

Card	Entry	Type	Name	Significance
4030 through 4039	1 to 8	F	EFPH $f_{average}$	Tabulation of effective full power hours (EFPH) as odd numbered values with average depletion as even numbered values. Up to 40 pairs can be given. This is used in the calculation of gas release.
4040 through 4049	1 to 8	F F	RELEASE EFPH	Tabulation of fraction of fission gas release to effective full power hours (EFPH). The release fractions are given as the odd numbered entries and the effective full power hours are given as even numbered entries. Up to 40 pairs can be given. This is used in the calculation of gas release.
5001	1	F	FISS/POW	Fission rate per unit heat generation rate. The fission rate is the product of nominal maximum power level, QMAX, from card 1000 and the relative power from cards $593+10 \cdot I_{cycle}$.
	4	F	T-OR-DES	Design factor multiplying the amount of swelling given in swelling tables. Does not affect the solid fission product component of swelling.
	5	F	VVZGR	Average fast flux for use in "steady state" cladding growth correlation.
	6	F	TZGRI	Nominal temperature for use in cladding growth correlation.

Card	Entry	Type	Name	Significance
5001 (Cont'd)	7	F	CZGR	Coefficient of fluence in cladding growth correlation.
	8	F	FLZGR	Nominal maximum fluence used in cladding growth correlation.
5002	1	F	FRATIO	Ratio of fast flux at full power to nominal values given on cards 2800 through 2824.
	2	F	PRATIO	Ratio of heat generation and fission rate at full power to the nominal values obtained from QGMAX on the 1000 card and FISS/POW on the 5001 card. This ratio also affects the input from card 5003 when that card appears.
5003	1	F	Q_{total}	Nominal surface heat flux at full power in Btu/hr ft^2. Supplants the average heat generation QGMAX on card 1000 if a positive value is given. Scaled by PRATIO on card 5002.
	2	F	$Q_{fission}$	Nominal surface heat flux at full power from heat generated by fissions. Used to compute fission rate at full power if a positive value is given, supplanting the results obtained by use of FISS/POW on card 5001. Scaled by PRATIO on card 5002.
	3	F	$E_{fission}$	Energy per fission event. Used in computing fission rate at full power. Program uses default value of 175 Mev/ fission unless a non-zero value of $E_{fission}$ is given.

Card	Entry	Type	Name	Significance
6001 through 6005				These cards correspond to the two primary fission gases (6001 and 6002), helium (6003) and the two primary volatiles outgassed by the fuel.
	1	F	A(I)	Coefficient of gas conductivity. Corresponds to a_i in Equation (II.28)
	2	F	N(I)	Exponent of temperature in gas conductivity. Corresponds to n_i in Equation (II.28).
	3	F	M(I)	Molecular weight of gas. Corresponds to M_i in Equation (II.29).
	4 (6004 and 6005 (cards only)	F	VV(I)	Volume of volatile gases released per unit volume of fuel. Appears only on cards 6004 and 6005. Used to calculate the number of atoms of volatiles.
6020	1	F	T(DEFECT)	Time at which clad perforation becomes effective. For times greater than $t_{perforation}$ the gap conductivity depends on the steam conductivity representation of Section II rather than the gas conductivity representation.
	2	F	STEAM JUMP	Temperature "jump distance" for steam filled gap. Corresponds to g_r in Equation (II.37).
	3	F	C(OXIDE)	Thermal conductivity of oxide film on surfaces of cladding. Used in calculation of ρ_{oxide} in Equation (II.23).

Card	Entry	Type	Name	Significance
603I	1,3,5,7	F	$TOXIDE_i$	Values of time in tabulation of oxide thickness at surfaces of cladding (I=1 for inner surface, I=2 for outer surface.)
	2,4,6,8	F	$OXIDETHICK_i$	Values of oxide thickness corresponding to preceding time. Used in calculation of ρ_{oxide} in Equation (II.23).
6040	1,3,5,7	F	$TCRACK_i$	Values of temperature in tabulation of relative cracking strength of fuel.
	2,4,6,8	F	$SCRACK_i$	Values of relative cracking strength of fuel (compared to 72°F) corresponding to preceding temperature. Used in definition of Equation (III.5).
6045	1	F	BETA	Spacing of pores at grain boundaries contributing to cracking. Corresponds to $\beta\ell$ in Equation (III.7).
	2	F	ASTAR	Rate coefficient in rate of closure of crack-like pores. Corresponds to a^* in Equation (III.8).
	3	F	QSUBA	"Activation energy" in rate of closure of crack-like pores. Corresponds to Q_a in Equation (III.8).
	4	F	QSUBB	"Activation energy" in rate of closure of crack-like pores. Corresponds to Q_b in Equation (III.8).
99998	1 and 2	A	I.D.	Identification parameter for Restart File allowing restart of the problem at a later date.
	3	A	SYNONYM	Synonym parameter for Restart File.
	4	I	VERSION	Version number parameter for Restart File.

Card	Entry	Type	Name	Significance
99999	1	I	NBEGIN	Case number at which to begin restart from a Restart File written in a previous job. Restart begins at beginning of case.
	2	I	NFIRSTCASE	Case number of first case stored on Restart File being restarted. The Restart File of a normal job starts at case number 2. A Restart File written on a restart may start with a later case number.
	3	I	VERSION	Version number of Restart File being restarted.

B. Reactor Operating Data

The reactor operating history can be thought of as being divided into a series of repeatable cycles. The input in an Input Package case then includes a number of reactor operating history cycles supplied on the "History Card" cards. These cards take the form $590 + 10 \cdot I_{cycle} + J_{data}$, where I_{cycle} runs from 1 to a maximum of 30 and J_{data} takes values from 0 through 9.

Cards $590 + 10 \cdot I_{cycle}$ contain two Integer entries: N_{cycle} and L_{cycle}. The cycle is repeated a number of times equal to the absolute value of N_{cycle}. A negative value of L_{cycle} indicates that this is the last cycle. The cycle consists of L_{cycle} data. L_{cycle} must be less than or equal to 30.

Cards $590 + 10 \cdot I_{cycle} + J_{data}$ $(J_{data} > 0)$ contain L_{cycle} Floating Point entries representing the reactor state at the reactor time. The contents of the cards are as indicated in Table VII.1. When the logical card $590 + 10 \cdot I_{cycle} + J_{data}$ contains fewer than L_{cycle} entries the

Table VII.1

Data on $590 + 10 \cdot I_{cycle} + J_{data}$ Cards

Card	Data
$591 + 10 \cdot I_{cycle}$	Coolant pressure
$592 + 10 \cdot I_{cycle}$	Coolant temperature
$593 + 10 \cdot I_{cycle}$	Relative power level
$594 + 10 \cdot I_{cycle}$	Time increment from last datum
$595 + 10 \cdot I_{cycle}$	Axial force applied at support
$596 + 10 \cdot I_{cycle}$	Support axial displacement
$597 + 10 \cdot I_{cycle}$	Not used in current version
$598 + 10 \cdot I_{cycle}$	Not used in current version
$599 + 10 \cdot I_{cycle}$	Pressure in fuel-clad gap for use in structural calculations

program assumes that the remainder of the entries are identical to the last entry. Thus, supplying a card of the form 605, 10.0 results in an applied axial force of 10 pounds throughout the history cycle and all repetitions of the cycle.

C. Output Provided to Facilitate Verification of Input

The program prints the data used in the analysis as determined from the input. The labels provided with this output are self-explanatory when they represent a recapitulation of the input data, but certain derived material property data presented at the end of the input edits merit discussion here.

The data contained on the I6JK card series is used to define the exact "thermally activated" creep representations for fuel and clad. The quantities labeled DED/DT correspond to $\dot{\epsilon}_d$ in Equation (A.I.5) and the quantities labeled DER/DT correspond to the second term in the definition of $\dot{h}$ in (A.I.6). These relationships are then used to calculate $\dot{\epsilon}_d$ at σ_g = 1000 psi for various values of strain hardening, h, and to calculate the "steady state" value of $\dot{\epsilon}_d$ (the value of $\dot{\epsilon}_d$ when $\dot{h} = 0$) at various values of σ_g.

The exact functional form of σ_g for the cladding based on Equation (VI.4) and the input data on card 909 is identified as clad "generalized stress" and labeled SG.

Finally, the thermal expansion defined by the input quantities ALPHA(J) on cards I6J0 is given for various significant values of temperatures generally encountered in CYGRO-4 analyses.

VIII. Results of Computations

The CYGRO-4 program provides output at each of the History Card Times - the times defined by the time increments given on cards $594 + 10 \cdot I_{cycle}$. The output summarizes the data at the end of the history step.

The first line of output includes the number of the history set in the Input Package case, the number of the cycle in the set, and the number of the step in the cycle.

This is followed by a recapitulation of the history input and the reactor data at the beginning and end of the history step. These comprise the quantities identified in Table VIII.1. This output can be suppressed by supplying a zero value for entry 3 on Card 2.

The next set of data provides information on the progress of the problem and gives the values of variables which have a single value at each rod cross section. These are enumerated in Table VIII.2. Values are given for the beginning and end of the history step. This output can be suppressed by supplying a zero value for entry 4 on Card 2.

The next set of data displays the results of computations for each of the finite element rings. This set is divided into fuel and clad regions (KM=1 and KM=2). Each finite element boundary is then indicated by its radius number - KR. The significance of the variables

Table VIII.1 Reactor Data

Variable	Units	Significance
DTIME	Hours	Length of history step.
QG/QGMAX	-	Relative power level. Actual power level is obtained as the product of the nominal power and the relative power.
T(WATER)	°F	Water temperature.
P(WATER)	Psi	Water pressure.
FSUP.MAX	Pounds	Magnitude of support force for which slipping will occur.
FIS.RATE	Fissions/cc•sec	Volume averaged fission rate in fuel. Stated in terms of original fuel volume.
DEPLTN	Fissions/cc	Volume averaged depletion of fuel. Stated in terms of original fuel volume.
F.FLUX	Neutrons/cm^2•sec	Fast flux.
FLUENC	Neutrons/cm^2	Time integrated fast flux.
P(GAP)	Psi	Pressure in the fuel-clad gap.
DP COLAPS	Psi	Current value of clad collapse pressure. See $P_{collapse}$ in Equation (IV.19).

Table VIII.2 Problem and Cross Section Data

Variable	Units	Significance
TIME	Hours	History time summed from the history time increments.
KTIME	-	The number of program controlled time steps.
DTROB	Hours	The most restrictive time step for which special purpose output can be provided with interpolation. This is controlled by the input variables EDOT-ROBOT and TAU-ROBOT on Card 913 and ROD SPAN and BOW ERROR on Card 914.
DD(M)	10^{-3} inch	Change in fuel diameter from initial size.
F/C GAP	10^{-3} inch	Fuel-clad radial gap.
DD(C)	10^{-3} inch	Change in clad diameter from initial size.
FCONR	Pounds/inch	Fuel-clad radial interaction force.
FCONZ	Pounds	Fuel-clad axial interaction force.
FSUP	Pounds	Rod-support axial interaction force.
EZ(M)	10^{-3} inch/inch	Fuel axial strain.
EZ(C)	10^{-3} inch/inch	Clad axial strain.
ESUP	10^{-3} inch/inch	Support axial strain.
ESLIP	10^{-3} inch/inch	Slippage between rod and support.
EPSGZ(C)	10^{-3} inch/inch	Axial component of flux induced growth in clad.

is indicated in Table VIII.3. This output is suppressed whenever the fifth entry on Card 2 is set to zero.

A second set of tabulated output follows. As before, the region is indicated by the value of KM, while the finite element index is KE. The variables reported in this set are listed in Table VIII.4. This set is not printed if entry 6 on Card 2 is set to zero.

For debug purposes, the results of densification calculations are printed when entry 7 on Card 2 is given the value 3. These data are explained in Table VIII.5.

When short, summary output is desired this can be obtained by assigning the value 3 to the eighth entry on Card 2. This results in the output of the variables labeled TEMP,RADIUS,TAVE,EV,VI,SR,SC,SZ,H, CREEP-R,CREEP-C, and CREEP-Z with the significance indicated in Tables VIII.3 and VIII.4. Since this data duplicates that given in other sets of data, the user will probably want to get it only when those sets are suppressed.

The final block of data includes the variables listed in Table VIII.6.

At the end of each Input Package Case the program reports a set of data for that case and for the rod history up to the end of the case.

Table VIII.3 Materials Data and Temperatures

Variable	Dimensions	Significance
TEMP	°F	Temperature at radius KR.
RADIUS	Inches	Radius of boundary KR between finite elements (KR-1) and KR.
DED/DT	Inches/inch·hour	Total generalized creep strain rate, $d\epsilon_d/dt$.
DH/DT	-	Rate of change of strain hardening dh/dt - see Equations (VI.16) and (A.I.6).
H	-	Strain hardening - see Equations (VI.15) and (A.I.5).
PB	Psi	"Hydrostatic" component of stress $(-[\sigma_1 + \sigma_2 + \sigma_3]/3)$.
TAVE	°F	Temperature at midradius of finite element.
EV	In^3/in^3	Volume strain - $(V/V_{original}) - 1$.
VH	In^3/in^3	Porosity (fuel only).
VH(i);i=1, ...5	In^3/in^3	Porosity in pore category i (fuel only).

Table VII.4 Stress and Strain Values in Finite Elements

Variable	Units	Significance
ET-6	Psi x 10^{-6}	Product of elastic modulus and factor 10^{-6}.
SR	Psi	Radial component of stress - σ_1.
SC	Psi	Circumferential component of stress - σ_2.
3Z	Psi	Axial component of stress - σ_3.
SG	Psi	Generalized stress - σ_g. See Equation (VI.4).
EPS-R	10^{-3} inch/inch	Radial component of total strain.
EPS-C	10^{-3} inch/inch	Circumferential component of total strain.
ELAS-R	10^{-3} inch/inch	Radial component of elastic strain.
ELAS-C	10^{-3} inch/inch	Circumferential component of elastic strain.
ELAS-Z	10^{-3} inch/inch	Axial component of elastic strain.
CREEP-R	10^{-3} inch/inch	Radial component of creep strain.
CREEP-C	10^{-3} inch/inch	Circumferential component of creep strain.
CREEP-Z	10^{-3} inch/inch	Axial component of creep strain.
A-THRML	10^{-3} inch/inch	Nominal thermal strain. To find individual thermal strain components, use factors given in entries one through three on Cards 1300 and 2300.
CRACK	-	For finite elements in the fuel, the presence of cracks on surfaces of constant radius, on surfaces of constant axiumthal angle, or on surfaces of constant z are indicated by 1's in the appropriate column - 1, 2, or 3.

Table VIII.5 Debug Output for Densification Calculations

Variable	Units	Significance
PA	Psi	The value of $-2\gamma/r_p$ from Equation (V.2).
SGAVE	Psi	The nominal average generalized stress $1.5(P_{hydrostatic} + 2\gamma/r_p)/\ln(1 + \epsilon_h/\epsilon_s)$, where ϵ_s is the volume of solid associated with the pore category.
SY	Psi	The value of S_y in Equation (V.2).
SC	Psi•sec•inch/inch	The value of η in Equation (V.2).

Table VIII.6 Miscellaneous Results of Computations

Variable	Significance
F-Z(COLLAPSE)	Portion of fuel-clad axial slip force arising from clad collapse. See Equations (IV.17) and (IV.18).
F-Z(HOURGLASSING)	Portion of fuel-clad axial slip force arising from pellet hourglassing. See Equation (IV.17) and (IV.18).
F-Z(EXCESS-T)	Portion of fuel-clad axial slip force arising from the "excess temperature" representation. See Equation (IV.18).
F-Z(PELLET ECCENTRICITY)	Portion of fuel-clad axial slip force arising from fuel eccentricity or lodged chips. See Equation (IV.18).
STRAIN COMPARISONS	Differences between strains and values of strain computed from the deformation mechanisms represented in the program. These provide indications of the adequacy of the convergence limits in use.
FORCE BALANCE	Difference between the computer finite element "node point" forces and the values of these forces computed from the stresses in the finite elements.
COMPARTMENT F/CC	Average depletion of cross-section expressed in terms of initial internal volume of cladding.
AVERAGE F/CC	Average depletion of rod expressed in terms of initial volume of fuel.
EFPH	Effective full power hours.
GAS RELEASED	Moles of gas released from fuel into fuel-clad gap and rod plenum.
HGAP	Conductance of fuel-clad gap.

Variables for the present case include the number of times each time step limit determined the size of the time step, the highest fuel centerline temperature, the largest value of generalized stress, σ_g, and the most positive value of hoop- (circumferential-) stress at the inside of the clad. Variables reported for the problem include the maximum diameter shrinkage, the maximum diameter increase, and the most extreme values of discrepancies between total strains and the values of total strain calculated from the combination of elastic-, thermal-, creep-, growth- or swelling- and cracking- strains. This provides an indication of the adequacy of convergence limits used in the problem.

Appendix I. CREEP RATE REPRESENTATION

This appendix compares the form of the CYGRO-4 creep rate representation with the representation of CYGRO-3 and presents certain limits on the values of the creep rate parameters. The important point to understand is that the CYGRO-4 creep rate model is the same as that in CYGRO-3. The only change is in the form of the equations.

As indicated in Reference (4), the CYGRO-3 computer program represents $\dot{\epsilon}_m$, the "thermal creep" component of deformation rate, by

$$\dot{\epsilon}_m = 10^{(C-3W-P[G-W]/Q)} \sigma_g^W h^{(G-W)/Q} \qquad (A.I.1)$$

where C, G, P, Q, and W are input material property parameters, σ_g is the effective stress, and h is the strain hardening. The strain hardening changes at the rate

$$\dot{h} = \dot{\epsilon}_M - \dot{\epsilon}_R \quad . \qquad (A.I.2)$$

Steady state creep occurs for $\dot{h} = 0$. By definition, this occurs at

$$\dot{\epsilon}_{(\text{steady state})} = 10^{C-3G} \sigma_g^G \quad . \qquad (A.I.3)$$

Solution of (A.I.1), (A.I.2) and (A.I.3) gives

$$\dot{\epsilon}_R = 10^{(C-PG/Q)} h^{G/Q} \quad . \qquad (A.I.4)$$

In CYGRO-4 the "thermal creep" component of deformation rate is represented by

$$\dot{\epsilon}_d = Q_d \, h^{R_d} \, \sigma_g^{W_d} \quad , \qquad \text{(A.I.5)}$$

where Q_d, R_d and W_d are input material property parameters, σ_g is the effective stress, and h is the strain hardening. The strain hardening changes at a rate

$$\dot{h} = Q_d \, h^{R_d} \, \sigma_g^{W_d} - Q_r \, h^{R_r} \quad . \qquad \text{(A.I.6)}$$

Comparison of (A.I.5) with (A.I.1) shows that Q_d, R_d, and W_d are related to C, G, P, Q, and W by

$$Q_d = 10^{(C-3W-P[G-W]/Q)} \qquad \text{(A.I.7)}$$

$$R_d = (G-W)/Q \qquad \text{(A.I.8)}$$

and

$$W_d = W \quad . \qquad \text{(A.I.9)}$$

Furthermore, comparison of (A.I.6) with (A.I.1), (A.I.2) and (A.I.4) shows that Q_r and R_r are related to C, G, P, Q, and W by

$$Q_r = 10^{(C-PG/Q)} \qquad \text{(A.I.10)}$$

and

$$R_r = G/Q \quad . \qquad \text{(A.I.11)}$$

Alternatively, C, G, P, Q, and W can be expressed in terms of Q_d, Q_r, R_d, R_r, and W_d by

$$C = \frac{(\log_{10} Q_d + 3W_d)R_r - R_d \log_{10} Q_r}{R_r - R_d} \qquad \text{(A.I.12)}$$

$$G = R_r W_d/(R_r - R_d) \qquad \text{(A.I.13)}$$

$$P = [\log_{10}(Q_d/Q_r) + 3W_d]/(R_r - R_d) \qquad \text{(A.I.14)}$$

$$Q = W_d/(R_r - R_d) \qquad \text{(A.I.15)}$$

and

$$W = W_d \qquad \text{(A.I.16)}$$

Equations (A.I.12) through (A.I.15) have a zero denominator for $R_r = R_d$. However, Equations (A.I.3) and (A.I.11) show that for finite Q this can occur only if $W = W_d = 0$. This corresponds to the case where $\dot{\epsilon}_m$ given by (A.I.1) and $\dot{\epsilon}_d$ given by (A.I.5) are independent of stress - a meaningless case.

Since $\dot{\epsilon}_{(\text{steady state})}$ given by (A.I.3) must increase as σ_g increases, G must be positive. Similar reasoning from (A.I.1) and (A.I.4) shows that

$$W_d - W > 0 \qquad \text{(A.I.17)}$$

Use of (A.I.17) in (A.I.13) thus leads to

$$R_r/(R_r - R_d) > 0 \qquad \text{(A.I.18)}$$

Returning to (A.I.5) and reasoning that a strain hardening material will have $\partial\dot{\epsilon}_g/\partial h \leq 0$, we see that

$$R_d \leq 0 \quad , \tag{A.I.19}$$

for materials of interest in practical fuel rod analysis. Moreover, since $\dot{\epsilon}_d \geq 0$,

$$Q_d > 0 \quad . \tag{A.I.20}$$

Finally, the fact that h will ordinarily not increase when $\dot{\epsilon}_d = 0$ implies

$$Q_r \geq 0 \quad . \tag{A.I.21}$$

The results presented in (A.I.17) through (A.I.20) constitute significant limits on the values Q_d, Q_r, R_d, R_r, and W_d. They form the basis for checks on the values of parameters used in computations.

```
      = TEST PROBLEM FROM CYGRO-3 DOCUMENT (REFERENCE 4)
0001  900,  0.0     , 2.6+20  ,2.0+21   , 1.0+12  , 2.0+13  , 1.0+14
0002  904,  0.0     , 0.0     ,0.0      , 1.0+5
0003  905,  5.0-1   , 0.0     ,0.0      , 0.0     , 0.0     , 0.0     ,0.0    ,1.0+4
0004  906,  0.0     , 0.0     ,0.0      , 0.0     , 0.0     , 0.0     ,0.0    ,0.0
0005  907,  1.0-3   , 0.0     ,0.0      , 0.0     , 0.0     , 0.0     ,1.0    ,2.0+3
0006  908,  2.5-1   , 1.0-5   ,2.5+4    , 1.0+4   , 0.0     , 0.0     ,0.0    ,0.0
0007  909,  2.3     , 3.3     ,2.3      , 3.3     , 0.0     , 0.0     ,0.2    ,1.0-8
0008  910,  0.0     , 0.0     ,1.0      , 0.0     , 0.0     , 0.0     ,0.0    ,0.0
0009  911,  1.0-6   , 1.0-6   ,1.0-2    , 1.0     , 1.0-6   , 1.0-9   ,1.0-5  ,4.8-4
0010  912,  1.0+2   , 1.0-1   ,4.0+3    , 9.5+4   , 7.0+2   , 5.2+4   ,5.0-1  ,5.0-1
0011  913,  1.0-1   , 1.0-4   ,2.5-1    , 1.0-1   , 1.0-5   , 1.0+3   ,1.0-2  ,1.5-1
0012  914,  1.5+1   , 5.0-5
0013  1000, 1.1+1   , 1.0     ,0.0      , 0.0     , 0.0     , 5.08+4  ,0.0    ,2.885-3
0014  1001, 1.0-2   , 1.0-10
0015  1100, 0.0
0016  1101, 5.0-3
0017  1102, 1.0-2
0018  1103, 1.5-2
0019  1104, 2.0-2
0020  1105, 3.0-2
0021  1106, 4.5-2
0022  1107, 6.0-2
0023  1108, 8.0-2
0024  1109, 1.0-1
0025  1110, 1.25-1
0026  1200, 0.0     , 0.0     ,0.0      , 1.0-2   , 1.0     , 0.0     ,0.0    ,1.64+8
0027  1300, 1.0     , 1.0     ,1.0      , 3.333-1 , 3.333-1 , 3.333-1
0028  1400, 0.0     , 6.0+3   ,1.0+4
0029  1610, 5.0+2   , 4.86-6  ,2.53+7   , 2.91-1  , 2.7-1   , 1.0     ,1.0    ,1.0
0030  1611, 0.0     ,-7.9     ,1.0      ,-4.5+1   , 2.0+1   , 1.85+1
0031  1612, 0.0     ,-6.6     ,1.0      ,-4.5+1   , 2.0+1   , 1.85+1
0032  1613, 0.0     ,-5.9     ,1.0      ,-4.5+1   , 2.0+1   , 1.85+1
0033  1620, 2.25+3  , 6.21-6  ,2.16+7   , 2.91-1  , 1.37-1  , 1.0     ,1.0    ,1.0
0034  1621, 0.0     ,-7.7     ,1.43     ,-3.4+1   , 2.0+1   , 1.85+1
0035  1622, 0.0     ,-6.8     ,1.23     ,-3.4+1   , 2.0+1   , 1.85+1
0036  1623, 0.0     ,-6.1     ,1.23     ,-3.4+1   , 2.0+1   , 1.85+1
0037  1630, 5.0+3   , 7.45-6  ,1.16+7   , 2.91-1  , 4.4-2   , 1.0     ,1.0    ,1.0
0038  1631, 0.0     , 2.64    ,4.81     ,-8.0     , 2.0+1   , 1.85+1
0039  1632, 0.0     , 1.56    ,5.06     ,-8.0     , 2.0+1   , 1.85+1
0040  1633, 0.0     , 7.2-1   ,5.06     ,-8.0     , 2.0+1   , 1.85+1
0041  1640, 5.0001+3, 7.45-6  ,1.16+7   , 2.91-1  , 4.4-2   , 1.0     ,1.0    ,1.0
0042  1641, 0.0     , 2.64    ,4.81     ,-8.0     , 2.0+1   , 1.85+1
0043  1642, 0.0     , 1.56    ,5.06     ,-8.0     , 2.0+1   , 1.85+1
0044  1643, 0.0     , 7.2-1   ,5.06     ,-8.0     , 2.0+1   , 1.85+1
0045  1700, 0.0     , 0.0     ,0.0      , 0.0     , 0.0     , 0.0     ,0.0    ,1.0
0046  2000, 6.0     , 0.0     ,0.0      , 0.0     , 0.0     , 0.0     ,0.0    ,0.0
0047  2100, 1.28-1
0048  2101, 1.3255-1
0049  2102, 1.3710-1
0050  2103, 1.4165-1
0051  2104, 1.4620-1
0052  2105, 1.5075-1
0053  2300, 1.0     , 1.0     ,1.0      ,-7.22-1  ,-1.1-2   , 7.33-1
0054  2610, 5.50+2  , 4.9-6   ,1.135+7  , 3.25-1  , 7.08-1
0055  2611, 0.0     ,-1.287+1 ,3.80     ,-6.95    , 2.04    , 1.85+1
0056  2612, 0.0     ,-1.414+1 ,4.71     ,-6.52    , 1.30    , 1.85+1
0057  2613, 0.0     ,-1.523+1 ,5.52     ,-6.07    , 6.4-1   , 1.85+1
```

```
0058    2620, 6.50+2  , 4.39-6  ,1.085+7  , 3.25-1  , 7.4-1
0059    2621, 0.0     ,-1.184+1 ,3.8      ,-5.58    , 2.04    , 1.85+1
0060    2622, 0.0     ,-1.336+1 ,4.71     ,-4.66    , 1.30    , 1.85+1
0061    2623, 0.0     ,-1.479+1 ,5.52     ,-3.78    , 6.4-1   , 1.85+1
0062    2630, 7.50+2  , 4.59-6  ,1.035+7  , 3.25-1  , 7.33-1
0063    2631, 0.0     ,-1.077+1 ,3.8      ,-4.61    , 2.04    , 1.85+1
0064    2632, 0.0     ,-1.257+1 ,4.71     ,-3.16    , 1.30    , 1.85+1
0065    2633, 0.0     ,-1.418+1 ,5.52     ,-1.85    , 6.4-1   , 1.85+1
0066    2640, 7.501+2 , 4.59-6  ,1.035+8  , 3.25-1  , 7.73-1
0067    2641, 0.0     ,-1.077+1 ,3.8      ,-4.61    , 2.04    , 1.85+1
0068    2642, 0.0     ,-1.257+1 ,4.71     ,-3.16    , 1.30    , 1.85+1
0069    2643, 0.0     ,-1.418+1 ,5.52     ,-1.85    , 6.4-1   , 1.85+1
0070    2700, 2.928-25, 2.46-37 ,4.0      , 1.0-4   , 0.0     , 0.0     , 0.0
0071    2800, 0.0     , 1.2+14  ,2.0+4    , 1.2+14
0072    4000, 9.5-1   , 2.1     ,3.0-1    , 9.6-1
0073    4001, 6.5219  ,-7.8628+1,2.13518+2, 1.1585-2, 1.4715-1,-3.6054-1,1.8+3 ,6.16+2
0074    4002, 3.2+1   , 4.4+1   ,1.2+5    , 3.5-1   , 3.0-1   , 0.0     ,0.0   ,0.96
0075    4010, 2.18-5  , 2.44-4  ,3.26-1   , 1.0     , 3.1-1   , 0.0     ,3.3-3 ,3.33-1
0076    4020, 0.0     , 0.0     ,1.20+21  , 1.20+21
0077    4030, 0.0     , 0.0     ,2.4+4    , 1.10+21
0078    4040, 0.0     , 0.0     ,1.0-3    , 4.0+3   , 4.0-3   , 8.0+3   ,1.0-2 ,1.2+4
0079    4041, 2.0-2   , 1.6+4   ,3.2-2    , 2.0+4   , 4.2-2   , 2.4+4
0080    5001, 2.987+8 , 0.0     ,0.0      , 1.0     , 1.0+14  , 5.0+2   ,1.0-24,2.0+21
0081    6001, 3.768-5 , 4.5703+1,1.313+2
0082    6002, 2.168-5 , 5.5121+1,8.38+1
0083    6003, 1.445-3 , 4.1659+1,4.003
0084    6004, 1.2514-3, 4.5068+1,2.0159   , 4.4-3
0085    6005, 1.5116-4, 4.6223+1,2.801+1  , 1.76-2
0086    6020, 1.0+5   , 0.0     ,0.79
0087    6031, 0.0     , 0.0     ,1.0+5    , 0.0
0088    6032, 0.0     , 0.0     ,1.0+5    , 0.0
0089    6040, 5000.0  , 1.0
0090    6045, 3.0-5   , 1.0+9   ,1.0+9    , 0.0
0091    600,  1       ,-4
0092    601,  0.0     , 2.0+3   ,2.0+3    , 2.0+3
0093    602,  7.8+1   , 6.536+2 ,6.536+2  , 6.543+2
0094    603,  0.0     , 1.002   ,1.120    , 1.202
0095    604,  1.0-2   , 3.654   ,8.60979+2, 7.88379+2
0096    609,  0.0
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661 LOCATIONS USED BY INP TABLE/LIST ARRAY
99 LINES WRITTEN TO POST PROCESSOR

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THIS JOB HAS IDENTIFICATION PARAMETERS--CSTR1 I.D. ACCCY4 , CSTR1 VERSION 1, CSTR1 DATE 770501, LINK DATE 05/12/77
JOB CARD I.D. CFANTMP, USER NEWMA, DATE 77/05/13 , TIME 14.05.27 , MACHINE A

RESTART FILE ID =*A77133 14.05.27 NEWM*, VERSION = 1

****

F(1)	F(2)	F(3)	FDOT(1)	FDOT(2)	FDOT(3)	SURFTM	SURFILM	900
0.	2.6000E+20	2.0000E+21	1.0000E+12	2.0000E+13	1.0000E+14	0.	0.	900
0	0	0	HWATER	0	0	0	0	904
0.	0.	0.	1.0000E+05	0.	0.	0.	0.	904
DENSRATIO	0	0	0	0	0	0	TCENTER-MAX	905
5.0000E-01	0.	0.	0.	0.	0.	0.	1.0000E+04	905
0	0	0	0	FPLEN	KPLEN	ALFAEND	ENDMU	906
0.	0.	0.	0.	0.	0.	0.	0.	906
SIZERATIO	0	0	0	0	0	MU	PCOLL	907
1.0000E-03	0.	0.	0.	0.	0.	1.0000E+00	2.0000E+03	907
FRCOL	DTMIN	SIGMA	SUPER	0	0	ECCEN	EMISS	908
2.5000E-01	1.0000E-05	2.5000E+04	1.0000E+04	0.	0.	0.	0.	908
RFACTT	PFACTT	RFACTC	PFACTC	XNPRF	KEEP COING	COMPRATIO	MINHARDNS	909
2.3000E+00	3.3000E+00	2.3000E+00	3.3000E+00	0.	0.	2.0000E-01	1.0000E-08	909
FMSUP	FSUP	KSUP	0	0	ASUP	TAXREF	MUAX	910
0.	0.	1.0000E+00	0.	0.	0.	0.	0.	910
GAPOVR	SLIPOVR	FGAPOVR	FZOVR	FZGMIN	FZSMIN	GAPOSC	GAPLIM	911
1.0000E-06	1.0000E-06	1.0000E-02	1.0000E+00	1.0000E-06	1.0000E-09	1.0000E-05	4.8000E-04	911
DTEMPMAX	RATEFRAC	TREF-F	Q-F	TREF-C	Q-C	CRJUMP	SGJUMP	912
1.0000E+02	1.0000E-01	4.0400E+03	9.5000E+04	7.0000E+02	5.2000E+04	5.0000E-01	5.0000E-01	912
CRAKOVR	CLOSOVR	EDOT-ROBOT	EDOT-CYGRO	NOM-CREEP	NOM-STRESS	JUMP STRN	TAU-ROBOT	913
1.0000E-01	1.0000E-04	2.5000E-01	1.0000E-01	1.0000E-05	1.0000E+03	1.0000E-02	1.5000E-01	913
ROD SPAN	BOW ERROR	0	0	0	0	0	0	914
1.5000E+01	5.0000E-05	0.	0.	0.	0.	0.	0.	914

FUEL PROPERTIES

NR	NG	0	0	0	QCMAX	0	S.TEN.	1000
1.1000E+01	1.0000E+00	0.	0.	0.	5.0800E+04	0.	2.8350E-03	1000
RSC	RSP							1001
1.0000E-02	1.0000E-10	0.	0.	0.	0.	0.	0.	1001
RADIUS(KR)	0	0	0	0	0	0	0	11JK
0.	0.	0.	0.	0.	0.	0.	0.	1100
5.0000E-03	0.	0.	0.	0.	0.	0.	0.	1101
1.0000E-02	0.	0.	0.	0.	0.	0.	0.	1102
1.5000E-02	0.	0.	0.	0.	0.	0.	0.	1103
2.0000E-02	0.	0.	0.	0.	0.	0.	0.	1104
3.0000E-02	0.	0.	0.	0.	0.	0.	0.	1105
4.5000E-02	0.	0.	0.	0.	0.	0.	0.	1106
6.0000E-02	0.	0.	0.	0.	0.	0.	0.	1107
8.0000E-02	0.	0.	0.	0.	0.	0.	0.	1108
1.0000E-01	0.	0.	0.	0.	0.	0.	0.	1109
1.2500E-01	0.	0.	0.	0.	0.	0.	0.	1110

0	0	0	VH(KG)	V(KG)	0	0	NHT(KG)	I2JK
			1.0000E-02	1.0000E+00			1.6400E+08	1200
RALPHA(R)	RALPHA(C)	RALPHA(Z)	RV(R)	RV(C)	RV(Z)	0	0	I300
1.0000E+00	1.0000E+00	1.0000E+00	3.3330E-01	3.3330E-01	3.3330E-01	0.	0.	1300
P(1)	P(2)	P(3)	0	0	0	0	0	1400
0.	6.0000E+03	1.0000E+04	0.	0.	0.	0.	0.	1400
TEMPT(J)	ALPHAT(J)	ET(J)	NUT(J)	CT(J)	RVP(1,J)	RVP(2,J)	RVP(3,J)	I6JK
0	CFT(K,J)	GFT(K,J)	PFT(K,J)	QFT(K,J)	WCS(K,J)	0	0	
5.0000E+02	4.8600E-06	2.5300E+07	2.9100E-01	2.7000E-01	1.0000E+00	1.0000E+00	1.0000E+00	
0.	-7.9000E+00	1.0000E+00	-4.5000E+01	2.0000E+01	1.8500E+01			
0.	-6.6000E+00	1.0000E+00	-4.5000E+01	2.0000E+01	1.8500E+01			
0.	-5.9000E+00	1.0000E+00	-4.5000E+01	2.0000E+01	1.8500E+01			
2.2500E+03	6.2100E-06	2.1600E+07	2.9100E-01	1.3700E-01	1.0000E+00	1.0000E+00	1.0000E+00	
0.	-7.7000E+00	1.4300E+00	-3.4000E+01	2.0000E+01	1.8500E+01			
0.	-6.8000E+00	1.2300E+00	-3.4000E+01	2.0000E+01	1.8500E+01			
0.	-6.1000E+00	1.2300E+00	-3.4000E+01	2.0000E+01	1.8500E+01			
5.0000E+03	7.4500E-06	1.1600E+07	2.9100E-01	4.4000E-02	1.0000E+00	1.0000E+00	1.0000E+00	
0.	2.6400E+00	4.8100E+00	-8.0000E+00	2.0000E+01	1.8500E+01			
0.	1.5600E+00	5.0600E+00	-8.0000E+00	2.0000E+01	1.8500E+01			
0.	7.2000E-01	5.0600E+00	-8.0000E+00	2.0000E+01	1.8500E+01			
5.0001E+03	7.4500E-06	1.1600E+07	2.9100E-01	4.4000E-02	1.0000E+00	1.0000E+00	1.0000E+00	
0.	2.6400E+00	4.8100E+00	-8.0000E+00	2.0000E+01	1.8500E+01			
0.	1.5600E+00	5.0600E+00	-8.0000E+00	2.0000E+01	1.8500E+01			
0.	7.2000E-01	5.0600E+00	-8.0000E+00	2.0000E+01	1.8500E+01			
CLIN	CPOW	XPOW	EINT	0	ATRAN	BTRAN	FREVSM	I700
0.	0.	0.	0.	0.	0.	0.	1.0000E+00	1700

CLAD PROPERTIES

NR	NG	0	0	0	QGMAX	0	S.TEN.	I000
6.0000E+00	0.	0.	0.	0.	0.	0.	0.	2000
RADIUS(KR)	0	0	0	0	0	0	0	I1JK
1.2800E-01	0.	0.	0.	0.	0.	0.	0.	2100
1.3255E-01	0.	0.	0.	0.	0.	0.	0.	2101
1.3710E-01	0.	0.	0.	0.	0.	0.	0.	2102
1.4165E-01	0.	0.	0.	0.	0.	0.	0.	2103
1.4620E-01	0.	0.	0.	0.	0.	0.	0.	2104
1.5075E-01	0.	0.	0.	0.	0.	0.	0.	2105
RALPHA(R)	RALPHA(C)	RALPHA(Z)	RV(R)	RV(C)	RV(Z)	0	0	I300
1.0000E+00	1.0000E+00	1.0000E+00	-7.2200E-01	-1.1000E-02	7.3300E-01	0.	0.	2300
TEMPT(J)	ALPHAT(J)	ET(J)	NUT(J)	CT(J)	RVP(1,J)	RVP(2,J)	RVP(3,J)	I6JK
0	CFT(K,J)	GFT(K,J)	PFT(K,J)	QFT(K,J)	WCS(K,J)	0	0	
5.5000E+02	4.9000E-06	1.1350E+07	3.2500E-01	7.0800E-01	0.	0.	0.	
0.	-1.2870E+01	3.8000E+00	-6.9500E+00	2.0400E+00	1.8500E+01			
0.	-1.4140E+01	4.7100E+00	-6.5200E+00	1.3000E+00	1.8500E+01			
0.	-1.5230E+01	5.5200E+00	-6.0700E+00	6.4000E-01	1.8500E+01			

```
6.5000E+02  4.3900E-06  1.0850E+07  3.2500E-01  7.4000E-01  0.          0.          0.
0.          -1.1840E+01  3.8000E+00 -5.5800E+00  2.0400E+00  1.8500E+01
0.          -1.3360E+01  4.7100E+00 -4.6600E+00  1.3000E+00  1.8500E+01
0.          -1.4790E+01  5.5200E+00 -3.7800E+00  6.4000E-01  1.8500E+01

7.5000E+02  4.5900E-06  1.0350E+07  3.2500E-01  7.3360E-01  0.          0.          0.
0.          -1.0770E+01  3.8000E+00 -4.6100E+00  2.0460E+00  1.8500E+01
0.          -1.2570E+01  4.7100E+00 -3.1600E+00  1.3000E+00  1.8500E+01
0.          -1.4180E+01  5.5200E+00 -1.8500E+00  6.4000E-01  1.8500E+01

7.5010E+02  4.5900E-06  1.0350E+08  3.2500E-01  7.7300E-01  0.          0.          0.
0.          -1.0770E+01  3.8000E+00 -4.6100E+00  2.0400E+00  1.8500E+01
0.          -1.2570E+01  4.7100E+00 -3.1600E+00  1.3000E+00  1.8500E+01
0.          -1.4180E+01  5.5200E+00 -1.8500E+00  6.4000E-01  1.8500E+01

CLIN        CPOW        XPOW        EINT        0           ATRAN       BTRAN       0           I700
2.9280E-25  2.4600E-37  4.0000E+00  1.0000E-04  0.          0.          0.          0.          2700
```

INPUT VALUES OF FAST FLUX AT FULL POWER WITH CORRESPONDING REACTOR TIMES.

```
0.          2.0000E+04
1.2000E+14  1.2000E+14
```

INPUT VALUES OF MAXIMUM SUPPORT SLIP FORCE WITH CORRESPONDING REACTOR TIMES.

```
0.
1.0000E-09
```

INPUT VALUES OF CLAD COLLAPSE PRESSURE WITH CORRESPONDING REACTOR TIMES,

```
0.
2.0000E+03
```

RADII FOR POWER DISTRIBUTION IN FUEL REGION ONE 3001
OPTIONAL CARD NOT SUPPLIED

RADII FOR POWER DISTRIBUTION IN FUEL REGION TWO 3002
OPTIONAL CARD NOT SUPPLIED

RADII FOR POWER DISTRIBUTION IN CLAD 3003
OPTIONAL CARD NOT SUPPLIED

TIME POWER DISTRIBUTION 301K
DEFAULT CASE. UNIFORM RADIAL POWER DISTRIBUTION.

```
ENG FACT    PORE CORR   WEIGHT PC   DENSITY                                             4000
9.5000E-01  2.1000E+00  3.0000E-01  9.6000E-01  0.          0.          0.          0.  4000

A0          A1          A2          B0          B1          B2          C           D   4001
6.5219E+00 -7.8628E+01  2.1352E+02  1.1585E-02  1.4715E-01 -3.6054E-01  1.8000E+03  6.1600E+02  4001

R1          R2          HARD        EMISSF      EMISSC      PCONIN      CCRACK      GASCON  4002
3.2000E+01  4.4000E+01  1.2000E+05  3.5000E-01  3.0000E-01  0.          0.          9.6000E-01  4002

GFG         GI          G/G+P       NATM        ALPHAFG     F RATIO     ALPHAHE     DELTAHE 4010
2.1800E-05  2.4400E-04  3.2500E-01  1.0000E+00  3.1000E-01  0.          3.3000E-03  3.3300E-01  4010
```

A(I)	N(I)	M(I)	VV(I)					600K
3.7680E-05	4.5703E+01	1.3130E+02	0.	0.	0.	0.	0.	6001
2.1680E-05	5.5121E+01	8.3800E+01	0.	0.	0.	0.	0.	6002
1.4450E-03	4.1659E+01	4.0030E+00	0.	0.	0.	0.	0.	6003
1.2514E-03	4.5068E+01	2.0159E+00	4.4000E-03	0.	0.	0.	0.	6004
1.5116E-04	4.6223E+01	2.8010E+01	1.7600E-02	0.	0.	0.	0.	6005
T(DEFECT)	STEAM JUMP	C(OXIDE)	0	0	0	0	0	6020
1.0000E+05	0.	7.9000E-01	0.	0.	0.	0.	0.	6020

THICKNESSES OF OXIDE LAYER AT INNER SURFACE OF CLADDING WITH CORRESPONDING REACTOR TIMES

0.	1.0000E+05
0.	0.

THICKNESSES OF OXIDE LAYER AT OUTER SURFACE OF CLADDING WITH CORRESPONDING REACTOR TIMES

0.	1.0000E+05
0.	0.

CRACKING STRESS RELATIVE TO ROOM TEMPERATURE VALUE AT SIGNIFICANT TEMPERATURES

5.0000E+03
1.0000E+00

BETA	ASTAR	QSUBA	QSUBB	0	0	0	0	6045
3.0000E-05	1.0000E+09	1.0000E+09	0.	0.	0.	0.	0.	6045
AV F/CC VS LOC F/CC								402K
EFPH VS AV F/CC								403K
FR GAS REL VS EFPH								404K
0.	0.	1.2000E+21	1.2000E+21	0.	0.	0.	0.	4020
0.	0.	2.4000E+04	1.1000E+21	0.	0.	0.	0.	4030
0.	0.	1.0000E-03	4.0000E+03	4.0000E-03	8.0000E+03	1.0000E-02	1.2000E+04	4040
2.0000E-02	1.6000E+04	3.2000E-02	2.0000E+04	4.2000E-02	2.4000E+04	0.	0.	4041

AV VS LOC F/CC TABLES CONVERTED FROM COMPARTMENT TO PELLET BASIS

0.	1.2583E+21
0.	1.2583E+21

EFPH VS AV F/CC TABLES CONVERTED FROM COMPARTMENT TO PELLET BASIS

0.	2.4000E+04
0.	1.1534E+21

FISS/POW	0	0	T-OR-DES	VVZGR	TZGRI	CZGR	FLZGR	5001
2.9870E+08	0.	0.	1.0000E+00	1.0000E+14	5.0000E+02	1.0000E-24	2.0000E+21	5001

EVS(MEAT) VS F/CC TABLE

0.	1.0000E+20	2.0000E+20	3.0000E+20	4.0000E+20	5.0000E+20	6.0000E+20	7.0000E+20
0.	5.0000E-03	1.4000E-02	2.0000E-02	2.7000E-02	3.6000E-02	4.5000E-02	5.5000E-02
8.0000E+20	9.0000E+20	1.0000E+21	1.1000E+21	1.2000E+21	1.3000E+21	1.4000E+21	1.5000E+21
6.3000E-02	7.1000E-02	8.5000E-02	9.5000E-02	1.0500E-01	1.2000E-01	1.3000E-01	1.4400E-01

1.6000E+21	1.7000E+21	1.8000E+21	1.9000E+21	2.0000E+21	2.1000E+21	2.2000E+21	2.3000E+21
1.5500E-01	1.7000E-01	1.8500E-01	2.0000E-01	2.1500E-01	2.3000E-01	2.4500E-01	2 5700E-01
2.4000E+21	2.5000E+21	2.6000E+21	2.7000E+21	2.8000E+21	2.9000E+21	3.0000E+21	
2.7300E-01	2.9000E-01	3.0500E-01	3.2300E-01	3.4000E-01	3.6000E-01	3.7700E-01	

END OF CARD PROCESSING**********

FUEL AND CLAD CREEP AND THERMAL EXPANSION REPRESENTATIONS.

FUEL THERMALLY ACTIVATED CREEP RATE, DED/DT, AND HARDENING RECOVERY RATE, DER/DT, REPRESENTATIONS AT TABULATED VALUES OF FISSION RATE AND TEMPERATURE

TEMPERATURE	FISSION RATE	1.00E+12 FISSIONS/CC.SEC	2.00E+13 FISSIONS/CC.SEC	1.00E+14 FISSIONS/CC.SEC
500	DED/DT	1.679-103*H**(-.875)*SG**(18.5)	3.350-102*H**(-.875)*SG**(18.5)	1.679-101*H**(-.875)*SG**(18.5)
	DER/DT	2.239E-06*H**(.050)	4.467E-05*H**(.050)	2.239E-04*H**(.050)
2250	DED/DT	6.039E-93*H**(-.853)*SG**(18.5)	2.193E-92*H**(-.863)*SG**(18.5)	1.099E-91*H**(-.863)*SG**(18.5)
	DER/DT	5.383E-06*H**(.071)	1.954E-05*H**(.061)	9.795E-05*H**(.061)
5000	DED/DT	4.613E-59*H**(-.684)*SG**(18.5)	4.831E-60*H**(-.672)*SG**(18.5)	6.982E-61*H**(-.672)*SG**(18.5)
	DER/DT	3.664E+04*H**(.240)	3.837E+03*H**(.253)	5.546E+02*H**(.253)
5000	DED/DT	4.613E-59*H**(-.684)*SG**(18.5)	4.831E-60*H**(-.672)*SG**(18.5)	6.982E-61*H**(-.672)*SG**(18.5)
	DER/DT	3.664E+04*H**(.240)	3.837E+03*H**(.253)	5.546E+02*H**(.253)

TRANSIENT CREEP AT 1000PSI EXPRESSED IN TERMS OF THE HARDENING PARAMETER H

TEMPERATURE	KF	EDOT(1.E-5)	EDOT(2.-E5)	EDOT(5.E-5)	EDOT(1.E-4)	EDOT(2.E-4)	EDOT(5.E-4)	EDOT(1.E-3)	EDOT(2.E-3)
500									
	1	1.259E-43	6.864E-44	3.079E-44	1.679E-44	9.154E-45	4.106E-45	2.239E-45	1.221E-45
	2	2.512E-42	1.370E-42	6.143E-43	3.350E-43	1.826E-43	8.192E-44	4.467E-44	2.436E-44
	3	1.259E-41	6.864E-42	3.079E-42	1.679E-42	9.154E-43	4.106E-43	2.239E-43	1.221E-43
2259									
	1	3.536E-33	1.957E-33	8.952E-34	4.955E-34	2.742E-34	1.254E-34	6.942E-35	3.842E-35
	2	1.440E-32	7.917E-33	3.589E-33	1.972E-33	1.084E-33	4.914E-34	2.701E-34	1.484E-34
	3	7.219E-32	3.968E-32	1.799E-32	9.886E-33	5.433E-33	2.463E-33	1.354E-33	7.440E-34
5000									
	1	3.859E+00	2.401E+00	1.282E+00	7.980E-01	4.965E-01	2.652E-01	1.650E-01	1.027E-01
	2	3.499E-01	2.196E-01	1.187E-01	7.447E-02	4.674E-02	2.525E-02	1.585E-02	9.947E-03
	3	5.058E-02	3.175E-02	1.715E-02	1.076E-02	6.756E-03	3.650E-03	2.291E-03	1.438E-03
5000									
	1	3.859E+00	2.401E+00	1.282E+00	7.980E-01	4.965E-01	2.652E-01	1.650E-01	1.027E-01
	2	3.499E-01	2.196E-01	1.187E-01	7.447E-02	4.674E-02	2.525E-02	1.585E-02	9.947E-03
	3	5.058E-02	3.175E-02	1.715E-02	1.076E-02	6.756E-03	3.650E-03	2.291E-03	1.438E-03

ASYMPTOTIC (STEADY STATE) CREEP RATES EXPRESSED AS A FUNCTION OF STRESS

TEMPERATURE	KF	EDOT(5.E+2)	EDOT(1.E+3)	EDOT(2.E+3)	EDOT(5.E+3)	EDOT(1.E+4)	EDOT(2.E+4)	EDOT(5.E+4)	EDOT(1.E+5)
500									
	1	6.295E-09	1.259E-08	2.518E-08	6.295E-08	1.259E-07	2.518E-07	6.295E-07	1.259E-06
	2	1.256E-07	2.512E-07	5.024E-07	1.256E-06	2.512E-06	5.024E-06	1.256E-05	2.512E-05
	3	6.295E-07	1.259E-06	2.518E-06	6.295E-06	1.259E-05	2.518E-05	6.295E-05	1.259E-04
2250									

	1	7.405E-09	1.995E-08	5.376E-08	1.993E-07	5.370E-07	1.447E-06	5.364E-06	1.445E-05
	2	6.757E-08	1.585E-07	3.718E-07	1.147E-06	2.692E-06	6.313E-06	1.949E-05	4.571E-05
	3	3.386E-07	7.943E-07	1.863E-06	5.751E-06	1.349E-05	3.164E-05	9.766E-05	2.291E-04
5000									
	1	1.556E+01	4.365E+02	1.224E+04	1.005E+06	2.818E+07	7.906E+08	6.487E+10	1.820E+12
	2	1.088E+00	3.631E+01	1.211E+03	1.250E+05	4.169E+06	1.391E+08	1.435E+10	4.786E+11
	3	1.573E-01	5.248E+00	1.751E+02	1.806E+04	6.026E+05	2.010E+07	2.074E+09	6.918E+10
5000									
	1	1.556E+01	4.365E+02	1.224E+04	1.005E+06	2.818E+07	7.906E+08	6.487E+10	1.820E+12
	2	1.088E+00	3.631E+01	1.211E+03	1.250E+05	4.169E+06	1.391E+08	1.435E+10	4.786E+11
	3	1.573E-01	5.248E+00	1.751E+02	1.806E+04	6.026E+05	2.010E+07	2.074E+09	6.918E+10

CLAD THERMALLY ACTIVATED CREEP RATE, DEM/DT, AND HARDENING RECOVERY RATE, DER/DT, REPRESENTATIONS AT TABULATED VALUES OF FLUENCE AND TEMPERATURE

TEMPERATURE		FLUENCE 0. NEUTRONS/CM**2	2.60E+20 NEUTRONS/CM**2	2.00E+21 NEUTRONS/CM**2
550	DED/DT	3.541-119*H**(-7.206)*SG**(18.5)	1.577-139*H**(-10.608)*SG**(18.5)	1.455-194*H**(-20.281)*SG**(18.5)
	DER/DT	1.191E+00*H**(1.363)	3.037E+09*H**(3.623)	1.330E+37*H**(8.625)
650	DED/DT	2.826-108*H**(-7.206)*SG**(18.5)	5.107-119*H**(-10.608)*SG**(18.5)	1.114-147*H**(-20.281)*SG**(18.5)
	DER/DT	3.582E-02*H**(1.363)	3.338E+03*H**(3.623)	5.494E+17*H**(8.625)
750	DED/DT	3.243-100*H**(-7.206)*SG**(18.5)	2.569-102*H**(-10.608)*SG**(18.5)	5.305-108*H**(-20.281)*SG**(18.5)
	DER/DT	6.565E-03*H**(1.863)	7.567E-02*H**(3.623)	5.974E+01*H**(8.625)
750	DED/DT	3.243-100*H**(-7.206)*SG**(18.5)	2.569-102*H**(-10.608)*SG**(18.5)	6.305-108*H**(-20.281)*SG**(18.5)
	DER/DT	6.565E-03*H**(1.863)	7.567E-02*H**(3.623)	5.974E+01*H**(8.625)

TRANSIENT CREEP AT 1000PSI EXPRESSED IN TERMS OF THE HARDENING PARAMETER H

TEMPERATURE	KF	EDOT(1.E-5)	EDOT(2.-E5)	EDOT(5.E-5)	EDOT(1.E-4)	EDOT(2.E-4)	EDOT(5.E-4)	EDOT(1.E-3)	EDOT(2.E-3)
550									
	1	1.198E-27	8.116E-30	1.101E-32	7.458E-35	5.052E-37	6.854E-40	4.643E-42	3.145E-44
	2	5.449E-31	3.492E-34	2.098E-38	1.345E-41	8.618E-45	5.178E-49	3.318E-52	2.127E-55
	3	1.172E-37	9.260E-44	7.818E-52	6.135E-58	4.814E-64	4.691E-72	3.210E-78	2.519E-84
650									
	1	9.563E-17	6.477E-19	8.788E-22	5.953E-24	4.032E-26	5.470E-29	3.705E-31	2.510E-33
	2	1.764E-10	1.131E-13	6.794E-18	4.354E-21	2.791E-24	1.677E-28	1.075E-31	6.886E-35
	3	8.977E+09	7.065E+03	5.985E-05	4.698E-11	3.686E-17	3.132E-25	2.458E-31	1.929E-37
750									
	1	1.097E-08	7.432E-11	1.003E-13	6.830E-16	4.626E-18	6.276E-21	4.251E-23	2.880E-25
	2	8.875E+06	5.687E+03	3.417E-01	2.190E-04	1.404E-07	8.433E-12	5.405E-15	3.464E-18
	3	5.081E+49	3.987E+43	3.383E+35	2.659E+29	2.087E+23	1.773E+15	1.391E+09	1.092E+03
750									
	1	1.097E-08	7.432E-11	1.003E-13	6.830E-16	4.626E-18	6.276E-21	4.251E-23	2.880E-25
	2	8.875E+06	5.687E+03	3.417E-01	2.190E-04	1.404E-07	8.433E-12	5.405E-15	3.464E-18
	3	5.081E+49	3.987E+43	3.388E+35	2.659E+29	2.087E+23	1.773E+15	1.391E+09	1.092E+03

ASYMPTOTIC (STEADY STATE) CREEP RATES EXPRESSED AS A FUNCTION OF STRESS

TEMPERATURE	KF	EDOT(5.E+2)	EDOT(1.E+3)	EDOT(2.E+3)	EDOT(5.E+3)	EDOT(1.E+4)	EDOT(2.E+4)	EDOT(5.E+4)	EDOT(1.E+5)
550									
	1	9.685E-15	1.349E-13	1.879E-12	6.111E-11	8.511E-10	1.186E-08	3.856E-07	5.370E-06
	2	2.768E-16	7.244E-15	1.896E-13	1.420E-11	3.715E-10	9.724E-09	7.280E-07	1.905E-05
	3	1.283E-17	5.888E-16	2.702E-14	4.249E-12	1.950E-10	8.947E-09	1.407E-06	6.457E-05
650									
	1	1.038E-13	1.445E-12	2.013E-11	6.548E-10	9.120E-09	1.270E-07	4.131E-06	5.754E-05
	2	1.668E-15	4.365E-14	1.142E-12	8.554E-11	2.239E-09	5.859E-08	4.387E-06	1.148E-04
	3	3.534E-17	1.622E-15	7.442E-14	1.170E-11	5.370E-10	2.464E-08	3.875E-06	1.778E-04
750									
	1	1.219E-12	1.698E-11	2.365E-10	7.693E-09	1.072E-07	1.492E-06	4.854E-05	6.761E-04
	2	1.028E-14	2.692E-13	7.045E-12	5.274E-10	1.380E-08	3.613E-07	2.705E-05	7.079E-04
	3	1.440E-16	6.607E-15	3.032E-13	4.768E-11	2.188E-09	1.004E-07	1.579E-05	7.244E-04
750									
	1	1.219E-12	1.698E-11	2.365E-10	7.693E-09	1.072E-07	1.492E-06	4.854E-05	6.761E-04
	2	1.028E-14	2.692E-13	7.045E-12	5.274E-10	1.380E-08	3.613E-07	2.705E-05	7.079E-04
	3	1.440E-16	6.607E-15	3.032E-13	4.768E-11	2.188E-09	1.004E-07	1.579E-05	7.244E-04

CLAD *GENERALIZED STRESS* REPRESENTATIONS

FOR SC.GE.SR, SC = (.697*(SR-SC)**2 + .303*(SC-SZ)**2 + .211*(SZ-SR)**2)**0.5

FOR SC.LT.SR, SC = (.697*(SR-SC)**2 + .303*(SC-SZ)**2 + .211*(SZ-SR)**2)**0.5

FUEL THERMAL EXPANSION CALCULATED FROM THERMAL EXPANSION COEFFICIENTS. (EXPANSION STRAINS MULTIPLIED BY 10**6)

TEMPERATURE	80	200	320	440	560	680	800	920	1040	1160	1280	1400	1520	1640
EXPANSION	38.9	622.1	1205.3	1788.5	2373.1	2967.4	3572.8	4189.3	4817.0	5455.7	6105.5	6766.5	7438.6	8121.8
TEMPERATURE	1760	1880	2000	2120	2240	2360	2480	2600	2720	2840	2960	3080	3200	3320
EXPANSION	8816.0	9521.4	10237.9	10965.5	11704.3	12452.2	13206.6	13967.4	14734.8	15508.7	16289.1	17075.9	17869.3	18669.2
TEMPERATURE	3440	3560	3680	3800	3920	4040	4160	4280	4400	4520	4640	4760	4880	5000
EXPANSION	19475.5	20288.3	21107.7	21933.5	22765.8	23604.6	24449.9	25301.7	26160.0	27024.8	27896.0	28773.8	29658.1	30548.8

CLAD THERMAL EXPANSION CALCULATED FROM THERMAL EXPANSION COEFFICIENTS. (EXPANSION STRAINS MULTIPLIED BY 10**6)

TEMPERATURE	80	100	120	140	160	180	200	220	240	260	280	300	320	340
EXPANSION	39.2	137.2	235.2	333.2	431.2	529.2	627.2	725.2	823.2	921.2	1019.2	1117.2	1215.2	1313.2
TEMPERATURE	360	380	400	420	440	460	480	500	520	540	560	580	600	620
EXPANSION	1411.2	1509.2	1607.2	1705.2	1803.2	1901.2	1999.2	2097.2	2195.2	2293.2	2390.9	2486.9	2580.8	2672.7
TEMPERATURE	640	660	680	700	720	740	760	780	800	820	840	860	880	900
EXPANSION	2762.5	2850.7	2939.3	3028.7	3118.9	3209.9	3301.6	3393.4	3485.2	3577.0	3668.8	3760.6	3852.4	3944.2

END OF INPUT**********

**********HISTORY SET 1, CYCLE NUMBER 1, STEP 1

DTIME	QG/QGMAX	T(WATER)	P(WATER)	FSUP.MAX	FIS.RATE	DEPLTN	F.FLUX	FLUENCE	P(GAP)	DP COLAPS
0.	0.	72.00	0.	.0	0.	0.	0.	0.	0.	2000
.0100	0.	78.00	0.	.0	0.	0.	0.	0.	0.	2000

TIME	KTIME	DT-ROB	DD(M)	F/C GAP	DD(C)	FCONR	FCONZ	FSUP	EZ(M)	EZ(C)	ESUP	ESLIP	EPSGZ(C)
0.	0		0.	3.00	0.	0.	0.	0.	0.	0.	0.	0.	0.
.010	1	1.0E+05	.01	3.00	.01	0.	-.0	.0	.03	.03	.03	-.00	0.

KM	KR	TEMP	RADIUS	DED/DT	DH/DT	H	PB	TAVE	EV	VH	VH(1)	VH(2)	VH(3)	VH(4)	VH(5)
1	1	78	0.	1678-169	-89125-06	10000-07	-0	78	-.00000	.01000	.01000	0.	0.	0.	0.
1	2	78	.00500	1678-169	-89125-06	10000-07	-0	78	-.00000	.01000	.01000	0.	0.	0.	0.
1	3	78	.01000	1678-169	-89125-06	10000-07	-0	78	-.00000	.01000	.01000	0.	0.	0.	0.
1	4	78	.01500	1678-169	-89125-06	10000-07	-0	78	-.00000	.01000	.01000	0.	0.	0.	0.
1	5	78	.02000	1678-169	-89125-06	10000-07	-0	78	-.00000	.01000	.01000	0.	0.	0.	0.
1	6	78	.03000	1678-169	-89125-06	10000-07	-0	78	-.00000	.01000	.01000	0.	0.	0.	0.
1	7	78	.04500	1678-169	-89125-06	10000-07	-0	78	-.00000	.01000	.01000	0.	0.	0.	0.
1	8	78	.06000	1678-169	-89125-06	10000-07	-0	78	-.00000	.01000	.01000	0.	0.	0.	0.
1	9	78	.08000	1678-169	-89125-06	10000-07	-0	78	-.00000	.01000	.01000	0.	0.	0.	0.
1	10	78	.10000	1678-169	-89125-06	10000-07	-0	78	-.00000	.01000	.01000	0.	0.	0.	0.
1	11	78	.12500												
2	1	78	.12800	1571-134	-14932-14	10000-07	1	78	0.						
2	2	78	.13255	1571-134	-14932-14	10000-07	1	78	0.						
2	3	78	.13710	1571-134	-14932-14	10000-07	1	78	0.						
2	4	78	.14165	1571-134	-14932-14	10000-07	1	78	0.						
2	5	78	.14620	1571-134	-14932-14	10000-07	1	78	0.						
2	6	78	.15075												

KM	KE	ET-6	SR	SC	SZ	SC	EPS-R	EPS-C	ELAS-R	ELAS-C	ELAS-Z	CREEP-R	CREEP-C	CREEP-Z	A-THRML	CRACK
1	1	25.3	0	0	1	1	.03	.03	-.00	-.00	.00	0.	0.	0.	.03	0 0 0
1	2	25.3	0	0	1	1	.03	.03	-.00	-.00	.00	0.	0.	0.	.03	0 0 0
1	3	25.3	0	0	1	1	.03	.03	-.00	-.00	.00	0.	0.	0.	.03	0 0 0
1	4	25.3	0	0	1	1	.03	.03	-.00	-.00	.00	0.	0.	0.	.03	0 0 0
1	5	25.3	0	0	1	1	.03	.03	-.00	-.00	.00	0.	0.	0.	.03	0 0 0
1	6	25.3	0	0	1	1	.03	.03	-.00	-.00	.00	0.	0.	0.	.03	0 0 0
1	7	25.3	0	-0	1	1	.03	.03	-.00	-.00	.00	0.	0.	0.	.03	0 0 0
1	8	25.3	0	-0	1	1	.03	.03	-.00	-.00	.00	0.	0.	0.	.03	0 0 0
1	9	25.3	0	0	1	1	.03	.03	-.00	-.00	.00	0.	0.	0.	.03	0 0 0
1	10	25.3	0	-0	1	1	.03	.03	-.00	-.00	.00	0.	0.	0.	.03	0 0 0
2	1	11.3	-0	-0	-2	2	.03	.03	.00	.00	-.00	0.	0.	0.	.03	0 0 0
2	2	11.3	-0	-0	-2	2	.03	.03	.00	.00	-.00	0.	0.	0.	.03	0 0 0
2	3	11.3	-0	0	-2	2	.03	.03	.00	.00	-.00	0.	0.	0.	.03	0 0 0
2	4	11.3	-0	0	-2	2	.03	.03	.00	.00	-.00	0.	0.	0.	.03	0 0 0
2	5	11.3	-0	0	-2	2	.03	.03	.00	.00	-.00	0.	0.	0.	.03	0 0 0

F-Z(COLLAPSE) = 0. F-Z(HOURGLASSING) = 0. F-Z(EXCESS-T) = 0. F-Z(PELLET ECCEN) = 0.
STRAIN COMPARISONS. FUEL-(R) 3.41E-13, (C) 3.27E-13, (Z)-1.07E-12 CLAD-(R)-1.90E-12, (C)-1.95E-12, (Z) 5.98E-12
FORCE BALANCE FUEL CIRC=-8.121E-26 FUEL AXIAL= 5.449E-05 CLAD CIRC= 5.447E-28 CLAD AXIAL=-1.358E-04
COMPARTMENT F/CC= 0. AVERAGE F/CC= 0. EFPH= 0. GAS RELEASED= 0. HGAP= 1.8273E+00
TIME STEPS MIN= 100000-01 MAX= 100000-01 AVG= 100000-01

**********HISTORY SET 1, CYCLE NUMBER 1, STEP 2

DTIME	QG/QGMAX	T(WATER)	P(WATER)	FSUP.MAX	FIS.RATE	DEPLTN	F.FLUX	FLUENCE	P(GAP)	DP COLAPS
0.		78.00	0.	.0	0.	0.	0.	0.	0.	2000
3.6540	1.0020	653.60	2000.00	.0	1.5204E+13	1.0000E+17	1.2024E+14	7.9084E+17	0.	2000

TIME	KTIME	DT-ROB	DD(M)	F/C GAP	DD(C)	FCONR	FCONZ	FSUP	EZ(M)	EZ(C)	ESUP	ESLIP	EPSGZ(C)
.010	1		.01	3.00	.01	0.	-.0	.0	.03	.03	.03	-.00	0.
3.664	60	3.1E+01	2.97	1.75	.64	0.	.0	.0	12.62	2.93	.00	2.93	.12

KH	KR	TEMP	RADIUS	DED/DT	DH/DT	H	PB	TAVE	EV	VH	VH(1)	VH(2)	VH(3)	VH(4)	VH(5)
1	1	2815	0.	43664-02	17161-02	15436-02	15069	2813	-.00002	.00998	.00998	0.	0.	0.	0.
1	2	2812	.00508	43181-02	18311-02	14912-02	14828	2607	-.00001	.00998	.00998	0.	0.	0.	0.
1	3	2802	.01016	42233-02	20321-02	14004-02	14318	2794	-.00001	.00998	.00998	0.	0.	0.	0.
1	4	2786	.01524	40817-02	22702-02	12684-02	13470	2774	-.00001	.00999	.00999	0.	0.	0.	0.
1	5	2763	.02032	37695-02	25879-02	10078-02	11168	2731	.00000	.00999	.00999	0.	0.	0.	0.
1	6	2699	.03047	30630-02	26411-02	53346-03	3399	2630	.00001	.01000	.01000	0.	0.	0.	0.
1	7	2560	.04569	14532-09	-54351-04	19969-05	3442	2467	.00001	.01000	.01000	0.	0.	0.	0.
1	8	2374	.06088	41145-05	-79647-06	52751-07	-7333	2221	.00001	.01000	.01000	0.	0.	0.	0.
1	9	2069	.08111	36275-72	-52528-05	10000-07	-2	1893	.00001	.01000	.01000	0.	0.	0.	0.
1	10	1718	.10130	11347-73	-63053-05	10000-07	-4	1484	.00001	.01000	.01000	0.	0.	0.	0.
1	11	1250	.12648												
2	1	742	.12823	16194-02	16188-02	39666-03	8111	732	.00035						
2	2	723	.13280	10108-02	10103-02	22657-03	7792	714	.00035						
2	3	705	.13737	60754-03	60710-03	12652-03	7300	696	.00035						
2	4	687	.14194	35657-03	35620-03	69953-04	6702	679	.00035						
2	5	670	.14650	20787-03	20757-03	38832-04	6046	662	.00035						
2	6	654	.15107												

KM	KE	ET-6	SR	SC	SZ	SC	EPS-R	EPS-C	ELAS-R	ELAS-C	ELAS-Z	CREEP-R	CREEP-C	CREEP-Z	A-THRML	CRACK
1	1	19.6	-8316	-8316	-28577	20261	16.20	16.20	.12	.12	-1.21	.73	.73	-1.46	15.33	0 0 0
1	2	19.6	-8168	-7873	-28444	20425	16.12	16.17	.12	.14	-1.21	.69	.73	-1.42	15.29	0 0 0
1	3	19.6	-7864	-6941	-28150	20763	15.96	16.12	.12	.18	-1.21	.62	.72	-1.33	15.21	0 0 0
1	4	19.7	-7383	-5420	-27607	21273	15.72	16.04	.11	.24	-1.21	.52	.70	-1.21	15.08	0 0 0
1	5	19.9	-6025	-1571	-25909	22445	15.24	15.85	.10	.39	-1.19	.33	.64	-.96	14.81	0 0 0
1	6	20.2	-2118	10560	-18639	25360	14.20	15.44	.01	.82	-1.04	.03	.44	-.47	14.16	0 0 0
1	7	20.8	-6	2	-10323	10321	13.27	14.95	.14	.14	-.50	-.01	.01	-.00	13.12	0 1 0
1	8	21.7	-4	3	22001	22001	11.29	14.25	-.30	-.30	1.02	-.00	-.00	.00	11.59	0 1 0
1	9	22.4	-2	4	3	6	9.60	13.40	-.00	.00	.00	.00	.00	-.00	9.60	0 1 1
1	10	23.2	-1	6	6	7	7.24	12.38	-.00	.00	.00	.00	-.00	-.00	7.24	0 1 1
2	1	10.4	-259	-14786	-9287	9505	4.26	1.87	.72	-1.12	-.42	.25	-.30	.06	3.18	0 0 0
2	2	10.5	-741	-14271	-8363	8977	3.97	1.94	.63	-1.07	-.33	.14	-.19	.05	3.09	0 0 0
2	3	10.6	-1170	-13436	-7294	8332	3.73	2.00	.52	-1.01	-.24	.07	-.11	.04	3.01	0 0 0
2	4	10.7	-1542	-12400	-6164	7647	3.51	2.06	.42	-.92	-.15	.04	-.07	.03	2.93	0 0 0
2	5	10.8	-1858	-11259	-5021	6987	3.31	2.10	.32	-.84	-.07	.02	-.04	.02	2.86	0 0 0

F-Z(COLLAPSE) = 0. F-Z(HOURGLASSING) = 0. F-Z(EXCESS-T) = 0. F-Z(PELLET ECCEN) = 0.
STRAIN COMPARISONS. FUEL-(R)-2.79E-07, (C) 1.18E-05, (Z) 1.29E-05 CLAD-(R) 1.80E-07, (C) 7.22E-07, (Z) 1.23E-07
FORCE BALANCE FUEL CIRC=-6.149E-02 FUEL AXIAL=-7.115E-02 CLAD CIRC=-3.154E-01 CLAD AXIAL=-2.510E-02
COMPARTMENT F/CC= 9.5369E+16 AVERAGE F/CC= 1.0000E+17 EFPH= 2.0808E+00 GAS RELEASED= 5.2020E-07 HGAP= 6.1800E+00
TIME STEPS MIN= 985711-03 MAX= 139537+00 AVG= 619322-01

**********HISTORY SET 1, CYCLE NUMBER 1, STEP 3

DTIME	QC/QCMAX	T(WATER)	P(WATER)	FSUP.MAX	FIS.RATE	DEPLTN	F.FLUX	FLUENCE	P(GAP)	DP COLAPS
	1.0020	653.60	2000.00	.0	1.5204E+13	1.0000E+17	1.2024E+14	7.9084E+17	0.	2000
860.9790	1.1200	653.60	2000.00	.0	1.6995E+13	5.0001E+19	1.3440E+14	3.9542E+20	0.	2000

TIME	KTIME	DT-ROB	DD(M)	F/C GAP	DD(C)	FCONR	FCONZ	FSUP	EZ(M)	EZ(C)	ESUP	ESLIP	EPSGZ(C)
3.664	60		2.97	1.75	.64	0.	.0	.0	12.62	2.93	.00	2.93	.12
864.643	222	3.1E+01	3.43	1.34	.31	0.	.0	.0	13.98	3.87	.00	3.87	.64

KM	KR	TEMP	RADIUS	DED/DT	DH/DT	H	PB	TAVE	EV	VH	VH(1)	VH(2)	VH(3)	VH(4)	VH(5)
1	1	3024	0.	11389-05	-42785-02	10000-07	1691	3022	.00041	.00666	.00656	0.	0.	0.	0.
1	2	3020	.00509	11017-05	-40317-02	10000-07	1716	3014	.00046	.00671	.00671	0.	0.	0.	0.
1	3	3008	.01018	10335-05	-35793-02	10000-07	1766	2999	.00056	.00681	.00681	0.	0.	0.	0.
1	4	2989	.01527	94295-06	-29926-02	10000-07	1840	2975	.00073	.00698	.00698	0.	0.	0.	0.
1	5	2962	.02036	77009-06	-19984-02	10000-07	2006	2924	.00113	.00738	.00738	0.	0.	0.	0.
1	6	2886	.03053	45678-06	-74993-03	10000-07	2434	2804	.00231	.00856	.00856	0.	0.	0.	0.
1	7	2721	.04578	51863-06	-14300-03	10000-07	3293	2611	.00304	.00928	.00928	0.	0.	0.	0.
1	8	2501	.06101	30045-09	-97441-05	10000-07	-3446	2322	.00375	.01000	.01000	0.	0.	0.	0.
1	9	2142	.08128	42627-71	-59987-05	10000-07	-2	1939	.00375	.01000	.01000	0.	0.	0.	0.
1	10	1736	.10150	55729-73	-76609-05	10000-07	-4	1471	.00375	.01000	.01000	0.	0.	0.	0.
1	11	1206	.12671												
2	1	752	.12805	20800-05	17683-05	34457-02	5486	742	.00064						
2	2	731	.13263	15858-05	11804-05	21760-02	6596	721	.00064						
2	3	711	.13720	12771-05	76219-06	13409-02	7455	701	.00064						
2	4	691	.14177	10240-05	40201-06	80938-03	7941	682	.00064						
2	5	672	.14634	82222-06	12957-06	48544-03	8038	663	.00064						
2	6	654	.15090												

KM	KE	ET-6	SR	SC	SZ	SG	EPS-R	EPS-C	ELAS-R	ELAS-C	ELAS-Z	CREEP-R	CREEP-C	CREEP-Z	A-THRML	CRACK
1	1	18.8	48	48	-5168	5215	18.21	18.21	.08	.08	-.28	1.27	1.27	-2.53	16.70	0 0 0
1	2	18.8	45	41	-5234	5277	18.14	18.19	.08	.08	-.28	1.23	1.28	-2.50	16.65	0 0 0
1	3	18.9	41	29	-5368	5403	17.99	18.14	.08	.08	-.29	1.14	1.29	-2.43	16.55	0 0 0
1	4	19.0	36	14	-5571	5596	17.77	18.06	.09	.09	-.29	1.02	1.31	-2.33	16.40	0 0 0
1	5	19.2	25	-7	-6037	6046	17.34	17.89	.09	.09	-.32	.78	1.33	-2.11	16.07	0 0 0
1	6	19.6	3	-63	-7242	7213	16.50	17.54	.11	.10	-.37	.32	1.35	-1.67	15.29	0 0 0
1	7	20.3	-7	2	-9875	9872	15.49	17.10	.14	.14	-.49	.28	.30	-.58	14.05	0 1 0
1	8	21.4	-4	3	10339	10339	13.33	16.39	-.14	-.14	.48	-.00	-.00	.01	12.23	0 1 0
1	9	22.3	-2	5	3	7	11.15	15.47	-.00	.00	.00	.00	.00	-.00	9.90	0 1 1
1	10	23.2	-1	7	6	7	8.43	14.30	-.00	.00	.00	.00	-.00	-.00	7.18	0 1 1
2	1	10.4	-175	-9982	-6301	6413	5.38	.49	.49	-.76	-.29	1.87	-2.16	.30	3.22	0 0 0
2	2	10.5	-542	-11994	-7252	7551	4.83	.65	.54	-.90	-.30	1.37	-1.77	.41	3.12	0 0 0
2	3	10.6	-945	-13725	-7695	8582	4.35	.78	.57	-1.03	-.28	.95	-1.43	.47	3.03	0 0 0
2	4	10.7	-1370	-14924	-7530	9402	3.94	.88	.55	-1.13	-.21	.65	-1.14	.49	2.95	0 0 0
2	5	10.8	-1794	-15421	-6900	9886	3.60	.97	.51	-1.17	-.12	.44	-.92	.49	2.86	0 0 0

F-Z(COLLAPSE) = 0. F-Z(HOURGLASSING) = 0. F-Z(EXCESS-T) = 0. F-Z(PELLET ECCEN) = 0.
STRAIN COMPARISONS. FUEL-(R)-2.78E-07, (C) 1.56E-05, (Z) 1.36E-05 CLAD-(R) 1.49E-07, (C) 7.25E-07, (Z) 4.00E-08
FORCE BALANCE FUEL CIRC=-6.436E-04 FUEL AXIAL= 7.433E-03 CLAD CIRC=-3.157E+00 CLAD AXIAL=-2.551E-01
COMPARTMENT F/CC= 4.7685E+19 AVERAGE F/CC= 5.0001E+19 EFPH= 1.0404E+03 GAS RELEASED= 2.6010E-04 HGAP= 7.7520E+00
TIME STEPS MIN= 100000-04 MAX= 298522+03 AVG= 531469+01

**********HISTORY SET 1, CYCLE NUMBER 1, STEP 4

DTIME	QC/QCMAX	T(WATER)	P(WATER)	FSUP.MAX	FIS.RATE	DEPLTN	F.FLUX	FLUENCE	P(GAP)	DP COLAPS
	1.1200	653.60	2000.00	.0	1.6995E+13	5.0001E+19	1.3440E+14	3.9542E+20	0.	2000
788.3790	1.2020	654.30	2000.00	.0	1.8239E+13	1.0000E+20	1.4424E+14	7.9084E+20	0.	2000

TIME	KTIME	DT-ROB	DD(M)	F/C GAP	DD(C)	FCONR	FCONZ	FSUP	EZ(M)	EZ(C)	ESUP	ESLIP	EPSGZ(C)
864.643	222		3.43	1.34	.31	0.	.0	.0	13.98	3.87	.00	3.87	.64
1653.022	229	1.0E+05	3.77	1.04	.06	0.	.0	.0	15.47	4.58	.00	4.58	1.05

KM	KR	TEMP	RADIUS	DED/DT	DH/DT	H	PB	TAVE	EV	VH	VH(1)	VH(2)	VH(3)	VH(4)	VH(5)
1	1	3165	0.	38383-07	-12809-01	10000-07	1518	3163	.00194	.00440	.00440	0.	0.	0.	0.
1	2	3161	.00510	33254-07	-12034-01	10000-07	1450	3154	.00199	.00445	.00445	0.	0.	0.	0.
1	3	3148	.01020	26803-07	-10618-01	10000-07	1296	3137	.00209	.00455	.00455	0.	0.	0.	0.
1	4	3126	.01529	24023-07	-87941-02	10000-07	1109	3111	.00226	.00472	.00472	0.	0.	0.	0.
1	5	3096	.02039	24813-07	-57434-02	10000-07	1128	3054	.00277	.00522	.00522	0.	0.	0.	0.
1	6	3011	.03057	25057-07	-20309-02	10000-07	1937	2919	.00435	.00680	.00680	0.	0.	0.	0.
1	7	2826	.04584	87118-07	-34572-03	10000-07	2600	2704	.00580	.00825	.00825	0.	0.	0.	0.
1	8	2581	.06110	74845-11	-18875-04	10000-07	-2619	2382	.00756	.01000	.01000	0.	0.	0.	0.
1	9	2182	.08140	10488-70	-66634-05	10000-07	-2	1959	.00756	.01000	.01000	0.	0.	0.	0.
1	10	1736	.10165	12698-72	-89546-05	10000-07	-4	1447	.00756	.01000	.01000	0.	0.	0.	0.
1	11	1159	.12688												
2	1	760	.12793	41931-05	39267-03	51651-02	4599	749	.00064						
2	2	738	.13250	28670-05	24738-05	33111-02	6097	727	.00064						
2	3	716	.13708	17118-05	11484-05	20347-02	7437	705	.00064						
2	4	695	.14165	12693-05	51637-06	11792-02	8377	684	.00064						
2	5	674	.14621	11314-05	22668-06	64648-03	8835	664	.00064						
2	6	654	.15078												

KM	KE	ET-6	SR	SC	SZ	SC	EPS-R	EPS-C	ELAS-R	ELAS-C	ELAS-Z	CREEP-R	CREEP-C	CREEP-Z	A-THRML	CRACK
1	1	18.3	-259	-259	-4035	3775	19.58	19.58	.05	.05	-.21	1.26	1.26	-2.51	17.60	0 0 0
1	2	18.3	-216	-130	-4002	3829	19.49	19.55	.05	.06	-.21	1.21	1.26	-2.47	17.54	0 0 0
1	3	18.4	-126	150	-3911	3930	19.32	19.49	.05	.07	-.21	1.12	1.27	-2.39	17.42	0 0 0
1	4	18.5	1	490	-3817	4085	19.07	19.41	.05	.09	-.21	.99	1.29	-2.28	17.25	0 0 0
1	5	18.7	157	538	-4078	4438	18.63	19.22	.06	.09	-.23	.76	1.31	-2.06	16.88	0 0 0
1	6	19.2	83	-466	-5427	5257	17.87	18.87	.09	.06	-.28	.32	1.34	-1.66	16.00	0 0 0
1	7	20.0	-7	2	-7796	7793	16.98	18.46	.11	.11	-.39	.32	.34	-.66	14.61	0 1 0
1	8	21.1	-4	3	7859	7860	14.97	17.80	-.11	-.11	.37	-.00	-.01	.01	12.57	0 1 0
1	9	22.2	-3	5	3	7	12.49	16.90	-.00	.00	.00	.00	.00	-.00	9.98	0 1 1
1	10	23.3	-1	7	7	8	9.53	15.69	-.00	.00	.00	.00	-.00	-.00	7.02	0 1 1
2	1	10.4	-149	-8451	-5197	5446	5.83	-.47	.41	-.65	-.23	2.76	-3.26	.51	3.25	0 0 0
2	2	10.5	-477	-11206	-6608	7102	5.18	-.27	.51	-.85	-.27	2.12	-2.76	.65	3.15	0 0 0
2	3	10.6	-871	-13780	-7659	8676	4.59	-.10	.58	-1.04	-.27	1.56	-2.31	.75	3.05	0 0 0
2	4	10.7	-1312	-15727	-8091	9922	4.08	.04	.60	-1.19	-.24	1.12	-1.93	.81	2.96	0 0 0
2	5	10.8	-1772	-16819	-7913	10702	3.66	.16	.58	-1.27	-.17	.81	-1.64	.83	2.87	0 0 0

F-Z(COLLAPSE) = 0. F-Z(HOURGLASSING) = 0. F-Z(EXCESS-T) = 0. F-Z(PELLET ECCEN) = 0.
STRAIN COMPARISONS. FUEL-(R)-2.96E-07, (C) 1.72E-05, (Z) 1.57E-05 CLAD-(R) 2.26E-07, (C) 8.31E-07, (Z)-3.67E-08
FORCE BALANCE FUEL CIRC= 6.363E-03 FUEL AXIAL=-8.247E-03 CLAD CIRC=-2.158E+00 CLAD AXIAL=-1.743E-01
COMPARTMENT F/CC= 9.5368E+19 AVERAGE F/CC= 1.0000E+20 EFPH= 2.0808E+03 GAS RELEASED= 5.2019E-04 HGAP= 9.4574E+00
TIME STEPS MIN= 122463+02 MAX= 340068+03 AVG= 112626+03

********** END OF INPUT PACKAGE CASE
CALCULATION COUNTS. KTIME = 229, NUMBER OF CALLS TO CTAC = 309, RUN TIME = .460, HIGHEST STRAIN DISCREPANCY 1.721E-05

SUMMARY COUNT OF LIMITING CRITERIA--HISTORY 4, TEMPERATURE (DTEMPMAX, TREF-X, Q-X) 14, SIZE CHANGE (SIZERATIO) 1
GAP (GAPOVR, FGAPOVR) 14, F-C SLIP (SLIPOVR, FZOVR) 19 R-S SLIP (SLIPOVR, FZOVR) 14, DENSIFICATION (DENSRATIO) 0
CRACKING (CRAKOVR, CLOSOVR) 1, CREEP RATE JUMP (CRJUMP) 9, STRESS CHANGE (EDOT-CYGRO) 50, (SGJUMP) 58
HARDENING (EDOT-CYGRO) 5, COMPLIANCE (COMPRATIO) 68, ROBOT TIME STEP (EDOT,TAU-ROBOT) 0

FOR THIS INPUT PACKAGE CASE SOME EXTREME VALUES WERE--FUEL CENTERLINE TEMPERATURE 3176.8 (AT T = 1626.6476), MID-CLAD GENERALIZED STRESS 8676.0 (AT T = 1653.0220), AND HOOP STRESS AT INSIDE OF CLAD -.0 (AT T = .0100)

SO FAR IN THIS PROBLEM SOME EXTREME VALUES ARE--CLAD DIAMETER SHRINKAGE .01 (AT T = .0100), CLAD DIAMETER INCREASE .64 (AT T = 3.6440) AND MINIMUM MID-CLAD RADIAL PLASTIC STRAIN -.00 (AT T = .08)

EXTREME VALUES OF STRAIN DISCREPANCIES--

	R	AT TIME	C	AT TIME	Z	AT TIME
FUEL ALGEBRAIC MAXIMUM	4.53E-07	1.573866	1.72E-05	1653.022000	1.57E-05	1653.022000
FUEL ALGEBRAIC MINIMUM	-3.00E-07	1626.647595	-4.27E-07	95.100230	-2.93E-07	1286.579638
CLAD ALGEBRAIC MAXIMUM	2.26E-07	1653.022000	8.31E-07	1658.022000	1.25E-07	3.681076
CLAD ALGEBRAIC MINIMUM	-5.43E-09	1.330990	-1.95E-12	.010000	-8.67E-08	1653.022000

PROBLEM COMPLETED

THIS PROBLEM INCLUDED 4 HISTORY CARD TIMES.

**********END OF OUTPUT**********OK-NORMAL END

THIS VERSION WILL RUN AT A MINIMUM FIELD LENGTH OF 137 OCTAL THOUSAND.

Appendix III. COMPUTER PROGRAM ABSTRACT

1. Program Name (and Title): CYGRO-4

2. Computer and Language(s): CDC-6600, FORTRAN IV

3. Problem Solved: CYGRO-4 (Ref. a) has been developed from the CYGRO-1, CYGRO-2, and CYGRO-3 (Ref. b) series of fuel rod analysis programs. It was derived from CYGRO-3 by the inclusion of improved numerical methods, improved programming procedures and revised physical models. Significant developments include time step control based on accuracy considerations, a new fuel-clad and rod-support interaction model, a new thermal conductivity model for the fuel-clad gap, and a new fuel cracking, crack deformation and crack healing model. The program calculates temperatures, deformations and stresses in cladded fuel rods. Axial and circumferential uniformity are assumed.

Temperature computations assume thermal equilibrium. The temperature distribution is thus a function of current reactor power level, gamma heating and nuclear self shielding, and the thermal conductivities of fuel, clad, fuel-clad gap and rod-coolant interface. Fuel conductivity is calculated from the input data as a function of the current, local temperature, porosity and cracking. Gap conductivity depends on gap size, thermal radiation, conduction at points of fuel-clad contact, and the conductivity of the fluid in the gap. This conductivity depends on composition on temperature and - where appropriate - on pressure.

Deformation of the fuel and clad, the stresses in them, and the forces of interaction between fuel and clad and rod and support are computed from a Large Deflection Finite Element model incorporating representations for thermo-elasticity, creep resulting from thermally activated mechanisms or mechanisms associated with bombardment by high energy particles, fuel swelling and densification, clad growth resulting from neutron bombardment, and fuel cracking and crack healing. In keeping with the Finite Element representation, the program accounts for the effects of spatial - as well as temporal - variation of temperature, stress, strain, cracking, and densification. This is an important feature of CYGRO-4.

Another important feature of CYGRO-4 is the comprehensive model for fuel-clad and rod-support interaction. In addition to interacting by radial stresses, the fuel and clad can interact through axial forces. Fuel and clad axial strain rates may be the same

or different depending on fuel-clad gap size, the direction and magnitude of the axial interaction force, and the effects of pellet "hourglassing", eccentric pellets, lodged chips, and clad collapse, where the effects are relevant. Similarly, the rod axial strain rate may be the same as, or different from, the rate of support extension. Relative motion between rod and support depends on the magnitude of the axial interaction force and the force needed for slippage. The support extension includes the effects of flexibility and thermal expansion, as well as user input extension as might arise, perhaps, from the representation of support extension associated with neutron flux induced growth. The comprehensive form of the fuel-clad and rod-support interaction model is particularly important in the correct representation of rod behavior in response to reactor power changes such as those encountered in "swing load" operation.

Input to CYGRO-4 is divided into two parts: rod data and operating history. The rod data comprises all quantities needed to characterize the geometry and physical properties of the rod, including such information as clad diameter and thickness, fuel diameter, and fuel and clad thermal conductivities, thermal expansion properties, creep behavior and response to the reactor environment. The operating history consists of the measured, or predicted, values of coolant pressure, coolant temperature, reactor power level, and the relevant relationships between power level, fission rate and neutron flux. The program computes the corresponding history of rod temperature, dimensional changes, and stresses.

4. Method of Solution: The fuel and clad are divided into a number of concentric, ring shaped finite elements. Equations for balance of forces and continuity of displacement between the rings determine the principal unknowns. Plastic flow (creep) is treated on an incremental basis. The required sizes of time steps are calculated internally on the basis of accuracy limits.

5. Restrictions on the Complexity of the Problem: A maximum of thirty rings (fuel plus clad) is permitted.

6. Related and Auxiliary Programs: CYGRO-4 uses Bettis programming environment routines (Ref. c) such as CARDS for input data, and FMG for writing binary data files for problem restarts and saving results for post-processing. The environmental routines are available with the CYGRO-4 program upon request.

7. Typical Running Time: Running time depends on complexities of history input and number of elements. The program completes about 450 time steps per c.p.u. minute. The number of steps per reactor hour depends on the complexity of the power history and the severity of the reactor environment. Practical problems display a range from about 0.5 reactor hours per step to about 10 hours per step.

8. Machine Requirements: CDC-6600 - 140 K of central memory
10 K of ECS

9. Operating System: CDC-6600 - SCOPE 3.3

10. Availability: Reference (a) is available from

U.S. Department of Commerce
National Technical Information Service
5285 Port Royal Road
Springfield, Virginia 22151

Copies of the computer program may be obtained by domestic users from

Argonne Code Center
Attention: Mrs. Margaret Butler
Argonne National Laboratory
9700 South Cass Avenue
Argonne, Illinois 60440

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