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$\sin^2\theta_W$ and RADIATIVE CORRECTIONS

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Precision measurements of $\sin^2\theta_W$ and the effects of radiative corrections are surveyed. A world average $\sin^2\theta_W = 0.229 \pm 0.004$ is obtained. Comparison of deep-inelastic $\nu_\mu N$ scattering and m_W or m_Z is shown to test the standard model at the quantum loop level and constrain new physics. Implications for grand unified theories are briefly discussed.

Introduction

Neutral current measurements of $\sin^2\theta_W$ and direct determinations of m_W and m_Z have now become precise enough to start testing the standard $SU(2)_L \times U(1)$ model at the level of its $O(\alpha)$ radiative corrections. Those tests confirm the standard model[1] and constrain new physics that might be appended to it. Furthermore, they now provide an accurate value for $\sin^2\theta_W$ which can be used to rule out or at least constrain some grand unified theories. In this talk, I will survey the present status of the most precise $\sin^2\theta_W$ experiments, focusing on the effect of radiative corrections and discuss some constraints on new physics implied by those measurements.

Listed in Table 1 are the values of $\sin^2\theta_W^{\text{unc.}}$ obtained from various types of experiments before taking electroweak radiative corrections into account. The numerical shift due to $O(\alpha)$ electroweak corrections and the final $\sin^2\theta_W \equiv 1-m_W^2/m_Z^2$ values are also given. Those radiative corrections were evaluated in the standard model assuming $m_t \simeq 36\text{GeV}$ and $m_H \simeq m_Z$. The effect of relaxing the assumption $m_t = 36\text{GeV}$ will be subsequently described. (Throughout this paper, I assume that ordinary QED corrections are separately accounted for by experimentalists.)

Let me begin by briefly commenting on each of the $\sin^2\theta_W$ measurements illustrated in Table 1.

Atomic Parity Violation: Cesium is the simplest heavy atomic system in which parity violation has been observed. Experiments in Paris and Boulder find for the so-called weak charge[2,3]

$$Q_W(Cs) = -71.7 \pm 5.8 \quad \text{(Exp. Ave.)} \qquad (1a)$$

where the error comes from experiment and atomic theory. That is to be compared with the standard model prediction (for $\sin^2\theta_W \equiv 1 - m_W^2/m_Z^2$)

$$Q_W(Cs) = -23 - 220\sin^2\theta_W^{\text{unc.}} \quad \text{(tree level)} \qquad (1b)$$

$$Q_W(Cs) = -22.5 - 216\sin^2\theta_W \quad \text{(inc. R.C.}^{14}\text{)} \qquad (1c)$$

Comparison of these quantities leads to the values for $\sin^2\theta_W^{\text{unc.}}$ and $\sin^2\theta_W$ in Table 1. The good agreement between $\sin^2\theta_W$ extracted in this way and the world average value provides one of the best constraints for models with extra neutral gauge bosons. Further measurements and continued scrutiny of the atomic theory may reduce the error in $Q_W(Cs)$ by a factor of 3 or 4.

eD Asymmetry: The asymmetry,

$$A \equiv (d\sigma_R - d\sigma_L)\,/\,(d\sigma_R + d\sigma_L),$$

in the scattering of polarized electrons on an unpolarized deuterium target was measured a number of years ago at SLAC. Without taking electroweak radiative corrections into account they found[4]

$$\sin^2\theta_W^{\text{unc.}} = 0.224 \pm 0.020 \qquad (2a)$$

Radiative corrections[15,16] reduce the predicted asymmetry by about 3% and thereby lead to

$$\sin^2\theta_W = 0.218 \pm 0.020. \qquad (2b)$$

$\overset{(-)}{\nu}_\mu e$ Scattering: Experiments at BNL[5] and CERN[6] have measured the quantity

Table 1: Values of $\sin^2\theta_W$ before and after electroweak radiative corrections are included for a variety of experiments. The values $m_t \simeq 36$GeV and $m_H \simeq m_Z$ were employed in the radiative correction.

Experiment	$\sin^2\theta_W^{\text{unc.}}$	Radiative Correction	$\sin^2\theta_W \equiv 1 - m_W^2/m_Z^2$
Cs Atomic P.V. (Paris[2]- Boulder[3])	0.221 ± 0.027	+0.007	0.228 ± 0.027
eD Asymmetry (Yale - SLAC[4])	0.224 ± 0.020	-0.006	0.218 ± 0.020
$\overset{(-)}{\nu}_\mu e$ (BNL[5]- CHARM[6])	0.212 ± 0.023	$\lesssim 0.001$	0.212 ± 0.023
$\overset{(-)}{\nu}_\mu P$ (BNL[7])	$0.220 \pm 0.016^{+0.023}_{-0.031}$	small	$0.220 \pm 0.016^{+0.023}_{-0.031}$
$\nu_\mu N$ deep-inel. (CDHS,[8] CHARM,[9]) (CCFRR,[10] FMM[11])	0.242 ± 0.006	-0.011	0.231 ± 0.006
$m_W = 80.8 \pm 1.4$GeV (UA1[12]- UA2[13])	0.213 ± 0.008	+0.016	0.229 ± 0.008
$m_Z = 92.3 \pm 1.7$GeV (UA1[12]- UA2[13])	0.205 ± 0.011	+0.022	0.227 ± 0.011
World Average			0.229 ± 0.004

$$R \equiv \frac{\sigma(\nu_\mu e \to \nu_\mu e)}{\sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e)} = \frac{3\left(1 - 4s^2 + 16/3 s^4\right)}{\left(1 - 4s^2 + 16 s^4\right)} \qquad (3a)$$

$$s^2 \equiv \sin^2\theta_W$$

to be

$$R = 1.38^{+0.40}_{-0.31} \pm 0.17 \quad \text{(BNL Exp.)} \qquad (3b)$$

$$R = 1.26^{+0.72}_{-0.45} \quad \text{(CHARM Collab.)} \qquad (3c)$$

Those measurements lead to the value of $\sin^2\theta_W$ in Table 1. Radiative corrections[17] are negligible ≤ 0.001. A CHARM II experiment at CERN hopes to collect several thousand $\overset{(-)}{\nu}_\mu e$ events and thereby lower the statistical error in $\sin^2\theta_W$ obtained from Eq. (3a) to ± 0.005.

$\overset{(-)}{\nu}_\mu P$ Elastic Scattering: Recent analysis of the ratio $\sigma^{el}(\nu_\mu P)/\sigma^{el}(\bar{\nu}_\mu P)$ by a BNL[7] experimental collaboration yields the value of $\sin^2\theta_W$ in Table 1. A complete analysis of the radiative corrections has not been carried out, but most corrections cancel in such a ratio. Continuing data analysis should lower the statistical error in $\sin^2\theta_W$ considerably.

Deep-Inelastic $\nu_\mu N$ Scattering: New high statistics results for

$$R_\nu \equiv \sigma(\nu_\mu N \to \nu_\mu X) / \sigma(\nu_\mu N \to \mu X)$$

have been reported at this conference.[8,9,10] That quantity must be corrected for sea quark effects, QCD corrections, charm thresholds, etc.[1] After all such corrections are made, the experimental value for R_ν can be compared with the standard model valence quark prediction[18,19]

$$R_\nu = \rho_{\nu N}^2 \left[\frac{1}{2} - \kappa \sin^2\theta_W + \frac{20}{27}\kappa^2 \sin^4\theta_W\right] \qquad (4a)$$

$$\rho_{\nu N}^2 = 1 + O(\alpha) \simeq 0.98 \qquad (4b)$$

$$\kappa = 1 + O(\alpha) \simeq 0.996 \qquad (4c)$$

and $\sin^2\theta_W$ can be extracted. Setting $\rho_{\nu N} = \kappa = 1$ in Eq. (4a) leads to $\sin^2\theta_W^{\text{unc.}}$ given in Table 1. (It is dominated by the recent results from CDHS[8] and CHARM.[9]) Electroweak radiative corrections[16,18,19] tend to reduce $\sin^2\theta_W$ by about 4% relative to $\sin^2\theta_W^{\text{unc.}}$. In fact, the shift $\Delta s^2 \equiv \sin^2\theta_W^{\text{unc.}}(\nu N) - \sin^2\theta_W$ can be approximated by

$$\frac{\Delta s^2}{s^2} \simeq -4\Delta\rho - \Delta\kappa \qquad (5)$$

where $\Delta\rho = \rho_{\nu N} - 1, \quad \Delta\kappa = \kappa - 1.$

$\underline{W^{\pm}\text{ and }Z\text{ Masses}}$: The natural relationship (with $\Delta r = 0$)

$$m_W^2 = m_Z^2 \cos^2\theta_W = \frac{\pi\alpha}{\sqrt{2}G_\mu \sin^2\theta_W\,(1-\Delta r)} \quad (6)$$

is modified by $O(\alpha)$ electroweak radiative corrections embodied in Δr. An evaluation of those corrections gave[18,20,21,22]

$$\Delta r = 0.0696 \pm 0.0020 \quad (7)$$

(A more recent analysis by F. Jegerlehner[23] gives $\Delta r = 0.0711 \pm 0.007$.) Setting $\Delta r = 0$ in Eq. (6) and using $\alpha = 1/137.036$, $G_\mu = 1.16636 \times 10^{-5}\text{GeV}^{-2}$ leads to the lowest order relations

$$\sin^2\theta_W^{\text{unc.}} = \left(\frac{37.281\text{GeV}}{m_W}\right)^2 \quad (8a)$$

$$\sin^2 2\theta_W^{\text{unc.}} = \left(\frac{74.562\text{GeV}}{m_Z}\right)^2 \quad (8b)$$

Including the Δr value in Eq. (7) modifies them to

$$\sin^2\theta_W = \left(\frac{38.65\text{GeV}}{m_W}\right)^2 \quad (9a)$$

$$\sin^2 2\theta_W^{\text{unc.}} = \left(\frac{77.30\text{GeV}}{m_Z}\right)^2 \quad (9b)$$

Employing the average m_W and m_Z masses in Table 1 then leads to the values of $\sin^2\theta_W^{\text{unc.}}$ and $\sin^2\theta_W$ given there.

Glancing at the $\sin^2\theta_W^{\text{unc.}}$ and $\sin^2\theta_W$ columns in Table 1, it is clear that the uncorrected $\sin^2\theta_W^{\text{unc.}}$ value obtained from deep-inelastic $\nu_\mu N$ scattering differs from the m_W and m_Z values, but the $\sin^2\theta_W$ results are all in good agreement after radiative corrections are accounted for. The standard model is, therefore, being tested at the level of its $O(\alpha)$ electroweak radiative corrections.

To make the above comparison more quantitative, consider the quantities

$$\Delta r + \frac{\Delta s^2}{s^2} \simeq 1 - \left(\frac{37.281\text{GeV}}{m_W \sin\theta_W^{\text{unc.}}(\nu N)}\right)^2 \quad (10a)$$

$$\Delta r + \frac{\Delta \sin^2 2\theta_W}{\sin^2 w\theta_W} \simeq 1 - \left(\frac{74.562\text{GeV}}{m_Z \sin 2\theta_W^{\text{unc.}}(\nu N)}\right)^2 \quad (10b)$$

where $\sin^2\theta_W^{\text{unc.}}(\nu N)$ specifies the uncorrected value extracted from deep-inelastic data. Theory predicts (for $m_t \simeq 36\text{GeV}$) that these radiative corrections are 0.11 and 0.10 respectively. Inserting the values of $\sin^2\theta_W^{\text{unc.}}(\nu N)$, m_W and m_Z from Table 1 into these expressions gives 0.12 ± 0.04 and 0.11 ± 0.04. The agreement between theory and experiment is very good. It illustrates quite nicely the need for radiative corrections.

New physics appendages to the standard model or changing the input m_t value would modify the theoretical predictions in Eq. (10). We can, therefore, use the present good agreement with experiment as a constraint. For example, if m_t were very large, the prediction for Δr would decrease whereas Δs^2 hardly changes. For $m_t^2/m_W^2 \gg 1$, one finds the leading term in the Δr shift is given by[21]

$$-\frac{3\alpha}{16\pi}\frac{\cos^2\theta_W}{\sin^4\theta_W}\frac{m_t^2}{m_W^2} \quad (11)$$

The exact dependence on m_t can be found in Refs. 21 and 22. From the present experimental value of Δr, A. Sirlin and I find the constraint[22]

$$m_t \lesssim 180\text{GeV} \quad (12)$$

That bound also applies to a 4th generation mass difference $m_{t'} - m_{b'}$. In the case of a heavy 4th generation charged lepton L with a massless neutrino partner, it becomes

$$m_L \lesssim 300\text{GeV} \quad (13)$$

These constraints are starting to become quite interesting. Future high precision measurements of m_W and m_Z should determine them to ± 0.1GeV and thus provide even better constraints (or perhaps a hint of new physics). Indeed, comparison of neutral current measurements with m_W and m_Z also provides a powerful probe of additional neutral gauge boson effects.[24]

As a final comment, I would like to call attention to the fact that the world average

$$\sin^2\theta_W = 0.229 \pm 0.004 \quad (14)$$

is now rather well determined and somewhat higher than in the past. The value in Eq. (14) has important implications for grand unified theories (GUTS). Indeed, it contradicts the minimal SU(5) model's prediction[25]

$$\sin^2\theta_W = 0.214 \pm 0.004 \quad (\text{Minimal SU}(5)) \quad (15)$$

Of course, that model has already been ruled out by proton decay experiments.[26] In the case of non-minimal GUTS, Eq. (14) can be used to constrain their additional mass scales. For example, it is in good agreement with supersymmetric GUTS with $m_{\text{SUSY}} \gtrsim 1$TeV.

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