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NOTES ON XENON REACTIVITY EFFECTS IN HANFORD REACTORS

By

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ABSTRACT

Equations are presented for the steady state xenon poisoning and potential xenon poisoning in a Hanford reactor, using conventional fuel. These equations contain the dimensions of the reactor, and the thermal utilization, so that they are not limited to present reactor design. The extension of the solution to the transient case, based on the presently used xenon equation, has been made. The use of pile fractions permits the combination of flattening effects and pile power into one parameter, which reduces the amount of calculation involved for a given range of reactor operation. Graphs are presented as an aid in determining the xenon poisoning in a Hanford reactor of present design.

INTRODUCTION

A method for calculating the equilibrium xenon poisoning in a Hanford reactor has been proposed based on pile fractions.[1] Since the dimensions of the reactor enter into the reactivity expressions, the significant parameters will be represented by symbols in this report. Slug constants will remain the same as those used in HW-28729, except for the concentration of U-235 atoms. Here, N_{25} will be taken to be 3.19×10^{20} atoms/cc, rather than 3.09×10^{20} atoms/cc as was used in HW-28729. This number results from correcting the density of uranium to account for the aluminum end caps of 8" slugs.

A method of applying pile fractions to the presently used transient xenon equation will also be developed. This method will not require a conversion from pile fractions to inhours flattening as required by the presently used method. Xenon poisoning will increase with flattening, for a given central tube power, rather than decrease at higher flattening values. This defect in the present method was shown in HW-28729.

Since this report is supplementary to HW-28729, the reader is referred to that document for the symbols used here.

EQUILIBRIUM XENON POISONING

Referring to HW-28729, equations (1) and (2) are valid for a pile of any length. The upper limit in (2) may be larger than 80° for a longer pile since the extrapolation length and charged length are more nearly equal for a long pile. If these two lengths should be equal, a change of 1.5% would result, giving the upper limit of the error.

Rather than (3) in HW-28729 we may write

$$L_s = \frac{-21,560\ f\ Y}{1 + \dfrac{0.739\ L\ W' \int Z^2 r\ dr}{10^6\ p \int Z^3 r\ dr}} \qquad (1)$$

where

$$W' = 9.265\ K\ N_{25}\ X(1 + \delta)\frac{\sigma_f}{\sigma_x} = 342 \text{ watts} \qquad (2)$$

Here, 3.19×10^{20} atoms/cc has been used for N_{25} rather than the value 3.09×10^{20} as in HW-28729. Using the same values for Y (0.064) and $\frac{\sigma_s}{\sigma_f}$ (6146) as before, we have

$$L_s = \frac{1380\ f}{1 + \dfrac{2.53 \times 10^{-4}\ L \int Z^2\ r\ dr}{p \int Z^3\ r\ dr}} \qquad (3)$$

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(1) HW-28729, Xenon Poisoning Calculations Based on Tube Power, J. O. Erkman, 7-15-53

[redacted]

As before, we let

$$\frac{P\int r\, Z^3\, dr}{r\, Z^2\, dr} = \frac{P}{N}\,\frac{\Sigma F_i^3}{\Sigma F_i^2}\quad , \tag{4}$$

so that

$$L_s = \frac{1380\ f}{1 + \dfrac{2.53 \times 10^{-4}\ L\ N \Sigma F_i^2}{P \Sigma F_i^3}}\quad , \tag{5}$$

where N is the number of tubes loaded with uranium, and P is the pile power. Equation (5) can be applied to a pile of any length L, and radius R. The pile radius enters into the limit of the integrals in (3) and the ratio of these integrals may be obtained from general curves in HW-29502, "Weighting Factors for Radially Flattened Piles," by J. O. Erkman.

Equation (5) may be compared with the old xenon equation

$$L_s = \frac{A\ P}{1 + B\ P} \tag{6}$$

where

$A = 4.43\ (1 - 0.00625F)$

$B = 0.00399\ (1 - 0.00625F)$

F = flattening in inhours

It is observed that for F = 1600 inhours A and B go to zero, giving $L_s = 0$ for any value of P. This defect has been pointed out in HW-28729. Rewriting (5) gives

$$L_s = \frac{\dfrac{1380\ fP \Sigma F_i^3}{2.53 \times 10^{-4}\ L\ N \Sigma F_i^2}}{1 + \dfrac{P \Sigma F_i^3}{2.53 \times 10^{-4}\ L\ N \Sigma F_i^2}} \tag{7}$$

so that A and B may be redefined as

$$A = \frac{5.46 \times 10^6\ f\ \Sigma F_i^3}{L\ N\ \Sigma F_i^2} \tag{8}$$

$$B = 3{,}954\ \frac{\Sigma F_i^3}{L\ N\ \Sigma F_i^2} \tag{9}$$

A and B now increase with flattening, overcoming the defect referred to above.

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APPLICATION TO TRANSIENT OPERATION

The transient xenon equation presently in use is(2)

$$L = \frac{AP}{1+BP} + \left[\frac{APX(1-s)-M_oI}{I-X(1+BP)}\right] e^{-It}$$

$$+\left[L_o - \frac{AP}{1+BP} - \frac{APX(1-s)-M_oI}{1-X(1+BP)}\right] e^{-X(1+BP)t} \qquad (10)$$

If a reactor has been operated in an equilibrium condition, A and B are known from (8) and (9). Hence, the transient behavior of the reactor can be determined from (10) without going through the intermediate step of converting pile fractions to inhours of flattening.

The ratio $\frac{\sum F^3}{\sum F^2}$ may be determined from the automatic recording devices in use at some of the reactors, or from the general curves in HW-29502 referred to before. This last method requires an estimation of the flattening in inhours as an intermediate step in the calculations.

Since A and B appear as a multiplier of P in (10), it may be convenient to construct tables, graph or nomographs based on the product $\frac{P}{N} \frac{\sum F^3}{\sum F^2}$. This should lead to a reduction in the amount of work involved in constructing the reference material.

FRACTION OF EQUILIBRIUM XENON POISON IN DISCHARGED METAL

In the development of the xenon reactivity effect, we have the expression(3)

$$L_s = -1380 f \frac{\int \frac{W\, dV}{W + W'}}{\int W^2\, dV} , \qquad (11)$$

where the integrals are evaluated over the loaded zone of the pile. To obtain the poisoning in a limited region of the pile, the integral in the numerator is evaluated over that region, while the denominator is evaluated over the loaded region of the pile as before.

We obtain

$$L_s(\text{discharged}) = \frac{-1380 f \dfrac{\int_{region} Z^2\, r\, dr}{\int_{pile} Z^2\, r\, dr}}{1 + \dfrac{2.53\, L \times 10^{-4}}{P} \dfrac{\int_{region} Z^2\, r\, dr}{\int_{region} Z^3\, r\, dr}} \qquad (12)$$

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(2) HW-25076, Calculation of Xenon in a Hanford Pile, R. O. Brugge, 8-11-52

(3) HW-23729, Review of the Xenon Problem, Appendix B, P. F. Gast

[redacted]

If we let n represent the number of tubes being discharged, we may replace integrals with pile fractions as before

$$L_s \text{ (discharged)} = \frac{-1380\ f \dfrac{\sum^{n} F^2}{\sum^{N} F^2}}{1 + \dfrac{2.53\ L \times 10^{-4}\ N \sum^{n} F^2}{P \sum^{N} F_i^{3}}} \qquad (13)$$

All the sums in (13) may be determined for those reactors with automatic recording equipment. For some discharges[4]

$$\frac{\sum^{n} F^2}{\sum^{n} F_i^{3}} = \frac{\sum^{N} F^2}{\sum^{N} F^3} \qquad (14)$$

In such a case (13) becomes the same as (5), except for the multiplier

$$\frac{\sum^{n} F_i^{2}}{\sum^{N} F_i^{2}} \qquad (15)$$

= fraction of equilibrium xenon discharged with metal

POTENTIAL XENON POISONING

We may use the potential xenon equation for equilibrium operation

$$M_s = \frac{A\ P\ X\ (1-s)}{I} \qquad (16)$$

since we have a knowledge of A, and for which we substitute

$$M_s = \frac{5.46 \times 10^{6}\ f\ P\ X\ (1-s) \sum F_i^{3}}{L\ N\ I \sum F_i^{2}} \qquad (17)$$

The potential poisoning in a group of tubes to be discharged becomes, after examining (14) of HW-28729

$$M_s \text{ (discharged)} = \frac{5.46 \times 10^{6}\ f\ P\ X\ (1-s) \sum^{n} F_i^{2}}{L\ N\ I \sum^{N} F_i^{2}} \qquad (18)$$

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(4) J. B. Czirr, Private communication

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Taking the ratio of (18) to (17) we have

$$\text{Fraction of potential poison discharge} = \frac{\sum^{n} F_i^3}{\sum^{N} F_i^3} \qquad (19)$$

TRANSIENT BEHAVIOR OF POTENTIAL XENON POISON

The equation for potential xenon poison is[5]

$$M = \frac{A\ PX\ (1-s)}{I}\ (1-e^{-It}) + M_o\ e^{-It} \qquad (20)$$

The constant A is defined by (8) so that M may be obtained for any radial flux configuration in the reactor.

SUMMARY AND CONCLUSIONS

The present xenon equations can be adapted so that pile fractions may be used directly in calculations. If the actual pile fractions are unknown, but the flattening is known, recourse may be made to the curves of HW-29502. Either method will overcome the chief defect inherent in the presently used tables which go to zero at a flattening of 1600 inhours. The use of only a single parameter, rather than the two now used (flattening and pile power) permits one table of values to cover the same range of reactor operation which formerly required several tables.

The flattening dependent quantities A and B which occur in the xenon equations may be calculated from

$$A = \frac{5.46 \times 10^6\ f\ \sum F_i^3}{L\ N\ \sum F_i^2} \qquad (21)$$

$$B = \frac{3{,}954\ \sum F_i^3}{L\ N\ F_i^2} \qquad (22)$$

Values of F, L and other quantities are given by D. K. McDaniel[6] for present reactors and K reactors as follows:

Quantity	Present Piles	K-Piles
f	0.865	0.88
L.	726 + 86 cm	864 + 86 cm
R_o	537.5 cm	609.7 cm
R	587.5 cm	659.7 cm
ih/μb	24.6	22.0

(5) HW-25076
(6) HW-30282, K-Pile Xenon Constants, D. K. McDaniel, 12-23-53

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Hence, all the quantities necessary for the calculation of A and B for any reactor using convential fuel are available. In Appendix I, graphs are presented which enable one to solve the xenon problem for wide ranges of power levels and flattenings for the present piles.

APPENDIX I

A and B are evaluated using the following: L = 812, f = 0.865.

$$A = 5.815 \times 10^3 \frac{\sum F_i^3}{N \sum F_i^2} \qquad (1)$$

$$B = 4.87 \frac{\sum F_i^3}{N \sum F_i^2} \qquad (2)$$

In all equations, A and B are multiplied by P, the pile power. The quantity

$$\frac{P}{N} \frac{\sum F_i^3}{\sum F_i^2}$$

will be used as a parameter, each value of which will specify a value for AP and BP, thereby specifying the xenon poisoning present in the pile. The quantities that must be calculated are given in Table I, for which s has been given the value 0.051. The significance of the symbols used in Table I are as follows:

$$L_s = \frac{AP}{1+BP} \qquad (3)$$

$$M_s = \frac{APX\ (1-s)}{I} \qquad (4)$$

$$C = \frac{APX(1-s)}{I-X(1+BP)} \qquad (5)$$

$$D = \frac{I}{I-X(1+BP)} \qquad (6)$$

$$E = e^{-X(1+BP)} \qquad (7)$$

$$e^{-I} = 0.914 \qquad (8)$$

[redacted]

The xenon equation may now be written

$$L = L_s + (C - M_o D)\, 0.914^t + \left[L_o - L_s - (C - M_o D) \right] E^t \qquad (9)$$

The iodine expression is

$$M = M_s - (M_s - M_o)\, 0.914^t \qquad (10)$$

The quantities L_s, M_s, C, D, and E may be determined either from Table I or from the attached graphs, for a considerable range of the parameter. t is time in units of hours.

TABLE I

Parameter	L_s	M_s	C	D	E
0.00	0	0	0	3.643	0.9275
0.10	391	400	-5076	-12.68	0.8942
0.15	504	600	-2347	- 3.908	0.878
0.20	589	801	-1852	- 2.313	0.862
0.25	655	1001	-1643	- 1.641	0.846
0.30	709	1201	-1529	- 1.273	0.831
0.35	752	1401	-1455	- 1.039	0.816
0.40	789	1601	-1406	- 0.878	0.801
0.45	820	1802	-1369	- 0.760	0.786
0.50	846	2002	-1341	- 0.670	0.772
0.55	869	2202	-1319	- 0.599	0.758
0.60	890	2402	-1301	- 0.542	0.744
0.70	923	2802	-1275	- 0.455	0.718
0.80	950	3203	-1255	- 0.392	0.692
0.90	972	3603	-1240	- 0.344	0.668
1.00	991	4003	-1228	- 0.307	0.643
1.10	1006	4404	-1219	- 0.277	0.620
1.20	1020	4804	-1211	- 0.252	0.598

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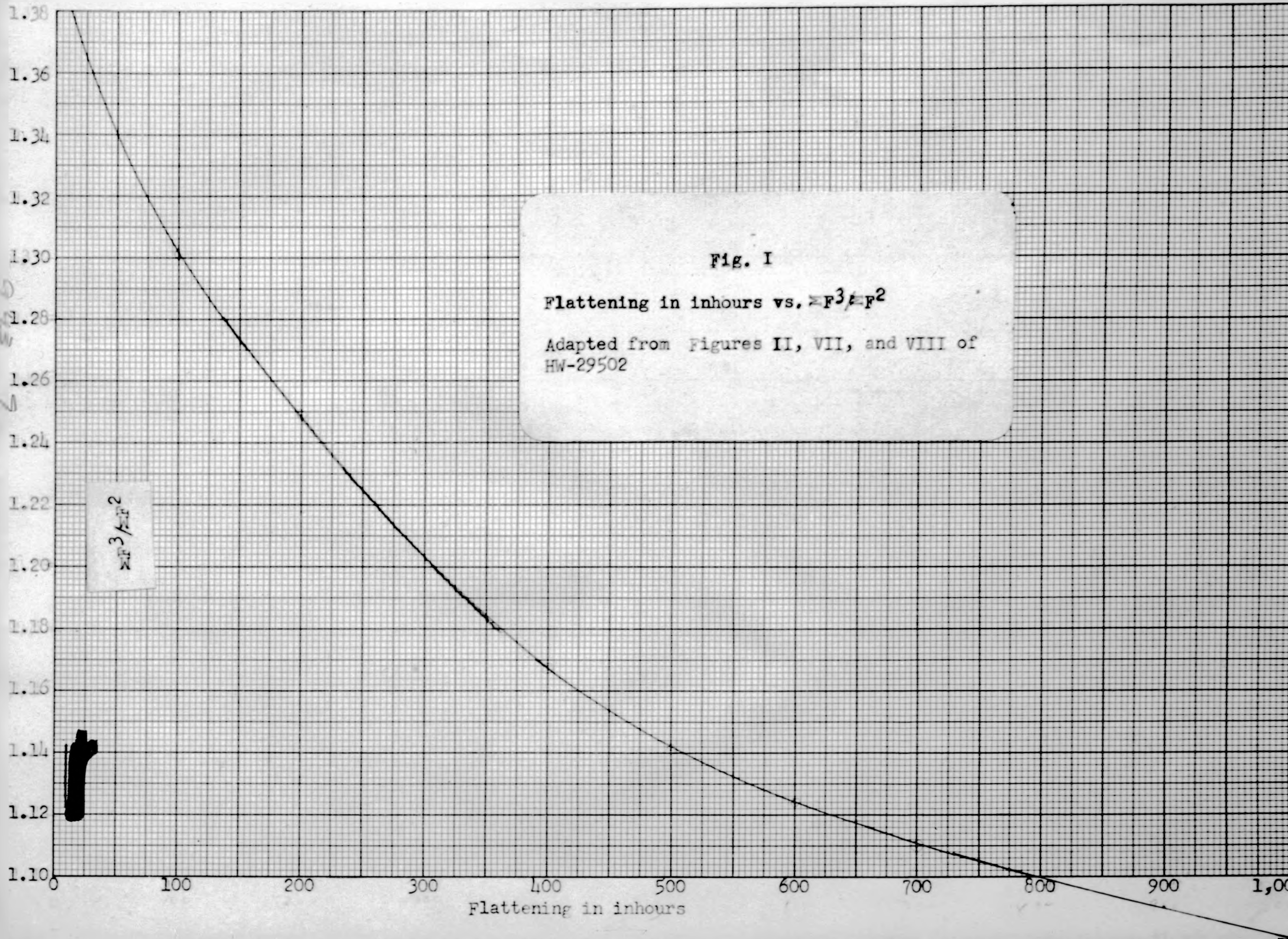

Fig. I

Flattening in inhours vs. $\Sigma F^3/\Sigma F^2$

Adapted from Figures II, VII, and VIII of HW-29502

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HW-31135

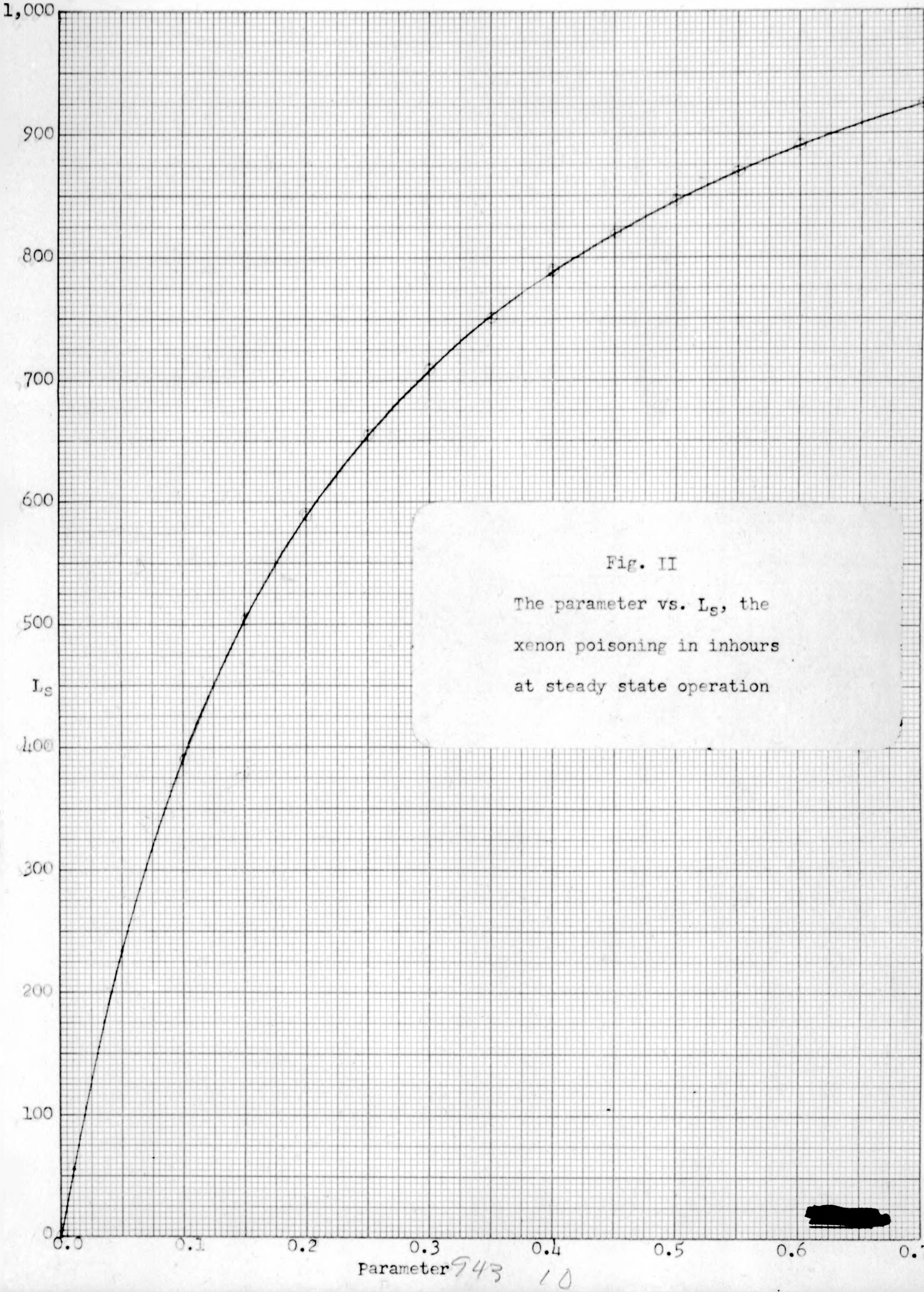

Fig. II

The parameter vs. L_s, the xenon poisoning in inhours at steady state operation

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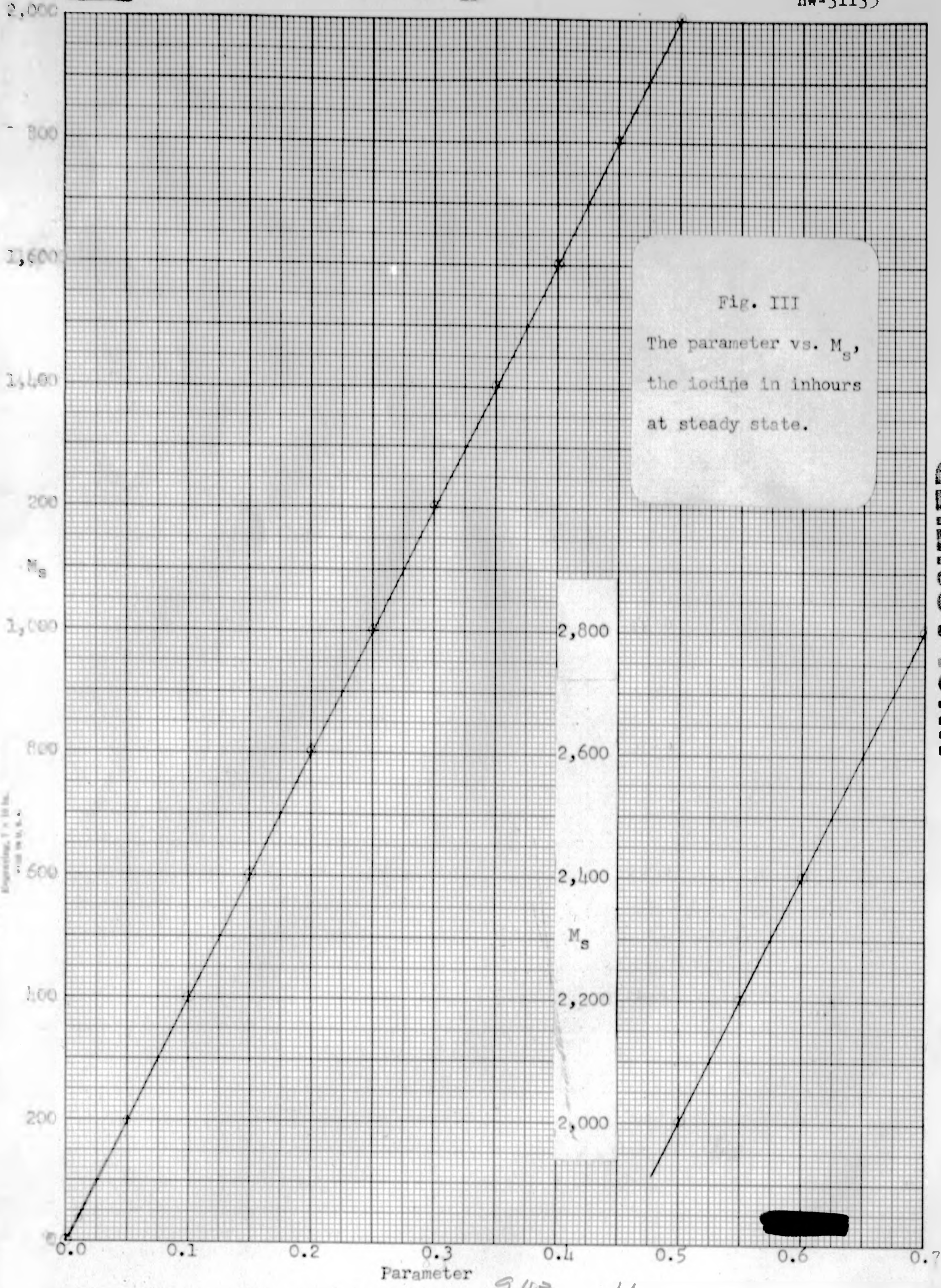

Fig. III
The parameter vs. M_s, the iodine in inhours at steady state.

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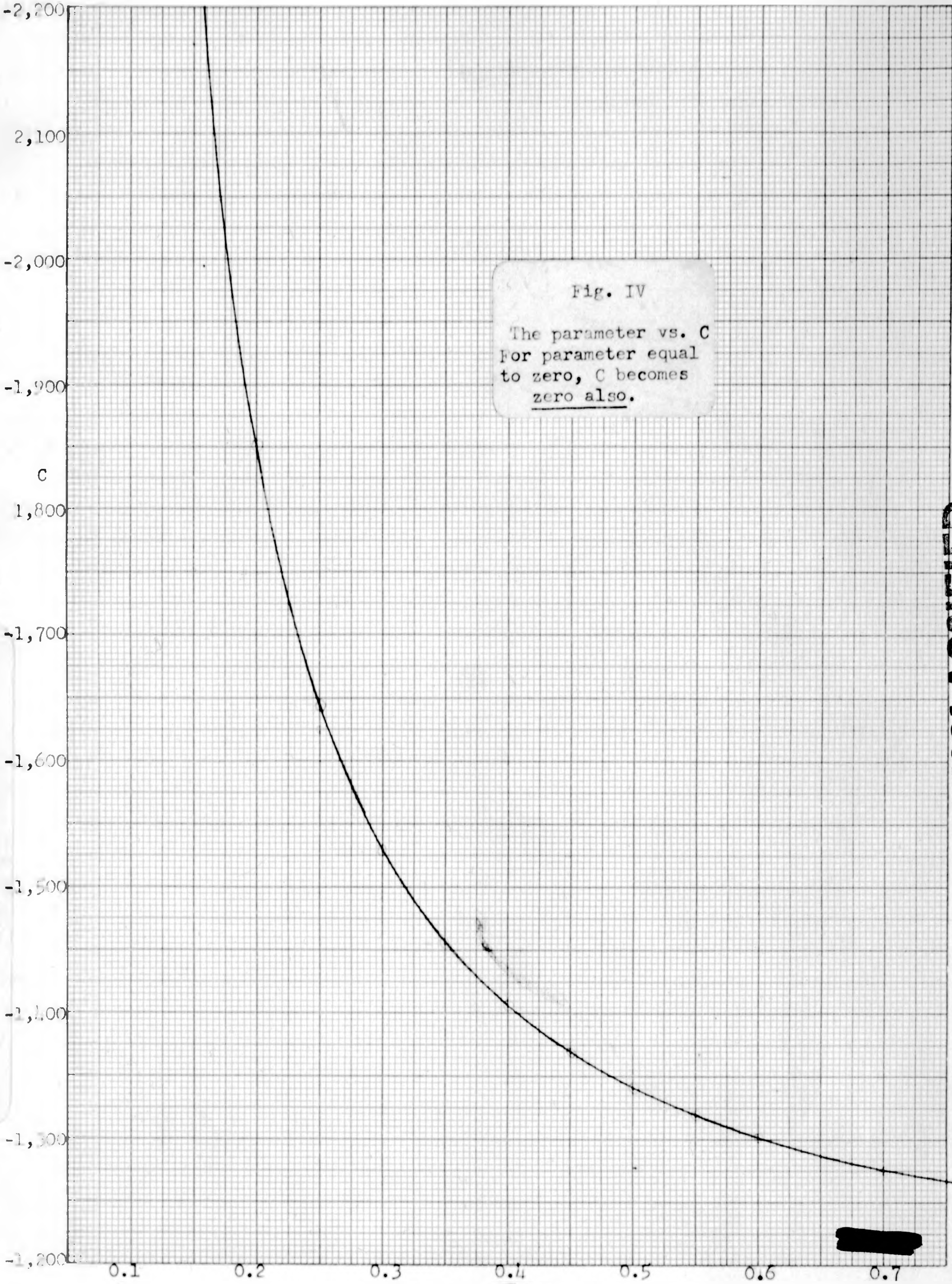

Fig. IV

The parameter vs. C
For parameter equal
to zero, C becomes
zero also.

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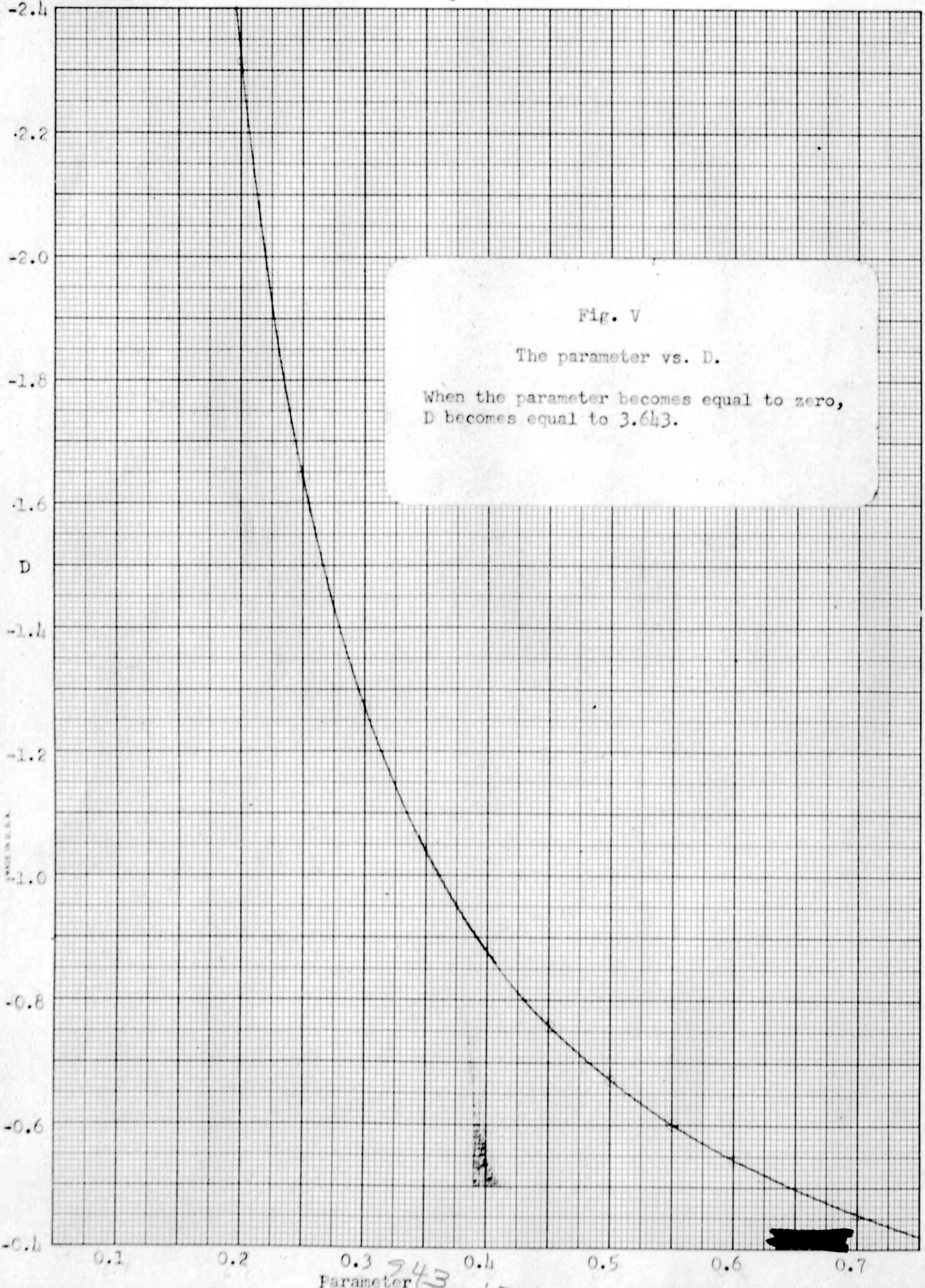

Fig. V

The parameter vs. D.

When the parameter becomes equal to zero, D becomes equal to 3.643.

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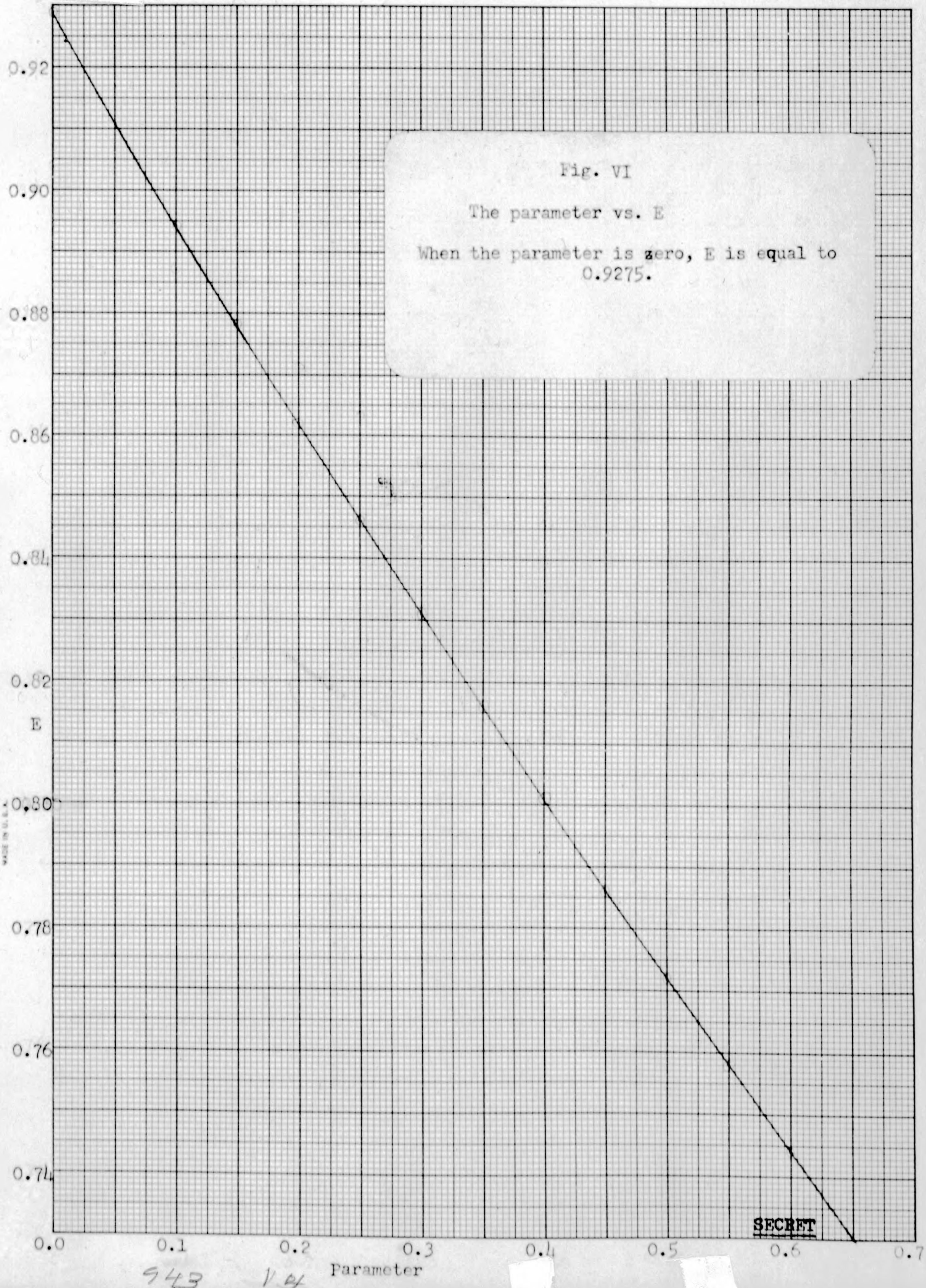

Fig. VI

The parameter vs. E

When the parameter is zero, E is equal to 0.9275.