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SU3 Shell Model Calculations for Light Nuclei*

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I. Introduction

One might ask "Why go to the trouble of performing conventional shell model calculations using the supermultiplet (SU4) and SU3 basis when there is in existence the Glasgow code [1] which requires no cfps, no coupling coefficients and can handle bases with tens of thousands of states?" Failing that "why doesn't one use the readily available Rochester-Oak Ridge jj-coupling code [2] ?"

A partial answer is that truncation according to the overall SU4 and SU3 symmetries of the many-body wave function is a systematic and relatively reliable procedure whereas truncation by restriction on the occupancy of single j orbits is often not satisfactory. When several major shells are active there is a much stronger reason for using SU3 since the problem of spurious centre of mass motion can only be handled satisfactorily in this basis. Also the SU3 wave functions are closely related to those of the weak-coupling, Nilsson and cluster models. This connection may be used to aid in the physical interpretation of the shell model wave function and to suggest how the basis should be extended to improve the description of specific classes of states (as opposed to

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fixing the model space and varying the interaction). The model is well suited to the computation of multinucleon spectroscopic amplitudes and many transition operators, such as the Gamow-Teller, E1 and isoscalar E2 operators, have simple transformation properties under SU3 or SU4 leading to useful selection rules. Finally it is important for our understanding of the physics that a simple hand calculation usually gives a good approximation to the full shell model calculation.

In the following the classification scheme and its physical roots in the properties of the effective interaction are briefly discussed. Then, to illustrate most of the points made in the previous paragraph, calculations for ^{16}O are presented as typical examples of calculations involving several major shells.

II. The Classification Scheme

The labelling scheme for the SU3/SU4 basis[23] is shown in Fig. 1 where the group chains which provide the labels are also indicated. α, β and κ label multiple occurrences of $(\lambda\mu)$, TS and L for the same $[f]$, $[\tilde{f}]$ and $(\lambda\mu)$ respectively. Only κ can be given any physical interpretation -- as a band quantum number. For more than one shell there is coupling in $(\lambda\mu)$, T and S. The SU6 and SU4 Clebsch-Gordan coefficients which make up the cfps have been tabulated -- e.g., for the p-shell by Jahn and van Wieringen[3] in the early 50's -- or are readily calculable. Full exploitation of the SU3 symmetry has been made possible through a very general code of Draayer and Akiyama[4] to calculate the Wigner and U coefficients for SU3.

The physical basis for the choice of labelling rests on the following two points.

(i) [f]: States of the highest symmetry [f] maximise the number of spatially symmetric pairs in the wavefunction and therefore lie lowest since the NN interaction is most strongly attractive in relative S states and is generally repulsive in odd states (strong space exchange force). E.g., in the p-shell the highest symmetry provides greater than 94% of the wave function for $A \leq 10$ and the worst case is A = 13 with 71% (for Cohen and Kurath's (8-16)2BME[5]). Similarly in the sd shell A = 18-21 are all round 90%.

(ii) $(\lambda\mu)$: Away from closed shells nuclei like to deform. Poor man's Hartree-Fock is the Nilsson model and if we go a step further and neglect the $\underline{\ell}\cdot\underline{s}$ and $\underline{\ell}^2$ terms in the Nilsson Hamiltonian the deformed single particle orbits are asymptotic Nilsson orbits. The SU3 intrinsic state is obtained by filling Slater determinants with the lowest available asymptotic Nilsson orbits and if necessary taking linear combinations of these to obtain good SU3 symmetry.[22] E.g., the #6 Nilsson orbit from Chi's tabulation[6] for $\delta = 0.3$ expanded in terms of asymptotic orbits reads $0.965[220]^{+}_{\frac{1}{2}} + 0.261[211]^{+}_{\frac{1}{2}} - 0.029[200]^{+}_{\frac{1}{2}}$ where $[220]^{+}_{\frac{1}{2}} = \frac{1}{\sqrt{3}}\, s_{1/2} + \sqrt{\frac{2}{5}}\, d_{5/2} - \sqrt{\frac{4}{15}}\, d_{3/2}$ etc. Now $(0.965)^8 \sim$ intensity of $(\lambda\mu) = (80)$ for $(\#6)^4 = 0.75$ to be compared with the $K + {}^{17}O$ shell model value = 0.78.

Typically the leading SU3 representation dominates the wave functions of the ground state band 70-80% throughout the first part of the sd-shell depending somewhat, of course, on the interaction. Excited bands are also often recognizable from the SU3 composition of their wave functions.

III. Positive Parity Levels of ^{16}O

The basis, which includes the full $2\hbar\omega$ space and a large class of $4\hbar\omega$ states, is shown in Fig. 2. Those representations marked with an

asterisk are free from spuriosity. In all other cases certain linear combinations of the shell model basis states (with the same overall quantum numbers) correspond to spurious states, as is discussed in Sec. 4, and must be eliminated from the basis. Crudely speaking working downwards through the list corresponds to building correlations within a major shell and across to allowing weak coupling to develop. In the weak coupling model of Ellis and Engeland[7] diagonalized single shell wave functions are coupled in J and T to form a basis. The individual components (one $(\lambda\mu)$ in each shell) may be expanded in the SU3 basis (strong coupling) by converting to LS coupling and using an SU3 $\supset$ R3 Wigner coefficient to obtain states with good total $(\lambda\mu)$.

The most important representations occur in the top left hand corner of each block and are marked with a box. They were chosen by Brown and Green[8] in an attempt to explain the first three 0^+ levels in ^{16}O. The off-diagonal matrix elements were estimated and the unperturbed energies chosen to fit experiment after diagonalization. The wave functions which resulted are compared in Fig. 3 with those from a shell model calculation in a much larger basis. The phase differences are not significant and the close agreement, the major effect being just some dilution, shows how good their physical intuition was.

The spectrum from the full calculation is shown in Fig. 4. The interactions are Cohen-Kurath (8-16)2BME[5] for the p-shell, Kuo-Brown[9] for the sd-shell, Millener-Kurath[10] for the ph and the Kuo bare G-matrix[11] elsewhere. The single particle energies are chosen to fit A = 15 and 17 (s and pf are not important here). The eigenstates have been separated out into those which are dominantly 2p-2h or 4p-4h, there being two well developed bands in the latter case.

The 4p-4h levels have been lowered by 2 MeV so that the 2p-2h and 4p-4h 6^+ levels at 14.8 and 16.2 MeV respectively could be reproduced (θ_α's for these levels are very sensitive to the mixing) i.e., with the intershell spacing such that the 2p-2h states are well reproduced the 4p-4h 0^+ comes ~ 3 MeV too high and the 8^+ at 21.7 MeV which is probably quite reasonable since there is strong alpha transfer strength just above the known 7^- at 20.9 MeV. The band is compressed which is a typical feature of truncated SU3 calculations. By choosing the high ($\lambda\mu$) representation there is a full or at least a relatively large basis for the high spin states. Inclusion of the omitted low ($\lambda\mu$) representations, which contain only low angular momenta, can serve only to depress the low spin members of the band.

Some details of the wavefunctions are shown in Fig. 5. The spectroscopic amplitudes for $^{12}C + \alpha$ show a very regular behaviour with J despite the K mixing apparent in the wavefunctions. To the ^{12}C ground state they are large for the $K^\pi = 0^+$ band and very small for the $K^\pi = 2^+$ band while to the first excited state of ^{12}C they are appreciable for the 0^+ and large for the 2^+ band. The alpha pickup to the 2^+ levels is predicted to favour the upper level by 2 or 3 to 1 whilst alpha stripping favours the lower level by ~ 80 to 1. This prediction is consistent with the experimental data and contrary to the expectations of the simple weak coupling model. The in-band E2's for the $K^\pi = 0^+$ band ($4 \rightarrow 2$ and $2 \rightarrow 0$) are large and well reproduced using the isoscalar effective charge of one half suggested by Mottelson's self consistent deformed harmonic oscillator model.[12] [The $K^\pi = 1^+$ 4p-2h band in ^{18}F where many more transition strengths are known provides a much better example of the ability of the model to reproduce strong in-band E2's]. The cross-band E2's are reproduced to within a factor of two.

In the full $2\hbar\omega$ calculation the 2p-2h energies are well reproduced. However, the energies for the low spin states converge rather slowly as the size of the basis increases. The 6^+ is obtained early in a highly truncated basis (non-spurious representations only) but the low spin $T = 0$ and the $T = 1$ levels move down at least 2 MeV as the basis expands. A general feature of the prescription for truncation is that it naturally works best and convergence is fastest when a 4n system occurs for either the particles or the holes. The lowest states are dominated by the highest spatial symmetry in agreement with the original Brown and Green assumption.[8] i.e., states with $T_h = 1$ and $T_p = 1$ do not dominate over $T_h = 0 \times T_p = 0$ although they generally do occur with somewhat greater intensity. The 1^+ $T = 0$ level is hard to reproduce: 1^+ doesn't occur for the leading symmetries and cluster model calculations indicate that we may well need to go outside the present shell model space.

The $T = 1$ levels provide a very sensitive test of the interaction. The energies look good but there are some deficiencies in the wave functions. As shown in Fig. 6 the weak coupling basis states lie close in energy and formation of the highest spatial symmetry requires strong mixing of the weak coupling basis states.

IV. Spurious States

To do the calculation a number of spurious states had to be eliminated from the basis; e.g., 18 from the 159 2^+ $T = 0$ basis. This is relatively easy to do in an SU3 basis. Consider first a trivial case. For S^4p^{12} the centre of mass (CM) must be in a 0S state. The CM can be raised to a p state simply by acting on the shell model ground state with the creation operator for a quantum of CM motion

$$A^{\dagger}_{(10)T=0\,S=0}\left(\begin{matrix}\text{totally}\\ \text{symmetric}\end{matrix}\right)|0> \;\rightarrow\; |(10)L=1\,S=0\;T=0>$$

which is identical with one of the $1\hbar\omega$ shell model basis states

$$\begin{matrix} p^{-1} & \times & sd \\ (01) & \times & (20) \rightarrow (21)\;\;(10) \end{matrix} \qquad \begin{cases} [4^4]\;(TS) = (00) \\ [4^3 31]\;(01)(10)(11) \end{cases}$$

This state must therefore just be left out of the basis. Singly spurious $2\hbar\omega$ states are obtained by operating on the non-spurious $1\hbar\omega$ states with $A^{\dagger}$ and doubly spurious states by operating on the $1\hbar\omega$ spurious state ($\equiv \frac{1}{\sqrt{2}} A^{\dagger 2}|0>$). This is a general procedure[13,10] by which spurious states may be constructed

$$|SPUR(\lambda'\mu')\kappa'L'TS> = |[A^{\dagger}(10)|SM(\lambda\mu)TS>](\lambda'\mu')\kappa'L'TS>$$

with $(\lambda'\mu') = (\lambda+1,\mu)\;\;(\lambda-1,\mu+1)\;\;(\lambda\mu-1)$ Overlap with SM states with the same transformation properties give an expansion in terms of the SM basis

$$< SM(\lambda'\mu')\kappa'L'TS|SPUR(\lambda'\mu')\kappa'L'TS> = < SM(\lambda'\mu')TS\|A^{\dagger}\|SM(\lambda\mu)TS>$$

which is the reduced matrix element of a very simple one body operator and as such is independent of κ' and L'. Each $(\lambda\mu)$ can be dealt with separately, some being automatically non-spurious since they can be reached only by adding a ph pair with (21) transformation properties [and not (10)]. Note that since the E1 operator also transforms as (10) [or (01)] E1's are usually forbidden between leading SU3 representations.

Consider the specific example shown in Fig. 7 of the space with $[4^4]$ symmetry and $(\lambda\mu) = (20)$. Since r^2 and r^2Y^2 connecting states differing by $2\hbar\omega$ transform as (20) this space contains the giant monopole

and quadrupole resonances. The $r^2Y_0^2$ piece in the Nilsson Hamiltonian also means that this space is important for Hartree-Fock corrections to the ground state. There are five shell model basis states in the space, three 2p-2h and one each of $p^{-1}pf$ and $s^{-1}sd$. Two linear combinations correspond to singly and doubly spurious states leaving three good states. Zamick's monopole state,[14] $|B>$, has no overlap with the singly spurious state but has a small one $\left(\sqrt{\frac{1}{24}}\right)$ with the doubly spurious state. If $|B>$ is orthogonalized to $|D>$ it picks up ~4% 2p-2h component and 1/24 of the monopole strength is lost to the doubly spurious state. The (20) states can cause a problem in the 0^+ $T=0$ space because if the interaction doesn't saturate properly $|B>$ comes too low. Also there are consistency problems with the Hartree-Fock corrections since the ground state gets depressed but the corresponding states are not in the basis to depress the 2p-2h and 4p-4h levels.

It is interesting to note that the 1p-1h strength dissolves into a large background of 2p-2h states with very little 1p-1h intensity in any one state.

V. Negative Parity States

There exists a band of intruder levels beginning with the 1^- level at 9.6 MeV, 3^- at 11.6... . There are difficulties with the Brown and Green interpretation[15] that these levels are mainly 3p-3h from the (63) representation. In my calculation the 7^- comes at 21.9 MeV, about 1 MeV above its supposed location, but the 3^- comes at 15 MeV and the 1^- an MeV or so below that. The Ellis and Engeland weak coupling calculation,[7] which is usually reliable since a major part of the diagonal energy is taken from experiment, produces a very similar result as does the ZBM calculation,[16] i.e., in all cases the 1^- comes around 13 MeV. Another

difficulty is that the 3p-3h states give a small θ_α for low J although quite a large value for J = 7. A similar band with large θ_α's begins at 5.8 MeV in ^{20}Ne suggesting that a corresponding band might begin around 12 MeV in ^{16}O. In the SM these states would be a mixture of 5p-5h and $p^{-4}sd^3pf$ configurations (the Pauli Principle favours getting the fourth p-shell particle out of the shell) and it is conceivable that their mixing with the 3p-3h states might produce the low-lying 1^- and 3^- levels. Some support for this idea comes from Suzuki's $^{12}C + \alpha$ cluster model calculations[17]; an alpha is coupled to the ground state band (04) representation of ^{12}C, the relative wave function being expanded in oscillators, (NO), with $N \leq 31$. Figure 8 shows that the intruder band is beautifully reproduced. Many N's contribute appreciable intensities but it is interesting that the 5^- and 7^- level have N = 7 ($\widehat{=}$ 3p-3h) as the largest contributor whilst the 1^- and 3^- are dominated by N = 9, 11, 13.

VI. Spectroscopic Amplitudes for Multinucleon Transfer

There have been detailed treatments of this subject by Ichimura et al.,[18] by Hecht and Braunschweig[19] and extensive applications by the Oxford Heavy Ion Group.[20] The basic idea, illustrated in Fig. 9, is that four or less nucleons are transferred in their lowest state of internal motion requiring that all the quanta associated with the individual orbits whence the nucleons came must be associated with their centre of mass coordinate and span out the (λ0) representation of SU3 with $\lambda = \sum_{i=1}^{k} N_i$. The cfp then simply refers to the splitting off of a single (λ0) configuration, e.g., an idealized example for $^{12}C + \alpha$ to a 4p-4h level.

$$\theta_\alpha = \left(\frac{16}{12}\right)^4 < (04)L = 0(80)L_\alpha \| (84)\kappa L > G(sd^4)$$

The heavy ion transfer reactions are particularly useful in that they tend to populate preferentially the high spin states which are best described by the SU3 calculations and thus provide a very good reference point.

VII. Remarks

It is clear that the SU3/SU4 basis provides a good starting point for shell model studies of light nuclei. The resulting wavefunctions usually have a simple physical interpretation, often through their relationship to the wave functions of the cluster, Nilsson or weak-coupling models. These models can often be used to suggest which configurations should be included in the basis (cf. Sec. IV). In this sense the philosophy differs somewhat from that of many conventional shell model calculations where the model space is fixed and the interaction is perhaps adjusted. Here there is more of a tendency to fix the interaction at some stage and then extend the basis to improve the description of certain levels. Generally one can be selective about the configurations one chooses to include and in such a way that spurious states can be dealt with.

At present a study is being made of negative parity states near the beginning fo the sd-shell where both $p \rightarrow sd$ and $sd \rightarrow pf$ excitations must be considered. Since in ^{21}Ne the "hole" and "particle" bands in the simple SU3 description are not mixed this nucleus can be used to help fix the single particle energies. One can then go on to try to reproduce the $K^\pi = 0^-$ "(90)" band in ^{20}Ne (bandhead; 1^- at 5.80 MeV) and the $K^\pi = \frac{1}{2}^-$ "(70)" band recently identified[21] in ^{19}F (bandhead; $3/2^-$ at 6.09 MeV). This represents a systematic approach towards a study of the importance of pf configurations in the ^{16}O region and, e.g., is obviously relevant to the theoretical description of giant resonance phenomena in the region.

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FIGURE CAPTIONS

Fig. 1 Labelling scheme for the SU3/SU4 basis states.

Fig. 2 Positive parity basis for ^{16}O. Representations marked with an asterisk are free from spuriosity. The representations most favoured by the truncation prescription are marked with a box.

Fig. 3 Comparison of Brown and Green's wave functions for the lowest 0^+ states in ^{16}O with those from a large basis shell model calculation.

Fig. 4 The spectrum of positive parity T = 0 levels from a calculation using the basis shown in Fig. 2. The 4p-4h levels have been shifted down by 2 MeV relative to the 0p-0h and 2p-2h levels.

Fig. 5 Details of the wave functions of the members of the lowest 4p-4h band in ^{16}O.

Fig. 6 (a) The spectrum of positive parity T = 1 and T = 2 levels from a calculation using the basis shown in Fig. 2.

(b) Unperturbed energies for a selection of weak-coupling basis states in ^{16}N and the spin-isospin decomposition of 2p-2h states of $[4^3 31]$ symmetry.

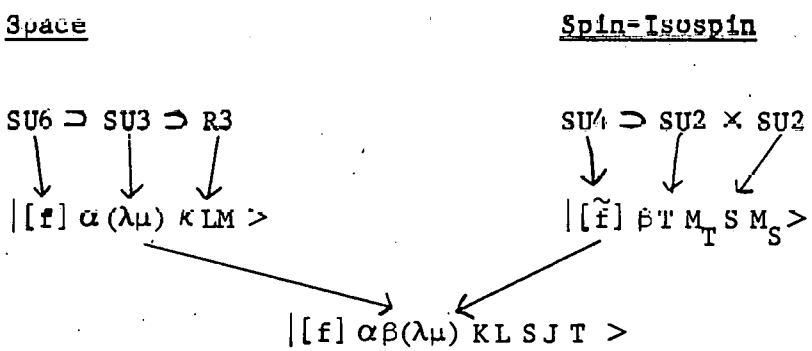
Fig. 7 Structure of the spurious and "monopole" states in the $[4^4]$ (20) subspace of $2\hbar\omega$ states.

Fig. 8 Some results from Suzuki's $^{12}C + \alpha$ cluster model calculations for ^{16}O. The shell model like states are dominated by N = 4 for the ground state and N = 5 for the "1p-1h" levels. The $^{12}C + \alpha$ states exhibit highly developed clustering and many N values are needed to describe them.

Fig. 9 The structure of multinucleon spectroscopic amplitudes under the assumption of cluster transfer.

SU3/SU4 SHELL MODEL BASIS

(i) Single major shell e.g.: sd^n



(ii) Two (or more) shells e.g.: $p^{n_1}\ sd^{n_2}$

$$A|p^{n_1}(\lambda_1\mu_1)\beta_1 T_1 S_1,\ sd^{n_2}[f_2]\alpha_2\beta_2(\lambda_2\mu_2)T_2S_2;\ (\lambda\mu)\ \kappa L\ S\ J\ T >_{\rho_{12}}$$

Fig. 1

POSITIVE PARITY BASIS FOR ^{16}O

	p^{12}			$\boxed{(00)^*}$				
	p^{10}	sd^2						
	(02)	[2] (40)	→	$\boxed{(42)^*}$	(31)	(20)		
		(02)	→	$(04)^*$	(12)	(20)		
		[11](21)	→	$(23)^*$	(31)	(12)	(20)	(01)
	(10)	(40)	→	$(50)^*$	(31)			
		(02)	→	(12)	(01)			
		(21)	→	(31)	(12)	(20)		
p^{11}	(01)	pf (30)	→	(31)	(20)			
s^3	(00)	sd (20)	→	(20)				
	p^8	sd^4						
	(04)	[4] (80)	→	$\boxed{(84)^*}$	(73)	(62)	(51)	(40)
		(42)	→	$(46)^*$	...			
		(04)	→	$(08)^*$	...			
		[31](61)	→	$(65)^*$	(73)	...		
	(12)	(80)	→	$(92)^*$	(73)	...		
		(61)	→	(73)	...			

Fig. 2

Mixing of three 0^+ states in ^{16}O

Brown and Green

	E	0p0h	2p2h	4p4h
0_1^+	0.00	0.874	0.469	0.130
0_2^+	6.07	-0.262	0.229	0.937
0_3^+	11.26	-0.410	0.853	-0.323

Shell Model

	0p0h	2p2h	404p
0_1^+	0.884	-0.376	0.164
0_2^+	-0.241	-0.116	0.872
0_3^+	0.349	0.798	0.131

Fig. 3

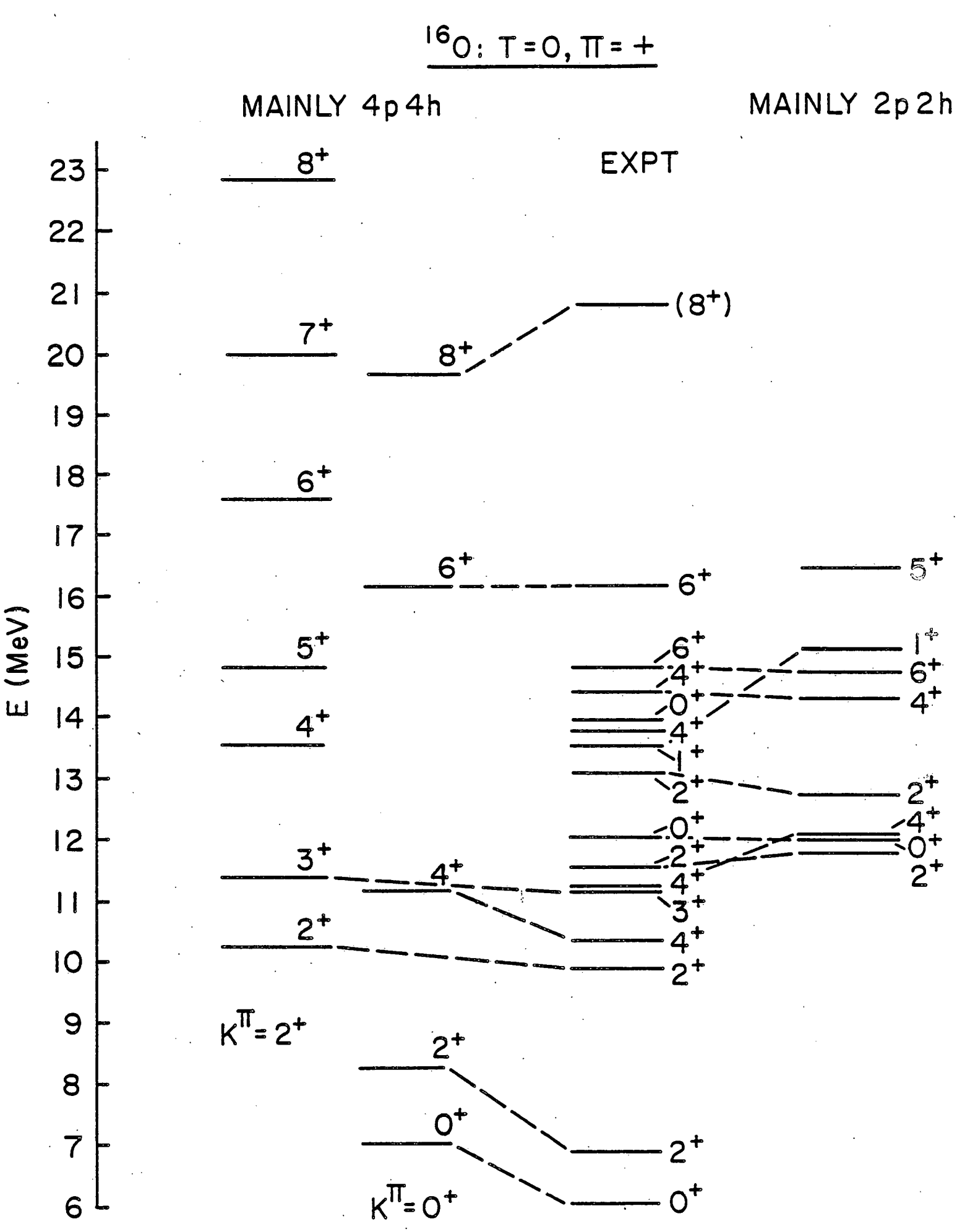

^{16}O: T=0, π=+
MAINLY 4p4h
MAINLY 2p2h
EXPT
E (MeV)
23
22
21
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
8+
7+
6+
5+
4+
3+
2+
(8+)
8+
6+
6+
6+
4+
0+
4+
1+
2+
0+
2+
4+
3+
4+
2+
5+
1+
6+
4+
2+
4+
0+
2+
$K^\pi = 2^+$
2+
0+
$K^\pi = 0^+$
2+
0+

Fig. 4

The "$K^{\pi} = 0^{+}$" Band

level	%2p2h	(84) K = 0	(84) K = 2	(84) K = 4	%(84)	%(92)
0_2^+	5.2	74.8	.	.	74.8	6.2
2_1^+	14.6	70.1	0.1	.	70.2	5.9
4_1^+	14.6	66.9	1.2	0.0	68.1	5.4
6_2^+	15.5	49.2	20.4	0.4	70.0	4.3
8_1^+	0.0	26.6	51.2	5.8	83.6	3.6

For the "$K^{\pi} = 2^{+}$" band the situation is very similar except that the 4^{+} strength is split between two levels.

Fig. 5

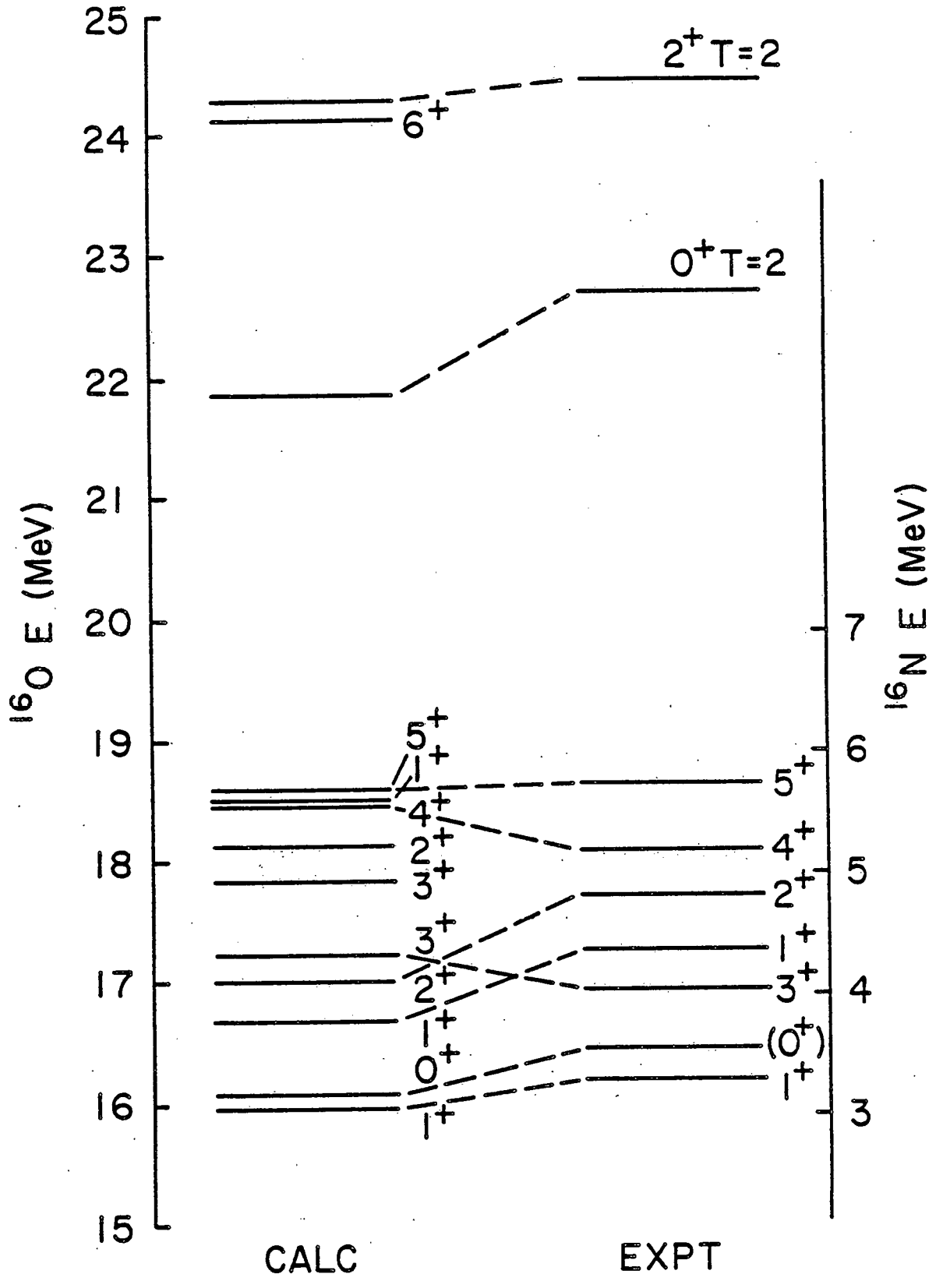

^{16}O: T=1 and T=2, π=+
25
24
23
22
21
20
19
18
17
16
15
^{16}O E (MeV)
^{16}N E (MeV)
7
6
5
4
3
2+ T=2
6+
0+ T=2
5+
1+
4+
2+
3+
3+
2+
1+
0+
1+
5+
4+
2+
1+
3+
(0+)
1+
CALC
EXPT

Fig. 6a

WEAK COUPLING BASIS FOR ^{16}N

3^+	5.57	^{14}C gs × ^{18}F (3^+) ← $(^3He,p)$ ⇒ lowest 3^+
$1^+,2^+,3^+$	5.38	^{14}N gs × ^{18}O (2^+)
1^+	4.63	^{14}C gs × ^{18}F gs $(^3He,p)$, (t,p) and ^{16}C β decay
1^+	3.36	^{14}N gs × ^{18}O gs ⇒ strong mixing

Highest spatial symmetry is $[4^3 31]$ and

$$|[2]^{-2}\times[2]:[31]\ T=1\ S=1> = \frac{1}{\sqrt{2}}\left[\underset{\substack{\uparrow\\(T_1S_1)}}{\overset{\downarrow}{(10)}}\times\underset{\substack{\uparrow\\(T_2S_2)}}{\overset{\downarrow}{(01)}} - (01)\times(10)\right]$$

p^{-2} and sd^2 over the first term

corresponding to strong mixing of the weak coupling basis states.

Fig. 6(b)

$[4^4]$ (20) BASIS

BASIS STATE		Doubly Spurious $\|D>$	Singly Spurious $\|S>$	Monopole $\|B>$
(02) × (40)	2p-2h	$\sqrt{\frac{15}{32}}$	$\sqrt{\frac{3}{16}}$	
(02) × (02)	2p-2h	$-\sqrt{\frac{3}{64}}$	$\sqrt{\frac{15}{32}}$	
(10) × (21)	2p-2h	$\sqrt{\frac{25}{64}}$	$-\sqrt{\frac{5}{32}}$	
p^{-1} pf		$\sqrt{\frac{5}{64}}$	$\sqrt{\frac{1}{32}}$	$\sqrt{\frac{5}{6}}$
s^{-1} sd		$-\sqrt{\frac{1}{64}}$	$-\sqrt{\frac{5}{32}}$	$\sqrt{\frac{1}{6}}$

$$< D|B > = \sqrt{\frac{1}{24}}$$

Fig. 7

SHELL MODEL-LIKE STATES

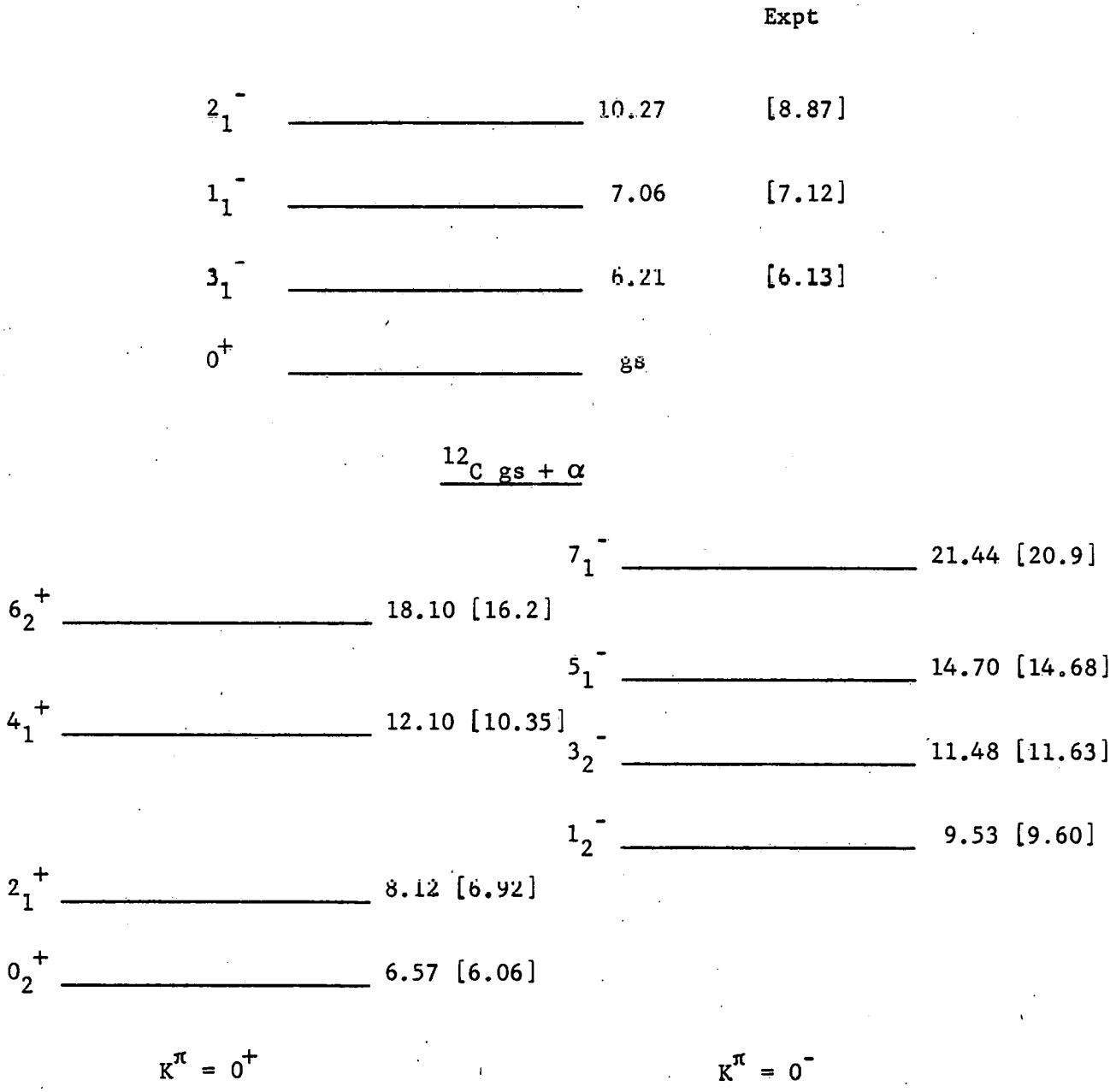


Fig. 8

SPECTROSCOPIC AMPLITUDES

For k particles where $k \leq 4$

$$\theta = \left(\frac{A}{A-k}\right)^{\frac{\lambda}{2}} \underbrace{\left\{ \sum_{\beta} \begin{pmatrix} A \\ k \end{pmatrix}^{\frac{1}{2}} < \psi_{A-k} \psi_{\beta} | \quad \psi_A > \right\}}_{\text{c.f.p.}} < \phi_k \phi_{n\ell} | \psi_{\beta} >$$

where for cluster transfer $\lambda = 2n+\ell = \sum_{i=1}^{k} N_i$ and $\beta \equiv [k](\lambda 0)\ell \; S_k J_k T_k$.

The overlap of the k particle wave function onto the 0s internal state is given by

$$G([N_i]^k) = < \phi_k \phi_{n\ell} | \psi_{\beta} > = \sqrt{\frac{k!}{k_1! k_2! \ldots}} \quad \frac{\sqrt{\lambda!}}{k^{\lambda/2}} \quad \prod_{i=1}^{k} \quad \frac{1}{\sqrt{N_i!}}$$

e.g.,

$$G(sd^4) = \sqrt{\frac{315}{8192}}$$

$$G(p^2sd^2) = \sqrt{\frac{48}{7}} \quad G(sd^4)$$

$$G(p^4) = \sqrt{\frac{256}{105}} \quad G(sd^4)$$

$$G(psd^2pf) = \sqrt{8} \quad G(sd^4)$$

Fig. 9