

COMPUTATION AND GRAPHICS
IN MATHEMATICAL RESEARCH

Progress Report
for Period June 1, 1989 - May 31, 1990

RECEIVED
MAR 1 0 1990

D.A. Hoffman and J. Spruck

University of Massachusetts
Amherst, MA 01003

January, 1990

Prepared for

THE U.S. DEPARTMENT OF ENERGY
AGREEMENT NO. DE-FG02-86ER25015

JSH **MASTER**

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

TABLE OF CONTENT

Cover Sheet i

Table of Content ii

Abstract 1

Description of Computation Facility 1

Research Activities 5

 Research of Joel Spruck 5

 Research of David Hoffman 6

 Research of Alexander Lifshits 8

 Research of William H. Meeks, III 9

Personnel Wholly or Partially Supported by this Grant 11

Research Visitors 11

Appendix 11

 Curriculum Vitae of Dr. Alexander Lifshits 12

 Geometry, Computation and Graphics Series List 16

Description of Activity in the First Seven Months of Funding

D. Hoffman J. Spruck

Abstract

We are actively developing and using interactive graphics software and numerical computation to study problems in global differential geometry and calculus of variations, the structure and motion of interfaces, and the simulation of plasma equilibrium and evolution. We have improved the hardware, software and design of our video animation facility and have begun to document our graphics software packages.

1. Description of the Computation Facility

Computation

The Center for Geometry, Analysis, Numerics, and Graphics maintains its computation facility in the University's Lederle Graduate Research Center. This facility serves the principal investigators along with collaborators and visitors (about 20 people in total). The GANG facility centers around a Silicon Graphics 4D/240GTX graphics computer. This machine has 4 20MIP processors, 32MB of memory, 1.2GB of disk storage and a 120MIPS graphics processor. The close coupling of high-speed processing with high-speed graphics has had a great effect on the Center's focus in differential geometry: researchers are now able to compute, and simultaneously visualize in near real-time, problems which required many minutes of time on earlier computers. Animation sequences are now produced as the problems are being calculated.

The Center uses a Star Technologies Graphicon 1700, a high-speed display list processor, for off-line animations. Two Sun 3/60 color workstations have received memory and display hardware upgrades, and are used for still color graphics and additional windows on the 4D/240GTX. Two Macintosh II color workstations are used by the principal investigators for Mathematica and access to the SGI and Sun servers.

The Center's Sun 4/260 continues to serve as the main file server for the Center's six client (dataless) workstations, and as the "core" development machine. This system has 2.0GB of disk storage, 32MB of memory, and a high-resolution monochrome console. The Center has purchased 1.2GB of additional disk capacity for the 4/260 to alleviate current storage problems, provide space for animation frames, and support smaller client machines.

Most of the Center's client workstations are Sun 3 machines. These machines are used chiefly as multi-window access points to the two servers, and for simpler (non-animated) graphics. Two Sun SPARCstation-1s were added in January 1990 under Sun's academic enhancement program, providing more high-performance access points to users. The Center also purchased two NeXT workstations. Early results porting one of the Center's software packages to the NeXT was quite successful. The Center runs the Thinking Machines, Inc. Connection Machine Simulator on the NeXT machine for experimentation with data parallel algorithms. The NeXT's removable optical disk is used for inexpensive on-line storage of the Center's picture library.

The Center's computer systems are connected to other research laboratories and computing facilities (such as the Engineering Computer Services, Math Research Computing Facility,

and University Computing Services) via the campus backbone Ethernet. The campus backbone has external connections to the INTERNet, NSFNet, CSNet, and BITNet. The University plans to upgrade this backbone to a high speed fiber trunk during 1990 - 1991. The University also plans to upgrade its NSFNet link to T1 speed (1.4Mb/s) in February 1990.

As vendor products become available during 1990, the Center's server and graphics machines will be placed on a new FDDI fiber-based network (100MB/sec), providing very high file system and graphics performance among the core machines. This network will be gated internally to the existing Ethernet (which will continue to support the smaller workstations).

A diagram of the computing facilities is attached.

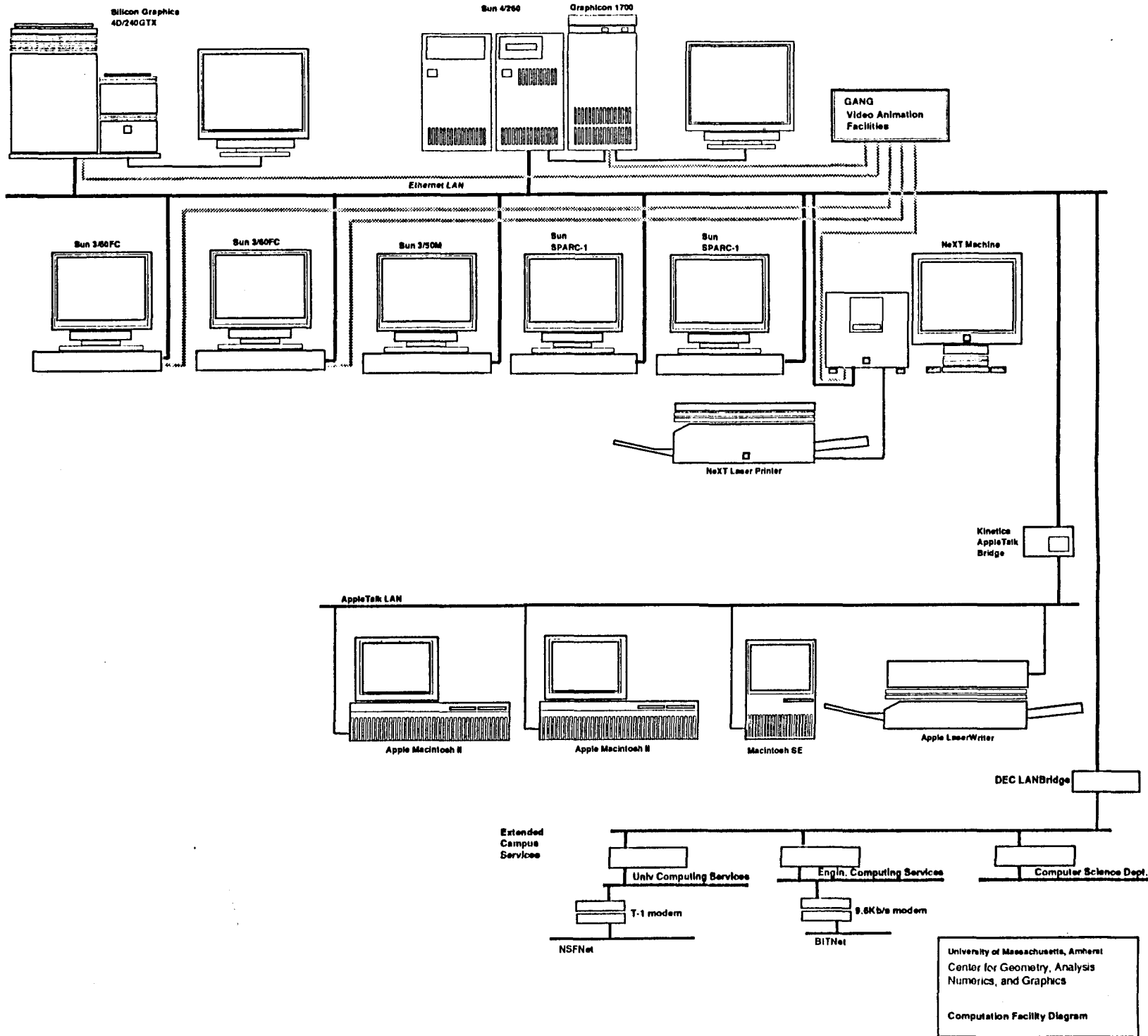
Video Animation

The increased performance of the Center's computers provided impetus to re-engineer the approach to the production of scientific video animations. We designed a new video production system which greatly improves video quality, meets archiving needs and, as software is developed in the future, makes the system easier to use and accessible from remote locations. A Lyon Lamb Real Time Converter (RTC) replaces the older RS-170 encoder, and generates very high-quality NTSC video (along with Super-VHS and RS-170 RGB) directly from high-resolution computer graphic video. The RTC has three "live" high-resolution inputs, and is computer-controllable. Using additional high-resolution video switching and distribution, all of the Center's graphics computers can be connected to the RTC. A Sony LVR5000 Laser Disk Recorder was added to the Center's existing Sony BVU-950 3/4" tape deck to record output from the RTC. Up to 50 minutes of video can be recorded on each LVR disk (write-once), either as still frames or sequences. Media life is expected to exceed 30 years, making the laser disk a far better archiving media than tape. During 1990, software will be designed to put these three units - the RTC, laser disk, and tape deck - entirely under computer control (the sole exception being the loading of media). This will integrate the video system with the computation system, and will make access to the equipment "safer" from a maintenance perspective. With the addition of broadcast switching, the new system will accept video from (and broadcast to) remote locations and, through the NSFNet, will be controllable from other sites. The Center will also add a 1/2" multi-standard deck so that video tapes can be exchanged with overseas colleagues.

A diagram of the video facilities is attached.

Software Development

Because the Center hosts a greater number of visitors and collaborators each year, additional effort has been directed toward documenting the Center's software. MESH, the adaptive mesh generator, has been completely re-worked for interactive use on the 4D/240GTX. Work began in December 1989 on the DISP array graphics package which runs on all the Center's hardware platforms. A version of DISP will be released to collaborating researchers in early 1990, with supporting documents, installation scripts and examples. Work will begin in 1990 on documenting the DIS (rasterfile display), VEW (polygon file display), and VRUN (VPL script execution) software packages. Further work is being done to document the several data formats used with these packages so that data can be more easily exchanged with visitors and collaborators who wish to use their own software tools in combination with these packages.



University of Massachusetts, Amherst
 Center for Geometry, Analysis
 Numerics, and Graphics
 Computation Facility Diagram

2. RESEARCH ACTIVITIES

Research of Joel Spruck

Joel Spruck worked on a variety of pure and applied problems. With a group of collaborators at the University of Massachusetts (Eydeland, Lifshits, Turkington) he has made substantial progress in modelling equilibria and slow evolution of plasmas. With L.C Evans, he has made substantial progress in developing a new level set approach to motion by mean curvature and with L. Caffarelli, he has developed new ideas for the study of geometric variational problems.

Spruck, in collaboration with Eydeland and Turkington completed a first paper on variational problems with many constraints that models semilinear problems where the profile function is determined implicitly from the constraints. This type of problem arises in many physical problems such as the ideal MHD equilibrium models of plasma physics, in the presence of one symmetry. This work has important applications to scientific computing which will be developed. The paper will appear as

Multiconstrained Variational Problems of Semilinear Eigenvalue Type: New Formulation and Algorithms. *Mathematics of Computation*.

Continuing this work, Spruck (with Eydeland and Turkington) showed that the algorithm of the previous paper extends to cover the case of an incompressible plasma with swirl (a system with three unknowns u, v, w). This work contains several new ideas relevant to nonconvex constrained optimization and will appear as

Computational methods for multiconstrained variational problems arising in equilibrium magnetohydrodynamics, preprint.

Continuing this work, Spruck in collaboration with A. Lifshitz (see his research summary and vita), Eydeland and Turkington, has developed a new method of studying slow evolution of a plasma that describes flux conserving and adiabatically compressed tokamaks. The method allows us to easily study magnetic island formation and annihilation. This work will appear as

A variational approach to adiabatic plasma evolution, in preparation.

In collaboration with L.C. Evans, Spruck completed the initial work on "Levelset motion by mean curvature" using a p.d.e. approach. They defined the appropriate motion of "viscosity solutions" and proved the existence and uniqueness of a global in time solution. This approach allows the definition of a weak solution even after the onset of singularities. Many surprising and interesting properties of the motion follow from a strong comparison principle for solutions. This work will appear in

Level set Motion by Mean Curvature I, submitted to *J. Diff. Geom.*

A second paper of Spruck and Evans gives a new proof of classical short time existence (as well as some other properties) by using the distance function to the motion. This work will appear as

Level set motion by Mean Curvature II, preprint.

Other work on these topics are being prepared for publication and new numerical experiments are underway.

Spruck, in collaboration with Caffarelli and Gidas completed his work on the classification of isolated singularities of the conformally invariant scalar field equation modelling critical Sobolev growth. In doing so, they developed a powerful new variant of the Alexandrov reflection technique that is measure theoretic and localizable. This work appeared in

Asymptotic Symmetry and the Local Behavior of Semilinear Elliptic Equations with Critical Sobolev Growth, CPAM 42, (1989) 271-297.

Spruck with Cafferelli continued the above mentioned work to study variational problems with critical Sobolev growth. In particular, they proved that the Dirichlet problem for positive data, not identically 0, is well-posed. This work will appear as

Variational Problems with Critical Sobolev Growth and Positive Dirichlet Data, Indiana University Math. J.

Research of David Hoffman

David Hoffman worked on a wide variety of research problems, and made fundamental progress in a variety of areas. He wrote with W. Meeks III the draft of

One-parameter families of embedded minimal surfaces (with W.H. Meeks);

He continued his collaboration with Edwin Thomas et. al., now at MIT:

Graphical simulation of 2-D TEM data from three dimensional surface information derived from mathematical models (with D. Anderson, J. Bellare, E. Thomas and J. Hoffman).

Hoffman is currently working on several research projects, whose working titles are:

Smooth limits of classical families of minimal surfaces (with M. Wohlgemuth);

New examples of singly periodic minimal surfaces (with M. Wohlgemuth);

One parameter families of singly -periodic minimal surfaces derived from exotic triply-periodic examples (with H. Karcher).

In the first paper, Wohlgemuth and Hoffman have investigated limiting behavior of families of embedded minimal surfaces. One striking example of what we prove is this: the smooth limit of Scherk's first family of embedded minimal surfaces is the helicoid. In the second project, we construct new embedded periodic minimal surfaces related to the examples of Callahan Hoffman and Meeks, and we are looking at the generic period problem for Abelian minimal surfaces. The third project centers on the discovery of new families of embedded singly periodic minimal surfaces, both of Scherk-type and with helicoidal ends, that were inspired by the examples found by the crystallographers Fischer and Koch.

Simulations of these surfaces using analytic methods and software written by James Hoffman with assistance from Eric Boix were used to determine their properties. Some of these surfaces exist in 1-parameter families. The curve that describes the solution lies in a space of three or four dimensions, where the curve is known implicitly as the intersection locus of the zero set of several real valued functions. These functions are quite "shallow" near their zero set, and this caused the usual simplex method to be unstable. Using automatic mesh generation and corner-finding programs written by Hoffman with the assistance of E. Boix, we were able to verify experimentally (using computer graphics) that these surfaces exist. It will be a much harder problem to provide a purely mathematical demonstration. The work with Wohlgemuth, Karcher and Callahan-Meeks seems to indicate that there is a general procedure for handle-adding on surface, which increases the genus while leaving the number of ends the same. This is poorly understood theoretically at the present time. A longstanding open question concerns the behavior of minimal surfaces that are properly embedded in a punctured neighborhood of a point. Is it possible that the puncture is a limit point? The constructive procedure developed for finding these new examples may shed some light on this question.

The research of Hoffman on periodic embedded minimal surfaces derived from crystallographic examples has been greatly aided by the work of James Riordan, an undergraduate, who has developed and implemented an algorithm for solving the Plateau problem by a method that is inherently parallelizable. Further work in the coming months will be done to adapt these programs to solve partially free boundary value problems for least-area and also to find CMC and Willmore surfaces. In addition, an attempt will be made to run the code on a massively parallel machine. Riordan also works as a systems programmer in our Center.

Eric Boix assisted in the application of MESH, written by Jim Hoffman, to a variety of problems concerning periodic and finite total curvature minimal surfaces. He was also instrumental in the testing and documentation of our library of basic graphics display routines, collectively called DISP.

References

During period 6/1/89 -12/31/89 the following papers of David Hoffman have appeared in print:

The asymptotic behavior of properly embedded minimal surfaces of finite topology. Journal of the AMS 2 No 4 (October, 1989), 667-681 (with W. H. Meeks) .

The construction and qualitative behavior of singly-periodic embedded minimal surfaces. To appear Contemporary Mathematics: Proceedings of the special session on geometry, UCLA. 1987 (R.Greene and C.Cheng, editors)

In the same period, the following papers have been accepted:

Embedded minimal surfaces of finite topology. (with W. Meeks) to appear in the Annals of Mathematics

The structure of singly-periodic minimal surfaces. To appear in Inventiones Math.(with M. Callahan and W.H. Meeks)

Limits of minimal surfaces and Scherk's fifth surface. To appear in Archive for Rational Mechanics and Analysis. (with W.H. Meeks)

The strong halfspace theorem for minimal surfaces. to appear in Inventiones Math.(with W. Meeks).

Hoffman also is a co-editor of conference proceedings of the 1988 MSRI Conference on Differential Geometry and Computer Graphics:

Computer Graphics and Geometric Analysis: Proceedings of the Conference on Differential Geometry, Calculus of Variations and Computer Graphics (with P. Concus and R. Finn) to be published in Math. Sci. Res. Inst. Conference Proceedings Series (Springer Verlag). Also contains the abstract

"Periodic area minimizing surfaces in microstructural science." (with E.Thomas, D. Anderson, D.Martin, and J.Hoffman)

Minimal surfaces bases on the catenoid. Invited article to appear in forthcoming special Geometry issue of the American Mathematical Monthly.(with W. Meeks)
This paper is completed and accepted for publication. GANG was asked to provide the cover illustration for this volume.

Research of Alexander Lifshits

Magnetohydrodynamics

We are interested in analytical and numerical studying of two-dimensional plasma equilibria and their evolution. In collaboration with A. Eydeland, J. Spruck and B. Turkington we developed a new

method of investigating plasma evolution. Combining ideas developed earlier by A. Eydeland, J. Spruck and B. Turkington for investigating equilibrium of a plasma with some novel ideas we were able to simulate slow motions of a plasma and observe a number of interesting bifurcations. Our method is based on characterization of a plasma equilibrium at a given moment of time as an energy minimizer subject to an infinite family of constraints derived

from conserved quantities associated with the governing evolution equations. It allows us to describe in a very natural manner evolution of flux conserving and adiabatically compressed tokamaks, as well as investigate the process of formation and annihilation of magnetic islands. Preliminary results are reported in a number of publications. We are planning to continue our study in order to incorporate some important physical processes in our model and be able to perform the modelling and numerical simulation of a plasma in Tokamaks, Stellarators and other experimental devices.

References:

A variational approach to adiabatic plasma evolution
(with A. Eydeland, J. Spruck and B. Turkington).
In preparation.

Multiconstraint variational problems in magnetohydrodynamics:
formulations and algorithms
(with A. Eydeland, J. Spruck and B. Turkington).
Sixth Beer - Sheva International Seminar on MHD Flows and Turbulence,
Israel, 1990.

A variational approach to adiabatic plasma evolution in ideal MHD

(with A. Eydeland, J. Spruck and B. Turkington).
The 13th Conference on the Numerical Simulation of Plasmas,
Santa Fe, New Mexico, USA, 1989.

Spectral Theory

The problem of studying the operator of multiplication by a, symmetric matrix-function defined in the space of square-integrable divergence-free vector fields in a bounded domain has important applications in geophysics, hydrodynamics (the classical Poincare' problem in the theory of rotating fluids) and magnetohydrodynamics. Spectral properties of this operator are

much more complicated than spectral properties of the corresponding operator defined in the whole space of square-integrable vector fields. Recently we considered the operator of multiplication by a function. Our results are reported in a paper accepted for publication. At present we investigate the general case. Part of this investigation is performed in collaboration with S. Friedlander.

References:

A new description of Alfvén modes. Physics Letters A. (Accepted for publication.)

Stability Theory for Symmetric Hyperbolic Systems

Recently I have realized that for a wide class of symmetric hyperbolic systems (including a number of systems of practical importance) it is possible to construct special solutions rapidly varying in space and slowly varying in time. Using these solutions as trial functions I was able to obtain effective stability criterion for such systems. In particular, I derived a new ballooning stability criterion in hydrodynamics and rederived the classical ballooning stability criterion in magnetohydrodynamics. The hydrodynamic ballooning instability causes some interesting hydrodynamic phenomena. In particular it can be responsible for the appearance of small scale structures on the large scale background. I am planning to apply the general ballooning stability criterion in order to investigate numerically stability of vortex rings with swirl which were studied earlier by A. Eydeland and B. Turkington.

Research of William H. Meeks, III

Recently, I have completed a number of research projects on minimal and constant mean curvature surfaces. In joint work with Korevaar, Kusner, and Solomon, we have been able to characterize the asymptotic behavior of embedded annuli of constant mean greater than 1 in hyperbolic three-space. We prove that such annuli are asymptotic to periodic surfaces of revolution and constant mean curvature.

In joint work with Charley Frohman, we have been able to prove the uniqueness of one-ended Heegaard surfaces in \mathbb{R}^3 . This fundamental result has important applications. For example, it follows that any two Fermi surfaces in an infinite crystal differ by a homeomorphism of space, and more generally, the regular neighborhoods of the one-skeleton of two triangulations of \mathbb{R}^3 are ambiently isotopic. Another application is to prove that any two one-ended embedded minimal surfaces of the same genus in \mathbb{R}^3 are ambiently isotopic; it then follows from earlier work of Callahan, Hoffman, and Meeks that any two doubly-periodic minimal surfaces are ambiently isotopic. Recently, we have made important progress on

generalizing this result to understanding the topology of embedded minimal surfaces with more than one end .

In joint work with Brian White we have been able to prove that given two convex planar curves that are extremal, in the sense that they lie in the boundary of their convex hull, bound zero one or two minimal annuli, and these annuli are always embedded. This result is proved by studying the stability properties of the annuli.

Recently I have completed a joint paper with Binto, Earp, and Rosenberg on the geometry of constant mean curvature surfaces bounded by a convex planar curve.

Other work, including a survey paper with David Hoffman and a survey of periodic minimal surface, appears in the list of references below.

References

A halfspace containment property for constant mean curvature surfaces bounded by a convex planar curve, preprint.(with F. Brito, R. Earp, H. Rosenberg)

The structure of singly periodic minimal surfaces, to appear in Inventiones Math.(with M. Callahan, D. Hoffman)

The ordering theorem for the ends of properly embedded minimal surfaces, preprint.(with C. Frohman)

The topological uniqueness of complete one-ended minimal surfaces and Heegard surfaces in \mathbb{R}^3 , preprint.(with C. Frohman)

The topology of complete minimal surfaces and Heegard surfaces in \mathbb{R}^3 , research announcement.(with C. Frohman)

Minimal surfaces based on the catenoid, to appear in the American Math Monthly.(with D. Hoffman)

Constant mean curvature surfaces in hyperbolic space, preprint. (with N. Korevaar, R. Kusner, B. Solomon)

The geometry of triply periodic minimal surfaces, preprint.

The geometry, topology and existence of periodic minimal surfaces, submitted.

The geometry of periodic minimal surfaces, preprint.(with H. Rosenberg)

Minimal surfaces bounded by a pair of convex planar curves, research announcement.(with B. White)

Minimal surfaces bounded by convex curves in parallel planes, preprint.(with B. White)

The space of minimal annuli bounded by an extremal pair of planar curves, preprint(with B. White)

3. Personnel Wholly of Partially Supported by this Grant

Ph.D students

Bo Guan - Motion of Interfaces and curvature(working with Spruck)

Guoghang Sun - Motion of interfaces by curvature functions.(working with Spruck)

Scientific and Graphics Programmers

James T. Hoffman- Graphics programming for all scientific projects, development of graphics interface for visual analysis of numerical simulations, development of algorithms for the analysis of data, graphics software development.

Eric Boix- Graphics and scientific programming, assisted Jim Hoffman and worked intensively with David Hoffman on research projects.

James Riordan- System programming and scientific programming.

Ramji Raghuan-(1/21/90-5/26/90) Replacement for Eric Boix.

Postdoctoral Research Associate

Alexander Lifshits- working with Spruck on the development of theoretical and numerical algorithms for modelling plasma equilibrium and evolution(curriculum vita in Appendix).

4. Research Visitors

9/89-1/1/90

Brian White (Stanford)

Takashi Kako (Saitama University, Japan)

Jon Pitts (Texas A&M University)

Richard Schoen (Stanford and Courant Institute)

Vladimir Oliker (Emory University)

Jurgen Jost (Harvard University)

Ivan Sterling (Technische Universitat, Berlin)

5. Appendix

1. Curriculum Vitae of Dr. Alexander Lifshits
2. Geometry, Computation and Graphics List

CURRICULUM VITAE OF DR. ALEXANDER LIFSHITS

Education

1982 Ph.D. in Mathematics, Moscow State University
Thesis: 'Spectral properties of degenerate of differential equations'

1979 M.S. in Mathematics, Moscow State University

Professional Experience

June 1989- Senior Research Associate (GANG),
University of Massachusetts at Amherst

Jan. - May 1989 Visiting Scientist, Physics Dept., M.I.T.

1982-1986-1988 Research Scientist, Senior Research Scientist
Laboratory of Mathematical Physics
Institute of Earth Physics, Academy of Sciences of the USSR

Invited Lectures

Since 1979 Invited lectures in about 20 universities and research institutions including New York University (Courant Institute), UCLA, University of Massachusetts, Northwestern University, University of Illinois, Moscow University, Leningrad University, Landau Institute for Theoretical Physics.

Awards

1981 First degree award of the Academy of Sciences to a young geophysicist.

1980 Second degree award of the Academy of Sciences to a young geophysicist.

Research Areas Functional analysis and its applications, spectral theory of self-adjoint and non-self-adjoint operators, partial differential equations of mathematical physics, magnetohydrodynamics, space physics, soliton theory.

List of Publications

Monographs

1. Magnetohydrodynamics and spectral theory. Kluwer Academic Publishers, Dordrecht, Boston, London. 1988. xii+445 pp.

Articles

1. A variational approach to adiabatic plasma evolution in ideal MHD (with A. Eydeland, J. Spruck and B. Turkington). In preparation.

2. A new description of Alfvén modes. Physics Letters A. Accepted for publication.

3. The echo phenomenon in hydrodynamics. Doklady Akademii Nauk SSSR 308 (1989), 340-344 (in Russian).

4. Stability of soliton superstructures (with S. Burkov). *Physics of the Earth and Planetary Interiors* 50 (1988), 56-59.
5. Continuous spectrum in general toroidal systems (ballooning and Alfvén modes). *Physics Letters A* 122(1987), 350-356.
6. Hydromagnetic oscillations in the magnetosphere - ionosphere cavity (with E. Fedorov). *Transactions (Doklady) of the USSR Academy of Sciences. Earth Science Sections* 287(1986), 11-13.
7. The Fermi Golden Rule for ideal magnetohydrodynamics. *Journal of Plasma Physics* 33(1985), 249-256.
8. Oscillation modes in ideal and resistive magnetohydrodynamic systems (with E. Fedorov). *Soviet Physics - Technical Physics* 30(1985), 458-459.
9. On the control of a stationary flow of matter in some chemical reactions (with M. Petrova). *Soviet Journal of Chemical Physics* 3(1985), 1113-1121.
10. Dissipative structures and Couette flow in a non-Newtonian fluid (with G. Rybnikov). *Soviet Physics - Doklady* 30(1985), 275-278.
11. Weakly connected systems with nonpropagating waves. *Functional Analysis and its Applications* 18(1984), 142-144.
12. On the continuous spectrum in some problems of mathematical physics. *Soviet Physics - Doklady* 29(1984), 625-627.
13. Stability of travelling vortex lattices in long Josephson junctions (with S. Burkov). *Physics Letters A*, 106(1984), 71-73.
14. Comment on the paper "On the influence of the Hall effect on the spectrum of the ideal magnetohydrodynamic cylindrical pinch" by U. Schaper (with E. Fedorov and U. Schaper). *Journal of Plasma Physics* 31(1984), 173-175.
15. Langmuir oscillations of a three - dimensional nonuniform plasma (with A. Krylov). *Soviet Physics - Doklady* 29(1984), 535-536.
16. Quasi - Alfvén oscillations of magnetic surfaces (with A. Krylov). *Planetary and Space Science* 32(1984), 481-492.
17. Spectral properties of degenerate systems of ordinary differential equations on a finite interval. *Functional Analysis and its Applications* 17(1983), 62-63.
18. Stability of moving soliton lattices (with S. Burkov). *Wave Motion* 5(1983), 197-213.
19. The shape of Alfvén modes in an axisymmetric adiabatic trap (the magnetosphere)(with A. Krylov). *Transactions (Doklady) of the USSR Academy of Sciences. Earth Science Sections* 268(1983), 32-35.

20. Localized waves in a cool nonuniform plasma (with A. Krylov). Soviet Physics - Doklady 28(1983), 621-622.
21. The form of resonant pulsations of the magnetic shells in the magnetosphere (with A. Krylov). Izvestiya of the Academy of Sciences of the USSR, Earth Physics (191983), 835-842.
22. Stability of moving soliton superstructures (with S. Burkov). JETP Letters 35(1982), 233-235.
23. Relation between the Alfvén velocity and phase delays in a one-dimensionally inhomogeneous plasma (with M. Gokhberg and A. Krylov). Geomagnetism and Aeronomy 22(1982), 337-339.
24. On the resonant properties of the magnetosphere (with A. Krylov and E. Fedorov). Izvestiya of the Academy of Sciences of the USSR, Earth Physics 17(1981), 429-434.
25. Oscillations of anisotropic resonators. Functional Analysis and its Applications 14(1980), 132-133.
26. On hyperbolic systems with nonpropagating perturbations (with A. Krylov). Soviet Mathematics - Doklady 21(1980), 868-871.
27. Resonance properties of magnetospheric lines of force (with A. Krylov and E. Fedorov). Geomagnetism and Astronomy 20(1980), 478-480.
28. Natural oscillations of anisotropic media (with A. Krylov). Soviet Physics - Doklady 24(1979), 542-543.
29. On the resonant properties of a plasma in a curvilinear magnetic field (with A. Krylov and E. Fedorov). Doklady Akademii Nauk SSSR 247(1979), 1094-1095 (in Russian).

Conference Proceedings

1. A variational approach to adiabatic plasma evolution in ideal MHD (with A. Eydeland, J Spruck and B. Turkington). The 13th Conference on the Numerical Simulation of Plasmas, Santa Fe, New Mexico, USA, 1989. Proceedings, PMA4.
2. Stability of moving soliton superstructures (with S. Burkov). International symposium "Synergetics and Cooperative Phenomena in Solids and Macromolecules", Tallin, USSR, 1982. Proceedings(1983), 169-175.

Scientific Meeting Abstracts

1. Multiconstraint variational problems in magnetohydrodynamics: formulations and algorithms (with A. Eydeland, J. Spruck and B. Turkington). Sixth Beer - Sheva International Seminar on MHD Flows and Turbulence, Israel, 1990. Accepted for publication.
2. Utilization of qualitative methods of spectral theory in magnetohydrodynamic stability theory. International Symposium "Nonlinear Seismology", Suzdal, USSR, 1986. Abstracts(1986), 122.

3. Stability of soliton superstructures (with S. Burkov). International Symposium "Nonlinear Seismology" Suzdal, USSR, 1986. Abstracts(1986), 94.
4. Some problems with continuous spectrum in inhomogeneous plasma theory (with A. Krylov). Uspekhi Matematicheskikh Nauk (Russian Mathematical Surveys) 39(1984), part 2, 218-219 (in Russian).
5. A class of problems with continuous spectrum (with A. Krylov). Uspekhi Matematicheskikh Nauk (Russian Mathematical Surveys) 39(1984), part 4, 131 (in Russian).
6. Degenerate systems of ordinary differential equations on $[a,b]$. Conference on Some Problems of Mathematics and Mechanics, Moscow State University, Moscow, USSR, 1981. Program and Abstracts 35(1983), (in Russian).
7. Stability of moving soliton superstructures (with S. Burkov). International Symposium "Synergetics and Cooperative Phenomena in Solids and Macromolecules", Tallin, USSR, 1982. Program and Abstracts 27(1982), (in Russian).
8. On resonance properties of magnetosphere. 7th Annual Meeting of European Geophysical Society, Budapest, Hungary, 1980. Program and Abstracts 51(1980).
9. Resonance properties of geomagnetic lines of force (with A. Krylov and E. Fedorov). International Symposium "Solar - Earth Physics", Ashkhabad, USSR, 1979. Program and Abstracts 32(1979), (in Russian).

The "Geometry, Computation and Graphics" series includes preprints and technical reports in Geometric Analysis, Numerical Computation, Scientific Graphics and Applications.

1. M. Callahan, D. Hoffman, and J. Hoffman. Computer graphics tools for the study of minimal surfaces. (Technical Report DOE/ER/25015-2)
2. M. Callahan, D. Hoffman, and W. H. Meeks III. Embedded minimal surfaces with an infinite number of ends. (Technical Report DOE/ER/25015-3)
3. D. Hoffman and W. H. Meeks III. The global theory of properly embedded minimal surfaces. (Technical Report DOE/ER/25015-24) (also see Technical Reports 26 - 28)
4. W. H. Meeks III. Regularity of the Albanese map for nonoriented surfaces. (Technical Report DOE/ER/25015-4)
5. A. Eydeland and J. Spruck. The inverse power method for semilinear elliptic equations. (Technical Report DOE/ER/25015-5)
6. J. Spruck. The elliptic sinh-Gordon equation and the construction of toroidal soap bubbles. (Technical Report DOE/ER/25015-6)
7. E. Barbanel. Complete minimal surfaces in R^3 of low total curvature. (Technical Report DOE/ER/25015-7)
8. Yisong Yang. Global spatially periodic solutions to the Ginzburg-Landau equation. (Technical Report DOE/ER/25015-8)
9. Yisong Yang. Almost-periodic solutions of nonlinear parabolic equations. (Technical Report DOE/ER/25015-9)
10. Yisong Yang. A nonlinear parabolic equation of a wavenumber selection model in cellular flows. (Technical Report DOE/ER/25015-10)
11. H.I. Choi, W. H. Meeks III and Brian White. A rigidity theorem for properly embedded minimal surfaces in R^3 . (Technical Report DOE/ER/25015-11)

12. A. Eydeland and B. Turkington. A numerical study of vortex rings with swirl. (Technical Report DOE/ER/25015-13)
13. Yisong Yang. Comparison of eigenvalues of the Sturm-Loisville problems. (Technical Report DOE/ER/25015-14)
14. Yisong Yang. Uniqueness without maximum principle for self-adjoint elliptic boundary value problems. (Technical Report DOE/ER/25015-15)
15. Yisong Yang. Numerical solution of a boundary value problem arising in plate deflection theory. (Technical Report DOE/ER/25015-16)
16. D.M. Anderson and E.L. Thomas. Microdomain morphology of star copolymers in the Strong-Segregation limit. (Technical Report DOE/ER/25015-17)
17. E.L. Thomas, D.M. Anderson, C.S. Henkee and D. Hoffman. Periodic area minimizing surfaces in block copolymers. (Technical Report DOE/ER/25015-18)
18. L.A. Caffarelli, B. Gidas and J. Spruck. Asymptotic symmetry and local behavior of semilinear elliptic equations with critical sobolev growth. (Technical Report DOE/ER/25015-19)
19. W.H. Meeks, III and H. Rosenberg. The global theory of doubly periodic minimal surfaces. (Technical Report DOE/ER/25015-20)
20. A. Eydeland and E. van Groesen. An extended self-organization principle for modelling and calculating the dissipation of 2D confined eulerian vortices. (Technical Report DOE/ER/25015-21)
21. D. Hoffman and W.H. Meeks, III. Embedded minimal surfaces of finite topology. (Technical Report DOE/ER/25015-26)
22. D. Hoffman and W.H. Meeks, III. The strong halfspace theorem for minimal surfaces. (Technical Report DOE/ER/25015-27)
23. D. Hoffman and W.H. Meeks, III. A variational approach for the existence of embedded minimal surfaces. (Technical Report DOE/ER/25015-28)

24. A. Eydeland and B. Turkington. An iterative method for computing steady vortex flow systems.
25. W.H. Meeks, III and Y. Fang. A note on the geometry of minimal annuli in R^3 .
26. D. Hoffman and W.H. Meeks, III. The asymptotic behavior of properly embedded minimal surfaces of finite topology.
27. M. Callahan, D. Hoffman and W.H. Meeks, III. The structure of singly-periodic minimal surfaces.
28. W.H. Meeks, III and H. Rosenberg. The maximum principle at infinity for minimal surfaces in flat three manifolds.
29. A. Eydeland, J. Spruck and B. Turkington. Multi-constrained variational problems of nonlinear eigenvalue type: New formulations and algorithms.
30. A. Baldes and O. Wohlrab. Computer graphics of solutions of the generalized Monge-Ampère equation.
31. D. Hoffman. New examples of singly-periodic minimal surfaces and their qualitative behavior.
32. D. Hoffman and W.H. Meeks, III. Limits of minimal surfaces and Scherk's second surface.
33. Yi Fang. A new family of Enneper type minimal surfaces.
34. W.H. Meeks, III and H. Rosenberg. The geometry of periodic minimal surfaces.
35. Y. Fang and W.H. Meeks, III. The geometry of minimal surfaces in a halfspace in R^3 .
36. W.H. Meeks, III. The theory of triply periodic minimal surfaces.
37. L.C. Evans and J. Spruck. Motion of level sets by mean curvature I.
38. L.C. Evans and J. Spruck. Motion of level sets by mean curvature II.

39. L.A. Caffarelli and J. Spruck. Variational problems with critical Sobolev growth and positive Dirichlet data.