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ABSTRACT

The reasons for the large discrepancies in the magnitude of the πN σ-commutator, $\sigma(\pi N)$, obtained by several authors are discussed using a dynamical theory of the πN scattering amplitude. With $\sigma(\pi N) \sim 25$ MeV this theory reproduces reasonably well both the experimental S-wave phase shifts at low energies and the amplitudes $\mathcal{C}^{(+)}(\nu=0, t\leq 0)$ determined by Langbein. In the method of Cheng and Dashen the value of $\sigma(\pi N)$ is obtained from these amplitudes by extrapolation to $t = 2m_\pi^2$. A study of the experimental D-wave "scattering lengths" implies that the coefficient of the term quadratic in t in the Höhler expansion of $\mathcal{C}^{(+)}(0,t)$ is negative. Adding such a term to the results of the S-wave theory will tend to improve the agreement for $\mathcal{C}^{(+)}(0,t\leq 0)$. These results suggest that the large "world value" $\sigma(\pi N) = 65 \pm 5$ MeV obtained using the Cheng-Dashen method is a consequence of errors in the extrapolation of amplitudes to the unphysical point $\nu=0$, $t=2m_\pi^2$. Only the smaller value $\sigma(\pi N) \sim 25$ MeV appears to be consistent with the experimental data and theoretical constraints.

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I. INTRODUCTION

The pion-nucleon σ-commutator, introduced by Weinberg,[1] is a measure of chiral symmetry breaking and a vital parameter of low energy πN scattering. Because of its importance there have been many attempts to evaluate this quantity[2] since the first effort of von Hippel and Kim.[3]

The work of Cheng and Dashen[4] (CD) provided a new method for determining the pion-nucleon σ-commutator, $\sigma(\pi N)$. The method involves extrapolation of the scattering amplitude in the variables $\nu = (s-u)/4M$ and t, the invariant momentum transfer, to unphysical values $\nu = 0$ and $t = 2m_\pi^2$ but keeping all particles on the mass shell. This appears to be superior to the methods based on the work of Fubini and Furlan[5] which uses amplitudes where one pion is off the mass shell. Only a dynamical theory can give these off-mass-shell amplitudes. On the other hand it is believed that the unphysical amplitudes required in the CD approach can be determined reasonably well through the use of sophisticated extrapolation methods and dispersion relations. One finds a significant difference between the results of the two methods. Early efforts based on the Fubini-Furlan theory gave values of[6] 25 to 30 MeV for $\sigma(\pi N)$ while the more recent work using the Cheng-Dashen approach has settled on a value between 60 and 70 MeV.[2,7-11]

Recently we have presented a dynamical theory of low energy pion-nucleon scattering.[12] This work is an extension of the Chew-Low theory[13] in which both nucleon recoil and the seagull terms are included. The isoscalar seagull term is evaluated with the help of the soft pion limit where one pion is made soft, and in the soft pion amplitude $\sigma(\pi N)$ appears as a parameter of the theory. Another major parameter is the coupling constant $\bar{g}_\pi = |g_\pi(4M^2)|$ appearing in the z-graphs, which are the main isovector terms. The calculated S-wave phase shifts are extremely sensitive to both $\sigma(\pi N)$ and $\bar{g}_\pi$. We determine

$$\bar{g}_\pi = 11.69 \text{ MeV},$$

$$\sigma(\pi N) = 25.5 \text{ MeV}$$

by requiring good agreement between calculated and experimental phase shifts. The agreement between this value of $\sigma(\pi N)$ and the values obtained from several Fubini-Furlan-type calculations[3,6] is, in a sense, accidental since there are two kinds of errors in these calculations which tend to cancel one another.[14]

In this paper we demonstrate that the discrepancy between our value of $\sigma(\pi N)$ and the very high value obtained from the CD approach is entirely due to extrapolation errors in the latter.

The extrapolation involves the variables ν and t. The current practice in the Cheng-Dashen approach is to deal with the amplitude[15]

$$\tilde{C}^{(+)}(\nu,t) = A^{(-)}(\nu,t) + \frac{\nu}{1-\frac{t}{4M^2}} B^{(+)}(\nu,t) - \frac{g_\pi^2(m_\pi^2)}{M(1-\frac{t}{4M^2})} \frac{\nu_B^2}{\nu_B^2-\nu^2}, \tag{1.1}$$

where A and B are the usual CGLN[16] amplitudes and

$$\nu_B = \frac{k\cdot k'}{2M} = \frac{k^2+k'^2-t}{4M} = \frac{2m_\pi^2-t}{4M}. \tag{1.2}$$

Using the values of the amplitude for physical $t \leq 0$ and $\nu \geq m_\pi\sqrt{(1-\frac{t}{4M^2})(1-\frac{t}{4m_\pi^2})}$ one first extrapolates in ν to $\nu = 0$ for fixed t. One then considers, in the notation of Höhler,[11] the expansion

$$\tilde{C}^{(+)}(0,t) = \tilde{C}_1^{(+)} + \tilde{C}_2^{(+)}\frac{t}{m_\pi^2} + \tilde{C}_7^{(+)}(\frac{t}{m_\pi^2})^2 + \ldots. \tag{1.3}$$

Most authors have retained only the first two terms on the right. Nielson and Oades[9] keep $\tilde{C}_7^{(+)}$, also. The form (1.3) is then used to evaluate $\tilde{C}^{(+)}(0,2m_\pi^2)$ from which $\sigma(\pi N)$ is obtained from the relation[10,17]

$$(\frac{\sqrt{2}m_\pi^2}{f_\pi})^2\sigma(\pi N) = \tilde{C}^{(+)}(0,2m_\pi^2) + \frac{g_\pi^2(m_\pi^2)}{2M^3}m_\pi^2. \tag{1.4}$$

The amplitude $\tilde{C}^{(+)}(0,t)$ can be evaluated using our theory. When we do this using exactly the same set of parameters which were used in (I) the resulting $\tilde{C}^{(+)}(0,t\leq 0)$ are in reasonable agreement with those of Langbein[10] in the range $0 \geq t \geq -0.1$ GeV^2. We point out that to improve the agreement one must <u>decrease</u> $\sigma(\pi N)$ slightly rather than increase it. To accomplish this while maintaining a good fit with the low energy S-wave phase shifts it is necessary to change the form factor mass μ which is used to parametrize the P-wave off-mass-shell amplitude. It must be increased from the value $8m_\pi$, used in (I), to a value close to $10m_\pi$. All other parameters remaining the same, but $\mu = 10m_\pi$, a good fit to the low energy S-wave phase shifts can be obtained with $\sigma(\pi N)$ slightly larger than 24 MeV and $\bar{g}_\pi$ slightly smaller than 11.9. The fit with Langbein's values of $\tilde{C}^{(+)}(0,t)$ is improved further by giving $\tilde{C}_7^{(+)}$ a small negative value. We discuss the plausibility of such a value for $\tilde{C}_7^{(+)}$. We cannot calculate $\tilde{C}_7^{(+)}$ reliably because our theory, designed for low energy pion-nucleon scattering, leaves out an important set of terms operative primarily in the D-wave. These terms are obtained by replacing the antinucleons by antideltas in the z-graphs.

With or without the negative $\tilde{C}_7^{(+)}$ our theoretically calculated $\tilde{C}_1^{(+)}$ and $\tilde{C}_2^{(+)}$ are invariably smaller than those of Langbein and various other authors. We find $\tilde{C}_1^{(+)} = -12.4(-10.8)$ GeV^{-1} and $\tilde{C}_2^{(+)} = +7.0(6.0)$ GeV^{-1} for $\mu = 8(10)m_\pi$, while the means of the values of Refs. 8-11 are $\tilde{C}_1^{(+)} = -10.2$ GeV^{-1} and $\tilde{C}_2^{(+)} = 7.5$ GeV^{-1}. The effects of these discrepancies are compounded in $\tilde{C}^{(+)}(0,2m_\pi^2) = \tilde{C}_1^{(+)} + 2\tilde{C}_2^{(+)}$ resulting in the large difference between our value of 25.5(24) MeV and a mean value of 58 MeV from the extrapolated results of Refs. 8-11.

Our success in getting good agreement with the low energy S-wave phase shifts while simultaneously getting quite close to Langbein's values of $\tilde{C}^{(+)}(0,t\leq 0)$

leaves us no choice but to raise doubt about the precision of the values of $\tilde{C}_1^{(+)}$ and $\tilde{C}_2^{(+)}$ obtained by other authors.

After reviewing our theory of the off-mass-shell πN amplitude in the following section, we present in Section III our method for computing $\tilde{C}^{(+)}(0,t)$. In Section IV our results for this amplitude are compared with the two sets of values of $\tilde{C}^{(+)}(0,t\leq 0)$ obtained by Langbein, and with the results for the Höhler parameters $\tilde{C}_1^{(+)}$, $\tilde{C}_2^{(+)}$ and $\tilde{C}_7^{(+)}$ obtained by Langbein and other authors. In Section V we summarize our findings.

II. Theory of the Off-Mass-Shell πN Amplitude

In this section we summarize the results of (I). We consider the amplitude,[18] schematically depicted in Fig. 1,

$$F_{\beta\alpha}(k) = i(k^2 - m_\pi^2)(k'^2 - m_\pi^2)\int d^4x\, e^{ik\cdot x}\langle p_f|T(\phi_\beta(x),\phi_\alpha(0))|p_i\rangle \qquad (2.1)$$

where we use PCAC and write

$$\phi_\alpha(x) = \frac{\sqrt{2}}{f_\pi}\frac{\partial A_\mu^\alpha(x)}{\partial x_\mu}, \qquad (2.2)$$

with $f_\pi = 0.939 m_\pi^3$.

The expression for $F_{\beta\alpha}(k)$ can also be rewritten in the form

$$F_{\beta\alpha}(k) = \int d^4x\, e^{ik\cdot x}\langle p_f|\delta(x_0)[\phi_\beta(x),j_\alpha(0)]k_0 + i\delta(x_0)[\dot{\phi}_\beta(x),j_\alpha(0)]|p_i\rangle$$
$$+ i\int d^4x\, e^{ik\cdot x}\langle p_f|T(j_\beta(x),j_\alpha(x))|p_i\rangle \qquad (2.3)$$

where

$$(\Box + m_\pi^2)\phi_\alpha(x) = j_\alpha(x)$$

is the pion source current. The first two terms are the so-called seagull terms. We <u>assume</u> that the isovector part of the seagull term is either exactly zero or at least negligibly small. We then eliminate the isoscalar part of the seagull term with the help of the soft pion limit of $F_{\beta\alpha}(k)$ obtained by first letting $\vec{k} \to 0$ and then $k_0 \to 0$ in Eq. (2.1). This limit is

$$F_{\beta\alpha}(0) = \left(\frac{\sqrt{2}}{f_\pi}\right)^2 m_\pi^2 (t - m_\pi^2)\langle p_f|\sigma(0)|p_i\rangle\delta_{\alpha\beta} - g_\pi(t)g_\pi(0)\bar{u}(p_f)$$
$$\times\left[\frac{M\gamma_0 - p_{f0}}{2M\, p_{f0}}\tau_\beta\tau_\alpha + \frac{M\gamma_0 - p_{i0}}{2M\, p_{i0}}\tau_\alpha\tau_\beta\right]u(p_i) \qquad (2.4)$$

where

$$g_\pi(0) = \frac{\sqrt{2}M\, m_\pi^2}{f_\pi}g_A(0) = 12.7 \qquad (2.5)$$

upon using $g_A(0) = 1.25$. The operator $\sigma(0)$ is the σ-commutator defined by

$$\langle p_f|\sigma(0)\delta_{\alpha\beta}|p_i\rangle = i\int d^4x\,\delta(x_0)\langle p_f|[A_0^\beta(x),\partial^\mu A_\mu^\alpha(0)]|p_i\rangle,$$

and we write

$$\langle p_f|\sigma(0)|p_i\rangle = \frac{\sigma(\pi N)\bar{u}(p_f)u(p_i)}{\left(1 - \frac{t}{\mu_1^2}\right)^2\left(1 - \frac{t}{\mu_2^2}\right)}. \qquad (2.6)$$

The second term in (2.4) arises from the soft pion limit of the nucleon pole terms.

We want to draw the reader's attention to the factor $(t - m_\pi^2)$ in the σ-commutator term, since there appears to be some confusion regarding the structure of this term. One finds in the literature,[19] for example, such forms as $\sigma(\pi N)[(1-\beta)(k^2 + k'^2 - m_\pi^2) + \beta(t - m_\pi^2)]$. In the absence of any dynamical theory such a form may appear possible because it satisfies the Adler consistency condition.[20] We want to emphasize that the correct value of $\beta = 1$.

The σ-commutator term appears through the soft pion limit $k \to 0$, which yields the form (2.4). Any other soft pion limit, such as $k + k' \to 0$, will involve, for non-forward scattering, unknown matrix elements $\langle p | A_0^\alpha(0) | p', q\gamma \rangle$ of the axial vector current between a one-nucleon state and a one-pion plus one-nucleon state. Thus our soft pion limit has two advantages. First, we avoid this unknown amplitude and second, the Adler condition is manifest.

After eliminating the seagull terms we get

$$F_{\beta\alpha}(k) = F_{\beta\alpha}(0) + i \int d^4x\, e^{ik\cdot x} \langle p_f | T(j_\beta(x), j_\alpha(0)) | p_i \rangle - i \int d^4x \langle p_f | T(j_\beta(x), j_\alpha(0)) | p_- \rangle \qquad (2.7)$$

where $F_{\beta\alpha}(0)$ is given by (2.4). Next we make Low expansions of the integrals. We truncate the expansions in the CM frame where the energy denominators are the largest. We retain intermediate states of one nucleon, one pion plus one nucleon, and the z-graph terms which occur with two nucleons plus one antinucleon intermediate states. As we will see later we can include inelastic intermediate states quite easily in many situations.

It is useful to introduce the partial wave expansion in the CM frame

$$F_{\beta\alpha}(k) = 4\pi \sum_\nu f_\nu(|\vec{p}_f|, |\vec{p}_i|)\, \Lambda_\nu(\vec{p}_f, \vec{p}_i, \beta, \alpha) \qquad (2.8)$$

where $\vec{p}_f$ and $\vec{p}_i$ are the final and initial nucleon CM momenta. The projection

operator Λ_ν is a product

$$\Lambda_\nu(\vec{p}_f, \vec{p}_i, \beta, \alpha) = \mathcal{P}_{L_\nu J_\nu}(\vec{p}_f, \vec{p}_i)\, \Pi_{I_\nu}(\beta, \alpha) \qquad (2.9)$$

of angular momentum and isospin projection operators.

$$\Pi_{1/2}(\beta,\alpha) = \frac{1}{3}\tau_\beta\tau_\alpha$$
$$\Pi_{3/2}(\beta,\alpha) = \delta_{\alpha\beta} - \frac{1}{3}\tau_\beta\tau_\alpha \qquad (2.10)$$

$$\mathcal{P}_{S_{1/2}}(\vec{p}_f, \vec{p}_i) = \frac{M}{4\pi\sqrt{(p_{f0}+M)(p_{i0}+M)}}\, \bar{u}(p_f)(\gamma_0+1)u(p_i)$$

$$\mathcal{P}_{P_{1/2}}(\vec{p}_f, \vec{p}_i) = \frac{M}{4\pi\sqrt{(p_{f0}-M)(p_{i0}-M)}}\, \bar{u}(p_f)(\gamma_0-1)u(p_i)$$

$$\mathcal{P}_{P_{3/2}}(\vec{p}_f, \vec{p}_i) = \frac{M}{4\pi\sqrt{(p_{f0}+M)(p_{i0}+M)}}\, 3\, \frac{\vec{p}_f\cdot\vec{p}_i}{|\vec{p}_f||\vec{p}_i|}\, \bar{u}(p_f)(\gamma_0+1)u(p_i) - \mathcal{P}_{P_{1/2}}(\vec{p}_f, \vec{p}_i). \qquad (2.11)$$

These angular momentum projection operators satisfy the idempotency condition

$$\int d\Omega_{\hat{n}}\, \mathcal{P}^\dagger_{LJ}(|p|\hat{n}, \vec{p}_f)\, \mathcal{P}_{L'J'}(|p|\hat{n}, \vec{p}_i) = \delta_{LL'}\delta_{JJ'}\, \mathcal{P}_{LJ}(\vec{p}_f, \vec{p}_i). \qquad (2.12)$$

The complete expression for the off-mass-shell amplitude is

$$\begin{aligned}
F_{\beta\alpha}(k) &= 4\pi \sum_\nu f_\nu(|\vec{p}_f|, |\vec{p}_i|)\, \Lambda_\nu(\vec{p}_f, \vec{p}_i, \beta, \alpha) \\
&= \frac{\sigma(\pi N)}{(1 - \frac{t}{\mu_1^2})^2 (1 - \frac{t}{\mu_2^2})}\, \delta_{\alpha\beta}\, \bar{u}(p_f)u(p_i) \left(\frac{\sqrt{2}}{f_\pi}\right)^2 m_\pi^2 (t - m_\pi^2) \\
&\quad - g_\pi(t) g_\pi(0)\, \bar{u}(p_f) \left[\frac{M\gamma_0 - p_{f0}}{2Mp_{f0}}\, \tau_\beta\tau_\alpha + \frac{M\gamma_0 - p_{i0}}{2Mp_{i0}}\, \tau_\alpha\tau_\beta\right] u(p_i) \\
&\quad + \frac{g_\pi((p_f-P)^2)\, g_\pi((p_i-P)^2)}{2(k_0 + p_{f0} - M)}\, \bar{u}(p_f)(1-\gamma_0)\tau_\beta\tau_\alpha\, u(p_i) \\
&\quad + \frac{g_\pi((p_f-\ell)^2)\, g_\pi((p_i-\ell)^2)}{2\ell_0(k_0 + \ell_0 - p_{i0})}\, \bar{u}(p_f)[(\ell_0 - p_{i0} - p_{f0})\gamma_0 + M]\tau_\alpha\tau_\beta\, u(p_i) \\
&\quad - \frac{\bar{g}_\pi((p_f+P)^2)\, \bar{g}_\pi((p_i+P)^2)}{2(k_0 + p_{f0} + M)}\, \bar{u}(p_f)(\gamma_0+1)\tau_\beta\tau_\alpha\, u(p_i) \\
&\quad - \frac{\bar{g}_\pi((p_f+\tilde{\ell})^2)\, \bar{g}_\pi((p_i+\tilde{\ell})^2)}{2\tilde{\ell}_0(p_{i0} + \tilde{\ell}_0 - k_0)}\, \bar{u}(p_f)[\gamma_0(\tilde{\ell}_0 + p_{i0} + p_{f0}) - M]\tau_\alpha\tau_\beta\, u(p_i)
\end{aligned}$$

$$+\frac{\bar{g}_\pi((p_f+\hat{p}_f)^2)\bar{g}_\pi((p_i+\hat{p}_f)^2)}{2p_{f0}}\,\bar{u}(p_f)\,\gamma_0\,\tau_\beta\,\tau_\alpha\,u(p_i)$$

$$+\frac{\bar{g}_\pi((p_f+\hat{p}_i)^2)\bar{g}_\pi((p_i+\hat{p}_i)^2)}{2p_{i0}}\,\bar{u}(p_f)\,\gamma_0\,\tau_\alpha\,\tau_\beta\,u(p_i)$$

$$+\frac{1}{\pi}\sum_a \xi_a \int_{M+m_\pi}^{\infty}\frac{dW}{\sqrt{W^2+L_a^2}-\varepsilon_a}\,\frac{Mq}{\sqrt{W^2+L_a^2}}\sum_\nu f_\nu^*(q,|\vec{p}_f^{\,a}|)f_\nu(q,|\vec{p}_i^{\,a}|)$$

$$\times J_\nu(\vec{p}_f^{\,a},\vec{p}_i^{\,a},\gamma_f^a,\gamma_i^a), \tag{2.13}$$

where W and q are the total CM energy and momentum of the intermediate state.

The first two terms of (2.13) have been discussed before. The third and the fourth terms are the direct and the crossed nucleon pole terms. They vanish when $\vec{k}\to 0$. The next two terms are the direct and the crossed z-graphs and the two subsequent ones are their soft pion limits.

For the pion nucleon vertex function $g_\pi(t)$ which appears in some of these terms we use for $t \le 0$ the form

$$g_\pi(t) = \frac{g_\pi(0)}{1+\dfrac{t(t-4M^2)}{4M^2m_0^2}}\ . \tag{2.14}$$

The quantity $\bar{g}_\pi(t)$ is an approximation to $|g_\pi(t\ge 4M^2)|$ and we represent it as

$$\bar{g}_\pi(t) = \frac{\bar{g}_\pi}{1+\dfrac{t-4M^2}{4m_0^2}}\ . \tag{2.15}$$

The four-momenta P, ℓ, $\tilde{\ell}$, $\hat{p}_i$ and $\hat{p}_f$ are defined by

$$P^2 = \ell^2 = \tilde{\ell}^2 = \hat{p}_i^2 = \hat{p}_f^2 = M^2$$

$$\vec{P} = 0,\ \vec{\ell} = \vec{p}_i + \vec{p}_f,\ \vec{\tilde{\ell}} = -\vec{p}_i - \vec{p}_f,\ \vec{\hat{p}}_i = -\vec{p}_i \text{ and } \vec{\hat{p}}_f = -\vec{p}_f. \tag{2.16}$$

The last term in (2.13) represents the four rescattering terms of the Low expansion. The label a runs from 1 through 4 and designates the hard right, the hard left, the soft right and the soft left terms, respectively. The hard right refers to the direct term of the first integral on the right-

hand side of (2.7), and so forth. The other quantities in the last term are defined in the Appendix. The variable ν denotes (2I,2J).

In (I) we discussed how the integral equations (2.13) are used to generate S-wave phase shifts at low energies. A large amount of computer time is required to obtain a reasonably accurate solution for a given set of parameters. Because of this the search of the various form factor masses was carried out by examining the [1,1] Padé approximants. The values we settled on are $\mu_1 = 8.24m_\pi$, $\mu_2 = 7.5m_\pi$ and $m_0 = 8.6m_\pi$. We also stressed the sensitivity of these phase shifts to the main parameters $\sigma(\pi N)$ and $\bar{g}_\pi$ for a given set of form factor masses. Comparison with Carter, Bugg and Carter[21] phase shifts yields the values $\bar{g}_\pi = 11.69$ and $\sigma(\pi N) = 25.5$ MeV. The reason for the discrepancy between this value of $\sigma(\pi N)$ and the values of 60-70 MeV obtained by several authors using the method of Cheng and Dashen follows from an analysis of the amplitude $\tilde{C}^{(+)}(\nu=0,t)$ of Eq. (1.1). We now discuss the evaluation of this amplitude in the context of this theory.

III. EVALUATION OF $\tilde{C}^{(+)}(0,t)$

Cheng and Dashen[4] considered the isoscalar amplitude

$$F^{(+)}(k^2,k'^2,\nu,\nu_B) \equiv \frac{1}{2}(F_{\beta\alpha}(k)+F_{\alpha\beta}(k)). \tag{3.1}$$

The amplitudes which appear on the right-hand side are the ones defined by (2.1). Of course, this equation defines the function for real vector k and physical p_i and p_f. The Adler consistency condition[20] requires that

$$F^{(+)}(k^2,k'^2,0,0) = (1-\frac{k^2+k'^2}{m_\pi^2})\, F^{(+)}(0,0,0,0,)+O(k^4) \tag{3.2}$$

where $O(k^4)$ stands for terms of the order of k^4, k'^4 and $k^2k'^2$. This means

$$F^{(+)}(m_\pi^2,m_\pi^2,0,0) = -F^{(+)}(0,0,0,0)+O(m_\pi^4)$$

$$= (\frac{\sqrt{2}}{f_\pi})^2\, m_\pi^4\, \sigma(\pi N)+O(m_\pi^4), \tag{3.3}$$

where the last line follows from (2.4) and (2.6) upon setting $\vec{p}_i = \vec{p}_f = 0$. Cheng and Dashen noted that the amplitude $F^{(+)}(m_\pi^2,m_\pi^2,0,0)$ involves on-mass-shell pions and nucleons with unphysical $s = M^2$ and $t = 2m_\pi^2$ and, therefore, can be evaluated with the help of dispersion relations and physical data.

Usually one evaluates the amplitude $\tilde{C}^{(+)}(\nu,t)$, defined in (1.1), instead of

$$F^{(+)}(m_\pi^2,m_\pi^2,\nu,\nu_B) = A^{(+)}(\nu,t) + \nu B^{(+)}(\nu,t) \tag{3.4}$$

because of the insensitivity of $\tilde{C}^{(+)}$ to the precise value of $g_\pi^2(m_\pi^2)$. The practice is to evaluate $\tilde{C}^{(+)}(\nu,t)$ for physical ν's and t's first, and then to extract $\tilde{C}^{(+)}(0,t)$ by using either dispersion relations or some sophisticated extrapolation technique. Finally most authors have used a linear extrapolation to obtain $\tilde{C}^{(+)}(0,2m_\pi^2)$ from which a value of $\sigma(\pi N)$ is obtained using Eq. (1.4).

We can evaluate the amplitude $\tilde{C}(0,t)$ from Eq. (2.13) by assigning values to the various CM variables in the following manner:

$$s = M^2 + 2M\nu_B,$$

$$k_0 = k_0' = \frac{m_\pi^2 + 2M\nu_B}{2\sqrt{s}},$$

$$p_{f0} = p_{i0} = \sqrt{s} - k_0,$$

$$\vec{k}^2 = \vec{k}'^2 = \vec{p}_f^2 = \vec{p}_i^2 = k_0^2 - m_\pi^2,$$

$$\bar{u}(p_f)\,\gamma_0\, u(p_i) = \frac{M}{\sqrt{s}}. \tag{3.5}$$

To facilitate identification of the Höhler parameters of Eq. (1.3), we expand the various terms of Eq. (2.13) in powers of t. The contribution to $\tilde{C}^{(+)}(0,t)$ from the σ-commutator, N-pole and z-graph terms is straightforwardly obtained. The contribution from the S-wave rescattering integrals results from two sources. First is the dependence on $|\vec{p}|$ of the S-wave amplitudes $f_\nu(|\vec{q}|,|\vec{p}|)$, where $\vec{q}$ is the CM momentum of an on-shell pion and nucleon. One may write, for small $|\vec{p}|$,

$$f_\nu(|\vec{q}|,|\vec{p}|) = f_{\nu 1}(|\vec{q}|) + \frac{\vec{p}^2}{m_\pi^2}\, f_{\nu 2}(|\vec{q}|) + \ldots . \tag{3.6}$$

Our S-wave calculations show that for S11 $f_{\nu 2}/f_{\nu 1} \sim 0.1$, while for S31 the ratio is ~ 0.01. From the last integral in (2.13) and the definition of $\vec{p}_i^{\,a}$ and $\vec{p}_f^{\,a}$ given in the Appendix one sees that the numerators of the rescattering integrals for the soft terms and the hard left term are t-dependent. A second and somewhat less important source is the t dependence of the energy denominators of the integrals.

Writing the S-wave rescattering contribution to $\tilde{C}^{(+)}(0,t)$ as $\tilde{C}_{1S}^{(+)} + \frac{t}{m_\pi^2}\tilde{C}_{2S}^{(+)}$, we find

$$\tilde{C}_{2S}^{(+)} = \frac{1}{6\pi^2}\int_{M+m_\pi}^{\infty} dW\, \frac{M^3 q}{W^3(W-M)}\Big[\{f_{112}^2(|\vec{q}|) + 2f_{312}^2(|\vec{q}|)\}\,\frac{(W-M)(2M-W)}{W^2}$$

$$+ \{f_{111}(|\vec{q}|)f_{112}(|\vec{q}|) + 2f_{311}(|\vec{q}|)f_{312}(|\vec{q}|)\}$$

$$- \{f_{11}^2(|\vec{q}|,im_\pi) + 2f_{31}^2(|\vec{q}|,im_\pi)\}\,\frac{2W-M}{4M^2(W-M)}\Big] \tag{3.7}$$

and,

$$\tilde{C}^{(+)}_{1S} = -2\tilde{C}^{(+)}_{2S} + \frac{1}{12\pi^2} \int_{M+m_\pi}^{\infty} dW \frac{M^3 q}{W^3(W-M)} \frac{M^2 + (2W-M)^2}{W^2} \{f^2_{112}(|\vec{q}|) + 2f^2_{312}(|\vec{q}|)\}, \tag{3.8}$$

where

$$f_\nu(|\vec{q}|, im_\pi) \equiv f_{\nu 1}(|\vec{q}|) - f_{\nu 2}(|\vec{q}|).$$

The integral term in (3.8) is the value of the S-wave rescattering integrals at the Cheng-Dashen point, $\nu = 0$, $t = 2m_\pi^2$. From (3.6) we see that $f_{\nu 2} = m_\pi^2 \frac{d}{d|\vec{p}|^2} f_\nu$ and so the term is $\sim m_\pi^4$. We did not include S-wave inelasticity in (I) and hence the values of the integrals in (3.7) and (3.8) are not as reliable as the low energy phase shifts. The values we get with the parameters of (I) are

$$\tilde{C}^{(+)}_{2S} = -1.12 \text{ GeV}^{-1},$$
$$\tilde{C}^{(-)}_{1S} + 2\tilde{C}^{(+)}_{2S} = 0.11 \text{ GeV}^{-1}. \tag{3.9}$$

To evaluate the P-wave rescattering contribution to $\tilde{C}^{(+)}(0,t)$ we use the ansatz

$$f_\nu(q,p) = f_\nu(q,q)\left(\frac{p}{q}\right) \frac{\phi_\nu(p)}{\phi_\nu(q)}, \tag{3.10}$$

similar to the one used in (1), where $f_\nu(q,q)$ is the physical elastic scattering amplitude for the channel ν and $\phi_\nu(p)$ is a suitably chosen form factor. The contribution of all P-wave inelastic intermediate states is included by replacing the products $f^*_\nu(q,p) f_\nu(q,p')$ in (2.10) by

$$\left(\frac{pp'}{q^2}\right) \frac{\phi_\nu(p)\phi_\nu(p')}{\phi^2_\nu(q)} \left(\frac{4\pi W}{M}\right)^2 \sigma_\nu = \left(\frac{pp'}{q^2}\right) \frac{\phi_\nu(p)\phi_\nu(p')}{\phi^2_\nu(q)} \left(\frac{4\pi W}{Mq}\right)^2 \frac{1 - \eta_\nu \operatorname{Cos} 2\delta_\nu}{2} \tag{3.11}$$

where σ_ν, η_ν and δ_ν are the partial wave cross section, inelasticity and phase shift for the channel ν.

By far the largest P-wave rescattering contribution to $\tilde{C}^{(+)}(0,t)$ comes from the P33 channel, for which our factorability ansatz seems eminently reasonable. We have included all P-wave channels and have used the same form factor for each channel, viz,

$$\phi(p) = \frac{1}{\left(1 + \frac{\vec{p}^2}{\mu^2}\right)^{5/2}}, \tag{3.12}$$

where the mass parameter $\mu = 8m_\pi$ was used in the numerical work discussed in (I).

For the P-wave rescattering contribution to $\tilde{C}^{(+)}(0,t)$ we use the form $\tilde{C}^{(+)}_{1P} + \frac{t}{m_\pi^2} \tilde{C}^{(+)}_{2P}(\mu) + \left(\frac{t}{m_\pi^2}\right)^2 \tilde{C}^{(+)}_{7P}$, thus retaining the term quadratic in t. We get

$$\tilde{C}^{(+)}_{2P}(\mu) = 4 \int dW \frac{W}{Mq^3} H(W)\left(1 + \frac{\vec{q}^2}{\mu^2}\right)^5 \left[\frac{1}{W-M} - \frac{M}{W^2} - \frac{1}{W}\right] \tag{3.13}$$

$$\tilde{C}^{(+)}_{7P}(\mu) = -\int dW \frac{Wm_\pi^2}{M^2 q^3} H(W)\left(1 + \frac{\vec{q}^2}{\mu^2}\right)^5 \left[\frac{1}{(W-M)^2} - \frac{M}{W^2(W-M)}\right] \tag{3.14}$$

$$\tilde{C}^{(+)}_{1P}(\mu) = -2\tilde{C}^{(+)}_{2P}(\mu) + 4\tilde{C}^{(+)}_{7P}(\mu) + 4\frac{m_\pi^4}{M^3} \int_{M+m_\pi}^{\infty} \frac{WdW}{W-M}\left(1 + \frac{\vec{q}^2}{\mu^2}\right)^5$$
$$\times \left\{1 - \frac{2}{3}\frac{M(W-M)}{W^2}\right\}\left(\frac{1 - \eta \operatorname{Cos} 2\delta}{2q^3}\right)_{P33}. \tag{3.15}$$

The symbol

$$H(W) = \frac{1}{3}\left[\left(\frac{1 - \eta \operatorname{Cos} 2\delta}{2}\right)_{P11} + 2\left(\frac{1 - \eta \operatorname{Cos} 2\delta}{2}\right)_{P31}\right.$$
$$\left. + 2\left(\frac{1 - \eta \operatorname{Cos} 2\delta}{2}\right)_{P13} + 4\left(\frac{1 - \eta \operatorname{Cos} 2\delta}{2}\right)_{P33}\right]. \tag{3.16}$$

The last term in (3.15) is the value of the P-wave rescattering term at the Cheng-Dashen point. It is manifestly $\sim m_\pi^4$.

Combining the terms we have

$$\tilde{C}^{(+)}(0,t) = \left(\frac{\sqrt{2}m_\pi}{f_\pi}\right)^2 \frac{\sigma(\pi N)(t - m_\pi^2)}{\left(1 - \frac{t}{\mu_1^2}\right)^2\left(1 - \frac{t}{\mu_2^2}\right)}$$
$$- (t - m_\pi^2)\frac{g_\pi^2(m_\pi^2)}{2M^3} + (t - 2m_\pi^2)\frac{\bar{g}_\pi^2}{4M^3}\left(1 + \frac{M^2}{m_0^2}\right)$$
$$+ \tilde{C}^{(+)}_{1S} + \tilde{C}^{(+)}_{1P}(\mu) + \frac{t}{m_\pi^2}\left[\tilde{C}^{(+)}_{2S} + \tilde{C}^{(+)}_{2P}(\mu)\right] + \left(\frac{t}{m_\pi^2}\right)^2 \tilde{C}^{(+)}_{7P}(\mu). \tag{3.17}$$

We note that the nucleon pole terms vanish at the Cheng-Dashen point and that the only contribution of the z-graphs at the CD point is a term $\sim \frac{m_\pi^4}{M^5}$ and is not included in (3.17). The other terms $\sim m_\pi^4$ at the CD point are also small. The σ-term has a m_π^4 piece

$$\left(\frac{\sqrt{2}m_\pi}{f_\pi}\right)^2 \sigma(\pi N)\; 2m_\pi^4\left(\frac{2}{\mu_1^2} + \frac{1}{\mu_2^2}\right) = 0.27 \text{ GeV}^{-1},$$

using $\sigma(\pi N) = 25.5$ MeV, $\mu_1 = 8.24m_\pi$ and $\mu_2 = 7.5m_\pi$. The P-wave integral with $\mu = 8m_\pi$ has an m_π^4 contribution of 0.16 GeV^{-1}, while the S-wave integral contribution has been given in (3.9) as 0.11 GeV^{-1}. Including the z-graph term mentioned above of −0.05 GeV^{-1}, we find the total magnitude of the m_π^4 terms to be 0.49 GeV^{-1}.[22] Since the main term in (3.18) (of order m_π^2) is

$$\left(\frac{\sqrt{2}m_\pi}{f_\pi}\right)^2 m_\pi^2\, \sigma(\pi N) = 2.89 \text{ GeV}^{-1},$$

we see that the basic idea of Cheng and Dashen that the terms $\sim m_\pi^4$ in (3.2) are small is quite reasonable as long as the object is to estimate σ(πN) within 16% or so. We should stress that a 16% change in the value of σ(πN), ceteris paribus, will result in a substantial change in the calculated S-wave phase shifts.

With Eq. (3.17) we can evaluate $\mathcal{C}^{(+)}(0,t)$ for either spacelike or timelike t. By comparing our predictions for $\mathcal{C}^{(+)}(0,t \le 0)$ and $\mathcal{C}^{(+)}(0,2m_\pi^2)$ with the values of Langbein and others we can determine the reason these authors have obtained such large values of σ(πN) using the method of Cheng and Dashen.

IV. COMPARISON WITH LANGBEIN'S $\mathcal{C}^{(+)}(0,t)$

In Figs. 2 and 3 we present Langbein's[10] solutions I and II for $\mathcal{C}^{(+)}(0,t \le 0)$. Solution I is based on the phase shifts of Refs. 21, 23, and 24, and solution II uses the phase shifts of Refs. 21, 24, and 25. The curve labelled a in each figure represents our theoretical $\mathcal{C}^{(+)}(0,t)$ as calculated with the parameters of (I).

The agreement between our calculated $\mathcal{C}^{(+)}(0,t)$ and Langbein's values is much better than what a comparison of our value of 25.5 MeV for σ(πN) with Langbein's value of ∼ 60 MeV may suggest. By making a linear fit to each of his solutions Langbein obtained two sets of values of the Höhler parameters $\mathcal{C}_1^{(+)}$ and $\mathcal{C}_2^{(+)}$. These and the resulting σ(πN) are listed in Table I. It is not possible to reproduce these values from our theory if we require reasonable agreement with the experimental low energy S-wave phase shifts. In other words, the results of our theory are not compatible with a linear fit.

A linear fit is strongly suggested if one assumes, as Langbein, that all the errors are purely statistical. However, it is likely that there are systematic errors in the phase shifts used and in the extrapolation in the variable ν from the physical region to ν = 0. Assuming this is so we may ask how the fit can be improved. An inspection of Figs. 2 and 3 suggests that we must decrease the slope, $\mathcal{C}_2^{(+)}$, and have a small negative curvature, $\mathcal{C}_7^{(+)}$.

To decrease $\mathcal{C}_2^{(+)}$ while maintaining good agreement with the S-wave phase shifts one must increase the P-wave form factor mass μ from the value $8m_\pi$. From (3.13) one sees that this will reduce $\mathcal{C}_{2P}^{(+)}$. The P-wave rescattering integrals have a small but non-negligible contribution to the S-wave equation. Increasing μ decreases the P-wave contribution to the S-wave. So to maintain a fit of the S-wave phase shifts we must readjust σ(πN) and $\bar{g}_\pi$. In Fig. 4 we show the phase shifts due to the set $\mu = 10m_\pi$, σ(πN) = 24 MeV and $\bar{g}_\pi = 11.64$, with all other parameters remaining the same, as the curves labelled b. Curves labelled a are the results of (I). The small differences between the two sets of phase shifts are of no consequence in the present discussion.

The values $\tilde{C}_{1S}^{(+)}$, $\tilde{C}_{2S}^{(+)}$, $\tilde{C}_{1P}^{(+)}$ and $\tilde{C}_{2P}^{(+)}$ for the two sets are shown in Table II. The curves labelled b in Figs. 2 and 3 represent the $\tilde{C}^{(+)}(0,t)$ due to the new set. The curves b have $\tilde{C}_7^{(+)} = 0.012$ GeV^{-1}. If we can change $\tilde{C}_7^{(+)}$ to -0.13 GeV^{-1} then curve b will pass through all the error flags of solution II. The repulsive character of the D-wave πN phase shifts suggests that, in fact, $\tilde{C}_7^{(+)}$ should be negative. Our failure to get a negative $\tilde{C}_7^{(+)}$ is due to the fact that we have not included in our theory the process $\pi \to N+\bar{\Delta}$. Its contribution to $\tilde{C}^{(+)}$ is diagrammatically represented by replacing the $\bar{N}$ in the z-graphs by $\bar{\Delta}$. This mechanism is operative mainly in the D-wave. The resulting contribution to $\tilde{C}_7^{(+)}$ is

$$-\frac{|g_{\pi N\Delta}((M-M_\Delta)^2)|^2}{12m_\pi^2(M-M_\Delta)M^2}\left[1+\frac{5}{9}\frac{M}{M_\Delta}+\frac{7}{9}\frac{M^2}{M_\Delta^2}+\frac{2}{9}\frac{M^3}{M_\Delta^3}\right] = -0.021\ \mathrm{GeV}^{-1}\ \frac{|g_{\pi N\Delta}((M+M_\Delta)^2)|^2}{4\pi}\ .$$

In the pole model one estimates

$$\frac{g_{\pi N\Delta}^2(m_\pi^2)}{4\pi} \simeq 0.3.$$

To reach the value -0.13 GeV^{-1} for $\tilde{C}_7^{(+)}$ we need $\frac{|g_{\pi N\Delta}((M+M_\Delta)^2)|^2}{4\pi} \simeq 6.2$. We do not know if this is a reasonable value. Further investigation of this point is profitable if there is assurance that the values of $\tilde{C}^{(+)}(0,t)$ are free from systematic errors. If indeed this is the case then the trend of our numerical results suggests the following. The actual value of μ is close to $10m_\pi$, $\sigma(\pi N)$ is $\simeq 24$ MeV and $\tilde{C}_7^{(+)}$ is $\simeq -0.1$ GeV^{-1}. We may draw the reader's attention to the fact that Langbein's estimate of the D-wave 'scattering lengths' gives[10]

$$m_\pi^4\ \frac{d^2}{dt^2}\ F^{(+)}(\nu,t) = -(0.9 \pm 0.7)\ \mathrm{GeV}^{-1},$$

at the elastic threshold, which is in agreement with this value of $\tilde{C}_7^{(+)}$.

In Table I we present the values of $\tilde{C}_1^{(+)}$ and $\tilde{C}_2^{(+)}$ due to various authors and our results for the sets a and b. The values obtained by these authors by least square fitting are consistently greater than ours. Naturally, the discrepancy for the amplitude $\tilde{C}^{(+)}(0,2m_\pi^2) = \tilde{C}_1^{(+)} + 2\tilde{C}_2^{(+)}$ is proportionately greater because of the cumulative effect. The reason for the discrepancy is two-fold. First, we have presented theoretical arguments for a small, negative $\tilde{C}_7^{(+)}$, but generally a term quadratic in t is not even included in the fitting calculations. And second, the errors in the values of $\tilde{C}^{(+)}(0,t\leq 0)$ of Ref. 10 are not purely statistical in nature. There are systematic errors present, and as a consequence, the least square fitting procedure gives erroneous results.

Though our analysis has been based on the work of Langbein, our conclusion concerning the value of $\sigma(\pi N)$ is also supported by a study of the work of Moir, Jacob, and Hite.[7] These authors evaluate the amplitude $\tilde{C}^{(+)}(0,t)$ using an interior dispersion relation. Their method involves first evaluating an amplitude $D(t,a_{CD})$, called the discrepancy function, for $t \leq 0$. An extrapolation in t is then carried out to $t = 2m_\pi^2$. The authors find the extrapolated value $D(2m_\pi^2,a_{CD}) = 5.77 \pm 1.06$ GeV^{-1}, which gives $\sigma(\pi N) = 67 \pm 8$ MeV. We note that $\sigma(\pi N) = 25.5$ MeV corresponds to $D(2m_\pi^2,a_{CD}) = 1.5$ GeV^{-1}.

In Fig. 5 we plot the values of $D(t,a_{CD})$ given in Fig. 1 of Ref. 7 for $-0.08\ \mathrm{GeV}^2 \leq t \leq 0$. We add to these points error flags of ± 1 GeV^{-1}.[27] (In Ref. 7 the error flags are placed only on the CD point.) On the $t = 2m_\pi^2$ line we indicate the points corresponding to the values 5.77 GeV^{-1} and 1.5 GeV^{-1}. It is clear that the lower value cannot be ruled out.

V. SUMMARY AND CONCLUSION

We obtained a value of $\sigma(\pi N)$ by fitting low energy S-wave phase shifts with a theory based on the notions of PCAC and soft pion limit and the assumption that the isovector seagull term is zero. The theory involves the Low expansion of the off-mass-shell πN amplitude. The expansion is truncated, but after subtraction, forming a nonlinear integral equation for the off-shell amplitude. The isovector and the isoscalar terms of this equation are both repulsive in the S31 channel, while they are of opposite sign in the S11 channel. Because of this the procedure of fitting both phase shifts simultaneously fixes fairly well the values of $\sigma(\pi N)$ and $\bar{g}_\pi$, which are the main parameters of the isoscalar and isovector terms, respectively. We estimate that possible numerical inaccuracies in our work can only cause errors of a few percent in these parameters. For a value of $8m_\pi$ for the P-wave form factor mass we get $\sigma(\pi N) = 25.5$ MeV. For $\mu = 10m_\pi$ a slightly lower value of $\sigma(\pi N)$ is obtained with nearly as good agreement with the S-wave phase shifts.

When we calculate $\tilde{C}^{(+)}(0, t \le 0)$ with either set of parameters we come very close to the values obtained by Langbein.[10] The agreement can be improved with $\mu = 10m_\pi$, $\sigma(\pi N) = 24$ MeV if we include, ad hoc, a t^2 term $\tilde{C}_7^{(+)} = -0.1\ \text{GeV}^{-1}$. The sign $\tilde{C}_7^{(+)}$ is what one expects from the repulsive character of the D-wave interaction. The magnitude is not incompatible with the value of $m_\pi^4 \frac{d^2}{dt^2} F^{(+)}$ at the elastic threshold.

The disagreement between our value of $\sigma(\pi N) = 25.5$ MeV and the larger "world value"[28] of 65 ± 5 MeV boils down to the question of whether a second derivative of the size and sign that we need can be ruled out from the accuracy of the currently available values of $\tilde{C}^{(+)}(0,t)$. We believe it cannot be.

Since the σ-commutator plays such an important role in the low-energy πN amplitude, it will also be important in pi-nuclear processes. It has been shown[29] that the form of the σ-term in Eq. (2.13) leads to a Laplacian term in the pi-nuclear optical potential. One expects that a careful study of low-energy pi-nuclear eleastic scattering, the many-body renormalization of the pion propagator in nuclear matter and other such processes can be used to determine the magnitude of $\sigma(\pi N)$, surely to an accuracy of better than ± 40 MeV, in spite of possible uncertainties of nuclear physics. We note that a preliminary study[29] of pi-nuclear scattering strongly favors the lower value of $\sigma(\pi N)$ mentioned above.

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We are very grateful to Dr. R. J. Jacob for providing us with the values of the discrepancy functions and Dr. W. Langbein for providing us with his values of $\tilde{C}^{(+)}(0,t)$. We thank Drs. V. S. Zidell, L. D. Roper, and R. A. Arndt for advance communication of their S31 phase shifts. We would also like to acknowledge some very helpful remarks by Dr. H. Rubinstein. We thank the University of Maryland Computer Science Center for providing computer time for this work. Part of this work was completed while one of us (JBC) participated in the Visitors Program of the University of Maryland's Nuclear Theory Group.

APPENDIX

Details of the derivation of the last term of (2.13) may be found in Ref. 30. Here we tabulate the values of the various quantities.

a	1	2	3	4
ξ_a	1	1	-1	-1
L_a^2	W^2	W^2	W^2	W^2
$\vec{L}_a$	0	$\vec{p}_i+\vec{p}_f$	$\vec{p}_f$	$\vec{p}_i$
ε_a	k_0+p_{f0}	$p_{i0}-k_0$	p_{f0}	p_{i0}
γ_f^a	β	α	β	α
γ_i^a	α	β	α	β

$$|\vec{p}_f^{\,a}| = \sqrt{(E_f^a)^2 - M^2}, \quad E_f^a = \frac{p_f \cdot L_a}{W}$$

$$|\vec{p}_i^{\,a}| = \sqrt{(E_i^a)^2 - M^2}, \quad E_i^a = \frac{p_i \cdot L_a}{W}$$

$$\mathcal{P}_{S_{1/2}}(\vec{p}_f^{\,a},\vec{p}_i^{\,a}) = \frac{M}{4\pi\sqrt{(E_i^a+M)(E_f^a+M)}}\,\bar{u}(p_f)\left(\frac{\not{L}_a}{W}+1\right)u(p_i)$$

$$\mathcal{P}_{P_{1/2}}(\vec{p}_f^{\,a},\vec{p}_i^{\,a}) = \frac{M}{4\pi\sqrt{(E_i^a-M)(E_f^a-M)}}\,\bar{u}(p_f)\left(\frac{\not{L}_a}{W}-1\right)u(p_i)$$

$$\mathcal{P}_{P_{1/2}}(\vec{p}_f^{\,a},\vec{p}_i^{\,a}) + \mathcal{P}_{P_{3/2}}(\vec{p}_f^{\,a},\vec{p}_i^{\,a}) = \frac{3}{|\vec{p}_f||\vec{p}_i|}\left(\frac{p_f\cdot L_a\; p_i\cdot L_a}{W^2} - p_i\cdot p_f\right)\mathcal{P}_{S_{1/2}}(\vec{p}_f,\vec{p}_i)$$

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TABLE CAPTIONS

TABLE I. Comparison of values of the Höhler parameters $\tilde{C}_1^{(+)}$, and $\tilde{C}_2^{(+)}$ obtained by several authors. In Refs. 8, 10 and 11 these are the only parameters quoted, while in Ref. 9 $\tilde{C}_7^{(+)}$ is included and found to be 0.25 ± 0.05 GeV^{-1}. The final column lists the values of $\sigma(\pi N)$ obtained by each author.

TABLE II. The S- and P-wave contributions (in GeV^{-1}) to $\tilde{C}_1^{(+)}$ and $\tilde{C}_2^{(+)}$ corresponding to the parameter sets a and b.

TABLE I

Authors	$\tilde{C}_1^{(+)}$ in GeV^{-1}	$\tilde{C}_2^{(+)}$ in GeV^{-1}	$\sigma(\pi N)$ in MeV
Chao, et al.[8]	-9.67 ± 0.30	8.19 ± 0.72	57 ± 12
Höhler, et al.[11]	-10.96 ± 1.43	7.00 ± 0.14	43 ± 12
Langbein Soln. I[10]	-10.27 ± 0.69	7.28 ± 0.70	57 ± 14
Langbein Soln. II[10]	-9.73 ± 1.11	7.52 ± 0.53	66 ± 14
Nielsen & Oades[9]	-10.39 ± 0.72	7.50 ± 0.36	66 ± 9
Ours a	-12.35	6.98	25.5
Ours b	-10.69	5.95	24.0

TABLE II

Parameter Set	$\tilde{C}_{1S}^{(+)}$	$\tilde{C}_{2S}^{(+)}$	$\tilde{C}_{1P}^{(+)}$	$\tilde{C}_{2P}^{(+)}$
a	2.34	-1.12	-11.02	5.87
b	1.36	-0.66	-8.56	4.56

FIGURE CAPTIONS

Fig. 1. Diagrammatic representation of the scattering amplitude for $\pi_\alpha(k') + N(p_i) \rightarrow \pi_\beta(k) + N(p_f)$.

Fig. 2. Comparison of theoretical curves of $\mathcal{C}^{(+)}(0,t)$ with the results of Langbein's solution I from Ref. 10, which are shown as points with error flags. The curve labelled a represents our theoretical $\mathcal{C}^{(+)}(0,t)$ as calculated with the parameters of (I); in particular $\mu = 8\,m_\pi$, $\sigma(\pi N) = 25.5$ MeV and $\bar{g}_\pi = 11.69$. Curve b results from the set $\mu = 10\ m_\pi$, $\sigma(\pi N) = 24$ MeV and $\bar{g}_\pi = 11.64$, with all other parameters the same as for curve a.

Fig. 3. Comparison of theoretical curves of $\mathcal{C}^{(+)}(0,t)$ for parameter sets a and b with Langbein's solution II from Ref. 10.

Fig. 4. The solid lines are our theoretical phase shifts corresponding to parameters sets a and b. The flagged points represent the energy-independent phase shift fit of Ref. 21, while the dashed line represents the energy-dependent fit of Ref. 26. The S31 phase shifts resulting from both set a and b parameters exactly reproduce the results of the energy-dependent fit for $T_\pi \lesssim 100$ MeV.

Fig. 5. Values of the discrepancy function $D(t,a_{CD})$ from Moir, Jacob and Hite, Ref. 7, are plotted for $-0.08 \leq t \leq 0$. On the $t = 2m_\pi^2$ line we show the value $D(2m_\pi^2, a_{CD}) = 5.77 \pm 1.06\ \mathrm{GeV}^{-1}$ obtained in Ref. 7 by extrapolation from $D(t \leq 0, a_{CD})$. This value of D yields $\sigma(\pi N) = 67 \pm 8$ MeV. Also shown is the value $D(2m_\pi^2, a_{CD}) = 1.5\ \mathrm{GeV}^{-1}$, which corresponds to $\sigma(\pi N) = 25.5$ MeV.

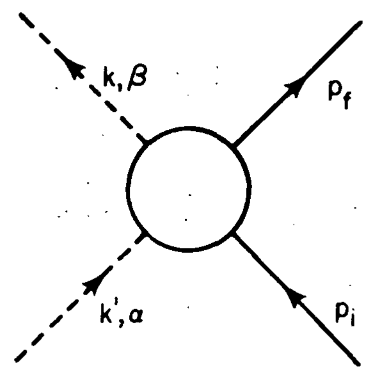
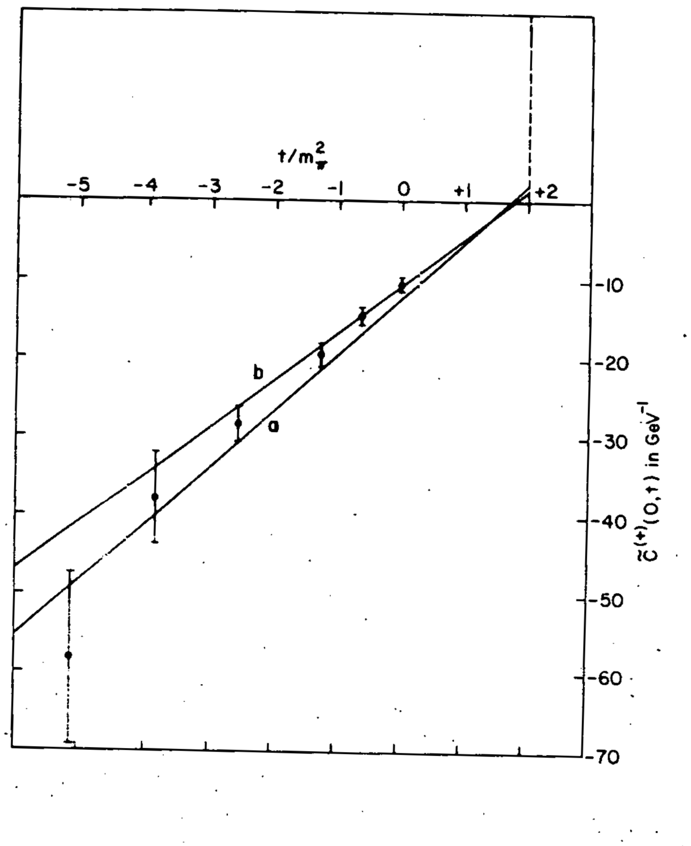
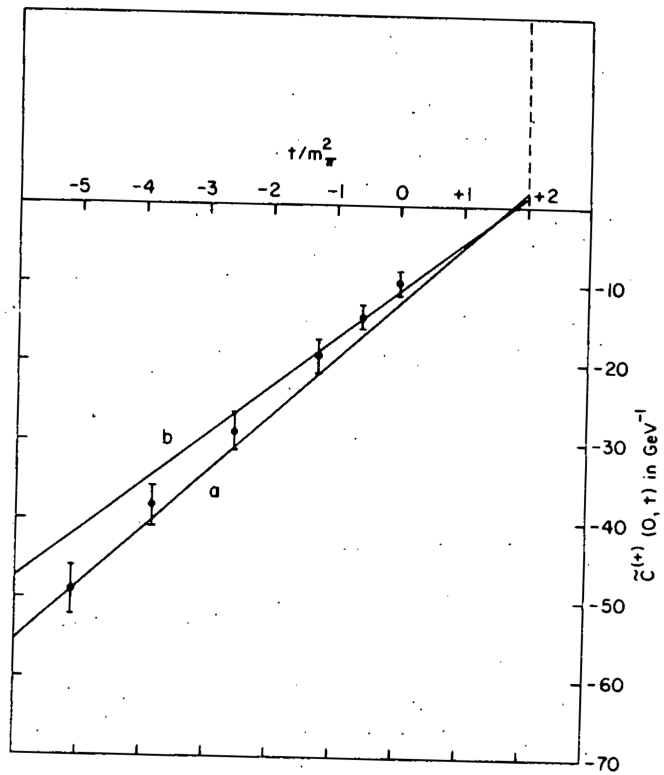


Fig. 1



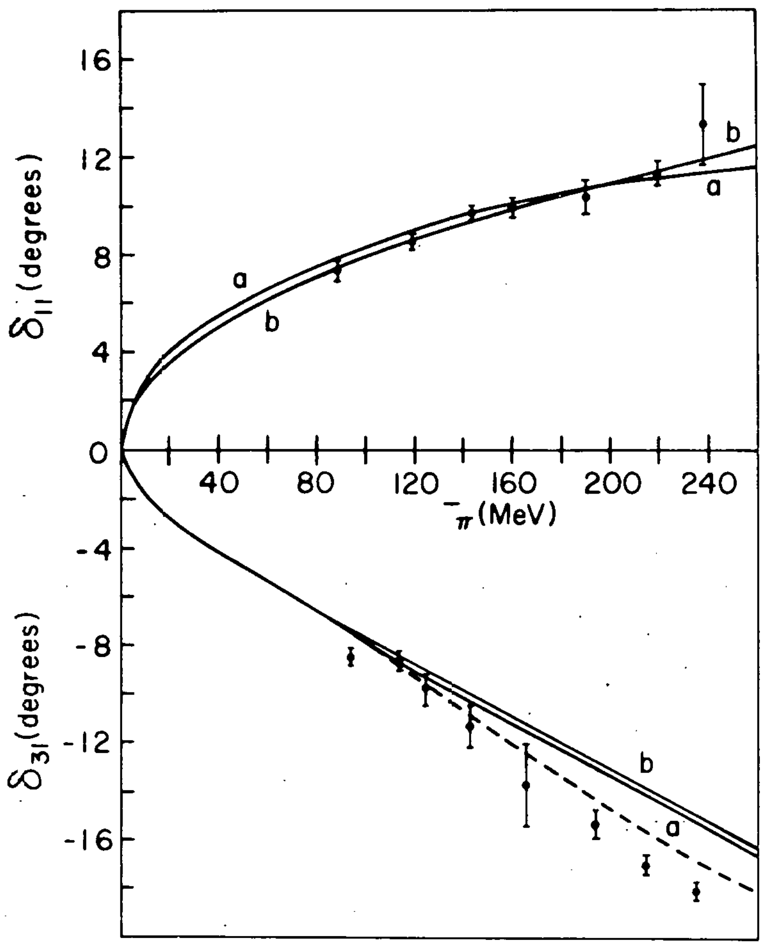
t/m$_\pi^2$
-5 -4 -3 -2 -1 0 +1 +2
-10 -20 -30 -40 -50 -60 -70
b
a
$\tilde{C}^{(+)}(0,t)$ in GeV^{-1}

Fig. 2



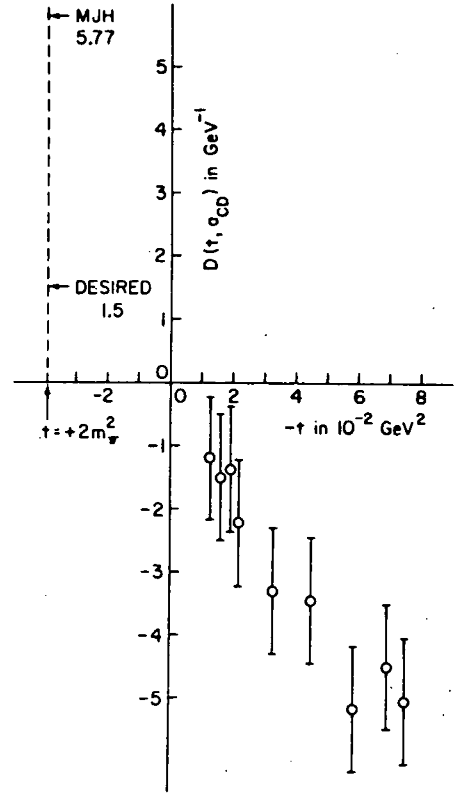
t/m$_\pi^2$
-5 -4 -3 -2 -1 0 +1 +2
-10 -20 -30 -40 -50 -60 -70
b
a
$\tilde{C}^{(+)}(0,t)$ in GeV^{-1}

Fig. 3



16
12
8
4
0
-4
-8
-12
-16
40
80
120
160
200
240
T_π(MeV)
δ_{11} (degrees)
δ_{31} (degrees)
a
b

Fig. 4



MJH
5.77
DESIRED
1.5
$t=+2m_\pi^2$
$D(t, a_{CD})$ in GeV^{-1}
$-t$ in 10^{-2} GeV^2

Fig. 5