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# Elastic-Plastic Response of Mild Steel Beams to Impulse Loads

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ELASTIC-PLASTIC RESPONSE OF MILD STEEL  
BEAMS TO IMPULSE LOADS

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ABSTRACT

The elastic-plastic response of 1018 steel beams is examined experimentally and analytically. Simply supported beams are loaded with half-sine distributed, short-duration magnetic pressure pulses and the response is monitored with a streak camera and strain gages. A closed-form, elastic-viscoplastic approximate theory for peak displacement is derived and compared with measurements. Good agreement is shown between predictions and measurements.

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## Introduction

Many investigations associated with the plastic response of structures to impulse loads have appeared in the literature and are discussed in the survey articles by Symonds [1]\* and Jones [2]. The discussion of other related work can also be found in the recently published paper by the authors [3]. In [3], the elastic-plastic response of rate-insensitive 6061-T6 aluminum beams was examined. This paper complements the work of Reference [3] in that 1018 steel exhibits strain rate effects which must be included for accurate response predictions. Predictions for peak beam deflections which include and neglect strain rate effects are compared with measurements. Good agreement is shown between measurements and predictions which include rate effects.

## Experiments

A sketch of the beam fixture is shown in Fig. 1. Beams are attached to 9.52 mm (0.375 in.) dia steel drill rod with flat head machine screws. The fixture was designed to simulate simply supported boundary conditions and minimize unwanted inertial and frictional effects.

An illustration of the experimental arrangement is shown in Fig. 2. Two 0.20 mm (0.0080 in.), 1100-0 aluminum conductors which are insulated from each other and the beam are placed on

\*Numbers in brackets designate References.

top of the beam. These conductors are connected to the transmission lines of a fast-discharge capacitor bank at one end and shorted together at the other end. When the switch is closed, a short-duration ( $\sim 8 \mu\text{s}$ ) magnetic pressure pulse is generated between the conductors which is proportional to the square of the local current density from capacitor discharge. As previously discussed [4], spatial pressure distribution can be achieved by varying the width of the conductors. Impulse per unit area is inversely proportional to the square of the conductor width, and for this application the conductor width is given by  $y(x) = Y(\sin \pi x/2\ell)^{-1/2}$  where  $2\ell$  is the beam length and  $Y$  is the conductor width at  $x = \ell$ . Conductor width is sketched in Fig. 2 which shows that at locations away from the beam center the conductors are wider than the beam. Wooden guard blocks are placed alongside the beam and underneath the conductors and a 15.9 mm (0.625 in.) nylon phenolic plate having the conductor's shape and cut at the beam center is placed on top of the conductors. The phenolic plate keeps the conductors positioned before the bank is switched and acts as a tamper during loading. When the capacitor bank is switched, the magnetic pressure loads the beam and throws the phenolic plates away from the deforming beam.

Beam specimens were cut from a 6.35 x 50.8 mm (0.250 x 2.00 in.) bar of 1018 steel into lengths of 0.419 m (16.5 in.). Holes were drilled 6.35 mm (0.25 in.) from either end of the bar so beams with 0.406 m (16.0 in.) span could be attached to the fixture shown in Fig. 1. The beam specimens were annealed

at 1600°F for 1 hour, cooled in a vacuum at a maximum rate of 100°F per hour down to 1250°F, and then furnace-cooled in a vacuum to room temperature.

Deflection-time was monitored at the beam centerline with an image motion streak camera and a typical record is shown in Fig. 3. This record shows two thick lines and two fine lines. The thick lines correspond to the rounded edges of the beam and the inner fine lines are scribe marks placed at 1.59 mm (0.0625 in.) from the lateral edges of the beam. Strain response was measured at the beam centerline with long lead strain gages (Micro-Measurements EP-08-125AV-120-B64) and a typical strain-time record is shown in Fig. 4.

#### Theory for Peak Deflection

An approximate elastic-viscoplastic solution is developed for the peak displacement of a simply supported beam of length  $2\ell$  loaded by the half-sine distributed impulse load  $I\delta(t) \sin(\pi x/2\ell)$  where  $x$  is measured from the pinned end. The stress-strain law is elastic-perfectly plastic with the rate sensitive yield stress law suggested by Cowper and Symonds [5]. The dynamic yield stress is given by

$$\sigma_y = \sigma_0 \left[ 1 + (\dot{\epsilon}/D)^{1/p} \right] \quad (1)$$

where  $\sigma_0$  is the static yield stress,  $\dot{\epsilon}$  is strain rate and  $D$ ,  $p$  are empirical constants. A comparison with the experimental data of [6,7] indicates that  $p = 5$ ,  $D = 40.4 \text{ s}^{-1}$  and  $\sigma_0 = 207 \text{ MPa}$  (30,000 psi) are reasonable choices of material constants for

mild steel. The elastic-viscoplastic stress-strain law used herein was used by Duffey [8] for expanding ring studies and is shown in Fig. 5.

For analysis the beam response is divided into two regions, the early time elastic response and the subsequent plastic deformation. Response in the elastic regime can be computed exactly and the beam deflection, stress and strain rate at the outer beam fibers are given by

$$w(x,t) = \frac{8(3)^{1/2}}{\pi^2} \left(\frac{\ell}{h}\right)^2 \frac{I}{\rho c} \sin\left(\frac{\pi x}{2\ell}\right) \sin\left[\frac{\pi^2 c h t}{8(3)^{1/2} \ell^2}\right] \quad (2a)$$

$$\sigma(x,t) = \frac{(3)^{1/2} c I}{h} \sin\left(\frac{\pi x}{2\ell}\right) \sin\left[\frac{\pi^2 c h t}{8(3)^{1/2} \ell^2}\right] \quad (2b)$$

$$\frac{\partial \epsilon}{\partial t}(x,t) = \frac{\pi^2 I}{8\ell^2 \rho} \sin\left(\frac{\pi x}{2\ell}\right) \cos\left[\frac{\pi^2 c h t}{8(3)^{1/2} \ell^2}\right] \quad (2c)$$

where  $E$  is Young's modulus,  $\rho$  is density,  $c^2 = E/\rho$ ,  $t$  is time, and  $h$  is the beam thickness. The time  $t_y$  at which the beam first yields at  $x = \ell$  can be found from equations (1) and (2). As indicated by Fig. 5, the beam response is linear-elastic until  $t = t_y$ . Values of  $t_y$  and the dynamic to static yield stress ratio  $\sigma_y/\sigma_0$  are given in Table I for the beam specimen and impulse range considered in this study.

I (Pa·s)	$t_y$ (ms)	$\sigma_y/\sigma_0$
200	2.495	1.32
300	1.448	1.46
450	0.942	1.50
600	0.710	1.56
750	0.573	1.60

TABLE I

Once the dynamic yield stress has been reached, it is assumed that the material is perfectly-plastic and rate-insensitive. For example; consider the loading  $I = 600$  Pa·s. From Table I,  $\sigma_y = 1.56 \sigma_0$  and the material is considered elastic-perfectly plastic with Young's modulus  $E$  and yield stress  $\sigma_y = 1.56 \sigma_0$ . This is the same approximation made by Perrone [9] and discussed by Duffey[8] for expanding ring analyses. Thus, the problem has been reduced to an elastic-perfectly plastic, rate-insensitive analysis where the yield stress is a function of the impulse loading. The formulas previously derived in [3] can then be used to predict peak displacement. This approximate method of solution will be compared with some experimental data in the next section.

#### Experimental Results and Comparison with Predictions

Seven beams were loaded with peak impulse intensities between 142 and 750 Pa·s. Values of peak impulse intensity  $I$  were determined from the early time portion of the streak records and equation (2a). For a static yield stress of  $\sigma_0 = 207$  MPA (30,000 psi) the beam response is entirely elastic until  $I = 150$  Pa·s.

A comparison of measured and predicted peak deflection is shown in Fig. 6. Predictions which neglect rate effects use the static yield stress  $\sigma_0$  and predictions with rate effects use the dynamic yield stress  $\sigma_y$ . Good agreement is shown between measurements and predictions which include rate effects; whereas, the predictions which neglect rate effects overestimate considerably the maximum deflection as the impulse intensity increases. Some displacement-time and strain-time data are presented in Figs. 7 and 8.

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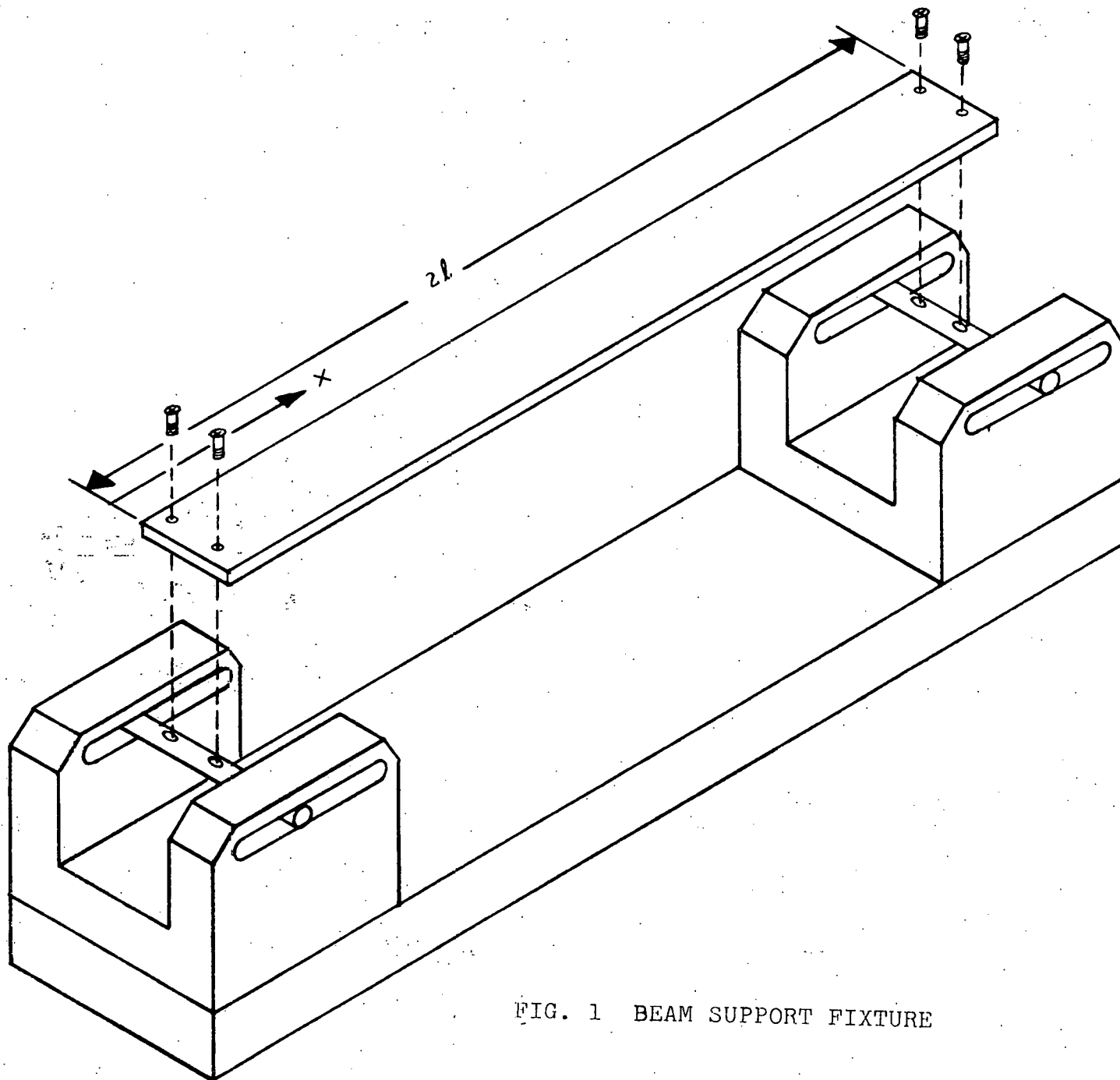


FIG. 1 BEAM SUPPORT FIXTURE

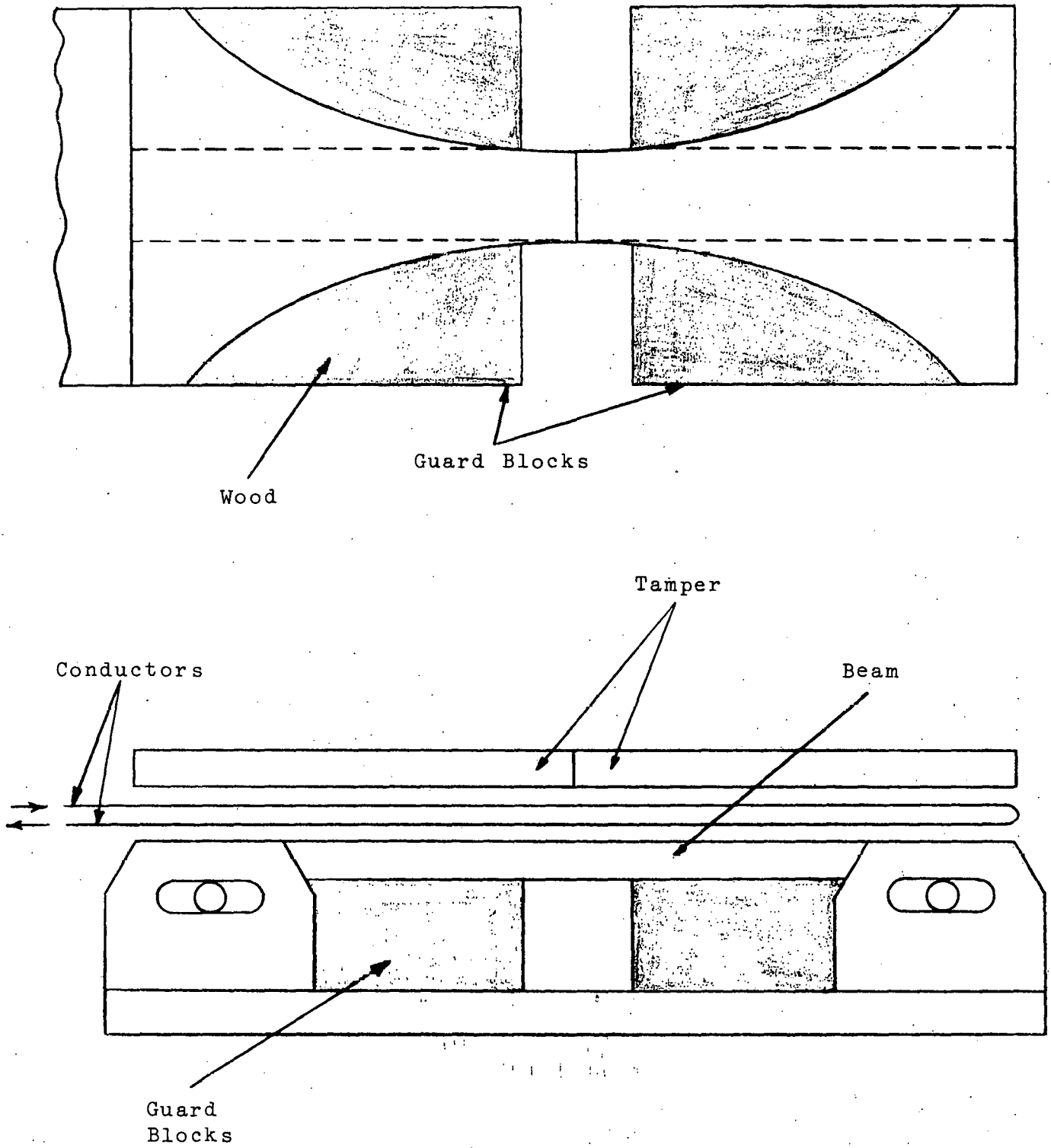


FIG. 2 EXPERIMENTAL ARRANGEMENT FOR BEAM EXPERIMENTS

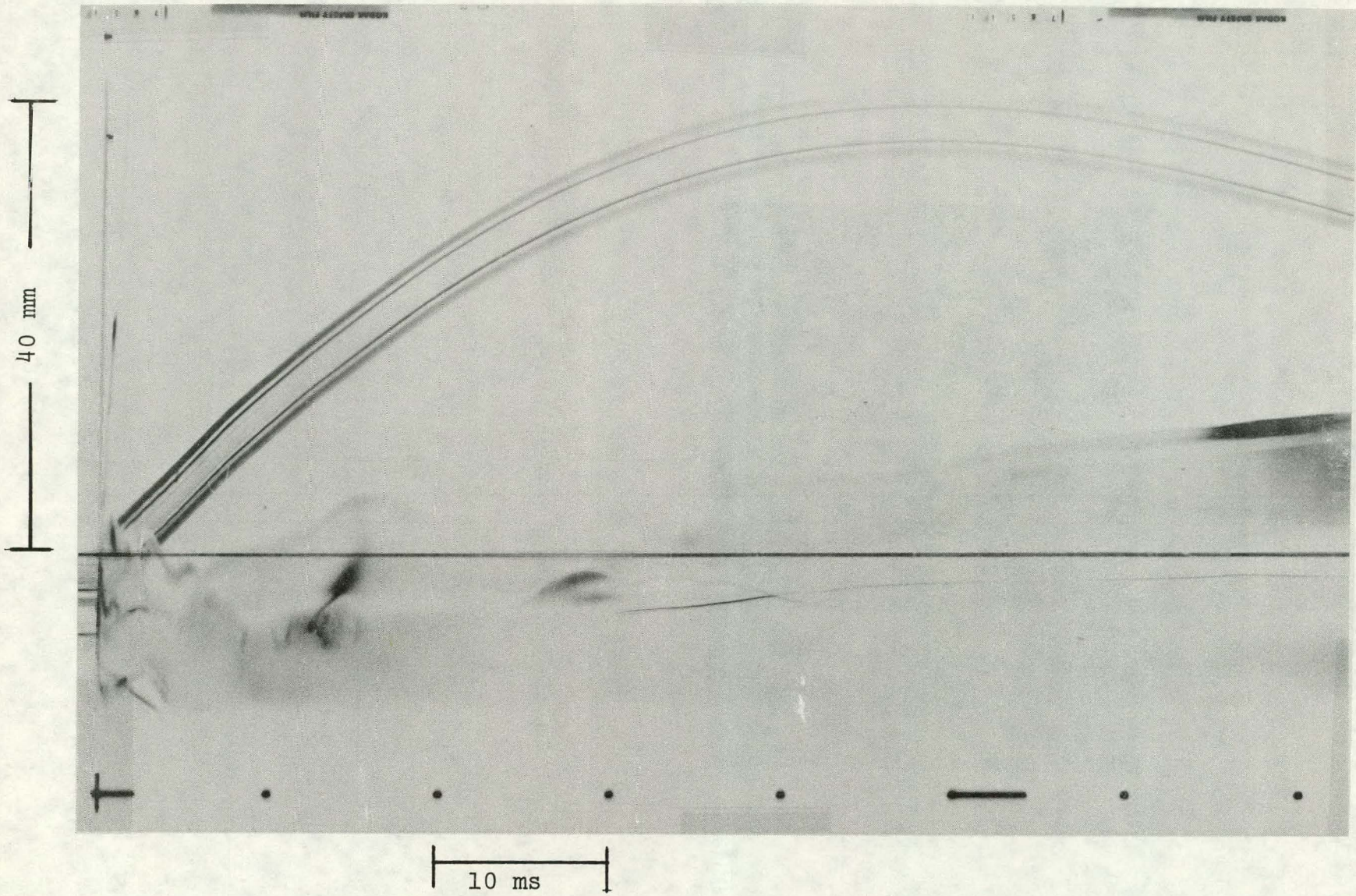


FIG. 3 STREAK RECORD OF DISPLACEMENT-TIME FOR  $I = 750 \text{ Pa}\cdot\text{s}$

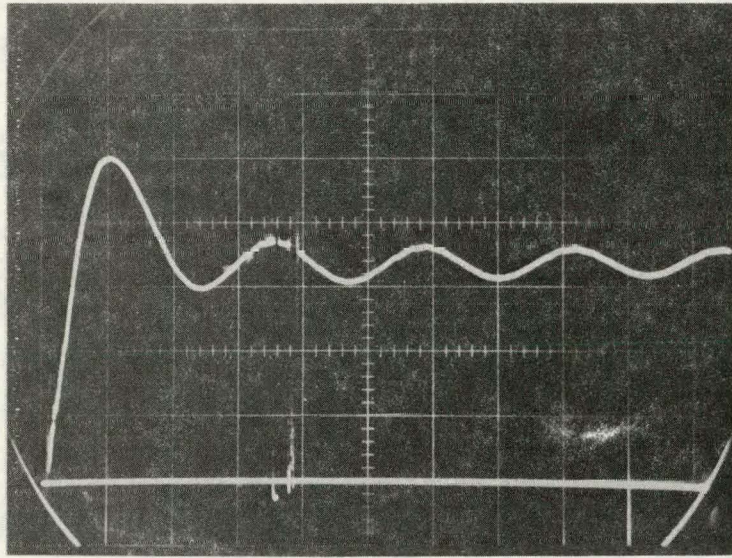


FIG. 4 STRAIN-TIME FOR  $I = 750 \text{ Pa}\cdot\text{s}$ ;  $2000 \mu\epsilon/\text{DIV}$ ,  
 $5 \text{ ms}/\text{DIV}$

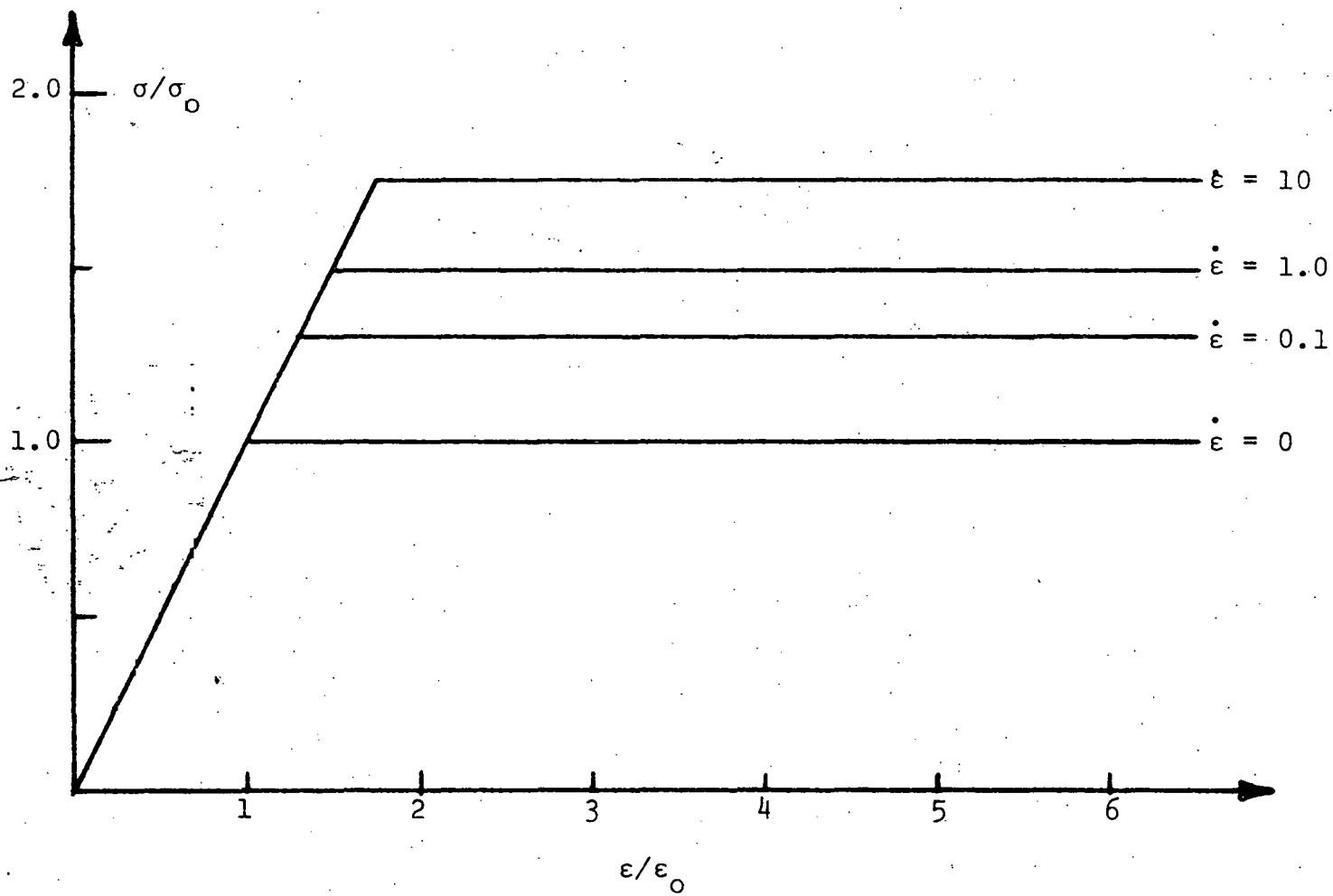


FIG. 5 STRESS-STRAIN;  $p = 5, D = 40.4 \text{ s}^{-1}$

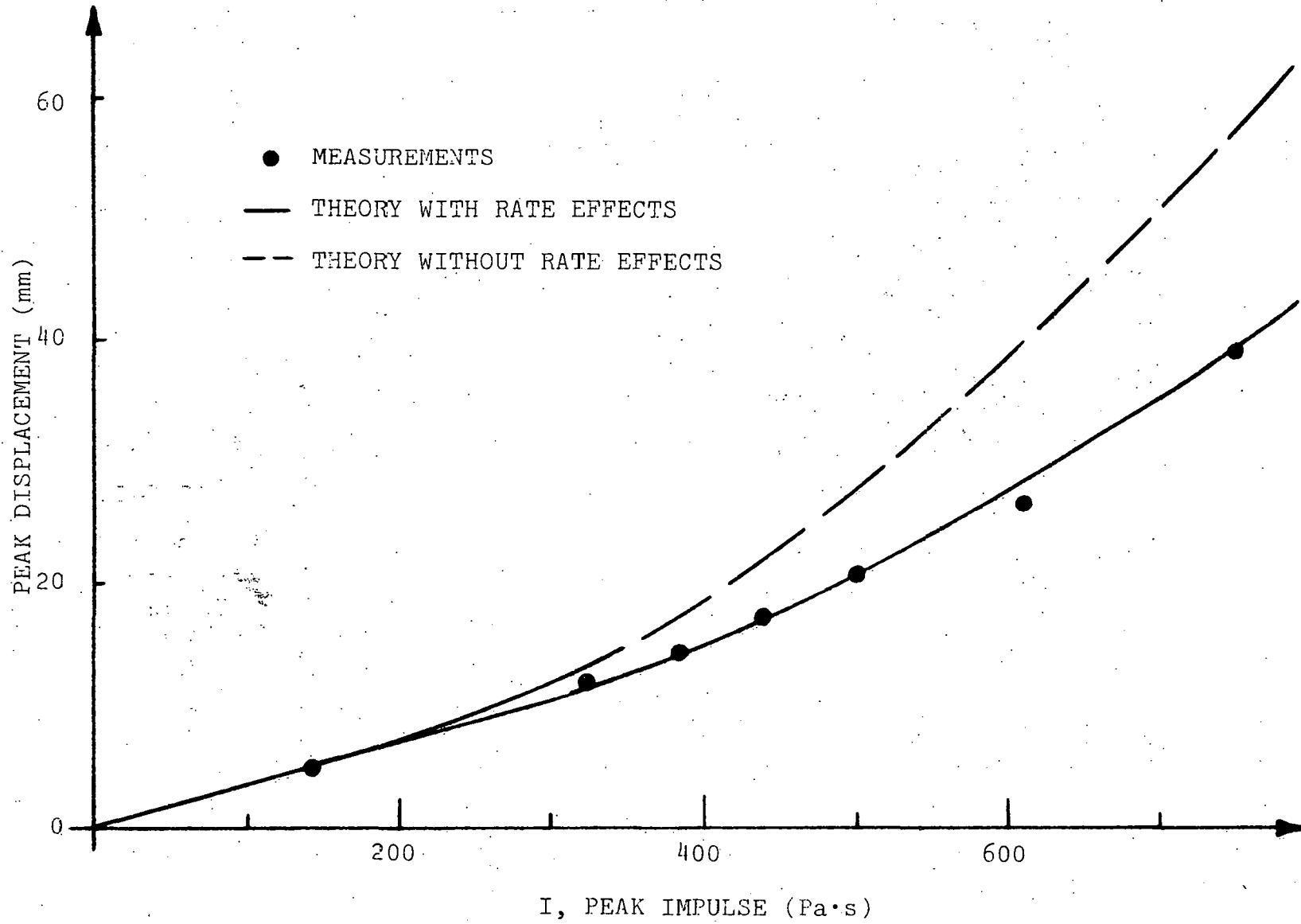


FIG. 6 PEAK DISPLACEMENT VERSUS IMPULSE

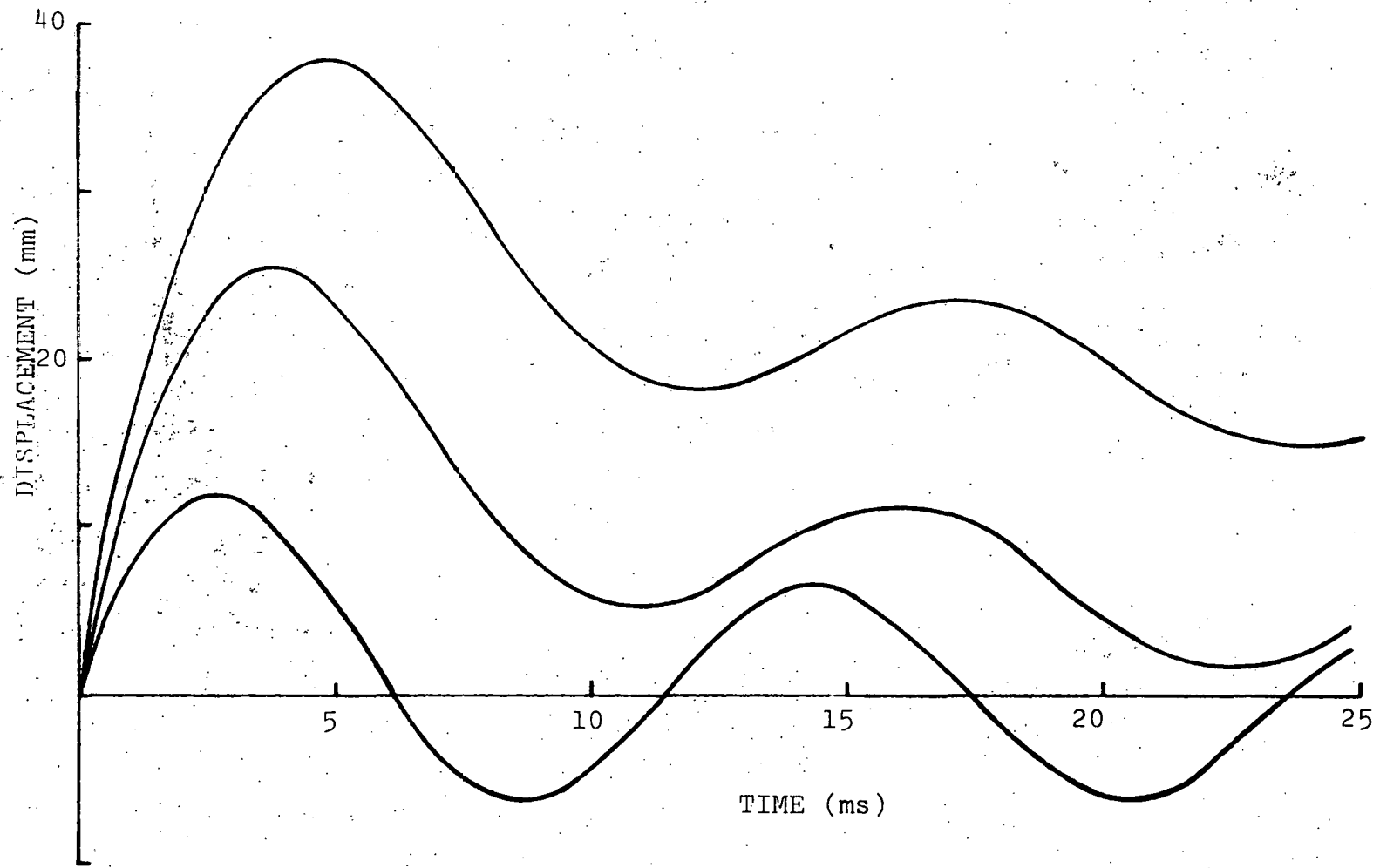


FIG. 7 DISPLACEMENT-TIME DATA FOR  $I = 325, 614$  AND  $750 \text{ Pa}\cdot\text{s}$

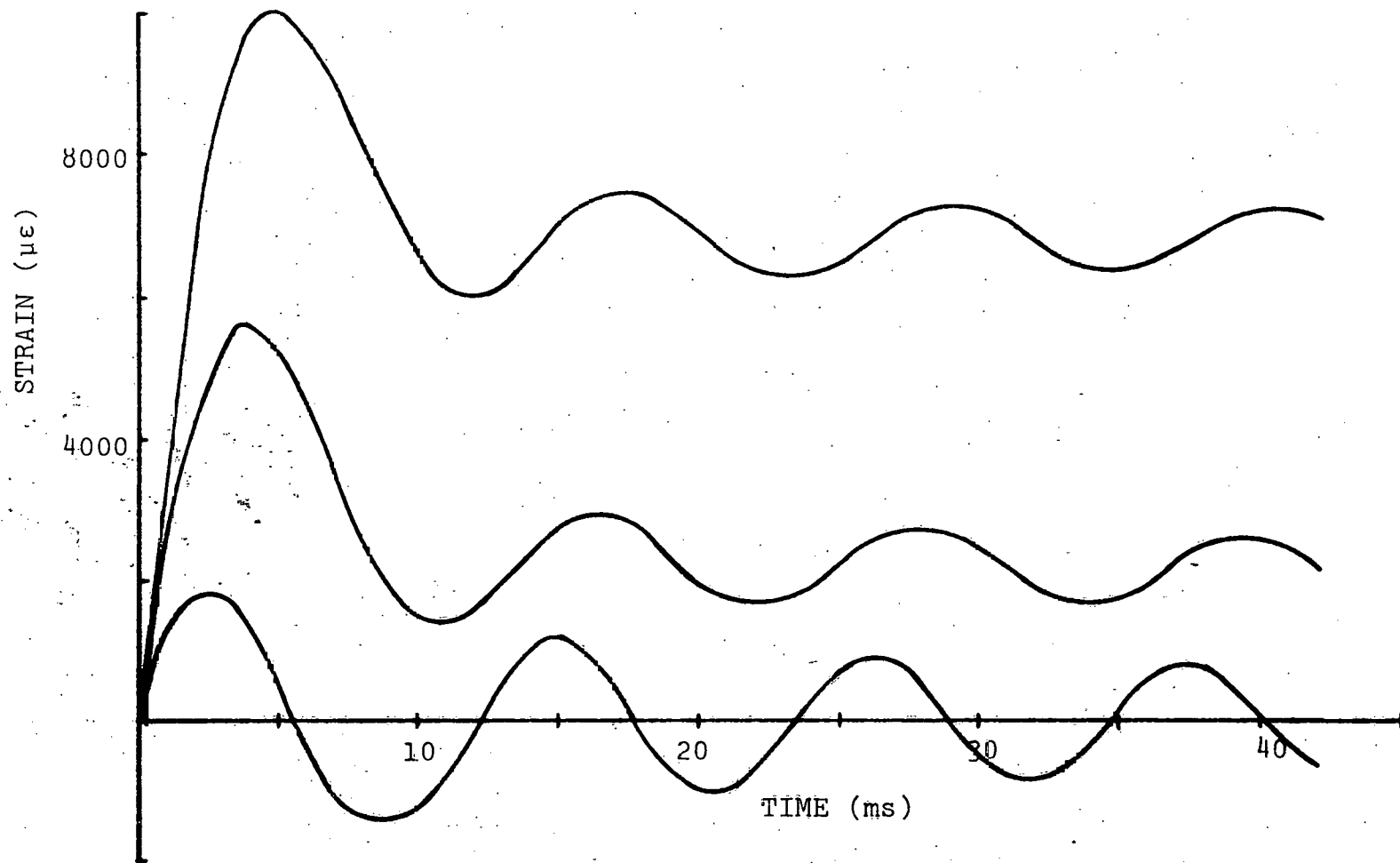


FIG. 8 STRAIN-TIME DATA FOR  $I = 325, 614$  and  $750$  Pa·s

Distribution:

5200 E. H. Beckner  
5230 M. Cowan  
5233 M. J. Forrestal (20)  
5233 M. J. Sagartz  
5233 D. L. Wesenberg (10)  
8266 E. A. Aas  
3141 C. A. Pepmueller (Actg.) (5)  
3151 W. L. Garner (3) for ERDA/TIC  
3171-1 R. P. Campbell (25) for ERDA/TIC