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Numerical and Physical Modelling of Bubbly Flow Phenomena

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by

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Abstract

The ultimate purpose of this study is to put the widely used engineering averaged-equations models of multi-phase flows on a firmer footing by comparing their results with those of direct numerical multi-particle simulations and rigorously-derived averaged equations. Since the averaged-equations models are comparatively well developed, in the first year of this project we have focused on the multi-particle simulation and ensemble averaging techniques. This report describes the progress to date on the several problems that are being studied.

The first problem addresses the derivation of ensemble-averaged equations for the propagation of pressure waves in bubbly liquids. The result is in the form of an expansion for small gas volume fraction. Previously known results are recovered at first order and in certain limits at higher order. However, a new insight into the structure of the expansion and the role of bubble-bubble interaction and surface deformations is obtained from this work.

In the second problem, by arranging N bubbles at random in a cubic cell and filling up the entire space with copies of this cell, we simulate the behavior of an infinite bubbly liquid. For this situation two alternative definitions of added mass are explored and the average viscous dissipation in the limit of large Reynolds number is studied.

In the third problem, we consider propagation of pressure waves in a layer of bubbles. The novel feature of this study is the fact that the volume fraction is arbitrary and the wavelength can be comparable to the inter-bubble distance.

The last problem involves the flow of a "plug" of bubbly liquid in a conduit. For the time being the bubbles are arranged on the axis of the conduit, but will later be placed at random.

The study of the first problem has been completed as of this writing. Work on the other three problems is currently in progress and will be completed in the near future. A summary of the work to be carried out during the period 7/90 to 6/91 is given in the last section.

1 Introduction

The ultimate objective of the present study is to shed light on the closure problem for the averaged equations of multi-phase flow by comparing their predictions with the results of direct numerical simulations of multi-particle systems. At present, techniques for such direct simulations are still relatively little developed and, therefore, in the first year of this study, we have mainly concentrated on them. The PI's, a post-doc, and several students are currently working on a number of such problems that are summarized in Sections 3 to 5.

Another possible approach to the validation of the heuristic averaged-equations models commonly used in engineering is their comparison with models more rigorously derived. We have also pursued this line of research for the relatively simple problem of linear pressure-wave propagation in a bubbly liquid. This study, which is summarized in Section 2, is near completion and will be submitted for publication shortly.

Plans for the work to be carried out during the second year of this contract are described in Section 7. A detailed budget for the continuation of this study has been submitted separately.

2 Pairwise interactions in dilute bubbly liquids

Author: A. S. Sangani

This study has been completed during the first year of the project and a manuscript based on this work is attached along with this report. Here we shall summarize the principal objectives and findings of the study. Our initial aim was to develop a rigorous approach for deriving averaged equations, i.e., equations that are satisfied by suitably-defined averaged pressure and velocity fields. There are, of course, already a large number of papers in the literature with the same general aim but a careful examination of these papers quickly suggested a need for a different approach. Many of the averaged equations suggested in the literature consist of writing averaged equations for the gas and liquid phases separately and then recommending the appropriate models for various terms for the interaction of the averaged fields of the individual phases via the common interface of the phases. An alternate approach to this, which has been employed with a considerable success in the study of sedimentation and rheology of dilute suspensions of small particles, is to derive the averaged equations for the entire mixture directly. For the case of gas-liquid mixtures such an approach was taken by Biesheuvel and van Wijngaarden (1984) who proposed a method based on a combination of volume and ensemble averaging methods and by Caflisch *et al.* (1985 a,b) who proposed a method of two-space homogenization. Keeping in mind that our eventual goal is to employ the results of large-scale direct numerical simulations from which typically the ensemble-averaged quantities are probably the easiest to evaluate even when two distinct mi-

cro and macro length scales do not exist, it seemed that the method of ensemble averages for the entire mixture was the most appropriate one to pursue.

We have developed a method that is somewhat similar to the one proposed by Biesheuvel and van Wijngaarden. Having developed this method, we wanted to apply it first to the case of dilute bubbly liquids with the anticipation that the results for these dilute mixtures will serve as an important check on our numerical calculations to be performed later. Some of these calculations are near completion and these are presented in Section 3.

We choose the simplest possible problem of linear pressure wave propagation through a dilute bubbly mixture and examined in detail the consequences of pairwise interactions among bubbles. This is the simplest possible problem in the sense that the nonlinear terms can be linearized and the problem of determining the microstructure of the mixture, i.e., the spatial configuration of the bubbles and their relative motions, which must be determined in many other problems as a part of the solution, need not be addressed. In spite of this considerable simplification, we found a number of interesting effects that arise solely from the interaction among the bubbles and in the remainder of this section we shall highlight some of these effects. A more detailed analysis and description may be found in the attached manuscript.

When the pressure wave propagates through a bubbly liquid, bubbles undergo volume, displacement, and shape oscillations because their compressibility and density are different from those of the liquid and because of the finite interfacial or surface tension. The ensemble averaged continuity and momentum equations in the absence of viscous and nonadiabatic effects require evaluation of mainly two coefficients, λ_p and λ_v, which may be defined by the relations

$$\lambda_p \frac{\partial p_0}{\partial t}(\mathbf{x}) = \int_{|\mathbf{x}-\mathbf{x_1}|\leq R} \frac{\partial p_1}{\partial t}(\mathbf{x}|\mathbf{x_1}) P(\mathbf{x_1})\, d^3\mathbf{x_1}, \tag{2.1}$$

$$\lambda_v \frac{\partial \mathbf{u}_0}{\partial t}(\mathbf{x}) = \int_{|\mathbf{x}-\mathbf{x_1}|\leq R} \frac{\partial \mathbf{u}_1}{\partial t}(\mathbf{x}|\mathbf{x_1}) P(\mathbf{x_1})\, d^3\mathbf{x_1}, \tag{2.2}$$

where p_0 and $\mathbf{u}_0$ are the unconditionally ensemble averaged pressure and velocity fields, p_1 and $\mathbf{u_1}$ are the corresponding conditionally averaged fields with the center of a bubble fixed at $\mathbf{x_1}$, t is the time, and $P(\mathbf{x_1})$ is the probability density function, i.e., the probability that a bubble may be found in the neighborhood of $\mathbf{x_1}$. In words, λ_p and λ_v can be defined roughly as the ratios of the average pressure and linear acceleration of a representative bubble in the neighborhood of $\mathbf{x}$ to the corresponding quantities for the mixture. The effective speed of the pressure wave propagation can be expressed in terms of these two coefficients as

$$\frac{1}{C_{ef}^2} = \frac{1}{C_L^2}[(1-\lambda_v\beta)(1-\lambda_p\beta)] + \frac{\beta\rho_L}{\gamma P_e}\lambda_p(1-\lambda_v\beta), \tag{2.3}$$

where C_L is the speed in the pure liquid, ρ_L is the density of the liquid, β is the volume fraction of the gas bubbles, P_e is the equilibrium pressure in the gas bubbles, and γ is the ratio of constant pressure and volume specific heats of the gas. For an air-water system at ambient pressure, the magnitude of the term containing $1/C_L^2$ in the above equation is typically very small and therefore, unless β is less than 10^{-4}, the first term on the right-hand side of (2.3) is small compared to the second term and therefore can be neglected. As a consequence of this, it is possible to evaluate $1/C_{ef}^2$ correct to $O(\beta^2)$ by determining λ_p correct to $O(\beta)$ and λ_v to $O(1)$. The $O(1)$ estimates of λ_v and λ_p are evaluated simply by examining the interaction of a single bubble with the incident pressure wave while the $O(\beta)$ correction to λ_p requires the examination of the pairwise interaction among bubbles. Thus, one assumes the presence of a second bubble at $\mathbf{x_2}$ and determines its effect on the pressure variation inside the bubble at $\mathbf{x_1}$ (to be referred to as the test bubble). The $O(\beta)$ correction is then obtained by multiplying this effect by the probability of finding a second bubble at $\mathbf{x_2}$ and then integrating this effect for all possible positions of the second bubble. Actually, the method just described is not quite correct as it turns out that this direct method of summing the contribution due to the second bubble does not converge but ways of overcoming such difficulties have been proposed in the literature and so we shall not elaborate them here. Instead we shall try to explain the phenomenon in words.

When β is small the effective wavelength in the mixture is $O(R\beta^{-1/2})$, where R is the radius of the bubbles, and so the two bubbles that are separated by a distance comparable to R see nearly the same pressure fluctuations around them and as a result undergo volume pulsations that are nearly in phase with each other. Since the liquid is nearly incompressible, a combined effect of many two-bubble interactions at a separation distances comparable to R is to have a conditionally averaged pressure or velocity field to diverge when the effective wavelength is infinite. The fact that the effective wavelength, however, is finite is important in preventing the cumulative effect of these pairwise interactions from diverging. Thus for each bubble that acts as a source of liquid during its volume expansion, there exists another bubble, located at half the effective wavelength away from the first bubble, that is undergoing a volume contraction and hence acts as a sink of liquid. The detailed analysis then shows that the correction to λ_p is $O(\beta \log_e \beta)$ followed by an $O(\beta)$ correction. These calculations also show that there is in fact also a relatively large $O(\beta^{1/2})$ correction to λ_p that is a purely imaginary quantity indicating that λ_p, and hence C_{ef}, are complex quantities, or that waves attenuate even in the absence of any viscous or nonadiabatic effects. This attenuation occurs because of the finite compressibility of the mixture and it is known as the acoustic radiation damping: The acoustic energy of the waves radiated from the test bubble is stored in the mixture and radiated later resulting in a phase difference between the pressure variations in the test bubble and the mixture. In all of the previous studies, which examined only the interaction of an isolated

bubble with the incident wave, the compressibility of the mixture was attributed to that of the pure liquid, whereas our calculations showed that one must use the compressibility of the mixture and not that of the pure liquid in evaluating the magnitude of the attenuation due to acoustic radiation. Since the compressibility of the mixture can be typically 10-30 times larger than that of the pure liquid, this is a significant finding.

The detailed calculation of the $O(\beta)$ correction to λ_p further showed that it, in fact, diverges at a frequency that is lower than the natural frequency of a single bubble. This is because a pair of bubbles resonate at frequencies that are different from the resonance frequency of the individual bubbles. Specifically, there are two modes in which a pair of bubbles can resonate. One corresponds to the two bubbles undergoing volume oscillations out of phase with each other and the other mode corresponds to the two bubbles undergoing volume oscillations in phase with each other. The former generally occurs at a frequency greater than the natural frequency of the isolated bubbles while the latter occurs at smaller frequencies. Now since the effective wavelength in the mixture is $O(R\beta^{-1/2})$, the two bubbles see different ambient pressures around them only when their separation distance is $O(R\beta^{-1/2})$ and, consequently, the first mode of out-of-phase resonance, which requires for its excitation unequal ambient pressures, is important only for pairs of widely separated bubbles. For such widely separated bubbles, the resonance frequency is only $O(\beta^{1/2})$ different from the natural frequency of the bubbles and therefore this mode does not make any contribution to the evaluation of λ_p up to $O(\beta)$ terms. This, however, is not the case with the second in-phase resonance mode which is excited by the nearly equal ambient pressures felt by the bubbles that are separated by a distance comparable to R. This resonance occurs at a frequency that depends, among other variables, on the separation distance and surface tension. When the latter is very large, the resonance occurs at a frequency ω_{cp} that is about 0.83 times the natural frequency ω_c for a pair of nearly touching bubbles. Thus for any frequency ω in the range (ω_{cp}, ω_c), there is always a pair of bubbles that resonates. For finite surface tensions, ω_{cp} for the pair of nearly touching bubbles is generally smaller than its value for large surface tension and in fact for certain (of which there are countably infinite numbers) surface tension values, the shape resonances make it nearly zero. Thus, depending upon surface tension and the separation distance between the bubbles, very large effects may be seen in the $O(\beta)$ correction to λ_p.

Although the resonance effects for pairs of bubbles described above may at first glance suggest that there will be a range of frequency values for which the bubbly mixture will exhibit a resonance like behavior, it turns out that the resonance of pairs of bubbles gives rise to a divergence in the estimate of λ_p only for ω very close to ω_{cp}. This is because over most of the frequency range (ω_{cp}, ω_c), for each pair of bubbles that is nearly resonating at any given instant, there is another pair of bubbles that is also nearly resonating but whose volume oscillations are out of phase with

those of the first pair, resulting in a near cancellation of the large resonance effects. This, however, does not occur for the pairs of nearly touching bubbles for which there are no counter pairs undergoing out of phase volume oscillations. The overall effect, after averaging over all pairs, is as a result, only a weak logarithmic divergence in the $O(\beta)$ correction to λ_p for ω near ω_{cp}. Since the effect of resonance near the natural frequency on λ_p is rather strong, being proportional to $1/(\omega_c^2 - \omega^2)$, compared with the contribution due to pairs of nearly touching bubbles, which is proportional to $\beta \log_e(|\omega_{cp} - \omega|)$, this divergence due to pairwise interaction will be overshadowed in practice by the more dominant isolated bubble resonances.

The numerical calculations for λ_p also show that the corrections are generally small when ω/ω_c is less than about 0.3. For such low frequencies, λ_p approaches nearly unity for all values of β since temporal variations occur slowly at small frequencies, and as a consequence the average pressure fluctuations in the test bubble are nearly the same as the average pressure fluctuations in the mixture. For such low frequencies it is possible in fact to determine $1/C_{ef}^2$ correct to $O(\beta^3)$ simply by determining λ_v correct to only $O(\beta)$ (cf. (2.3)). Therefore the $O(\beta)$ correction to λ_v for small ω was also determined using the method of pairwise interactions. In this case, since the frequency is small, the volume oscillations are absent and therefore there is no resonance effect of the kind described above. However, when the surface tension is not large, as is the case in many experimental conditions for bubbly liquids, the shape deformation resonances are significant and these in turn make a large effect on the displacement oscillation of the test bubble and consequently on λ_v. Thus we found that the $O(\beta)$ correction to λ_v diverges for a countably infinite number of frequencies owing to the shape deformation resonances. Each of these divergences, being logarithmic, is weak but also, since the frequency range between each successive divergence gradually becomes narrower, the overall effect is difficult to estimate. It may be noted here that in previous theories, which examined only the interaction of an isolated bubble with the incident wave, such resonances were not important because when the wavelength is large, the shape resonances are not excited by the pressure distribution around the bubble which is nearly uniform. However, in the presence of a second bubble, the pressure distribution on the surface of the test bubble becomes asymmetric and therefore the shape deformation resonances are excited even in the limit of infinite effective wavelength.

Since the $O(\beta)$ estimates of both λ_v and λ_p diverge at several frequencies owing to the resonance of pairs of bubbles at frequencies different from the natural frequency of the bubbles, it is important to include the damping mechanisms due to viscous and thermal effects. A new method was developed for determining the corrections to λ_p and λ_v when the viscosity of the liquid is small but nonnegligible. Similarly a method was developed for determining the corrections due to nonadiabatic changes inside the gas bubbles. It was found that thermal effects only dampen the spherically symmetric part of the pressure distribution around the test bubbles and

thus play an important role in preventing the divergence of λ_p from occurring. On the other hand, the divergence in λ_v, which occurs even at low frequencies and is due to the shape deformation resonances, can only be damped by the viscous effects.

The $O(\beta^3)$ theory for small frequency and the $O(\beta^2)$ theory for arbitrary frequencies were compared with the experimental data available in the literature and it was found that the agreement between the theories and experiments was generally satisfactory except for frequencies close to and above the natural frequencies of the bubbles. For these frequencies the coefficient of $O(\beta)$ correction to λ_p as calculated from the theory became very large. Thus the problem of predicting the phase speed and attenuation of waves for such high frequencies still remains unsolved.

This study of pairwise interaction among bubbles has pointed out the essentially very complex nature of the multiparticle interactions among the bubbles and their role on the macroscopic behavior of the bubbly liquids. For relatively dense mixtures it is clear that the resonance effects due to three or more bubbles will also begin to play a significant role. Also, in view of the fact that in the absence of any damping mechanism, these resonances cause the overall properties to diverge, it seems that the magnitude of viscosity, even when small, may play an important role in determining the overall behavior. It seems that large-scale numerical simulations with selected values of viscosity, gas thermal diffusivity, etc. may be needed for obtaining meaningful predictions. Such numerical calculations will be carried out in the future. The case of slightly nonlinear deformations also seems fruitful to pursue. Although the resulting analysis at present appears algebraically complicated, it seems that pursuing it will indicate the proper nature of various weakly nonlinear terms that arise in the ensemble-averaged equations.

3 Drag and added mass in dense bubbly liquids

Authors: D.Z. Zhang and A. S. Sangani

In this study, which is not yet completed, we simulate the potential incompressible flow around an infinite number of massless bubbles. The calculation is made feasible by imposing a periodic structure to the system. This is obtained by arranging N bubbles at random in a cubic cell, and then filling up the whole space with copies of this cell. For each value of the bubbles' volume fraction, several arrangements within the cell are generated and the results ensemble-averaged.

Our purpose is to investigate several problems in which the infinite lattice of bubbles is immersed in a flow that would have uniform velocity in the absence of the bubbles. We therefore write the solution to the Laplace equation satisfied by the velocity potential ϕ,

$$\nabla^2\phi = 0. \tag{3.1}$$

in the form

$$\phi = x_1 + \sum_{p=1}^{N} \sum_{n=1}^{\infty} \sum_{m=0}^{n} [A_{nm}^{p} \partial_1^{n-m} \triangle_m + \tilde{A}_{nm}^{p} \partial_1^{n-m} \tilde{\triangle}_m] S_1(\mathbf{r} - \mathbf{r}^p), \qquad (3.2)$$

where $\partial_1 \equiv \partial/\partial x_1$, $\mathbf{r}^p$ denotes the position of the center of the p-th bubble $(p = 1, 2, \ldots, N)$ in the fundamental cell, and

$$\triangle_m = \left(\frac{\partial}{\partial \xi}\right)^m + \left(\frac{\partial}{\partial \eta}\right)^m, \qquad (3.3)$$

$$\tilde{\triangle}_m = i\left[\left(\frac{\partial}{\partial \xi}\right)^m - \left(\frac{\partial}{\partial \eta}\right)^m\right], \qquad (3.4)$$

$$\xi = x_2 + ix_3, \qquad \eta = x_2 - ix_3, \qquad (3.5)$$

and $S_1(\mathbf{r})$ is given by

$$S_1(\mathbf{r}) = \frac{1}{\pi V_c} \sum_{\mathbf{k}} \frac{1}{\mathbf{k} \cdot \mathbf{k}} \exp(-2\pi i \mathbf{k} \cdot \mathbf{r}), \qquad (3.6)$$

in which the sum is extended to all the vectors $\mathbf{k}$ of the reciprocal lattice (except $\mathbf{k} = \mathbf{0}$), and V_c denotes the volume of the fundamental cell. As shown in Sangani and Yao (1988), the function S_1 and its derivatives can be evaluated by combining Ewald's technique (Hasimoto 1959) and the direct summation method.

The summation in (3.6) represents the disturbance induced in the flow by the presence of the bubbles and the use of the particular form of S_1 shown ensures the periodicity of the disturbance potential. Since the problem for ϕ is linear, we have taken the coefficient of the first term x_1 in (3.2) to be unity without loss of generality.

Near the surface of the p-th bubble, the potential can be expanded in spherical harmonics as

$$\phi = \sum_{n=0}^{\infty} \sum_{m=0}^{n} \Big[(C_{nm}^{p} r^n + D_{nm}^{p} r^{-n-1}) \cos m\varphi + (\tilde{C}_{nm}^{p} r^n + \tilde{D}_{nm}^{p} r^{-n-1}) \sin m\varphi\Big] P_n^m(\cos\theta). \qquad (3.7)$$

The relations between the coefficients of these local expansions and those of the global expansion (3.2) are given in Sangani and Yao (1988). The application of suitable boundary conditions on the surface of the bubbles then determines the coefficients.

The liquid velocity $< \mathbf{u}^L >$ averaged over the volume of the cell is given by

$$< \mathbf{u}^L > = \frac{1}{V_L} \int_{V_L} \nabla\phi \, dV_L, \qquad (3.8)$$

where V_L is the volume of the cell occupied by the liquid. Similarly, the average velocity of the bubbles, $< \mathbf{u}^G >$, is given by

$$< \mathbf{u}^G >= \frac{v_s}{(V_c - V_L)} \sum_{p=1}^{N} \mathbf{V}^p, \tag{3.9}$$

where $v_s = \frac{4}{3}\pi R^3$ is the volume of the bubbles, all assumed to have the same radius R, and $\mathbf{V}^p$ is the velocity of the p-th bubble. In terms of $< \mathbf{u}^L >$ and $< \mathbf{u}^G >$, the average mixture velocity $< \mathbf{v} >$ is

$$< \mathbf{v} >= (1 - \beta) < \mathbf{u}^L > + \beta < \mathbf{u}^G >, \tag{3.10}$$

where $\beta = N v_s / V_c$ is the volume fraction of the bubbles.

These average quantities are readily evaluated. The velocities $\mathbf{V}^p$ can be determined by substituting (3.7) into the kinematic boundary condition

$$\mathbf{n} \cdot \nabla \phi|_R = \mathbf{n} \cdot \mathbf{V}^p, \tag{3.11}$$

where we have assumed that the surface tension is large so that the small deformations of the bubble shape can be neglected. With this relation, the component of the bubble velocity in the x_1 direction, for example, can be evaluated from

$$V_1^p = \frac{3}{4\pi R^2} \int_{S_p} \frac{\partial \phi}{\partial r} P_1(\cos\theta)\, dS_p = C_{10}^p - 2D_{10}^p R^{-3}. \tag{3.12}$$

Substituting (3.12) into (3.9), we obtain the x_1-component of the average bubble velocity as

$$< u_1^G >= \frac{1}{N} \sum_{p=1}^{N} (C_{10}^p - 2D_{10}^p R^{-3}). \tag{3.13}$$

To calculate the liquid average velocity we use the generalized divergence theorem to write

$$\int_{V_L} \nabla \phi \, dV_L = \int_{S_c} \phi \, \mathbf{n} \, dS_c - \sum_{p=1}^{N} \int_{S_p} \phi \, \mathbf{n} \, dS_p, \tag{3.14}$$

where S_c is the surface of the cell and $\mathbf{n}$ is the outward normal with respect to S_c and the bubbles' surfaces S_p. The contribution from the bubbles' surfaces is readily evaluated from (3.7). For example, the component in the direction of x_1 is given by,

$$\int_{S_p} \phi \, n_1 \, dS_p = v_s (C_{10}^p + D_{10}^p R^{-3}). \tag{3.15}$$

To evaluate the first integral in (3.14), it is convenient to use the form (3.2) of the velocity potential because the sum, being periodic, contributes zero so that, again in the direction of x_1,

$$\int_{S_c} \phi \, n_1 \, dS_c = V_c. \tag{3.16}$$

Substituting (3.15) and (3.16) into (3.14) and (3.8) we find that the average liquid velocity in the x_1 direction is given by

$$< u_1^L >= \frac{1}{1-\beta} \left[1 - \frac{\beta}{N} \sum_{p=1}^{N} (C_{10}^p + D_{10}^p R^{-3}) \right], \tag{3.17}$$

from which the volume average velocity (3.10) is obtained as

$$< v_1 >= 1 - \frac{3\beta}{N} \sum_{p=1}^{N} D_{10}^p R^{-3}. \tag{3.18}$$

The approach just described allows us to calculate now several quantities of interest.

Added Mass

For an assembly of particles several non-equivalent definitions of the added mass coefficient are possible. We have begun to explore this matter.

For example, one possible definition is the following. Suppose that the bubbles undergo an oscillatory motion in the direction of x_1 with a velocity of amplitude $< u_1^G >$ and frequency ω. Then, for each particle, one may define an added mass coefficient C_m corresponding to the force in the x_1 direction for motion in the x_1 direction by

$$- \int_{S_p} p \, n_1 \, dS_p = \frac{1}{2} v_s \rho C_m i\omega < u_1^G > . \tag{3.19}$$

The left-hand side is the x_1-component of the force on the p-th bubble and the coefficient $1/2$ has been introduced so that $C_m \to 1$ in the dilute limit. For oscillations of small amplitude the Bernoulli equation reduces to

$$\frac{\partial \phi}{\partial t} + \frac{p}{\rho} = F(t), \tag{3.20}$$

so that, with the change $\partial/\partial t \to i\omega$, we obtain from (3.19)

$$\int_{S_p} \phi \, n_1 \, dS_p = \frac{1}{2} v_s C_m < u_1^G > .$$

If the bubbles are stationary while the liquid oscillates with a velocity $< u_1^L >= - < u_1^G >$, the previous definition must be modified by adding to the velocity potential a term $\mathbf{x} \cdot < \mathbf{u}^G >$ corresponding to the inertia force. Furthermore, following Biesheuvel and Spoelstra (1989), we relate C_m to the average volume velocity $< v_1 >$ rather than to the liquid velocity. These considerations motivate the following definition of C_m

$$\int_{S_p} \phi \, n_1 \, dS_p = \frac{1}{2} v_s (C_m + 2) < v_1 > . \tag{3.21}$$

Since now the bubbles are stationary, we have from (3.12)

$$C_{10}^p - 2 D_{10}^p R^{-3} = 0. \tag{3.22}$$

Using this relation in (3.21), we then find

$$C_m = \frac{3 C_{10}^p}{< v_1 >} - 2. \tag{3.23}$$

For small β the quantities appearing in this definition can be calculated analytically and the result is

$$C_m \simeq 1 + 3.32 \, \beta,$$

in agreement with Biesheuvel and van Wijngaarden (1984). For arbitrary β, we have calculated numerically C_m according to (3.23) for each bubble in the fundamental cell. For each value of β, the calculation was repeated for a number of different configurations. The results of such computations are represented in Figure 1 by the symbols # and the small-β asymptote by the dash-and-dot line.

Another possible definition of the added mass coefficient is the following. Suppose that the mixture oscillates with an average velocity $< v_1 > e^{i\omega t}$. For a single body in a liquid, classical hydrodynamics gives the following relation between the velocity u_1^G of the body and the velocity u_1^L of the liquid at large distance from the body

$$u_1^G = \frac{C_m + 2}{C_m} u_1^L,$$

where again C_m has been defined so as to equal 1 for a single isolated sphere. One can adapt this definition to the case at hand by identifying u_1^L with the average velocity $< v_1 >$ of the mixture so that

$$< u_1^G >= \frac{C_m + 2}{C_m} < v_1 > . \tag{3.24}$$

The difference with the previous case is that now the total force on each bubble vanishes so that

$$C_{10}^p + D_{10} R^{-3} = 0. \tag{3.25}$$

With this relation, the added mass coefficient for the p-th bubble is calculated from

$$C_m = \frac{2}{3C_{10}^p(1-\beta)-1}. \tag{3.26}$$

Again, for small β, one finds

$$C_m \simeq 1 + 3.32\,\beta,$$

in agreement with van Wijngaarden (1976). Results for larger values of β can be obtained numerically as before. They are shown in Figure 1 by the asterisks. The dashed line is the small-β asymptote. It is interesting to observe that the two results are very close to each other up to at least $\beta = \frac{1}{2}$.

The two different definitions of C_m can be treated together by writing the x_1-component of the total force $\mathbf{F}$ on a bubble in the form

$$F_1 = \rho\, v_s \left[\frac{1}{2} C_m \left(\frac{d < u_1^G >}{dt} - \frac{d < v_1 >}{dt}\right) - \frac{d < v_1 >}{dt}\right]. \tag{3.27}$$

Then the first definition (3.21) of C_m is obtained by setting $< u_1^G >= 0$, while the second one is obtained by setting $F_1 = 0$. An interesting consequence of our unified approach to these two different definitions is the explicit proof that the results thus obtained are not equivalent but correspond to different physical quantities.

The two definitions given above are not the only ones possible. A further exploration of these issues will be carried out in the continuation of this study.

Drag

It is well known that, at large Reynolds number, the drag on a bubble on the surface of which the tangential stress vanishes can be estimated by calculating the energy dissipation in the liquid using the potential-flow velocity distribution. In applying this procedure it would be necessary, for consistency, to allow the spheres to deform so as to approach more closely the behavior of actual free surfaces. For simplicity, however, here the bubbles are assumed to maintain the spherical shape.

In this calculation we keep the spheres fixed while the liquid has an average velocity $< u_1^L >$. We define an average drag coefficient per sphere, C_d, by

$$12\, N\, \pi\, \mu\, R\, C_d\, (< v_1 >)^2 = \dot{E} \tag{3.28}$$

where μ is the dynamic viscosity and $\dot{E}$ is the energy dissipation rate in the fundamental cell given by

$$\dot{E} = 2\mu \int_{V_c} \left(\frac{\partial^2 \phi}{\partial x_i \partial x_j}\right)^2 dV_c \tag{3.29}$$

$$= 2\mu \left[\int_{S_c} (\mathbf{n} \cdot \nabla)(\nabla\phi)^2 \, dS_c - \sum_{p=1}^{N} \int_{S_p} (\mathbf{n} \cdot \nabla)(\nabla\phi)^2 \, dS_p \right].$$

It is easy to show that the integral on the surface S_c of the fundamental cell in the above expression vanishes while the integrals over the surface of the bubble can be evaluated from (3.7) by making use of the orthogonal properties of the Legendre polynomials and the relations among D^p_{nm}, $\tilde{D}^p_{nm}$, A_{nm}, and $\tilde{A}_{nm}$ to yield

$$\dot{E} = 8\pi\mu \sum_{n=1}^{\infty} \sum_{m=0}^{n} \frac{(n+1)(2n+1)}{nR^{2n+3}2^{2-2m}} (n+m)!(n-m)! \times \left\{ (1+\delta_{m0}) \left(A^p_{nm}\right)^2 + (1-\delta_{m0}) \left(\tilde{A}^p_{nm}\right)^2 \right\}. \tag{3.30}$$

Some results obtained in this way are shown in Fig. 2 as a function of the spheres' volume fraction. The asterisks indicate the mean value obtained with different simulations and the crosses the variance from the mean.

4 Acoustic wave propagation in dense bubbly liquids

Authors: A. Lezzi and A. S. Sangani

In this study we examine the situation in which the effective wavelength of sound waves becomes comparable to the size of bubbles to determine when the theories for predicting the phase speed or attenuation based on averaged equations and the long wavelength assumption begin to breakdown and to devise, if possible, an alternate averaged-equation theory which will yield reasonably accurate estimates. For this purpose we choose a somewhat idealized configuration for the spatial distribution of the bubbles, viz., when their centers are located on N parallel equidistant planes arranged with their normal pointing in the direction of wave propagation. The arrangement of bubbles within each plane is periodic. The volume fraction of bubbles is large and the frequency of the waves is comparable to, or larger than, the resonance frequency of the bubbles. In this case the effective wavelength is comparable to the size of the bubbles.

We have determined an exact (numerical) solution of this problem in terms of planar periodic singular solutions of the Helmholtz equation. Briefly, the fundamental planar periodic singular solution of the Helmholtz equation corresponds to an array of point scatterers on a plane $x_1 = 0$, i.e.,

$$\nabla^2\psi + k^2\psi = -4\pi \sum_{\mathbf{x_L}} \delta(\mathbf{x} - \mathbf{x_L}) \tag{4.1}$$

where ψ is the amplitude of the pressure variation in the liquid, k is the wavenumber for the pure liquid, i.e., $k = \omega/C_L$, ω is the frequency, C_L is the speed of sound in the pure liquid, and $\mathbf{x_L}$ are the lattice points defining

the position of the bubbles' centers on the plane $x_1 = 0$. The solution of (4.1) is given by

$$\psi = \sum_{\mathbf{x_L}} \frac{e^{-ik|\mathbf{x}-\mathbf{x_L}|}}{|\mathbf{x}-\mathbf{x_L}|}, \tag{4.2}$$

which appears as outgoing planar waves from $x_1 = 0$ for $|x_1| \to \infty$ and as spherically outgoing waves from $\mathbf{x_L}$ near each scatterer. The solution given by (4.2) is actually not very useful for computational purposes as the sum in it converges very slowly when k is small. Therefore this sum is rewritten in a different form using Ewald's technique which converts it into a suitable sum in the reciprocal lattice. The details of this technique may be found in Sangani and Behl (1989) where it was employed in the calculation of the fundamental planar singular solutions of the Stokes and Laplace equations. In fact, when k is small, it is possible to obtain a uniformly valid approximation for ψ in terms of the solutions derived by Sangani and Behl.

Now, since any spatial derivative of the fundamental solution ψ also satisfies the Helmholtz equation, it is possible to generate an infinite number of planar periodic singular solutions of the Helmholtz equation simply by successive differentiation of the fundamental solution. A general solution is then constructed by adding all the singular solutions to the incident wave. Thus if a plane wave in the x_1 direction is incident upon the mixture, then the amplitude of the pressure variation is given by

$$\hat{p} = \hat{p}_a e^{-ikx_1} + \sum_{\alpha=1}^{N} \sum_{n=0}^{\infty} \sum_{m=0}^{n} A_{nm}^{\alpha} Y_n^m \left(\frac{\partial}{\partial x_1}\right) \psi(\mathbf{x} - \mathbf{x}_1^{\alpha}) \tag{4.3}$$

where $\hat{p}_a$ is the amplitude of the pressure variation in the incident wave, Y_n^m is a solid spherical harmonic of degree n, x_1^{α} is the x_1 coordinate of the α plane, and A_{nm} are constants that can be determined from the appropriate boundary conditions on the surface of the bubble and at infinity.

The constants A_{nm} can be determined either by the method of successive approximations or by the method of direct substitution. The former is useful for the case of small volume fractions and large wavelengths. According to this method each of the constants is first expressed as consisting of two parts corresponding to two effective planar waves in the mixture. Thus

$$A_{nm}^{\alpha} = a_{nm}\exp(-i\Gamma x_1^{\alpha}) + b_{nm}\exp(i\Gamma' x_1^{\alpha}) \tag{4.4}$$

where Γ and Γ' can be shown to be related to the effective wavenumbers of two planar waves travelling in opposite directions at $x_1 = x_1^{\alpha}$. Actually, the decomposition shown in (4.4) is valid only for bubbles well within the bulk of the mixture, i.e., for $1 << \alpha << N$. Edge effects will be important for α close to unity or N and the modifications required for the bubbles near the edge are taken into account separately once the bulk solution is determined. The method of successive approximations then consists of expanding a_{nm},

b_{nm}, Γ, and Γ' in powers of $\beta^{1/3}$ (corresponding to the bubble radius-to-spacing ratio) and determining the successive approximations to Γ and Γ' and thereby determine the phase speed, attenuation, and the coefficients of the reflected and transmitted waves. This method is somewhat similar to that employed by Twersky (1962).

In this way we have obtained estimates of the phase speed that are in agreement with the $O(\beta^{1/3})$ correction obtained by Rubinstein (1985) and the $O(\beta)$ estimate for the phase speed when the frequency is small as given by Caflisch *et al.* (1985 a,b). These investigators employed the method of homogenization to derive the effective equations and thus it is interesting that their results can be recovered by an independent analysis for the situation examined here which is somewhat different. More interesting, however, is to examine the situation where the homogenization method will no longer be useful. As mentioned before, this corresponds to the case of large β and ω. Although the method of successive approximations can be carried out, in principle, to evaluate the next few approximations beyond those determined by Rubinstein and Caflisch *et al.*, the resulting algebra quickly becomes cumbersome and it is advantageous to use instead the method of direct substitution. In this method, the phase speed, attenuation, and reflected and transmitted wave coefficients are determined numerically for selected values of N, ω, and β. The infinite sum in n in (4.3) is first truncated to include only a finite number of singular solutions per plane of bubbles and then the same number of boundary conditions on the surface of the bubble are derived suitably and the resulting set of linear algebraic equations is solved directly. The number of singularities are then increased and the calculation repeated until the calculated values of the effective speed, attenuation, etc. do not change significantly. Using this method it is also easy to examine the edge effect. These calculations are currently in progress.

5 Multi-phase flow in a nozzle

Authors: H.S. Kim, A. Prosperetti, A. S. Sangani

The ultimate aim of this study is the numerical simulation of the flow of a number of particles or bubbles in a duct of variable geometry. The work carried out to date concerns a very simplified situation which is being studied to perfect the numerical techniques. We consider the buoyancy-induced motion of a number of rigid massless spheres on the axis of symmetry of a nozzle or pipe. The liquid is assumed to be incompressible and inviscid and the velocity field irrotational.

The starting point is Green's identity for the velocity potential ϕ,

$$2\gamma\pi\phi(\mathbf{x}) = \int_S \left[\frac{1}{|\mathbf{x}-\mathbf{x}'|}\frac{\partial\phi}{\partial n'} - \phi(\mathbf{x}')\frac{\partial}{\partial n'}\frac{1}{|\mathbf{x}-\mathbf{x}'|} \right] dS', \qquad (5.1)$$

where $\gamma = 2$ if $\mathbf{x}$ is inside the domain occupied by the liquid and $\gamma = 1$ if $\mathbf{x}$

belongs to the boundary S of this domain. For our purposes this relation will be applied to points on the boundary. Here and in the following the time dependence of ϕ is understood.

In the present problem the domain occupied by the fluid is bounded by N spherical surfaces S^α, $\alpha = 1, 2, \ldots, N$, the lateral wall of the pipe W, and two surfaces at infinity upstream and downstream of the region occupied by the bubbles. Since, in the present simple situation, the motion of the liquid is solely due to that of the spheres, these surfaces can be disregarded. On the pipe wall, the normal velocity vanishes while, on the sphere α,

$$\frac{\partial \phi}{\partial n} = \mathbf{V}^\alpha \cdot \mathbf{n}. \tag{5.2}$$

The preceding equation then becomes

$$2\pi\phi(\mathbf{x}) = -\int_W \phi(\mathbf{x}') \frac{\partial}{\partial n'} \frac{1}{|\mathbf{x} - \mathbf{x}'|} \, dS_w \tag{5.3}$$

$$+ \sum_{\alpha=1}^{N} \int_{S^\alpha} \left[\frac{1}{|\mathbf{x} - \mathbf{x}'|} \mathbf{V}^\alpha \cdot \mathbf{n}' - \phi(\mathbf{x}') \frac{\partial}{\partial n'} \frac{1}{|\mathbf{x} - \mathbf{x}'|} \right] dS^\alpha.$$

To solve (5.3) we start by setting, on the surface of each sphere,

$$\phi|_{S^\alpha} = \sum_{n=0}^{\infty} B_n^\alpha P_n(\cos\theta^\alpha). \tag{5.4}$$

Here the axial symmetry of the problem has been used and θ^α is the azimuthal angle in a spherical coordinate system centered at the center $\mathbf{x}^\alpha$ of the sphere α and having as polar axis the axis of symmetry of system. Using this expansion of the unknown potential on the surface of each sphere, for $\mathbf{x}$ on the pipe wall W, (5.3) gives

$$2\pi\phi(\mathbf{x}) = -\int_W \phi(\mathbf{x}') \frac{\partial}{\partial n'} \frac{1}{|\mathbf{x} - \mathbf{x}'|} \, dS_w + \sum_{\alpha=1}^{N} \frac{2\pi a^2}{|\mathbf{x} - \mathbf{x}^\alpha|^2} \left[-\frac{2}{3} a V^\alpha \cos\delta^\alpha \right. \tag{5.5}$$

$$\left. + \sum_{n=0}^{\infty} \frac{2n}{2n+1} B_n^\alpha \left(\frac{a}{|\mathbf{x} - \mathbf{x}^\alpha|} \right)^{n-1} P_n(\cos\delta^\alpha) \right],$$

where δ^α is the angle between the axis and the line joining the points $\mathbf{x}$ and $\mathbf{x}^\alpha$. For the purpose of calculating the integral over the pipe wall appearing in this equation it is convenient to make use of a system of cylindrical coordinates (r, z, η) having the axis of symmetry as z axis. We then take $\mathbf{x}$ on the meridian plane $\eta = 0$, so that $\mathbf{x} = (r, z, 0)$, and decompose the integration over W into a line integral over the trace w that the pipe wall marks on a meridian plane and an angular integration over all meridian

planes, $0 \leq \eta \leq 2\pi$. Due to the axial symmetry of the problem, the latter integration can be carried out explicitly and leads to

$$\phi(\mathbf{x}) = -\int_w \phi(\mathbf{x}')H(\mathbf{x},\mathbf{x}')\,dw' + \sum_{\alpha=1}^{N} \frac{a^2}{|\mathbf{x}-\mathbf{x}^\alpha|^2}\left[-\frac{2}{3}aV^\alpha \cos\delta^\alpha \right. \tag{5.6}$$

$$\left. + \sum_{n=0}^{\infty} \frac{2n}{2n+1} B_n^\alpha \left(\frac{a}{|\mathbf{x}-\mathbf{x}^\alpha|}\right)^{n-1} P_n(\cos\,\delta^\alpha)\right],$$

where

$$H(\mathbf{x},\mathbf{x}') \equiv H(r,z;r',z') = \frac{2}{\pi} r' \frac{\partial}{\partial n'} \frac{K(m)}{\sqrt{A}}, \tag{5.7}$$

with K the complete elliptic integral of the first kind and

$$m = \frac{4rr'}{A}, \qquad A = (r+r')^2 + (z-z')^2. \tag{5.8}$$

We now write (5.3) for a point $\mathbf{x}$ on the spherical surface S^β, and take the scalar product of the resulting equation with $P_k(\cos\theta^\beta), k = 0,1,2,\ldots$. The result is

$$\frac{4\pi}{2k+1} B_k^\beta = -\int_W \phi(\mathbf{x}') \frac{\partial}{\partial n'} \left\langle P_k(\cos\,\theta^\beta), \frac{1}{|\mathbf{x}-\mathbf{x}'|} \right\rangle dS_w \tag{5.9}$$

$$+ \sum_{\alpha=1}^{N} \left[V^\alpha \int_{S^\alpha} \left\langle P_k(\cos\,\theta^\beta), \frac{\cos\,\theta^\alpha}{|\mathbf{x}-\mathbf{x}'|} \right\rangle dS^\alpha - \right.$$

$$\left. - B_n^\alpha \int_{S^\alpha} \frac{\partial}{\partial n'} \left\langle P_k(\cos\,\theta^\beta), \frac{P_n(\cos\,\theta^\alpha)}{|\mathbf{x}-\mathbf{x}'|} \right\rangle dS^\alpha \right],$$

where $<,>$ denotes the scalar product over θ^β,

$$< f,g > \equiv \int_0^\pi f(\cos\,\theta^\beta) g(\cos\,\theta^\beta)\,\sin\,\theta^\beta\,d\theta^\beta. \tag{5.10}$$

Some of the scalar products appearing in this relation can be evaluated analytically in closed form, while for others the use of a sufficiently high-order Gaussian quadrature is more efficient.

Equations (5.6) and (5.9) relate the values of ϕ on the pipe wall, the coefficients B_n^α, $n = 0,1,2,\ldots$; $\alpha = 1,2,\ldots N$, and the velocities of the spheres $V^\alpha, \alpha = 1,2,\ldots,N$. To close the system of equations, we need additional relations between the velocity potential and the translational velocity of the spheres. To this end we use the fact that, since the spheres are massless, the total force on each one of them must vanish so that, for $\alpha = 1,2,\ldots N$,

$$\int_{S^\alpha} p\,\mathbf{n}\,dS^\alpha = 0, \tag{5.11}$$

where p is the pressure. To calculate p, we use the Bernoulli integral

$$p = -\rho \left[\frac{\partial \phi}{\partial t} + \frac{1}{2}(\nabla\phi \cdot \nabla\phi) + gz + F(t) \right] \tag{5.12}$$

where g is the acceleration of gravity and $F(t)$ a constant with respect to the spatial variable $\mathbf{x}$. On S^α we may write

$$\nabla\phi \cdot \nabla\phi = (\mathbf{V}^\alpha \cdot \mathbf{n})^2 + \frac{1}{a^2}\left(\frac{\partial \phi}{\partial \theta^\alpha}\right)^2 . \tag{5.13}$$

It is therefore clear from (5.4) that (5.12) will then lead to a system of equations involving V^α and the time derivatives of the B_n^α's.

The closure of the system requires now a set of equations to calculate the position of the spheres' centers, and these are evidently given by

$$\frac{d\mathbf{x}^\alpha}{dt} = \mathbf{V}^\alpha. \tag{5.14}$$

We are currently in the process of writing a computer problem for the solution of the system of equations presented above.

6 Personnel

The work described in the previous section has involved two graduate students (A. Lezzi and D.Z. Zhang) at half time each, one post-doctoral fellow (H.S. Kim) at half time, 5% of the time of the PI (A.P.), and 50% of the time of the Co-PI (A.S.). Recently, a third graduate student (C.Y. Fei) has also started to work in this area. It is expected that these levels of effort will be maintained until the end of the present first year of funding on June 30, 1990.

7 Future work

At the end of June, 1990 Dr. Ashok Sangani will return to Syracuse and, from that point on, his collaboration in this study will be covered by a separate contract the proposal for which has already been submitted to DOE. Accordingly, the work described in this section is the part that will be carried out at Johns Hopkins.

The first task will consist of the completion of the studies in progress summarized in sections 3 to 5. Secondly, it will be necessary to choose one or several averaged-equations models of two-phase flows and to write codes for their numerical integration. When this will have been accomplished, we shall be in the position to compare the averaged-equations results with

those of the direct numerical simulation and to improve the closure relations included in the averaged models.

It is estimated that these objectives can be met by the summer of 1991. A close connection with the work being carried out at Syracuse by Prof. Sangani and his students will be maintained throughout this project.

8 References

Biesheuvel, A. & van Wijngaarden L. 1984 Two-phase flow equations for a dilute dispersion of gas bubbles in liquid. *J. Fluid Mech.* **148**, 301.

Biesheuvel, A. & Spoelstra, S. 1989 The added mass coefficient of a dispersion of spherical gas bubbles in liquid. *Int. J. Multiphase Flow* **15**, 911.

Caflisch, R. E., Miksis, M. J., Papanicolaou, G. C. & Ting, L. 1985a Effective equation for wave propagation in bubbly liquids. *J. Fluid Mech.* **153**, 259.

Caflisch, R. E., Miksis, M. J., Papanicolaou, G. C. & Ting, L. 1985b Wave propagation in bubbly liquids at finite volume fraction. *J. Fluid Mech.* **160**, 1.

Hasimoto, H. 1959 On the periodic fundamental solutions of the Stokes equations and their applications to viscous flow past a cubic array of spheres. *J. Fluid Mech.* **5**, 317.

Rubenstein, J. 1985 Bubble interaction effects on waves in bubbly liquids. *J. Acoust. Soc. Am.* **77**, 2061.

Sangani, A. S. & Behl, S. 1989 The planar singular solutions of Stokes and Laplace equations and their application to transport processes near porous surfaces. *Phys. Fluids A*, **1**, 21.

Sangani, A. S. & Yao, C. 1988 Bulk conductivity of composites with spherical inclusions. *J. Appl. Phys.* **63**, 1334.

Twerski, V. 1962 On scattering of waves by random distributions. I. Free-space scatterer formalism. *J. Math. Phys.* **3**, 700

van Wijngaarden, L. 1976 Hydrodynamic intercation between gas bubbles in liquid. *J. Fluid Mech.* **77**, 27.

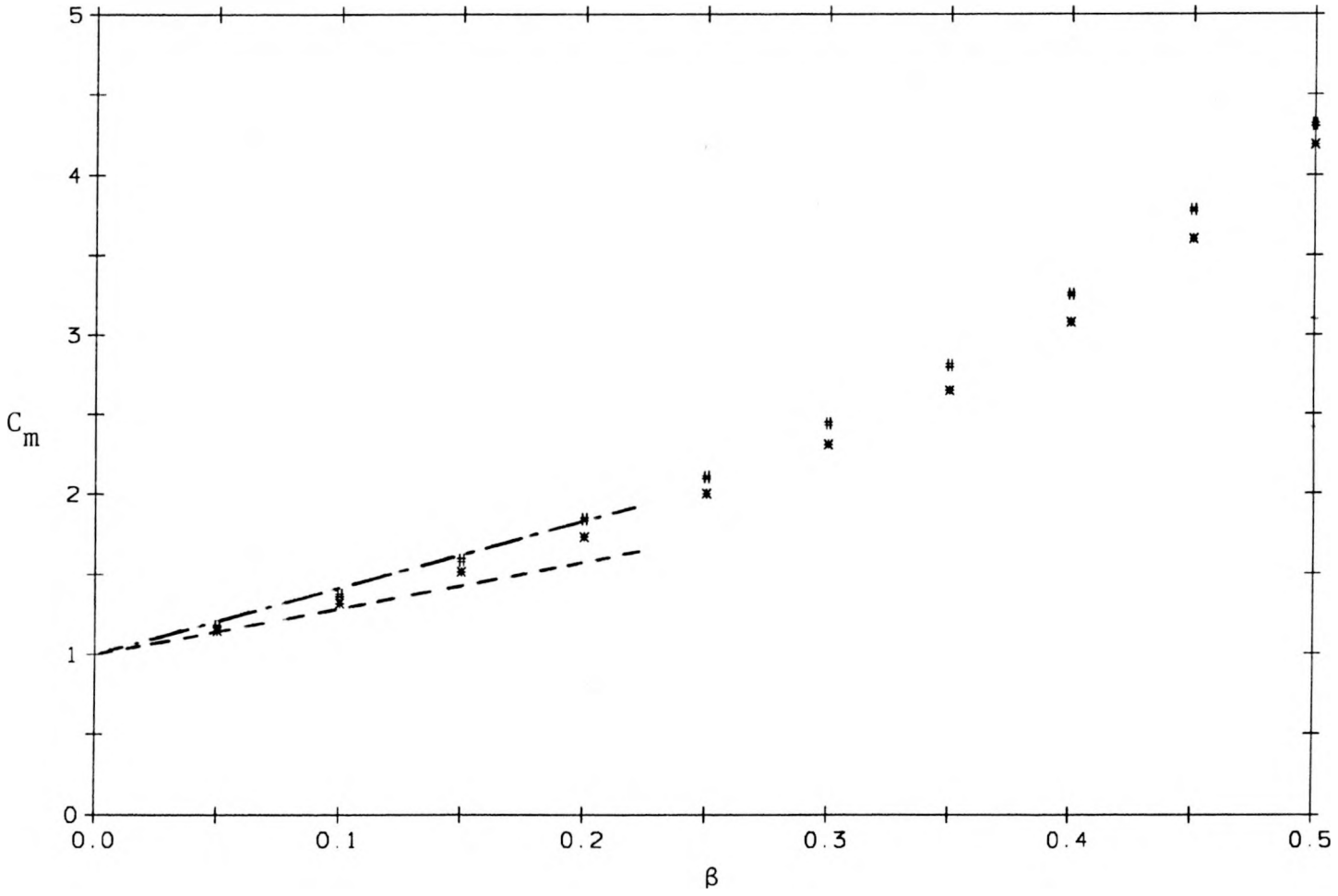

Fig. 1

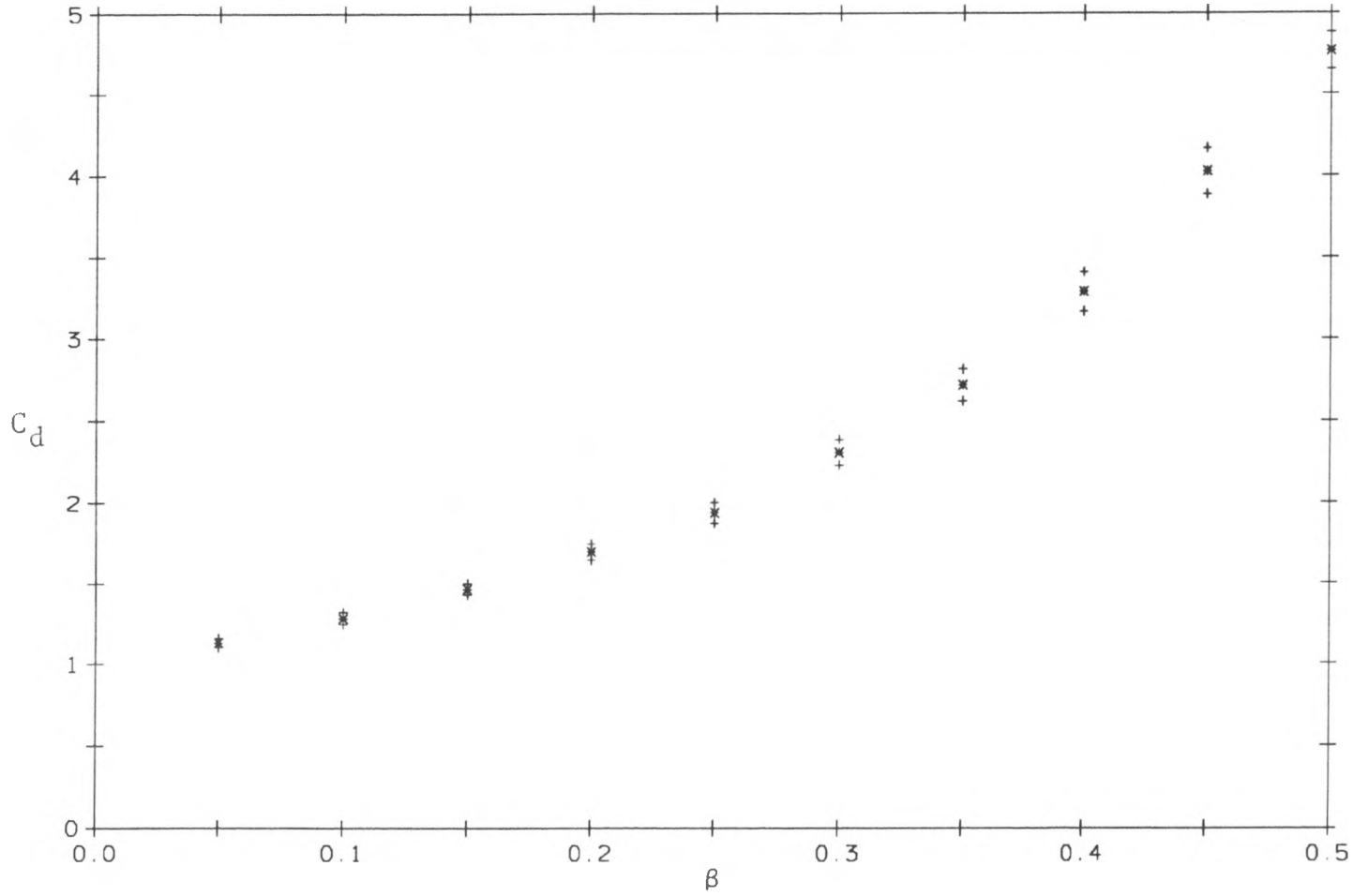

Fig. 2