CONF-8803113--2

To be published in the Proceedings of the 8th Moriond Astrophysics Meeting "Dark Matter", *held in Les Arcs, France, from March 6th to March 12th 1988.*

UCRL--98783

DE88 011724

DYNAMICAL EVOLUTION OF COSMIC STRINGS

FRANÇOIS R. BOUCHET
Department of Astronomy, University of California, Berkeley;
and The Institute of Geophysics and Planetary Physics, Lawrence Livermore National Laboratory, Livermore;
on leave from The Centre de Physique Théorique de l'Ecole Polytechnique, Palaiseau, France;
and from The Institut d'Astrophysique de Paris, Paris, France.

In collaboration with D. P. BENNETT, Fermilab and U. of Chicago.

ABSTRACT

We have studied by means of numerical simulations the dynamical evolution of a network of cosmic strings, both in the radiation and matter era. Our basic conclusion is that a scaling solution exists, *i.e.* the string energy density evolves as t^{-2}. This means that the process by which long strings dump their energy into closed loops (which can gravitationally radiate away) is efficient enough to prevent the string domination over other forms of energy. This conclusion does not depend on the initial string energy density, nor on the various numerical parameters. On the other hand, the generated spectrum of loop sizes does depend on the value of our numerical lower cutoff (*i.e.* the minimum length of loop we allow to be chopped off the network). Furthermore, the network evolution is very different from what was assumed before[12], namely the creation of a few horizon sized loops per horizon volume and per hubble time, which subsequently fragment into about 10 smaller "daughter" loops. Rather, many tiny loops are directly cut from the network of infinite strings, and it appears that the only fundamental scale (the horizon) has been lost. This is probably because a fundamental ingredient had been overlooked, namely the kinks. These kinks are created in pairs at each intercommutation, and very rapidly, the long strings appear to be very "kinky". Thus the number of long strings per horizon is still of the order of a few, but their total length is fairly large. Furthermore, a large number of kinks favors the formation of small loops, and their size might well be governed by the kink density along the long strings. Finally, we computed the two-point correlation function of the loops and found significant differences from the work of Turok[20].

Cosmic Strings are topologically stable linear defects that form in many grand unified theories (GUT) during a symmetry breaking phase transition in the early Universe. They might also be fundamental string remnants of an earlier phase (*e.g.* Piran, this volume). Contrary to monopoles and domain walls (the zero- and two-dimensional defects), they are not obviously a disaster for Cosmology. In fact, the idea that they might account for the formation of galaxies and large scale structure has recently generated a lot of interest. If the string tension μ is at the GUT scale (*i.e.* $\mu \sim (10^{16} GeV)^2$), they could provide appropriate seeds for the matter accretion in the matter era (or for the Ostriker-Thomson-Witten[1] explosions, if they are superconducting). Furthermore they have interesting observable signatures, like a non-zero residual of the millisecond pulsar timing measurements[2], or their gravitational lensing effects[3], or the expected step-like discontinuities in the microwave background[4]. For the value of μ aforementioned (corresponding to $G\mu/c^2 \sim 10^{-6}$, G being Newton's constant), these might soon be detectable.

The physics of these objects is rather simple on a macroscopic scale. The equation of motion can be derived from the grand unified field theory, and is equivalent to the equations derived from the Nambu action (which is appropriate for fundamental strings), up to terms of the order of the transverse dimension of the string (typically 10^{-30} cm) divided by the local curvature radius (in the kiloparsec range for cosmological applications). In the following we use units such that $c \equiv 1$. But we occasionally include the c's in order to make dimensions stand out more clearly, or to quote final results. In the Robertson-Walker metric $ds^2 = a^2(d\tau^2 - d\mathbf{x}^2)$ where a is the expansion factor, the equation is[5]

$$\ddot{\mathbf{x}} + 2\left(\frac{\dot{a}}{a}\right)\dot{\mathbf{x}}(1 - \dot{\mathbf{x}}^2) = \left(\frac{1}{\epsilon}\right)\left(\frac{\mathbf{x}'}{\epsilon}\right)' , \tag{1}$$

in the gauge where $\dot{\mathbf{x}} \cdot \mathbf{x}' = 0$ (*i.e.* the velocity is perpendicular to the string). Dots denote derivatives with respect to conformal time τ, primes denote partial derivatives with respect to the string length parameter σ, and $\epsilon = \sqrt{\mathbf{x}'^2/(1 - \dot{\mathbf{x}}^2)}$ ($E = \mu a \int \epsilon \, d\sigma$ is thus the string energy). In the limit of zero expansion ($\dot{a} = 0$), Eq. 1 simply describes the evolution of a usual oscillating string. The term due to the expansion of the metric damps these oscillations by redshifting the velocities, and is efficient for structures of size comparable to, or larger than the horizon. The characteristic velocity is of order unity.

The only time when the Nambu equations are not sufficient to describe string evolution is when two strings interact, *i.e.* when they cross each other. This is the only time when the details of the fundamental field theory might play an important role. Numerical calculations[6,7] for the simplest string theories showed that in almost every case (*i.e.* for almost all relative velocities and angles) strings intercommute rather than pass through each other. Furthermore, it is expected on theoretical grounds that the intercommuting probability P controls the rate of relaxation of the string system but has no bearing on the existence of a scaling solution (see below), provided P is non zero. In the following we assumed P=1 to speed up the numerical calculation.

Since the strings oscillate, they gravitationally radiate, and closed loops can thus completely disappear. Various estimates[8,9,10] showed that the decay rate seemed to depend only weakly on the loop shape, so $\dot{E} = \mu \dot{l} \simeq 50 G\mu^2$, where l is the length of the loop. For $G\mu \sim 10^{-6}$, the loop lifetime is of the order of $10^4 l$. This effect should thus be negligible in studying the string system evolution at the horizon scale, and is not incorporated in our numerical modeling.

Finally, at the time t_0 of the phase transition, the Higgs field orientations cannot be correlated on scales larger than the horizon, and thus the relative orientation of string segments should accordingly be uncorrelated at scales larger than ct_0. One thus expects the string network at formation to be a collection of random walks with step size smaller than ct_0. Most of the string length[11] ($\simeq 75\%$) is in the form of infinitely long strings, which cannot subsequently radiate away, and are thus expected to survive indefinitely.

If one ignores the strings interaction, the evolution for the string energy per unit length $\epsilon = dl/d\sigma$ is obtained from Eq. 1

$$\dot{\epsilon} = -2\,(\dot{a}/a)\,\epsilon\,\dot{x}^2, \tag{2}$$

and by integration $\dot{E}/E = (\dot{a}/a)(1 - 2 < v^2 >)$, where $< v^2 >$ is the velocity dispersion. Hence the evolution without interactions of the strings energy density $\rho_S \propto E a^{-3}$ is

$$\dot{\rho}_S/\rho_S = -2(\dot{a}/a)(1+ < v^2 >), \text{ and thus } \rho_S \propto a^{-2(1+<v^2>)}. \tag{3}$$

Therefore ρ_S behaves as radiation only if $< v^2 >= 1$, while $< v^2 >= 1/2$ in flat space, and is even smaller in the expanding case. A non-interacting string system would rapidly come to dominate the energy density of the Universe, altering the growth rate of the expansion factor, and among other things, would disrupt the usual nucleosynthesis. The "Standard" scenario[12] holds that there is a scaling solution, *i.e.* $\rho_S \propto t^{-2}$, with a few length of rather straight strings per horizon volume, and horizon-size loops are produced at a rate of a few per horizon volume per Hubble time, this rate being large enough to insure a sufficient energy transfer onto loops. These "parent" loops then fragment into about ten daughter loops.

Kibble[13] and Bennett[14,15] studied this process analytically, by modeling the energy transfer mechanisms in terms of loop production and destruction functions. This is difficult, because the efficiency of the inverse process, namely the loop reconnections onto the long string network depends on the size of the objects formed. Larger loops have a larger cross section, and are more likely to reconnect. On the other hand, if their self-intersection probability is large enough, they will rapidly fragment, and daughter loops will reconnect less efficiently, which enhances the energy transfer onto loops that can ultimately radiate away. Even though the fate of the string system cannot be decided on pure analytical grounds, one can show that the strings will either come to dominate, or will converge toward a scaling solution. Misleading transients are likely to occur only when the reconnection process is important. Also, the formalism provides a framework to analyze numerical results. As a matter of fact, by using this formalism, Bennett[14] showed that the simulation results of Albrecht and Turok[16] were inconsistent.

Thus, although much work has already been devoted to the cosmological implications of the strings existence, most of it is on fairly uncertain grounds, since the existence of a scaling solution must be postulated, as well as its expected properties (*e.g.* the loop size spectrum). Unfortunately, galaxy formation scenarios are rather dependent on the detail of the strings distribution. One would thus like to devise an accurate numerical code enabling to check the existence of the scaling solution, and obtain its characteristics, in particular the size spectrum, the loops velocities, and their correlation properties. Obviously the ultimate goal is to build a self-consistent kinetic model describing properly the strings network's evolution.

The numerical approach taken here is first to generate "reasonable" initial conditions, *i.e.* to create a string configuration which is a collection of random walks of given step-size ξ_0 in a box with periodic boundary conditions. To do so, we simply follow the procedure introduced by Vashaspati and Vilenkin[11] for their numerical study of the phase transition. Nevertheless, the generated configurations do not have to be considered as faithful representations of the outcome of the phase transition. The procedure is rather a convenient device to generate initial conditions that embody the essential characteristics of a string configuration at any time, *i.e.* the absence of correlations on large scales, and the subsequent presence of infinite strings. Thus one should think of these conditions as out of equilibrium states used to study the possible relaxation toward a universal scaling solution. Indeed, this does not fix the horizon-size, which enables us to freely set the initial string energy density per horizon. The one improvement we made over the standard procedure was to round-off the corners (Fig. 1) in order to diminish the number of discontinuous derivatives the numerical program of evolution will have to cope with.

To evolve the strings, we start by sampling the strings with points initially regularly spaced and linked by "upstring" and "downstring" pointers so that neighboring points can be determined. The string position anywhere is then obtained by linear interpolation between the sampling points (a higher order interpolation scheme would be inappropriate, since the strings are far from smooth due to the kinks generated at each intercommutation). We then proceed by discretizing the equation of motion (Eq. 1). The system is evolved in time by a modified leapfrog scheme and spatial derivatives at mid-points are obtained by finite differences. We also evolve independently (with a semi-implicit scheme) the local energy density ϵ according to Eq. 2. The accuracy of the calculations is then checked by comparison with direct estimates of ϵ using the computed $\dot{\mathbf{x}}$ and $\mathbf{x}'$. Each loop carries its own timestep satisfying the Courant condition, and our time-step halving routine preserves the O(2) accuracy of the overall calculation.

To determine if two string segments crossed during the time step, we check the volume of the tetrahedron spanned by the four points on the two segments. If it changed sign during the step, the configuration is checked at the time the volume is zero, to see if a crossing did really occur (the positions of the points are extrapolated linearly between time steps). Also, the internal dynamics of loops is on a much smaller timescale than the displacement of complete loops. A given loop is thus checked for self-crossings at each of its individual timesteps, while the crossings between loops are checked only at each system timestep, when all loops are synchronized. When two segments have been determined to cross, we interchange partners, and average the positions and the velocities in the crossing region to help reduce the gauge condition violation.

One of the simulations we did was in a box $36\xi_0$ on a side, with an initial horizon size $H_0 = 2ct_0 = 14\xi_0$. It was evolved for 3.2 expansion factors (more than a factor of 10 in physical time), so that the final horizon size was $H = 45\xi_0$. The total number of strings increased from about 1000 at the beginning to about 16,500 at the end. The strings were sampled by about 350,000 points (which corresponds to 10 sampling points per initial correlation length ξ_0), and the calculation took about 40 cpu hours on a Cray-2. The evolution of the energy density in long strings ρ_{LS} (defined arbitrarily as being longer than $3.2ct$) multiplied by $(ct)^2/\mu$ is plotted in Fig. 2 as a function of the Horizon size in units of ξ_0. At first strings are merely stretched. Then the loop production starts off, and a plateau is rapidly reached after an expansion by less than two ($a = H/H_0$). Such a fast relaxation justifies *a posteriori* the omission of gravitational decay corrections.

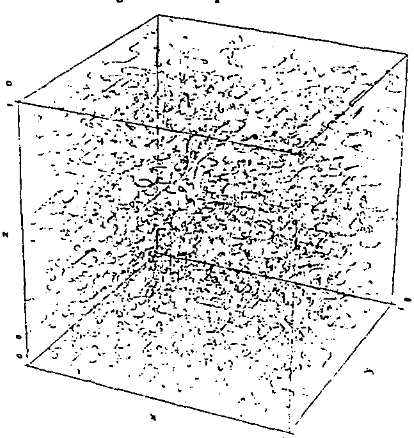


Figure 1

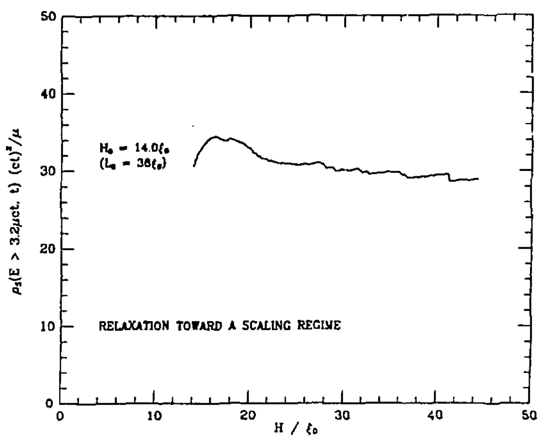


Figure 2

A lot of information can already be gained simply by looking at a carved out volume of the previous simulation. Since the only meter stick available to an observer is the horizon, we chose to display a volume of side ct (Fig. 3), which is thus an increasing fraction of the computational volume (*i.e.* from 0.7% initially to 23% finally). The snapshot presented is the final one when $H = 2ct = 45\xi_0$ ($L = 36\xi_0$). One notices immediately that long strings do appear relatively straight on the horizon scale, which does not mean they are smooth. On the contrary, one can easily see their extremely "kinky" nature. It is also apparent that the majority of energy (almost 75% actually) is in the form of very small loops. Since the initial state had very little energy in small loops, this is graphic evidence that loop production actually suceeds in transfering large amounts of energy from the longs strings to loops. Another striking feature is the total absence of horizon-sized loops; all the loops are much smaller than this.

In order to confirm the theoretical prediction that the scaling solution is stable, we thus evolved configurations with larger and smaller initial horizon-size H_0, *i.e.* with different initial string energy densities. As is shown in Fig. 4, configurations with larger initial energy densities chop off many loops in order to lower the energy density in long strings ρ_{LS}, while in string-poor configurations more string-stretching and lower loop production rate yield an energy density increase. In other words, the scaling solution appears to be a stable point, and perturbed configurations quickly relax to a stationary state[17,18].

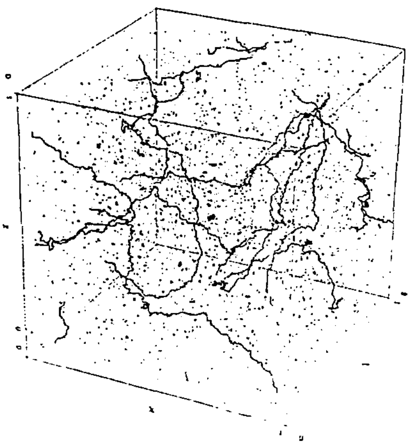


Figure 3

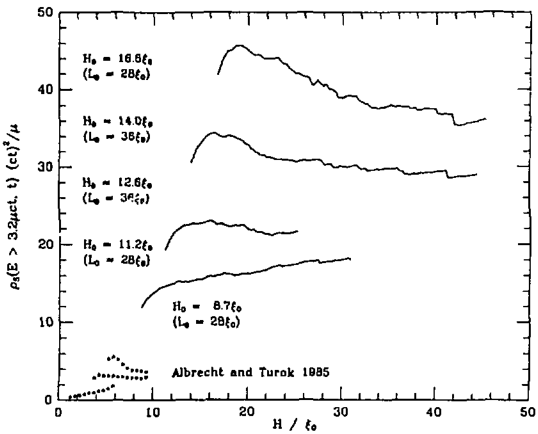


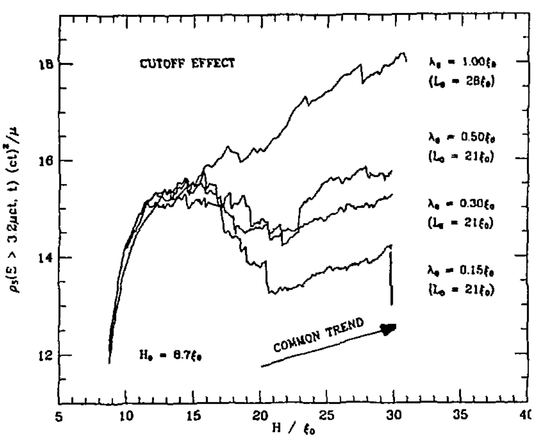
Figure 4

We checked the dependence of these results on our various numerical parameters, in particular the size of the computation box to check for boundary effects, the number of points per initial correlation length to determine possible sampling effects, as well as the time step requirements, or the value of our numerical lower cutoff. None of those but the last matters. This cutoff is implemented by requiring a minimal number of points for a loop to be allowed to chop off the network. We ran the simulation corresponding to the bottom curve of Fig. 4 ($H_0 = 8.7\xi_0$) with various cutoffs λ measured in units of the initial number of points per correlation length, *i.e.* per ξ_0, since only their ratio is relevant. The results are shown in Fig. 5. Although the evolutions are different (after the first stretching period, the loop production burst is much more pronounced with a lower cutoff), it is reassuring to see the common trend toward larger energy densities appear. Since we checked that the energy in created loops is essentially independent of λ, the variations can be accounted for by the reconnection efficiency, which cannot indefinitely decrease. We therefore conclude that a scaling solution exists, and we can confirm the theoretical prediction that it is stable. However, there is the posibility of significant systematic errors in our determination of ρ_{LS}, so we allow generous error bars[19] $\rho_{LS} = \zeta_{rad}\,\mu(ct)^{-2}$, with $\zeta_{rad} = 20 \pm 10$.

What does this imply in terms of energy transfer? Let us call $\dot{\rho}_T$ the rate of net energy transfer from the long strings into loops. One can thus modify Eq. 3 to take into account the total effect of the interactions

$$\dot{\rho}_{LS}/\rho_{LS} = -2(\dot{a}/a)(1+ < v^2 >) - \dot{\rho}_T/\rho_{LS}, \tag{4}$$

and we know that the scaling is established in the radiation era when $\dot{\rho}_{LS}/\rho_{LS} = -4(\dot{a}/a)$. Thus $\dot{\rho}_{Trad} = \mu\zeta_{rad}(1- < v^2 >_{rad})t^{-3}$. We measure $< v^2 >_{rad} \simeq 0.45$, and the system thus transfers a length of about $11ct$ per ct^3 and per expansion time t.



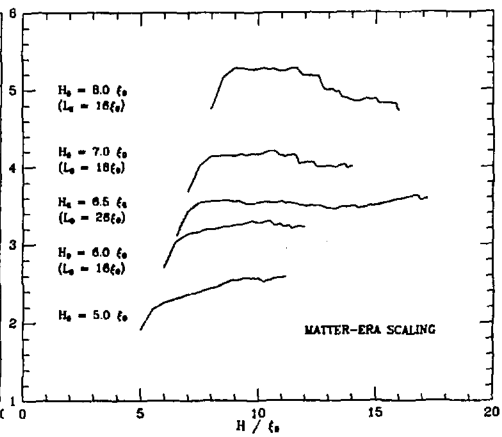


Figure 5 Figure 6

We performed a similar study of ρ_{LS} in the matter era (see Fig. 6). In this case, the relaxation is extremely fast, and we estimate the density at scaling to be such that $\zeta_{mat} = 3.5 \pm 1$, which is nearly a factor of ten smaller than in the radiation era. Since the scaling is now established for $\dot{\rho}_{LS}/\rho_{LS} = -3(\dot{a}/a)$, we have $\dot{\rho}_{Tmat} = (2/3)\mu\zeta_{mat}(1-2 < v^2 >_{mat})t^{-3}$. We measure $< v^2 >_{mat} \simeq 0.40$. The system thus only transfers a length of about $0.45ct$ per ct^3 and per expansion time. Is this much lower energy density really surprising? Let us assume that the long string system may be characterized by a single scale L such that $\rho_{LS} = \mu L^{-2}$, *i.e.* the long strings are typically separated by L, and their characteristic interaction time is of the order of L^{-1}. One thus expects $\dot{\rho}_{Tmat}/\dot{\rho}_{Trad} = L_{mat}/L_{rad} = (\zeta_{rad}/\zeta_{mat})^{1/2}$. By comparison with our previous estimation, we get

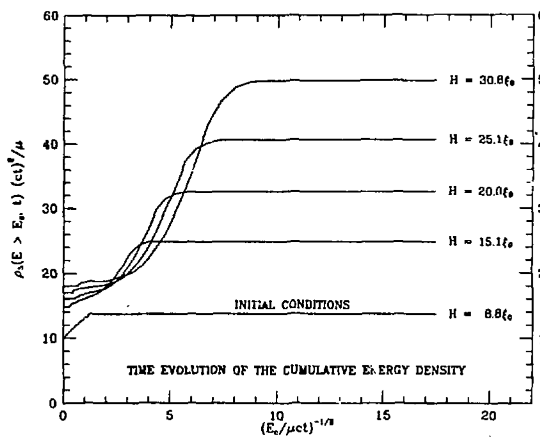
$$\zeta_{mat} = \left(\frac{2}{3}\,\frac{1-2 < v^2 >_{mat}}{1- < v^2 >_{rad}}\right)^2 \zeta_{rad}, \tag{5}$$

which yields $\zeta_{mat} \sim 0.06\zeta_{rad} = 1.2$ in reasonable agreement with our direct measurement. Although quite simple, the model demonstrates the main difference between the scaling solutions in the matter and radiation eras: in the radiation era ζ must be large so that loop production proceeds at a rate high enough for $\rho_{LS} \sim a^{-4}$ while in the matter era a much smaller ζ and lower loop production rate are required.

We now turn to the question of the loops distribution. What do we expect, once the scaling is established? Let us first assume that the network emits at time t_i in the radiation era χ loops of energy $E = \alpha\mu c t_i$ per ct_i^3 and per expansion time t_i, *i.e.* $dn(E, t_i) = \chi c dt_i/(ct_i)^4$. At a later time t, comoving volumes have expanded by $(t/t_i)^{3/2}$ and the number density has been redshifted accordingly: $dn(E,t) = \chi\alpha^{3/2}(E/\mu)^{-5/2}(ct)^{-3/2}d(E/\mu)$. The cumulative energy distribution in loops larger than some cutoff $E_c/\mu ct$ (corresponding to a length larger than some fixed fraction of ct) is then $\rho(E > E_c, t) = \int_{E_c}^{\infty} dE\, E\, dn(E,t)/dE$. In fact, one expects that loops are created in a range of sizes, so that one has to take $\chi = \chi(\alpha)$, and integrate over α. Thus

$$\rho(E > E_c, t)(ct)^2/\mu = \zeta_{rad} + \gamma(E_c/\mu ct)^{-1/2}, \text{ with } \quad \gamma = 2\int \chi(\alpha)\alpha^{3/2}d\alpha \, . \tag{6}$$

The energy density in long strings has been included in order to obtain the total cumulative energy density.



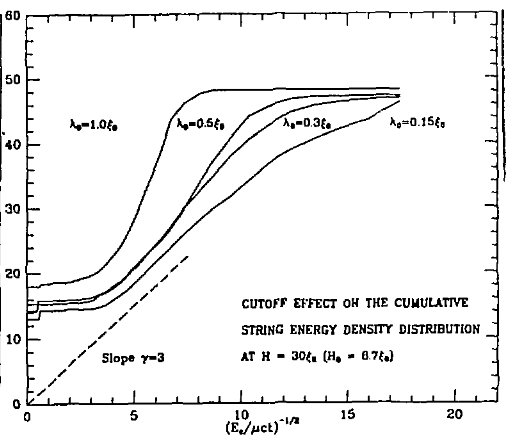


Figure 7 Figure 8

The time evolution of this cumulative energy distribution for the simulation corresponding to the lower curve of Fig. 4 is shown in Fig. 7. Times are labeled on the right by the current value of the horizon, and the bottom curve corresponds to the initial distribution, which is devoid by construction of loops smaller than $4\xi_0$, *i.e.* $(E_c/\mu ct_o)^{-1/2} \simeq 1$ since $H_0 = 2ct_0 = 8.7\xi_0$. One can recover on this graph the growth of the long string energy density as wéll as the appearance of a gap (flat curves on the left) which only translates our visual impression of the absence of loops of size comparable to the horizon. One also notes that the loop number density is indeed proportional to $E^{-5/2}$, as expected from Eq. 6. Nevertheless this is somewhat coincidental since our loop distribution is not yet in a true scaling regime. At smaller scales the straight lines turn over at the location of our cutoff (which is a decreasing fraction of the horizon since it requires a fixed number of points for a loop to be emitted).

Finally we note that on this graph the gravitational cutoff due to the finite lifetime of the loops would appear at $(E_c/\mu ct)^{-1/2} \simeq 100$.

To investigate the effect of the cutoff, we consider the distributions in simulations with the same initial conditions ($H_0 = 8.7\xi_0$), but different cutoff λ (the same simulations as in Fig. 5). The cumulative density distributions are compared at $H \simeq 30$ in Fig. 8. It is apparent that the slope γ depends on λ, although we might be close to cutoff independence since the slope variation seems to saturate for the lowest λ. This dependence is expected since the cutoff affects the size distribution of loops at birth $\chi(\alpha)$, and thus γ. How can it be then that ρ_{LS} is not affected more drastically? It turns out that the total energy dumped ($\propto \int \chi(\alpha)d\alpha$) is approximately constant. Everything happens as if, when the formation of small loops is forbidden, the network simply chops off larger loops in smaller number. If for instance the loop emission is controlled by the kinks (*e.g.* a piece of string chops off when it contains enough kinks to have a chance of looping back), then the kinks might travel a little further and cause the emission of a somewhat larger loop. Incidentally, two kinks are left over on the long string network, while more than two are taken away by the formed loop, thereby limiting the kink density on the long strings (but apparently at a rather high value).

Even though we do see a spectrum of loop sizes at birth, it is interesting to see what is the characteristic energy of the loops we form as well as how numerous they are. (The size defined as the *rms* radius R of the loops is simply a fixed fraction of the loop energy E, *i.e.* $E = \beta\mu R$, with $\beta \simeq 20$.) Approximating $\chi(\alpha)$ by $\chi(\alpha) = \bar{\chi}\delta(\alpha - \bar{\alpha})$, then $\gamma = 2\bar{\chi}\bar{\alpha}^{3/2}$. Our present results suggest $\gamma < 3$ (which implies $\nu \simeq \gamma/\beta^{3/2} < 0.03$ in the notation of Turok and Brandenberger[12]). By taking $\gamma = 3$, and recalling that the net energy transferred times $(ct)^2/\mu$ is $\bar{\chi}\bar{\alpha} = \zeta(1- < v^2 >) \simeq 11$, we get $\bar{\alpha} = (\gamma/2\zeta(1- < v^2 >))^2 \simeq 0.02$ and $\bar{\chi} \simeq 600$, *i.e.* we generate many tiny loops in contrast to the "standard scenario". This is in qualitative agreement with our visual impression of Fig. 3 and the size gap in Fig. 8. Nevertheless the gap in Fig. 8 ends at $(E_c/\mu ct)^{-1/2} \simeq 4$ which corresponds to $\alpha \simeq 0.06$. This discrepancy simply denotes the oversimplification in approximating the spectrum by a δ function. One should also recall at this point that this cutoff study bears on simulations which have not yet reached the scaling regime and are thus still somewhat exploratory.

Even though the results concerning the loop sizes are still quantitatively quite uncertain, it is interesting to look at their correlation properties since it has been claimed[20] that the scenario naturally accounts for the correlation of clusters of galaxies and their scaling with richness[21,22]. The richness scaling is such that the correlation functions ξ are similar for different cluster classes when distances are measured in units of the mean separation between the objects considered. This inter-object distance increases for rarer richer clusters thus enhancing the correlations. While difficult to explain otherwise, this scaling is a natural byproduct of the existence of only one fundamental scale (the horizon), in the cosmic string scenario (apart from the horizon size at equal matter and radiation density). Indeed, the correlation function at scaling should be a constant when distances are expressed in units of ct, while the number of loops of a given size per horizon $(ct)^3 dn/dE$ should also be constant, which yields $\xi = f(r/(dn/dE)^{-1/3})$. What is not obvious on the other hand is the nature of $f(y)$, which should be $\sim 0.27y^{-1.8}$ to agree with the observations (note that this slope is not well constrained by the data, and was fixed to 1.8 to agree with the galaxy correlation function slope).

We have computed the correlation properties in the radiation era, and it turns out that the main conclusions does not seem to be affected by our cutoff λ. Our results are shown in figure 9 for the smallest cutoff simulation of Fig. 5 and 8. The open circles correspond to the correlation of loops at birth, *i.e.* their subsequent displacement was ignored. It follows quite closely the dash-dot line which corresponds to an y^{-2} fit to the cluster data and Turok's[20] results. If tiny loops are created along lines, dimensional analysis naturally yields the y^{-2} dependence. On the other hand, the match in amplitude came as a surprise, in accordance with Turok's[20] result. When we compute instead the correlations between all loops present at a given time in the simulation box, we obtain the curves labeled by squares (after 3.2 expansion factor) and filled circles (after 4.2 expansion factors), still in reasonable agreement with the observations. Nevertheless, when we extrapolated the loop positions to $a = 6.5$ by using their center of mass velocities, all correlations were pretty much washed out (filled triangles). This is because the loops have typically traveled further than their mean seperation. We also tried to lower the loop center of mass velocities by an arbitrary factor to test the dependence of this negative conclusion on the precise computed velocity values. The correlations still tended to be washed out even after we decrease the velocities by a factor as large as two. Furthermore, our loops are born tiny and do not fragment very much, thus it is unlikely that spurious fragmentations might result in anomalously large velocities.

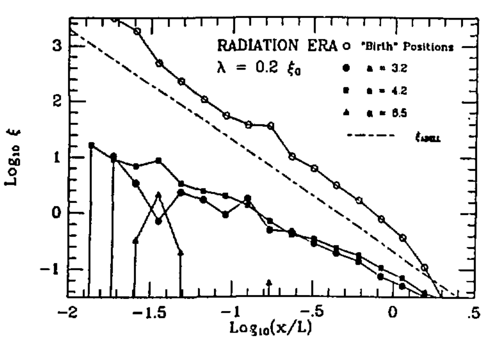


Figure 9

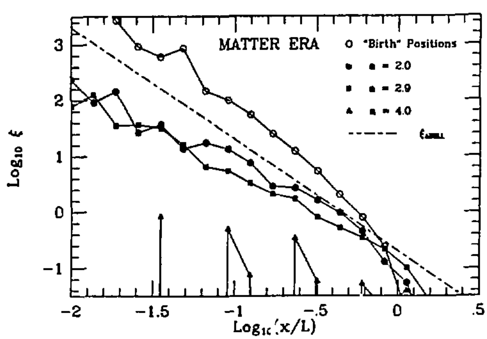


Figure 10

At this point, we might be tempted to conclude that the cosmic string model for galaxy formation does not do very well with the correlation functions for galaxies or Abell clusters. This would be premature, however, because most of the loops in our simulation are close enough to the lower cutoff in loop size so that they are artificially inhibited from fragmenting further. Thus, many of these loops may be unphysical. When an attempt is made to use only the loops that are large enough so that they are probably physical, the correlation function does not seem to wash out quite so quickly, but the statistics are much worse. We also performed the same kind of analysis in the matter era (Fig. 10), and the results were slightly better (but the same caveat still applies).

In conclusion, we have proven that a scaling solution exists both in the radiation and the matter era. The long string energy density is measured to be $\rho_{LS} = \zeta(ct)^2/\mu$, with $\zeta_{rad} = 20 \pm 10$, and $\zeta_{mat} = 3.5 \pm 1$, and about $11ct$ of string length is transferred onto loops per $(ct)^3$ and per expansion time in the radiation era, this number being reduced to about $0.45ct$ in the matter era. Our results outline a scenario very different from the standard one: many tiny loops are directly chopped off the network with a dispersion of sizes. Correlations at birth are consistent with the observed ones in Abell's clusters. Nevertheless many issues remain to be resolved, in particular the effect of further lowering our numerical cutoff. Unfortunately, this cannot be done with our present algorithm, since at the end of a large simulation, we have only a few discretizing points per kink, which prevents any further reasonably accurate calculations. We are presently devising an improved algorithm to evolve the kinks analytically and later correct the evolution of the smooth component of the strings, hoping to be then able to settle the remaining questions. One might nevertheless speculate that this improvement will rather strengthen our present conclusions, since kinks will become even more prevalent.

This work was supported in part by the Theoretical Astrophysics Center at the University of California at Berkeley, and under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contractNo. W-7405-Eng-48.

REFERENCES

1) Ostriker, J.P., Thompson, C., and Witten, E. 1986, *Phys. Lett.* , **116b**, 141.
2) Hogan, C.J., and Rees, M.J. 1986, *Nature*, **311**, 109.
3) Vilenkin, A. 1981, *Phys. Rev.* **D**, **23**, 852.
4) Kaiser, N., and Stebbins. A. 1984, *Nature*, **310**, 391.
5) Turok, N., and Bhattarcharjee, P. 1984, *Phys. Rev.* **D**, **29**, 1557.
6) Shellard, E.P.S. 1987, *Nucl. Phys.* **B**, **283**, 624.
7) Matzner 1988, *Texas U. preprint.*
8) Vashaspati, T., and Vilenkin, A. 1985, *Phys. Rev.* **D**, **31**, 3052.
9) Garfinkle, D., and Vashaspati, T. 1987, *Phys. Rev.* **D**, **36**, 2229.
10) Burden, C.J. 1985, *Glasgow U. Preprint 85-0745.*
11) Vashaspati, T., and Vilenkin, A. 1984, *Phys. Rev.* **D**, **30**, 2036.
12) Turok, N., and Brandenberger, R.H. 1986, *Phys. Rev.* **D**, **33**, 2175.
13) Kibble, T.W.B. 1985, *Nucl. Phys.*, **252**, 227.
14) Bennett, D.P. 1986a, *Phys. Rev.* **D**, **33**, 872.
15) Bennett, D.P. 1986b, *Phys. Rev.* **D**, **34**, 3592.
16) Albrech, A., and Turok, N. 1985, *Phys. Rev. Lett.*, **54**, 1868.
17) Bouchet, F.R., and Bennett, D.P. 1987, Proceedings of the IAU Symposium 130, *"Dark matter in the Universe"*, Balatonfured, Hungary.
18) Bennett, D.P., and Bouchet, F.R. 1987, Proceedings of the NATO-ASI meeting *"The post-recombination Universe"*, Cambridge, England.
19) Bennett, D.P., and Bouchet, F.R. 1988, *Phys. Rev. Lett.*, **60**, 257.
20) Turok, N. 1985, *Phys. Rev. Lett.*, **55**, 1801.
21) Bahcall, N.A., and Soneira, R.M. 1986, *Astrophys. J.*, **270**, 20.
22) Klypin, A.A., and Kopylov, A.I. 1986, *Sov. Astron. Lett.*, **9**, 41.