CONF-751130--32

1207

MARCH 1976 MATT-1207

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*Pages	NTIS Selling Price
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51-150	5.45
151-325	7.60
326-500	10.60
501-1000	13.60

Two Plasmon Parametric Decay in a Slightly Inhomogeneous Plasma

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ABSTRACT

The convective parametric decay of an incident electromagnetic wave (ω_o, k_o) into two plasmons at $\omega = \omega_p$ in a slightly inhomogeneous plasma of scale length L is considered. Asymptotic solutions for the fields are obtained which show that for $L/\lambda_o >> (\omega_p \lambda_D / \nu \lambda_o)^2$, where ν is the plasmon damping rate, the homogeneous-plasma decay criterion must be satisfied by the pump field for instability.

Presented at the 17th Annual Meeting of the Plasma Physics Division of the American Physical Society, St. Petersburg, Florida, 10-14 November 1975.

There has been much interest recently in parametric instabilities in inhomogeneous media, especially in relation to the possibility of absorbing an incident laser beam by parametric decay into two plasmons.[1-4] Rosenbluth[5] solved the general inhomogeneous problem and found that in order for the decay waves to grow to an appreciable level, $\gamma_o^2/V_1V_2\kappa$ must be much greater than unity, where V_1 and V_2 are respectively the group velocities of the two decay waves, $\kappa x \equiv \sum_i k_i$ is the sum of the wave vectors of the three waves, and γ_o is the coupling coefficient. In this paper we solve Rosenbluth's equations for the two plasmon decay case ($\omega_o \simeq 2\omega_p$) in the limit of $\gamma_o^2/V_1V_2\kappa >> 1$, and we include the damping terms. We only consider the convectively unstable case, since Rosenbluth showed that no absolutely unstable waves arise for finite κ. The resulting solutions shed light on the transition between the homogeneous and inhomogeneous regimes and predict a damping determined threshold condition when $L/\lambda_o >> 3\pi(\omega_p\lambda_D/\nu\lambda_o)^2$, where ν is the plasmon damping coefficient, and $L \equiv [(1/n)(dn/dx)]^{-1}$.

We arrive at Rosenbluth's form of the relevant equations by using the equations of Lee and Kaw[6] and expressing $\phi_+ = a_+(x)\cdot\exp\,(ik_{\|+}x + ik_\perp y - i\omega_+ t)$, where ϕ_+ is the decay wave potential at frequency $\omega = \omega_+ \approx \omega_p$ and $|\partial a_+/\partial x| << |k_{\|+}a_+|$. A similar form for the other wave ϕ_- holds. We obtain

$$\frac{\partial a_+}{\partial x} + \Gamma a_+ = \gamma_o a_- \exp\left(\frac{i\kappa x^2}{2}\right) \tag{1a}$$

$$\frac{\partial a_-}{\partial x} - \Gamma a_- = -\gamma_o a_+ \exp\left(\frac{-i\kappa x^2}{2}\right) \tag{1b}$$

where $\Gamma = \dfrac{\nu}{3k_{\|o}\lambda_D{}^2\omega_p}$,

$$\gamma_o = \frac{ek_o k_\perp E_o}{3\lambda_D{}^2 m_e \omega_o{}^2 k^2} ,$$

$$k^2 = k_\perp{}^2 + k_{\|o}{}^2 ,$$

$$E_o(\bar{x},t) = \frac{E_o}{2}\left(e^{ik_o x - i\omega_o t} + e^{-ik_o x + i\omega_o t}\right) ,$$

$$\sum_i k_i = k_{\|-} + k_o - k_{\|+} \approx - \frac{x k_o}{6\lambda_D{}^2 L k_{\|o}{}^2} \equiv \kappa x$$

$$\omega_p{}^2 = \omega_{po}{}^2 \left(1 + \frac{x}{L}\right) , \qquad k_{\|o} = k_{\|} \quad \text{at} \quad x=0 .$$

γ_o is the coupling coefficient and E_o is the incident wave field. We have required $\sum_i k_i = 0$ at $x = 0$. In order for this WKB approximation to be valid, the cutoff layer (where $k_{\|} = 0$) must be far away from the coupling region.

If we let $a_+(x) = b_+(x) \cdot \exp(i\kappa x^2/4)$ and $a_-(x) = b_-(x) \cdot \exp(-i\kappa x^2/4)$, we get

$$\frac{\partial^2 b_+}{\partial X^2} + b_+ \left(\frac{i\kappa}{2} + \gamma_o{}^2 + \frac{\kappa^2 X^2}{4}\right) = 0 \qquad (2)$$

where $X = x - 2i\Gamma/\kappa$. Letting $b_+(X) = g(X) \exp(i\kappa X^2/4)$, we get

$$\frac{\partial^2 g}{\partial X^2} + i\kappa X \frac{\partial g}{\partial X} + (i\kappa + \gamma_o{}^2) g = 0 \qquad (3)$$

By fourier transforming, we find the exact solution

$$g(X) = \int dk \exp\left(ikX + \frac{ik^2}{2\kappa} - \frac{i\gamma_o{}^2}{\kappa} \ln k\right) \qquad (4)$$

This integral in k space must be taken between two points at which the integrand is zero. There is also a branch point at the origin which must not be circled. Figure 1 shows how the path of integration may be picked.

For large $|X|$ we may evaluate $g(X)$ asymptotically. We have two saddle points, one at $k_1 = -\kappa X$ and another at $k_2 = \gamma_o^2/\kappa X$. The appropriate integration paths are shown in Fig. 2. Using these paths, we can approximately evaluate the integral for $|\kappa X^2| >> 1$, and we get the connection formulas for the region $x << 0$ to that of $x >> 0$:

$$\exp\left[-\Gamma x + \frac{i\Gamma^2}{\kappa} - i\left(\frac{\gamma_o^2}{\kappa}\right)\ln|\kappa X|\right] +$$
$$\left(\frac{\gamma_o^2}{|\kappa x|^2}\right)^{1/2} \exp\left(\frac{i\kappa X^2}{4} + \frac{i\kappa x^2}{4} + \frac{i\gamma_o^2}{\kappa} + \frac{i\gamma_o^4}{2\kappa^3 X^2} - \frac{i\gamma_o^2}{\kappa}\ln\left|\frac{\gamma_o^2}{\kappa X}\right| - \frac{\pi\gamma_o^2}{\kappa}\right)$$
$$\longleftrightarrow \exp\left[-\Gamma x + \frac{i\Gamma^2}{\kappa} - i\left(\frac{\gamma_o^2}{\kappa}\right)\ln|\kappa X| - \frac{\pi\gamma_o^2}{\kappa}\right] \tag{5}$$

where $\kappa < 0$. The first term (in the left side region) for $x << 0$ corresponds to a plasmon propagating toward the coupling region. The $x << 0$ solution shows that this wave is amplified by $\exp(\pi\gamma_o^2/|\kappa|)$ in crossing to $x >> 0$, in agreement with Rosenbluth's work.[5] The second term for $x << 0$ is the reaction of the other wave ϕ_- on ϕ_+ and decreases like $1/x$. For the region $x >> 0$ we have ignored this term, since it is smaller by $\exp(\pi\gamma_o^2/\kappa)$, where $|\gamma_o^2/\kappa| >> 1$. We again note that Eq. 5 is only valid when the region of large parametric coupling is far away from the cutoff layer. The cutoff layer is located at $x_c = 3\lambda_D^2 Lk_{\|o}^2$, and the parametric-coupling region is limited by $\kappa x^2 < \gamma_o^2/\kappa$ or equivalently by $x_p = (2\gamma_o/k_o)\, 3\lambda_D^2 Lk_{\|o}^2$. Thus, $x_c > x_p$ when $2\gamma_o < k_o$. This condition is equivalent to

$$\frac{ek_{\perp}E_o}{\lambda_D{}^2 m_e \omega_o{}^2 k^2} < 1 \quad . \tag{6}$$

Or, letting $k\lambda_D \sim 0.2 \equiv \varepsilon$,

$$\frac{1}{2\varepsilon} \frac{V_o}{v_{th}} < 1 \quad , \tag{7}$$

where $V_o = eE_o/m\omega_o$ is the electron velocity due to the pump, and $v_{th} = T_e/m_e$.

To find the threshold condition for net growth of the decay waves, we rewrite Eq. (2) as

$$\frac{\partial^2 b_+}{\partial \xi^2} + b_+ \left(-\frac{i}{2} + \gamma^2 + \frac{\xi^2}{4} - i\Gamma_1 \xi\right) = 0 \tag{8}$$

where

$$\gamma^2 = \gamma_1{}^2 - \Gamma_1{}^2$$

$$\Gamma_1 = \frac{\Gamma}{|\kappa|^{1/2}}$$

$$\gamma_1 = \frac{\gamma_o}{|\kappa|^{1/2}}$$

$$\xi = -|\kappa|^{1/2} x$$

We now require $\gamma^2 << \Gamma_1{}^2$ and $\gamma_1{}^2$ (i.e. $\Gamma_1 \approx \gamma_1$) , and neglect the $\xi^2/4$ and $i/2$ terms. This is valid for large ξ when Γ_1 , $\gamma_1 >> 1$ and when $\xi << 4\Gamma_1$. We then get as a solution

$$b_+(\xi) = \int dk \exp\left(ik\, y - ik^3/3\right) \tag{9}$$

where $y = i\Gamma_1^{1/3}\,\xi - \gamma^2/\Gamma_1^{2/3}$. In Fig. 3 we show the paths in the complex k plane used to evaluate $b_+(\xi)$. We now assume that $\gamma^2/\Gamma_1^{2/3} >> 1$, so that we may asymptotically evaluate $b_+(\xi)$ for $|y|$ large for all ξ . The integral in Eq. (9) has two saddle points at $k = \pm y^{1/2}$. For simplicity we choose path 2 in Fig. 3. Figure 4 shows how the saddle points move in the complex plane as ξ varies for both $\gamma^2 > 0$ and $\gamma^2 < 0$. If path 2 crosses a given saddle point, it picks up a contribution

$$b_+(\xi) \sim \exp\left[i\theta + \frac{2}{3}iy_o^{3/2} - \Gamma_1^{1/3}y_o^{1/2}(\xi-\xi_o)\right] \times \left(\frac{\pi}{|y_o^{1/2}|}\right)^{1/2} \quad (10)$$

where ξ is close to ξ_o , $y = y_o$ when $\xi = \xi_o$, and θ is the angle of the steepest descent path.

Figure 4a shows the behavior of the integral of path 2 for $\gamma^2 < 0$ as ξ changes. The angular position of the relevant saddle point rotates between $-\pi/4 < \theta < \pi/4$. Equation (10) then shows that this solution always decays and is not parametrically unstable. Figure 4b shows the behavior of this solution for $\gamma^2 > 0$. For $\xi > 0$, the solution picks up the saddle point at $\theta \sim \pi/4$ and the solution decays with growing ξ . As ξ crosses zero, the lower saddle point becomes exponentially small compared to the upper one. The upper saddle point then changes the behavior of $b_+(\xi)$ from damping to growth as ξ crosses zero. For $\xi < 0$, path 2 picks up the lower saddle point and $b_+(\xi)$ again damps with ξ . Thus, for $\gamma^2 > 0$ the decay waves travelling toward $x=0$ first grow, picking up energy from the pump, and then decay as they travel away from $x=0$, dumping their energy into the plasma. It is also apparent that for convective instability when $\gamma_1^2 >> 1$ (homogeneous limit), γ_o must be greater than Γ .

This threshold condition $\gamma_o > \Gamma$ is relevant for slightly inhomogeneous plasmas. Using Eq. (1), we see that

$$\frac{\gamma_o^2}{\kappa} \approx \frac{1}{6} \frac{V_o^2}{c^2} \frac{1}{k_o \lambda_D} \frac{L}{\lambda_D} \tag{11}$$

and that $\gamma_o = \Gamma$ requires

$$k_o V_o = 4\nu \tag{12}$$

Thus, when Eq. (12) is satisfied, we find that the condition $\gamma_o^2/\kappa >> 1$ is equivalent to

$$\frac{L}{\lambda_o} >> 3\pi \left(\frac{\omega_p \lambda_D}{\nu \lambda} \right)^2 \tag{13}$$

When Eq. (13) is satisfied, Eq. (12) automatically becomes the more stringent threshold condition which must be satisfied for instability. For example, a plasma with $n = 2.5 \times 10^{18}$ cm^{-3} , $T_e = 50$ eV , and L = 10 cm has $L/\lambda_o \sim 10^3 \ (\omega_p \lambda_D/\nu\lambda_o)^2$ for $\lambda_o = 10.6$ microns; Eq. (12) is then the relevant decay threshold.

Finally, we may estimate the spatial extent of the instability. To do this, we use Eq. (5) and consider only the case $\gamma_o >> \Gamma$ and $\gamma_o^2/\kappa >> 1$. We assume that the plasmon grows from the noise by a factor $\exp\ (\pi\gamma_o^2/|\kappa|)$ in a region near $x = 0$. The plasmon then continues to propagate until damping brings it back to the noise level at $x = x_w$. To find

x_w , we must match the growth factor, $\exp(\pi\gamma_o^2/|\kappa|)$, to the decay factor, $\exp(-\Gamma x)$. We obtain

$$\frac{\pi\gamma_o^2}{|\kappa|} \approx \Gamma x_w \qquad (14a)$$

or

$$x_w \approx \frac{\pi\gamma_o^2}{\Gamma|\kappa|} \qquad (14b)$$

where x_w is approximately the width of the region where the decay waves are strong. x_w thus increases as γ_o^2 increases and as Γ decreases.

ACKNOWLEDGMENT

This work was supported by U. S. Energy Research and Development Administration Contract E(11-1)-3073. J. J. Schuss was supported by a National Science Foundation Graduate Fellowship.

REFERENCES

[1] E. A. Jackson, Phys. Rev. 153, 235 (1967).

[2] P. K. Kaw and J. M. Dawson, Phys. Fluids 12, 2586 (1969).

[3] R. White, P. Kaw, D. Pesme, M. N. Rosenbluth, G. Laval, R. Huff, R. Varma, Nuclear Fusion 14, 45 (1974).

[4] E. Fabre and C. Stenz, Phys. Rev. Lett. 32, 823 (1974).

[5] M. N. Rosenbluth, Phys. Rev. Lett. 29, 565 (1972).

[6] Y. C. Lee and P. K. Kaw, Phys. Rev. Lett. 32, 135 (1974).

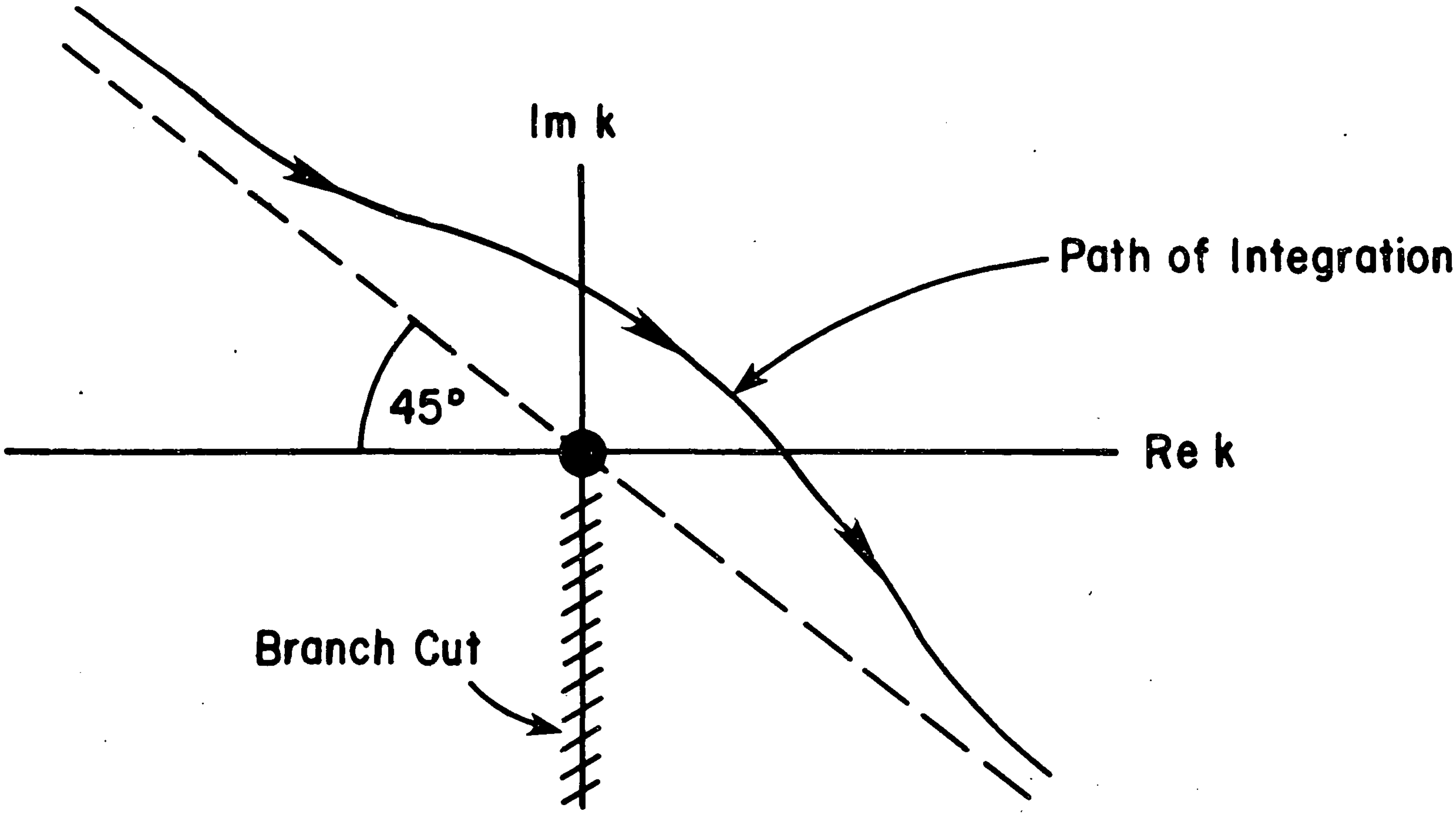

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Fig. 1. Path of integration along which Equation (4) is evaluated.

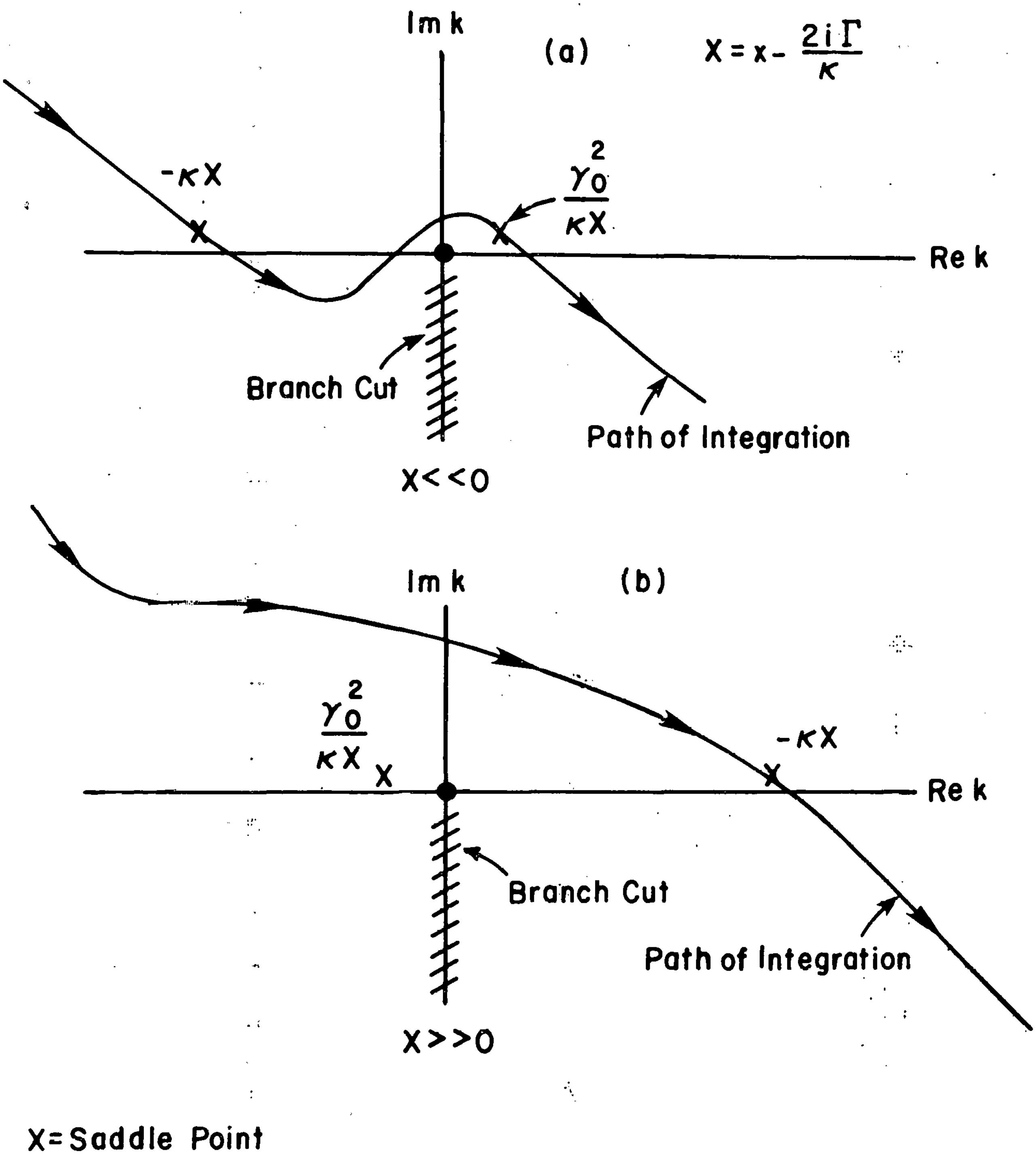

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Fig. 2. Paths of integration for finding the connection formula of Equation (5), for $x >> 0$ and $x << 0$.

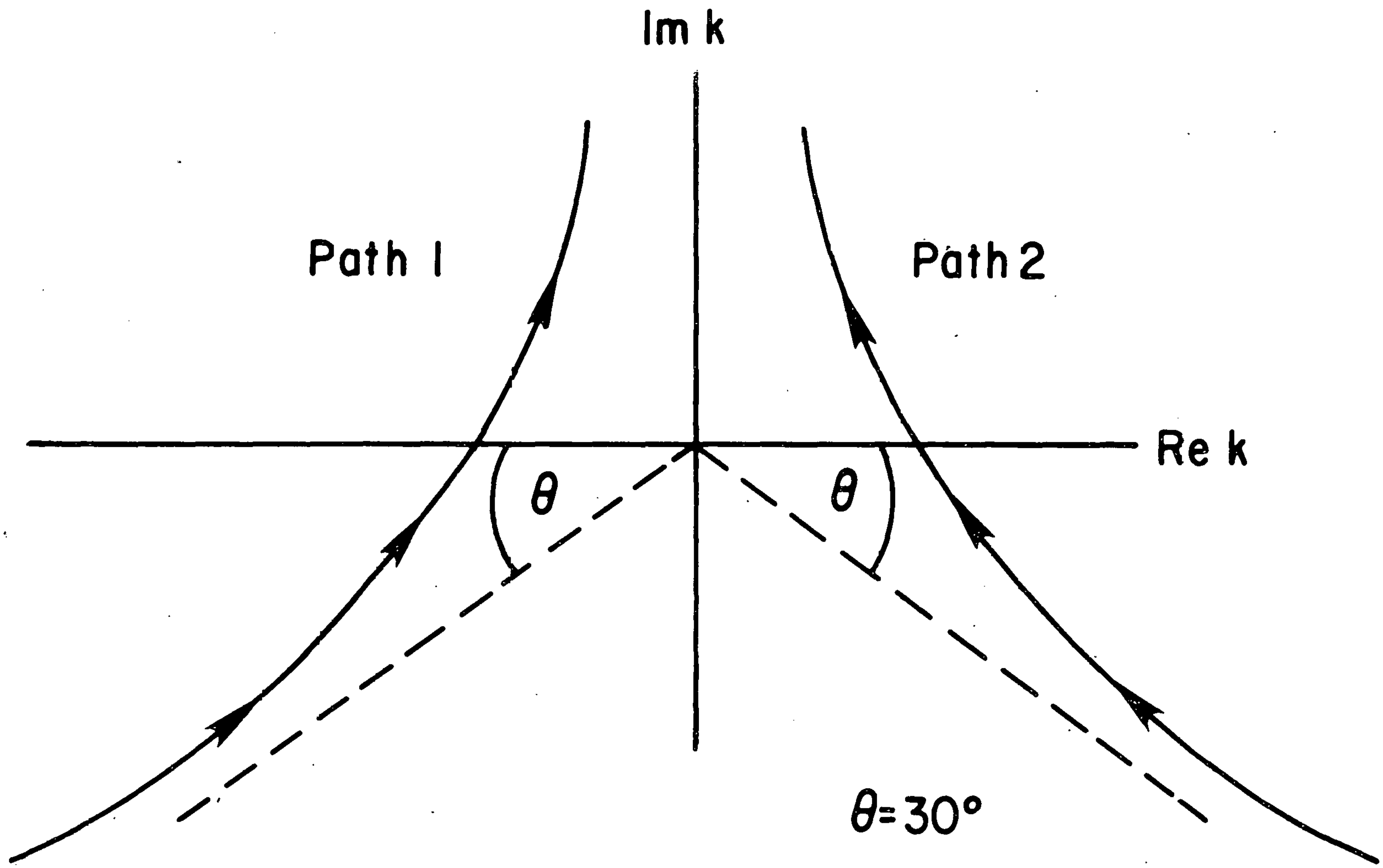

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Fig. 3. Paths of integration for solving Equation (9).

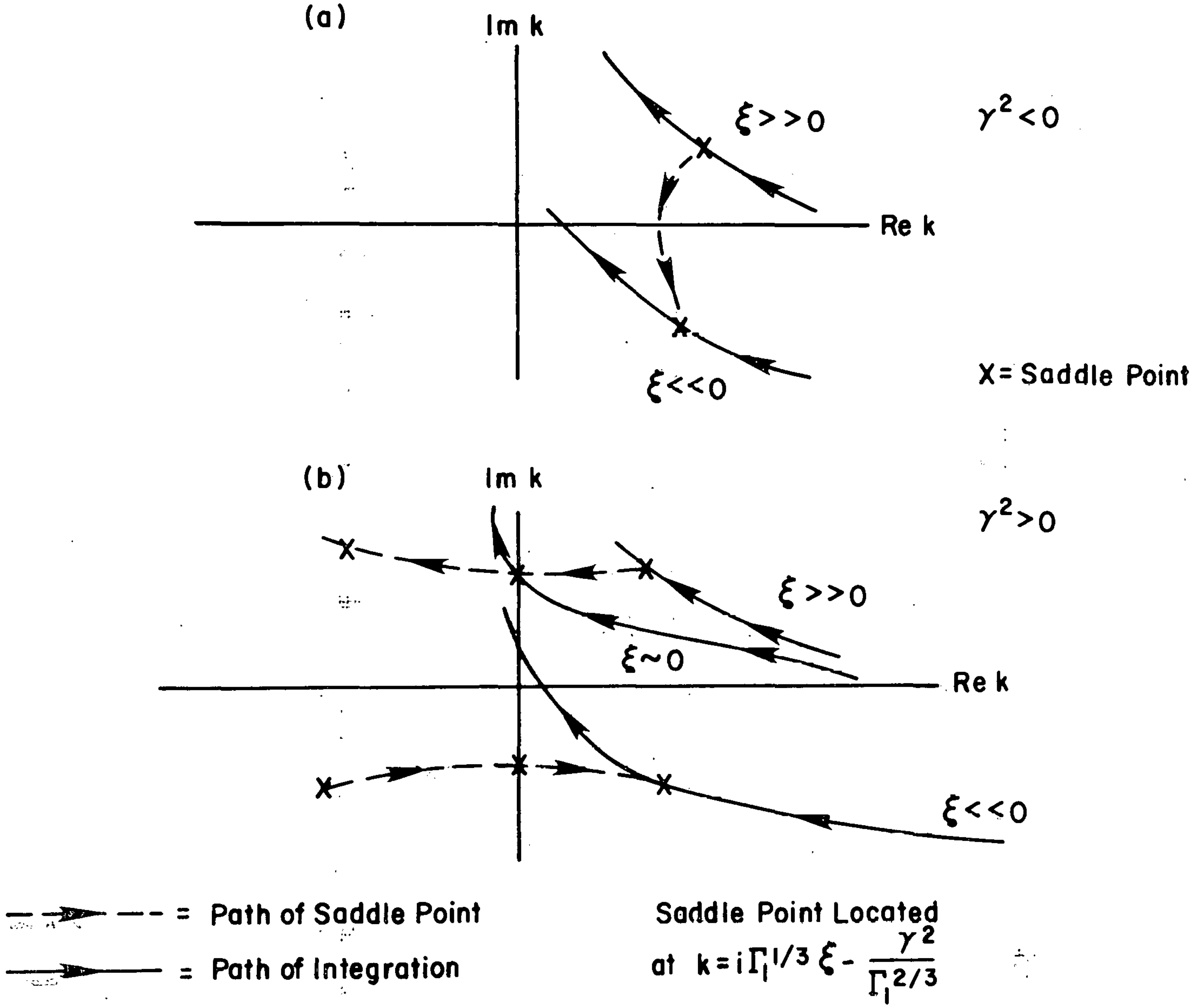

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Fig. 4. Asymptotic evaluation of Equation (9). The dotted line shows how the saddle points move as ξ goes from $+\infty$ to $-\infty$. In (a) the real part of $y^{1/2}$ at the saddle point is always >0 and the solution always damps with increasing ξ. In (b) the real part of $y^{1/2}$ at the upper saddle point changes from positive to negative as ξ crosses zero, indicating net growth and instability.