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TITLE: SUPERSYMMETRY AND MODEL BUILDING

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SUPERSYMMETRY AND MODEL BUILDING

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ABSTRACT

We present an introductory review of supersymmetry and supersymmetric model building. The topics discussed include, a brief introduction to the formalism of supersymmetry, the gauge hierarchy problem, the minimal supersymmetric standard model and supersymmetric grand unified theories.

1. INTRODUCTION TO SUPERSYMMETRY [SUSY]

The extended rigid Poincaré SUSY algebra includes the following generators

$$Q_\alpha{}^i,\ \bar{Q}_{\dot{\alpha} i}\ ,\ P_\mu\ ,\ M_{\mu\nu}\ ,\ G_a \tag{1.1}$$

where Q are the supersymmetry charges, P and M are Poincaré generators and G are generators of an internal symmetry group $\mathbb{G}$. The indices $\alpha, \dot{\alpha}=1,2$; $\mu,\nu=0,\ldots,3$; $a,b=1,\ldots,d_{adj}$ (where d_{adj} is the dimension of the adjoint representation of $\mathbb{G}$) and $i=1,\ldots,N$. Q and $\bar{Q}$ are anti-commuting Weyl spinors with $\bar{Q}_{\dot{\alpha} i} \equiv (Q_\alpha{}^i)^\dagger$. For a general review of SUSY, including more details of topics discussed in these lectures, see refs. [1-5] and the original papers referenced therein. (We use the spinor notation of Haber and Kane[4]).

The generators of (1.1) satisfy the SUSY algebra

$$[Q_\alpha{}^i, P_\mu] = 0$$

$$\{Q_\alpha{}^i,\ \bar{Q}_{\dot{\alpha} j}\} = 2\,\delta^i{}_j\, \sigma^\mu{}_{\alpha\dot{\alpha}} P_\mu \tag{1.2a}$$

$$\{ Q_\alpha{}^i , Q_\beta{}^j \} = 2\, \varepsilon_{\alpha\beta}\, Z^{ij}$$

$$[P_\mu , P_\nu] = 0$$

$$[G_a , G_b] = i\, f_{ab}{}^c\, G_c \qquad (1.2b)$$

$$[G_a , Q_\alpha{}^i] = (T_a)^i{}_j\, Q_\alpha{}^j$$

$$[M_{\mu\nu} , P_\rho] = i\,(\eta_{\rho\nu} P_\mu - \eta_{\mu\rho} P_\nu) \qquad (1.2c)$$

$$[M_{\mu\nu} , M_{\rho\sigma}] = i\,(\eta_{\rho\nu} M_{\mu\sigma} - \eta_{\mu\rho} M_{\nu\sigma} + \eta_{\sigma\nu} M_{\rho\mu} - \eta_{\mu\sigma} M_{\rho\nu})$$

$$[M_{\mu\nu} , Q_\alpha] = -i\,(\sigma_{\mu\nu})_\alpha{}^\beta\, Q_\beta$$

$$[M_{\mu\nu} , \bar{Q}_{\dot\alpha i}] = i\, \bar{Q}_{\dot\beta i}\,(\bar\sigma_{\mu\nu})^{\dot\beta}{}_{\dot\alpha}$$

[Our conventions are $\eta_{\mu\nu} = (1,-1,-1,-1)$; $P^\mu = (E,\vec{p})$; $\bar\sigma^{\mu\,\dot\alpha\beta} = (1,-\vec\sigma)$, $\sigma^\mu{}_{\alpha\dot\beta} = (1,\vec\sigma)$ where $\vec\sigma$ are the standard Pauli matrices given by $\sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\sigma^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$, $\sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. The Lorentz generators in this spinor representation are given by $(\sigma^{\mu\nu})_\alpha{}^\beta = \frac{1}{4}(\sigma^\mu \bar\sigma^\nu - \sigma^\nu \bar\sigma^\mu)$ and $(\bar\sigma^{\mu\nu})^{\dot\alpha}{}_{\dot\beta} = (\bar\sigma^\mu \sigma^\nu - \bar\sigma^\nu \sigma^\mu)$.]

The first equation of (1.2a) indicates that the SUSY charges are conserved and indeed this requires that the Lagrangian be invariant under SUSY transformations which take bosons into fermions and vice versa. In the third equation of (1.2a) the central charge Z only exists for N > 1. Equations (1.2b) define the internal symmetry algebra $\mathbb{G}$. The maximal symmetry group is $\mathbb{G} = U(N)$. For N = 1 this U(1) symmetry is commonly referred to as R invariance. Finally equations (1.2c) define the Lorentz properties of the generators.

Prior to SUSY it had been shown that the maximal symmetry group of the S-matrix containing the Poincaré group $\mathbb{P}$ and a compact Lie group $\mathbb{G}$ was the direct product group $\mathbb{P}\times\mathbb{G}$. This is the Coleman-Mandula theorem which considered the possibility of an algebra of commutation relations only. Subsequently Haag, Lopuszanski and Sohnius generalized the algebra to include anti-commutators as well and showed that the Poincaré superalgebra of (1.2) is the maximal symmetry algebra of the S-matrix in four dimensions, of course).

The algebra (1.2) already exhibits one of the remarkable features of SUSY, i.e. the vacuum energy in a theory of rigid (or global) SUSY is an order parameter for spontaneous SUSY breaking. Consider the second equation of (1.2a), we infer the following result

$$2E \equiv Tr(\sigma^{\mu} P_{\mu}) = \Sigma_{\alpha=1}^{2} \left| Q_{\alpha}{}^{i} \right|^{2} \geq 0 \tag{1.3}$$

Taking the vacuum expectation value of (1.3) we obtain

$$E_{vac} \equiv \langle E \rangle_0 = \frac{1}{2} \Sigma_{\alpha} \left| Q_{\alpha}{}^{i} \, |0\rangle \right|^{2} \geq 0 \tag{1.4}$$

Thus $E_{vac} = 0$ iff $Q_{\alpha}{}^{i} \, |0\rangle = 0 \;\; \forall \; i$. Note that for rigid SUSY if one (of N) supersymmetries are broken, they all are broken. For local SUSY, on the otherhand, it is possible to break an N extended SUSY to any N' < N.

Let us now find the massless irreducible representations of N SUSY. Consider a massless particle moving in the -z direction with four momentum $P_{\mu} = (E,0,0,E)$ satisfying $P^2 = 0$. From eq.(1.2) we find

$$\{ Q_{\alpha}{}^{I}, \overline{Q}_{\dot{\alpha} J} \} = 2\delta^{I}{}_{J} (1_{\alpha\dot{\alpha}} E + \sigma_{\alpha\dot{\alpha}} E) = 4\delta^{I}{}_{J} E \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \tag{1.5}$$

$$\{ Q_{\alpha}{}^{i}, Q_{\beta}{}^{J} \} = 0$$

where we've taken $Z^{IJ} = 0$. We then define raising and lowering operators

$$a^{I} \equiv -\sqrt{E} \; Q_{1}{}^{I} \quad , \quad a^{\dagger}{}_{I} = \frac{1}{2\sqrt{E}} \overline{Q}_{1I} \tag{1.6}$$

satisfying

$$\{ a^{I}, a^{\dagger}{}_{J} \} = \delta^{I}{}_{J} \quad , \quad \{ a^{I}, a^{J} \} = 0.$$

We take $Q_{2}{}^{I} \equiv 0$. The raising and lowering operators of (1.6) act on the single particle states with helicity λ, $|E,\lambda\rangle$. Let us also define the Pauli-Lubanski spin operator

$$W_{\mu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} P^{\nu} M^{\rho\sigma} \tag{1.7}$$

Then $W_0 = \vec{M} \cdot \vec{P}$ ($M_{ij} \equiv \varepsilon_{ijk} M^{k}$) is the helicity operator satisfying

$$W_0 |E, \lambda\rangle = \lambda E |E, \lambda\rangle \; . \tag{1.8}$$

It is now easy to check that the state $a^{\dagger}{}_{I} |E, \lambda\rangle$ has helicity $\lambda + {}^{1}/_{2}$.

Thus the raising and lowering operators raise and lower the helicity of the state by half a unit. Explicitly we consider

$$W^0\ \overline{Q}_{\dot{\alpha}i}|E,\lambda\rangle = ([W^0,\ \overline{Q}_{\dot{\alpha}i}] + \lambda E\ \overline{Q}_{\dot{\alpha}i})|E,\lambda\rangle$$

$$= (iE(\overline{\sigma}_{12})^{\dot{\beta}}{}_{\dot{\alpha}}\ \overline{Q}_{\dot{\beta}i} + \lambda E\ \overline{Q}_{\dot{\alpha}i})|E,\lambda\rangle$$

using (1.2c) (1.9)

$$= (\tfrac{1}{2}(\sigma^3)^{\dot{\beta}}{}_{\dot{\alpha}} + \lambda\,\delta^{\dot{\beta}}{}_{\dot{\alpha}})E\ \overline{Q}_{\dot{\beta}i}|E,\lambda\rangle$$

using the identity $i(\overline{\sigma}_{12})^{\dot{\beta}}{}_{\dot{\alpha}} \equiv (\sigma^3)^{\dot{\beta}}{}_{\dot{\alpha}}$.

We then find

$$W^0 a^\dagger_i|E,\lambda\rangle = E(\lambda + \tfrac{1}{2})\,a^\dagger_i|E,\lambda\rangle \tag{1 10}$$

or $|E,\lambda+\tfrac{1}{2},i\rangle \equiv a^\dagger_i|E,\lambda\rangle$.

We can now use the raising and lowering operators (1.6) to construct the massless irreducible representations of N SUSY. We just start with any state $|E,\lambda\rangle$ and act with the raising operators until we reach the highest helicity state which is annihilated by all the raising operators. We generate in this way all states in one irreducible representation of N SUSY. In table 1 we give a few examples.

Table 1

	helicity	degeneracy	helicity	degeneracy	
N = 4	1	1	2	1	
	$^1/_2$	4	$^3/_2$	4	
	0	6	1	6	+ CPT
	$-^1/_2$	4	$^1/_2$	4	conj.
	-1	1	0	1	
N = 2					
	$^1/_2$	1	1	1	
	0	2	$^1/_2$	2	+ CPT
	$-^1/_2$	1	0	1	conj.
	2	1			
	$^3/_2$	2	+ CPT conjugates		
	1	1			

N = 1

$^1/_2$	1	+ CPT	1	1	+ CPT
0	1	conj.	$^1/_2$	1	conj

2	1	+ CPT conjugates
$^3/_2$	1	

The CPT conjugate states are necessary in order to construct Lorentz covariant quantum fields. In the N = 1 case, the multiplet whose highest weight is $^1/_2$ is called a chiral multiplet, the vector multiplet has highest weight 1 and the gravity multiplet has highest weight 2.

Each of the above multiplets can also transform as an irreducible representation of some internal symmetry group $\mathbb{G}$. The gravity multiplets will be taken to be singlets in order to have only one graviton. The vector multiplets will be taken to be in the adjoint representation of $\mathbb{G}$ and the chiral multiplets can be in an arbitrary representation. For N ≥ 2 all multiplets are necessarily real representations of $\mathbb{G}$; where a reducible representation r is real if it contains an equal number of left-handed and right-handed states. Since the observable low energy states of quarks and leptons are <u>not</u> in real representations of the gauge group SU(3)×SU(2)×U(1) we shall henceforth only consider N = 1 SUSY.*

Massive irreducible representations of N = 1 SUSY may be obtained by combining certain massless representations. For example, a massive gauge multiplet includes the states in a massless chiral and vector multiplet; a massive Dirac multiplet includes two massless chiral multiplets and a massive Weyl or Majorana multiplet includes the states in one chiral multiplet.

* Phenomenological low energy models with N = 2 SUSY have been constructed[6]. They necessarily contain so-called mirror fermions. It seems difficult to obtain a realistic spectrum of masses in these theories

As a precursor to constructing supersymmetric Lagrangians, we feel it might be instructive to construct the Lagrangian for QED using two component Weyl spinor notation. We shall choose, by convention, to work with left-handed Weyl spinor fields only. If ψ is a left-handed Weyl spinor field, it annihilates a left-handed particle and creates a right-handed (anti-)particle. The word (anti-) is in parentheses since in order to be able to distinguish particle and anti-particle one must have a conserved particle number. For Majorana fields this distinction doesn't exist. $\psi^\dagger$ creates a left-handed particle and annihilates a right-handed (anti-)particle.

The free equations of motion for ψ are

$$(\overline{\sigma}_\mu P^\mu)^{\dot{\alpha}\beta}\psi_\beta = 0 \Rightarrow (E + \vec{\sigma}\cdot\vec{p})\psi = 0 \qquad (1.11)$$

or $(\vec{\sigma}\cdot\vec{p}/E)\psi = -\psi$.

If ψ is left-handed, then $i\sigma^2\psi^*$ is right-handed. To see this one just takes the complex conjugate of equation (1.11) $(E + \vec{\sigma}^*\cdot\vec{p})\psi^* = 0 \Rightarrow (E - \vec{\sigma}\cdot\vec{p})\,i\sigma^2\psi^* = 0$ or $(\vec{\sigma}\cdot\vec{p}/E)\,i\sigma^2\psi^* = i\sigma^2\psi^*$.

QED

With these preliminaries aside, let us now consider QED. The electron is represented by two Weyl spinors. e_α annihilates a left-handed electron and creates a right-handed positron; $\overline{e}_\alpha$ annihilates a left-handed positron and creates a right-handed electron. The photon field is given by A_μ. The QED Lagrangian is then given by

$$\mathfrak{L}_{QED} = e^\dagger\overline{\sigma}\cdot(i\partial - eA)e + \overline{e}^\dagger\overline{\sigma}\cdot(i\partial + eA)\overline{e} + [m_e(\overline{e}\,e) + \text{h.c.}]$$
$$-\tfrac{1}{4}\;F_{\mu\upsilon} \qquad (1.12)$$

where $F_{\mu\upsilon} = \partial_\mu A_\upsilon - \partial_\upsilon A_\mu$ and the Lorentz scalar $(\overline{e}\,e) \equiv -\overline{e}^T(i\sigma^2)e$. Sometimes it is convenient to define the two component matrix $\varepsilon^{\alpha\beta} \equiv (i\sigma^2)^{\alpha\beta}$ which can be used to raise and lower spinor indices, i.e. $\psi^\alpha \equiv \varepsilon^{\alpha\beta}\psi_\beta$. In this notation $(\overline{e}\,e) = \overline{e}^\alpha e_\alpha$.

We can define the following global transformations acting on the fields e and $\overline{e}$.

$$e \rightarrow \exp(-i\alpha)\, e \quad ; \quad \bar{e} \rightarrow \exp(+i\alpha)\, \bar{e} \qquad (1.13a)$$

$$e \rightarrow \exp(+i\beta)\, e \quad ; \quad \bar{e} \rightarrow \exp(+i\beta)\, \bar{e} \qquad (1.13b)$$

(1.13a) is a symmetry of $\mathfrak{L}_{QED}$ generated by the conserved charge $N_e \equiv -Q$ where N_e counts electron number and Q is electric charge. (1.13b) is commonly called a chiral transformation. This symmetry is explicitly broken by the electron mass term $m_e(\bar{e}\, e)$. Note, a term in the Lagrangian of the form $(e\, e)$ is what is called a Majorana mass term. It breaks the symmetry of (1.13a). Since this symmetry is also a local gauge symmetry, such a term would spoil the renormalizability of the theory.

It is often convenient to convert Weyl notation into Dirac notation. We thus include the following dictionary. We define a four component Dirac spinor for the electron Ψ_e . Then given

$$\Psi_e \equiv \begin{matrix} e \\ i\,\sigma^2\, \bar{e}^* \end{matrix} \qquad \gamma^\mu \equiv \begin{matrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{matrix} \qquad (1.14)$$

and

$$\gamma^5 \equiv i\,\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

the equations of motion (1.11) are

$$(\gamma^\mu P_\mu - m_e)\ \Psi_e = 0. \qquad (1.15)$$

We also have

$$\mathfrak{L}_{QED} = \bar{\Psi}_e \gamma_\mu (i\,\partial^\mu - e\,A^\mu)\,\Psi_e + m_e\ \bar{\Psi}_e\Psi_e - \tfrac{1}{4} F^2{}_{\mu\upsilon}$$

where $\bar{\Psi}_e \equiv \Psi_e^{\dagger}\gamma^0$ and m_e is real. (1.16).

PROBLEM

Define the CP transformation U_{CP} such that

$$e'(x) \equiv U_{CP}\, e(x)\, U_{CP} = \sigma^2\, e(x') \qquad (1.17)$$

$$A'_\mu(x) \equiv U_{CP}\, A_\mu(x)\, U_{CP} = (-A_0(x'),\, A_i(x'))$$

where $x^\mu = (x^0, \vec{x})$ and $x'^\mu = (x^0, -\vec{x})$.

1) Show that $U_{CP}\,\mathfrak{L}_{QED}\,U_{CP} = \mathfrak{L}_{QED}$ iff m_e is real.

2) Show that if $m_e = |m_e|e^{i\epsilon}$, one can always remove the phase by the field redefinition $e \rightarrow e^{-i\epsilon} e$. Therefore no physical CP violating phase exists in QED.

SUSY QED

We define the left-handed chiral superfields

$$E(y,\Theta) \equiv \tilde{e}(y) + \sqrt{2}\,(\Theta\, e(y)) + (\Theta\Theta)\,F(y) \qquad (1.18)$$

$$\overline{E}(y,\Theta) \equiv \tilde{\bar{e}}(y) + \sqrt{2}\,(\Theta\, \bar{e}(y)) + (\Theta\Theta)\,\overline{F}(y)$$

where $y^\mu = x^\mu + i\,\Theta\sigma^\mu\overline{\Theta}$ and Θ_α, α=1,2 is a complex Grassman left-handed Weyl spinor. [$(\Theta\Theta) \equiv -i\,\Theta^T\sigma^2\Theta$]. They include the electron e and positron $\bar{e}$ fields defined previously. They also include $\tilde{e}$ and $\tilde{\bar{e}}$, a pair of complex scalar fields for the scalar electron (selectron) and scalar positron (supositron) ,respectively. F and $\overline{F}$ are complex auxiliary fields. E and $\overline{E}$ are commuting Lorentz scalar superfields.

We also define the real gauge prepotential

$$V(x,\Theta,\overline{\Theta}) = C(x) + i\,(\Theta\chi(x)) + 1/_2\,(\Theta\Theta)\,[M(x) + iN(x)] - \Theta\sigma^\mu\overline{\Theta}\,A_\mu(x)$$
$$+\, i\,(\Theta\Theta)\,(\overline{\Theta}[\overline{\lambda}(x) + 1/_2\,\overline{\sigma}^\mu\,\partial_\mu\chi(x)]) + \tfrac{1}{2}(\Theta\Theta)(\overline{\Theta}\overline{\Theta})\,[D(x) + \tfrac{1}{2}\Box C(x)] + \text{h.c.} \qquad (1.19)$$

where $\overline{\Theta}_{\dot\alpha} \equiv \Theta_\alpha{}^*$ and $(\overline{\Theta}\overline{\Theta}) \equiv \overline{\Theta}_{\dot\alpha}\overline{\Theta}^{\dot\alpha}$.

A_μ is the photon field, λ_α is the photino and D is a real scalar auxiliary field. V is a real [$V = V^\dagger$] commuting Lorentz scalar superfield. The other fields C, χ, M, and N are gauge artifacts, as we will soon see.

SUSY TRANSFORMATION

The infinitesimal supersymmetry transformation law [by a constant Grassman parameter ε_α] for a general superfield $\Phi(x,\Theta,\overline{\Theta})$ is given by

$$\delta_\varepsilon\Phi = \left[\,(\,(\varepsilon Q) + (\overline{\varepsilon}\,\overline{Q})\,),\,\Phi\,\right] \qquad (1.20)$$

with the supersymmetry generators

$$Q_\alpha = [\partial_\theta - i\sigma^\mu\bar\theta\partial_\mu]_\alpha \quad , \quad \bar Q_{\dot\alpha} = [\partial_{\bar\theta} - i\theta\sigma^\mu\partial_\mu]_{\dot\alpha}$$

satisfying $\{Q_\alpha, \bar Q_{\dot\alpha}\} = 2i\sigma^\mu{}_{\alpha\dot\alpha}\partial_\mu$. A finite SUSY transformation is given by

$$\Phi(x,\theta,\bar\theta) = \Phi(x',\theta',\bar\theta') \qquad (1.21)$$

with $x' = x - i(\varepsilon\sigma^\mu\bar\theta - \theta\sigma^\mu\bar\varepsilon)$, $\theta' = \theta+\varepsilon$ and $\bar\theta' = \bar\theta + \bar\varepsilon'$.

CHIRAL MULTIPLET

The infinitesimal SUSY transformation for a general chiral superfield $\Phi(x,\theta) = \varphi(x) + \sqrt{2}(\theta\psi(x)) + (\theta\theta)F(x)$ [chiral superfields satisfy the constraint $\bar D\Phi = 0$ where $\bar D \equiv \partial_{\bar\theta} + i\theta\sigma^\mu\partial_\mu$] is given by

$$\delta_\varepsilon\varphi(x) = \sqrt{2}(\varepsilon\psi(x))$$

$$\delta_\varepsilon\psi(x) = i\sqrt{2}\sigma^\mu\bar\varepsilon\partial_\mu\varphi(x) + \sqrt{2}\varepsilon F(x) \qquad (1.22)$$

$$\delta_\varepsilon F(x) = i\sqrt{2}\bar\varepsilon\bar\sigma^\mu\partial_\mu\psi(x) \quad .$$

The last equation demonstrates that the third component of a chiral superfield, the so-called F-term, transforms as a total derivative. Therefore the F-term can be used to construct Lagrangian densities. The resulting action is then SUSY invariant. Moreover, we note that the product of any two chiral superfields is again a chiral superfield.

Taking the vacuum expectation value of the second equation, we find

$$\langle\delta_\varepsilon\psi(x)\rangle = \sqrt{2}\varepsilon\langle F(x)\rangle. \qquad (1.23)$$

Thus a non-vanishing expectation value for F indicates spontaneous SUSY breaking. Moreover , if $\langle F\rangle \neq 0$, then ψ is a massless Goldstone fermion (in the case of rigid SUSY) or Goldstino. The mechanism whereby SUSY is spontaneously broken via $\langle F\rangle \neq 0$ is called the O'Raifeartaigh mechanism.

GAUGE MULTIPLET

The infinitesimal SUSY transformation for the vector (or gauge) multiplet is given by

$$\delta_\epsilon F_{\mu\nu}(x) = i\,[\,\varepsilon\,\sigma_\nu\,\partial_\mu\,\bar\lambda(x) + \bar\varepsilon\,\bar\sigma_\nu\,\partial_\mu\,\lambda(x) - (\mu \longleftrightarrow \nu)\,]$$

$$\delta_\epsilon \lambda(x) = i\,\varepsilon\,D(x) + \sigma^{\mu\nu}\,\varepsilon\,F_{\mu\nu}(x) \qquad (1.24)$$

$$\delta_\epsilon D(x) = \bar\varepsilon\,\bar\sigma^\mu\,\partial_\mu\,\lambda(x) - \varepsilon\,\sigma^\mu\,\partial_\mu\,\bar\lambda(x) \quad .$$

The last equation demonstrates that the so-called D-term (or last component of a vector superfield) transforms as a total derivative and thus can also be used to construct an invariant action. Again, the product of two real vector superfields is a real vector superfield. Moreover, the product of a chiral superfield and it's hermitian conjugate is also a real vector superfield.

The vacuum expectation value of the second equation gives

$$\langle\, \delta_\epsilon\,\lambda(x)\rangle = i\,\varepsilon\langle D(x)\rangle \quad . \qquad (1.25)$$

Thus $\langle D\rangle$ is also an order parameter for the spontaneous breaking of SUSY. If $\langle D\rangle \neq 0$ then SUSY is spontaneously broken and λ is a Goldstino (in the case of rigid SUSY). The mechanism of breaking SUSY via a non-vanishing D-term is called the Fayet-Iliopoulos mechanism.

GAUGE INVARIANCE

The SUSY generalization of the local U(1) gauge transformation is given by

$$E'(x,\Theta) = e^{2ie\,\Lambda(x,\Theta)}\,E\,(x,\Theta)$$

$$\bar E'(x,\Theta) = e^{-2ie\,\Lambda(x,\Theta)}\,\bar F\,(x,\Theta) \qquad (1.26)$$

$$e^{2e\,V'(x,\Theta,\bar\Theta)} = e^{-2ie\,\Lambda\,(x,\Theta)}\;e^{2e\,V(x,\Theta,\bar\Theta)}\;e^{2ie\,\Lambda(x,\Theta)}$$

$$E^{\dagger\prime}(x,\Theta) = E^\dagger(x,\Theta)\,e^{-2ie\,\Lambda(x,\Theta)}$$

etc. and Λ satisfies $\bar D_{\dot\alpha}\,\Lambda = 0$. In equation (1.24) we considered only those fields in V which transform covariantly under the local U(1) gauge transformation. All the other fields (C,χ,M,N) are gauge artifacts, meaning that by a suitable choice of gauge they can be transformed to zero. This is the so-called Wess-Zumino [WZ] gauge. The WZ gauge is not manifestly supersymmetric i.e. a general SUSY transformation must be compensated by a gauge transformation in order to return to the WZ gauge. The WZ gauge has the nice feature that $V^n = 0$ for $n \geq 3$.

The gauge and SUSY covariant field strength is given by

$$W_\alpha = -\frac{1}{4}\overline{D}\overline{D}(\; e^{-2eV} D_\alpha \; e^{2eV}\;) \;. \qquad (1.27)$$

Under a local U(1) gauge transformation it transforms as follows

$$W'_\alpha = -\frac{1}{4}\overline{D}\overline{D}(\; e^{-2ie\Lambda}\, e^{-2eV} e^{2ie\Lambda} D_\alpha \; e^{-2ie\Lambda}\, e^{2eV} e^{2ie\Lambda}\;)$$

$$= -\frac{1}{4}\overline{D}\overline{D}(\; e^{-2ie\Lambda}\, e^{-2eV} D_\alpha\, e^{2eV} e^{2ie\Lambda}\;)$$

$$= e^{-2ie\Lambda} W_\alpha\, e^{2ie\Lambda} \; -\frac{1}{4}\, e^{-2ie\Lambda}\; \overline{D}\overline{D} D_\alpha\, e^{2ie\Lambda} \;.$$

(The last term vanishes using $\overline{D}\overline{D} D_\alpha e^{2ie\Lambda} = \overline{D}\{\overline{D}, D_\alpha\} e^{2ie\Lambda}$ $\approx \overline{D}\partial e^{2ie\Lambda} = 0$ and $\overline{D}\Lambda = 0$.) Thus we have

$$W'_\alpha = e^{-2ie\Lambda} W_\alpha\, e^{2ie\Lambda} \;. \qquad (1.28)$$

In the WZ gauge, we find

$$W(x,\Theta) = -i\,\lambda(x) + [\,D(x) - i\,\sigma^{\mu\upsilon} F_{\mu\upsilon}(x)\,]\,\Theta + (\Theta\Theta)\,\sigma^\mu \partial_\mu \overline{\lambda}(x)$$

where $F_{\mu\upsilon} = \partial_\mu A_\upsilon - \partial_\upsilon A_\mu$. (1.29)

It is now possible to write down the invariant Lagrangian for SUSY QED . We have

$$\begin{aligned} \mathfrak{L}_{SQED} = & \int d^4\Theta\, (\, E(y,\Theta)\, e^{-2e\,V(x,\Theta,\overline{\Theta})}\, E^\dagger(y,\Theta) \\ & \qquad + \overline{E}^\dagger(y,\Theta)\, e^{2e\,V(x,\Theta,\overline{\Theta})}\, \overline{E}(y,\Theta)\,) \qquad (1.30) \\ & + \;(\int d^2\Theta\, m_e\, \overline{E}(x,\Theta)\, E(x,\Theta) + \text{h.c.}\,) \\ & + \;(\int d^2\Theta\, \frac{1}{4e^2}\, W^\alpha W_\alpha + \text{h.c.}\,)\;. \end{aligned}$$

Note that the terms in equation (1.30) are in one to one correspondence with similar terms in (1.12). As we shall soon show the terms in (1.12) are included in (1.30) in addition to novel supersymmetric contributions. Also note that the first and last terms in (1.30) are in fact D-terms and the middle term is an F-term. [$\int d^4\Theta = \int d^2\Theta\, d^2\overline{\Theta}$].

In order to do perturbative calculations with $\mathfrak{L}_{SQED}$ one must choose a gauge. The SUSY R_ξ gauges are manifestly supersymmetric and thus SUSY Ward identities will be satisfied in these gauges[7]. One may also use the WZ gauge with the standard non-SUSY R_ξ gauges but these calculations will not be manifestly supersymmetric[8]. Nevertheless any gauge invariant quantity will be supersymmetric.

GENERAL GAUGE INVARIANT N=1 RIGID SUSY LAGRANGIAN

Let us now write down the most general renormalizable SUSY Lagrangian with non-abelian gauge symmetry group G.

$$\mathfrak{L} = \left(\int d^2\theta \, \frac{1}{16 k g^2} \mathrm{Tr}\, W^{\alpha} W_{\alpha} + \text{h.c.} \right) \tag{1.31a}$$

$$+ \int d^4\theta \, \Phi_i^{\dagger} e^{2g V(i)} \Phi_i \tag{1.31b}$$

$$+ \left(\int d^2\theta \, W(\Phi_i) + \text{h.c.} \right) \tag{1.31c}$$

where $V(i) = V_a(x,\theta,\bar{\theta})\, T^a(i)$; $T^a(i)$ are the hermitian generators of G in the representation of Φ_i , g is the gauge coupling constant and the constant k is fixed by the relation $\mathrm{Tr}(T^a(f)\, T^b(f)) = k\, \delta^{ab}$ with f denoting the fundamental representation. Unless otherwise stated, we will take $k=1$ for $G=U(1)$ and $k=\frac{1}{2}$ otherwise. The quantity $W(\Phi_i)$ is a gauge invariant holomorphic function of Φ_i and is commonly called the superspace potential.

The Lagrangian $\mathfrak{L}$ can now be expanded in terms of the component fields. This corresponds to doing the integrations over the Grassman coordinates θ and $\bar{\theta}$. We find (using the convention $\int d^2\theta\,(\theta\theta) = 1$) in the WZ gauge

$$\mathfrak{L} = -\frac{1}{4}(F^a{}_{\mu\nu})^2 + i\,\bar{\lambda}^a \bar{\sigma}^{\mu} \mathfrak{D}_{\mu} \lambda^a + \frac{1}{2}(D^a)^2 \tag{1.32a}$$

$$+ |\mathfrak{D}_{\mu} \varphi_i|^2 + i\,\bar{\psi}_i \bar{\sigma}^{\mu} \mathfrak{D}_{\mu} \psi_i + |F_i|^2 \tag{1.32b}$$

$$+ i\sqrt{2}\, g\, (\varphi_i^{\dagger} T^a(i)(\psi_i \lambda^a) - (\bar{\lambda}^a \bar{\psi}_i) T^a(i) \varphi_i)$$

$$+ g D^a \varphi_i^{\dagger} T^a(i) \varphi_i$$

$$+ F_i \left[\partial W / \partial \Phi_i \right]\Big|_{\theta=0} - \psi_i \psi_j \left[\partial^2 W / \partial \Phi_i \partial \Phi_j \right]\Big|_{\theta=0} \tag{1.32c}$$

$$+ \text{h.c.} \quad .$$

where the terms in (1.32a–c) are derived from those in (1.31a–c), respectively. $\mathfrak{D}_{\mu} = i\,\partial_{\mu} + g A^a{}_{\mu} T^a(i)$ and it is now clear why F_i and D^a are called auxiliary fields, since they have no kinetic terms in their equations of motion. One may use the equations of motion for F_i and D^a to eliminate these fields from the Lagrangian. The equations of motion are

$$\left[\delta\mathfrak{L}\backslash\delta F_i^{\dagger}\right] = F_i + \left[\partial W/\partial\Phi_i\right]|^{\dagger}_{\theta=0} = 0 \qquad (1.33)$$
$$\left[\delta\mathfrak{L}\backslash\delta D^a\right] = D^a + g D^a \varphi_i^{\dagger} T^a(i)\varphi_i = 0 \quad .$$

Eliminating F and D using (1.33) we obtain the scalar potential

$$V(\varphi_i) = \frac{1}{2}(D^a)^2 + |F_i|^2 \geq 0 \quad . \qquad (1.34)$$

Note the ground state is supersymmetric iff the vacuum energy vanishes, and $V=0$ iff $F_i = D^a = 0$, as discussed previously in eqns. (1.4, 1.23, 1.25).

NON-RENORMALIZATION THEOREM AND ITS CONSEQUENCES

Supersymmetric theories have been around for about 15 years now. It was realized quite early that many miraculous cancellations ocurred in perturbative calculations. We want to recall these properties of SUSY perturbation theory and discuss their consequences. We shall discuss the non-renormalization theorem in the context of a simple model ,i.e. SUSY QED. Recall the Lagrangian $\mathfrak{L}_{SQED}$ discussed previously in equation (1.30). In addition we shall add the following new terms

$$\begin{aligned}\overline{\mathfrak{L}}_{SQED} = \mathfrak{L}_{SQED} &+ (\lambda \int d^2\theta\, \Phi \overline{E} E + {}^{m}/_{2} \int d^2\theta\, \Phi^2 + \text{h.c.}) \\ &+ e\xi \int d^4\theta\, V \qquad (1.35)\end{aligned}$$

In components we have (in the WZ gauge)

$$\begin{aligned}\overline{\mathfrak{L}}_{SQED} = &-\frac{1}{4} F^2_{\mu\upsilon} + i\,\tilde{\gamma}^{\dagger}\overline{\sigma}^{\mu}\partial_{\mu}\tilde{\gamma} + \frac{1}{2} D^2 \\ &+ |\mathfrak{D}_{\mu}\tilde{e}|^2 + i\, e^{\dagger}\overline{\sigma}^{\mu}\mathfrak{D}_{\mu} e + |F_e|^2 \\ &+ |\mathfrak{D}_{\mu}\overline{e}|^2 + i\,\overline{e}^{\dagger}\overline{\sigma}^{\mu}\mathfrak{D}_{\mu}\overline{e} + |F_{\overline{e}}|^2 \qquad (1.36) \\ &+ i\sqrt{2}\, e\,(-\tilde{e}^{\dagger}(e\,\tilde{\gamma}) + \overline{e}^{\dagger}(\overline{e}\,\tilde{\gamma}) - \text{h.c.}) \\ &+ e D(-|\tilde{e}|^2 + |\overline{e}|^2 + \xi) \\ &+ [F_e(m_e\overline{e} + \lambda\varphi\overline{e}) + F_e(m_e\tilde{e} + \lambda\varphi\tilde{e}) \\ &+ F_{\varphi}(\lambda\overline{e}\tilde{e} + m\varphi) - m_e(\overline{e}\, e) - {}^{m}/_{2}\psi\psi \\ &- \lambda(\varphi(\overline{e}e) + (\psi\overline{e})\tilde{e} + (\psi e)\overline{e}) + \text{h.c.}]\end{aligned}$$

λ, m and ξ are a Yukawa coupling, mass and Fayet Iliopoulos parameter, respectively. The components of the new neutral chiral superfield Φ are (φ, ψ, F_{φ}).

The Feynman diagrams for this theory are given in figure 1.

Figure 1

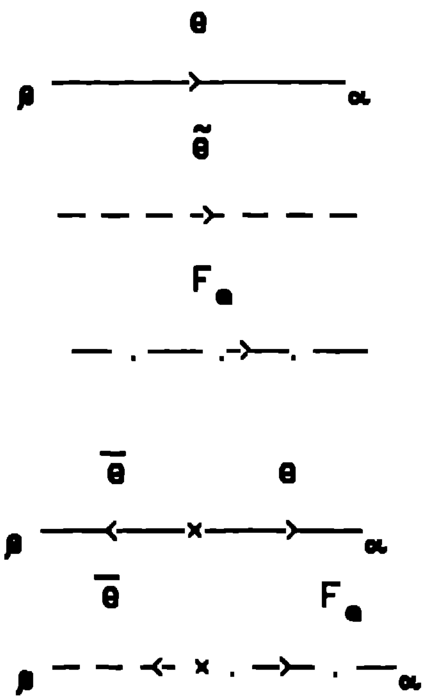
$\langle T(e_\alpha e^\dagger_\beta)\rangle = i\,\sigma_\mu p^\mu/[p^2-m_e^2]$

$\langle T(\tilde{e}\,\tilde{e}^\dagger)\rangle = i/[p^2-m_e^2]$

$\langle T(F_e F_e^\dagger)\rangle = i\,p^2/[p^2-m_e^2]$

$\langle T(e_\alpha \bar{e}_\beta)\rangle = -i\,m_e/[p^2-m_e^2]$

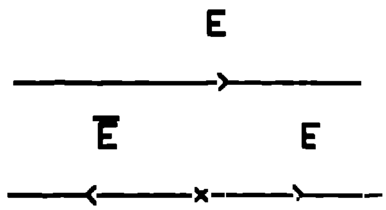
$\langle T(F_e\,\bar{e})\rangle = i\,m_e/[p^2-m_e^2]$



The above propagators are all included in the following superpropagators

$\langle T(E(x,\theta)\,E^\dagger(x',\theta'))\rangle$

$\langle T(E(x,\theta)\,\bar{E}(x',\theta'))\rangle$



Propagators for other chiral fields are similar. In addition, there are gauge propagators.

STATEMENT OF THE NON-RENORMALIZATION THEOREM. [NRT][1-5]

1) F terms receive no radiative corrections to any order in perturbation theory.

2) In superfield language, all radiative corrections to the effective action are of the form

$$\delta\Gamma = \int d^4\theta \int d^4x_1 \ldots d^4x_n\, F_1(x_1,\theta,\bar{\theta}) \ldots F_n(x_n,\theta,\bar{\theta})\, G(x_1,\ldots,x_n) \qquad (1.57)$$

where the product $F_1 \ldots F_n$ is real and F_i are products of superfields and their derivatives.

Let us look at one example of the theorem. Consider corrections to the operator $\int d^2\theta\, \bar{E} E$. The one-loop graphs are given in fig. 2a in terms of both supergraphs and component graphs. According to the theorem these graphs vanish. This result is trivially satisfied at the component level. At two-loops (see fig.2b) the result is nontrivial and requires the cancellation of several nonvanishing contributions.

Figure 2

a)

b)

CONSEQUENCES OF THE NON-RENORMALIZATION THEOREM

1) There can be no SUSY breaking in any finite order of perturbation theory, _if_ SUSY is unbroken at the tree level.

2) The parameters λ, m_e and m are only renormalized by wave function renormalization.

3) The so-called Fayet-Iliopoulos D-term, $e\xi \int d^4\theta\, V$, obtains quadratic radiative corrections of the form $\delta\xi \cong (\Sigma_i q_i)\Lambda^2$ where q_i are the U(1) gauge charges of chiral fields Φ_i. This result is obtained at one loop but is valid to all orders in perturbation theory.

4) There are no quadratic scalar mass corrections, iff $\delta\xi = 0$.

5) It is "technically natural" to set the parameters λ, m_e and m to zero in the bare Lagrangian.

In the next few paragraphs we shall elaborate on these five points.

1) As a result of the NRT the perturbative effective action is of the form

$$\Omega_{eff} = \int d^4\theta\, K(\Phi_i, \Phi_j^\dagger) + \int d^2\theta\, W(\Phi_i) + \text{h.c.} \qquad (1.38)$$

$$= F_{\Phi_i} F_{\Phi_j}^\dagger \left[\partial^2 K/\partial\Phi_i \partial\Phi_j^\dagger\right]\Big|_{\theta=0} + F_{\Phi_i}\left[\partial W/\partial\Phi_i\right]\Big|_{\theta=0}$$

$$+ \text{h.c.} + \text{D-terms}$$

where $W(\Phi_i)$ is the tree level superspace potential and $K(\Phi_i, \Phi_j^\dagger)$ includes all the radiative corrections. The equations of motion for F_{Φ_i} are given by the expression

$$\left[\delta\Omega_{eff} \backslash \delta F_{\Phi_i}\right] = F^\dagger_{\Phi_j}\left[\partial^2 K/\partial\Phi_i\partial\Phi_j^\dagger\right]\Big|_{\theta=0} + \left[\partial W/\partial\Phi_i\right]\Big|_{\theta=0} = 0$$

$$(1.39)$$

If at the tree level the equation $\left[\partial W/\partial\Phi_i\right]\Big|_{\theta=0} = 0$ has a solution, then the equation $F^\dagger_{\Phi_i} = 0$ has a solution to all orders in perturbation theory. Thus SUSY is preserved to all orders. One realizes that a SUSY vacuum is extremely stable. The same analysis can be applied to the D terms with the same conclusion.

2) Consider the general renormalizable Lagrangian of equation (1.31) with g interpreted as a bare coupling and assume the superspace potential W includes the following terms,

$$W \propto \lambda_{ijk}\,\Phi_{ui}\,\Phi_{uj}\,\Phi_{uk} + m_{ij}\,\Phi_{ui}\,\Phi_{uj} \quad . \qquad (1.40)$$

Then, as a result of the NRT, λ and m are renormalized by wave function renormalization only. Specifically we find

$$\lambda^r_{ijk} = \lambda_{ijk}\,(Z_i Z_j Z_k)^{1/2} \quad \text{and} \quad m^r_{ij} = m_{ij}\,(Z_i Z_j)^{1/2} \quad . \qquad (1.41)$$

The renormalized Lagrangian of (1.31) is given by

$$\mathfrak{L} = \left(\int d^2\theta\, \frac{1}{16k\, g_r^2}\, Z_g\, \mathrm{Tr}\, W_r^{\alpha} W_{r\alpha} + \text{h.c.} \right) \qquad (1.42a)$$

$$+\; Z_i \int d^4\theta\; \Phi_{ri}^{\dagger}\, e^{2g_r V_r^{(i)}}\, \Phi_{ri} \qquad (1.42b)$$

$$+\; \left(\int d^2\theta\; W(Z_i^{1/2}\,\Phi_{ri}) + \text{h.c.} \right) \qquad (1.42c)$$

where the renormalized fields are defined by the expressions $\Phi_{ri} = Z_i^{-1/2}\Phi_i$, $V_r = Z_g^{-1/2} f(V)$ and $g_r = Z_g^{1/2} g$. $f(V)$ is in general a nonlinear function of V. Note, that $Z_\varphi = Z_\psi$ in a SUSY gauge, which we have implicitly assumed. In a non-SUSY gauge $Z_\varphi \neq Z_\psi$, nevertheless, the renormalization group equations for λ and m remain unchanged[8].

3) The radiative correction to ξ at one-loop is depicted in figure 3 We find

figure 3

D

$\delta\xi \;\sim$ - - - -o

q_i

$$\delta\xi \sim \;{}_{i}\, q_i \int^{\Lambda} d^4p/[p^2 - m_i^2] \sim \left({}_{i}\, q_i \right) \Lambda^2 \quad . \qquad (1.43)$$

This result is true to all orders in perturbation theory[9]. Charged scalar fields obtain a contribution to their mass proportional to $g(\xi + \delta\xi)$. As we shall soon discuss, this is the only origin for radiative mass corrections to scalar fields which is quadratic in the cut-off. Thus if one wants to eliminate these quadratic corrections, one must demand that $\Sigma_i q_i = 0$ for any U(1) gauge sector of the theory. This condition is of course satisfied for $U(1)_{\text{hypercharge}}$ in the standard model.

4) If $\delta\xi = 0$ then there are no quadratic mass corrections for scalars. The radiative corrections to m discussed in (2) are logarithmic corrections proportional to $\ln \Lambda$. The only other possible quadratic corrections to scalar masses are usually generated via propagator corrections as in figure 4. However, since these corrections must yield a D-term, we find that the potential quadratic mass correction necessarily vanishes.

figure 4

The sum of Feynman graphs in fig. 4 generates the following expression at zero momentum

$$\lambda^2 \left[-\mathrm{Tr}\,[\sigma_\mu p^\mu (i\sigma^2) \sigma^T{}_\nu p^\nu (-i\sigma^2)] /(p^2 - m_e{}^2)^2 + 2p^2 /(p^2 - m_{\tilde e}{}^2)^2 \right] = 0 \qquad (1.44)$$

using $\mathrm{Tr}(\sigma_\mu \bar\sigma_\nu) = 2\eta_{\mu\nu}$, $\sigma^2 \sigma^T{}_\mu \sigma^2 = \bar\sigma_\mu$ and $m_{\tilde e}{}^2 = m_e{}^2$. The resulting expression for non-zero momentum is of the form

$$\lambda^2 \ln\Lambda \left[\int d^4\theta\, E^\dagger E \right] . \qquad (1.45)$$

Note, if SUSY is broken and $m_{\tilde e}{}^2 \neq m_e{}^2$, then the above cancellation no longer occurs and one obtains

$$\delta m_{\tilde e}{}^2 \sim \lambda^2 (m_{\tilde e}{}^2 - m_e{}^2) . \qquad (1.46)$$

[We shall use the symbol Λ_{ss} to denote the SUSY breaking scale defined by $m_{\tilde{e}}^2 - m_e^2 = \Lambda_{ss}^2$.]

5) In a renormalizable field theory, any term in the effective action will typically be generated in perturbation theory <u>if</u> it is not forbidden by an exact symmetry. If however one sets a parameter to zero and the symmetry increases such as to keep that parameter zero to all orders in perturbation theory, then one says that it is "natural" to leave such a term out of the Lagrangian. In a SUSY theory , any F-term with parameter λ or m may be set to zero and they will not be generated in any finite order of perturbation theory, even though the symmetry of the theory may never ave increased. We say that, in this case, it is "technically natural" to leave such terms out of the Lagrangian.

SOFT SUSY BREAKING OPERATORS

These are a set of explicit SUSY breaking operators which one may add to a SUSY Lagrangian, <u>preserving</u> the renormalizability of the theory <u>and</u> the absence of quadratic divergences. They are typically generated in the effective action once SUSY is spontaneously broken. They are useful in parametrizing SUSY breaking effects in realistic models. As an example we give the possible soft SUSY breaking operators which may be added to SUSY QED.

$$\delta\Omega_{SQED} \sim [M/2](\tilde{\gamma}\tilde{\gamma}) + m_{\tilde{e}}^2|\tilde{e}|^2 + m_{\tilde{e}}^2|\overline{e}|^2 + m_{\varphi}^2|\varphi|^2$$
$$+ A\lambda\overline{e}\,\tilde{e}\,\varphi + B m_e \overline{e}\,\tilde{e} + \text{h.c.} \qquad (1.47)$$

An example of an operator which is <u>not</u> a soft SUSY breaking term is $\varphi^2\varphi^*$.

To conclude this section, we present two examples of the radiative corrections to parameters in (1.47).

1) <u>Corrections to $m_{\tilde{e}}$</u>

In figure 5 we give the three contributions to $m_{\tilde{e}}$ at one loop.

figure 5

We obtain the following corrections, respectively

$$\delta m_{\tilde e}^2 \sim -\lambda^2/(16\pi^2)\ m_{\tilde e}^2 \ln(\Lambda/m) + e^2/(16\pi^2) M^2 \ln(\Lambda/m)$$
$$-\lambda^2 A^2/(16\pi^2) \ln(\Lambda/m) \quad . \qquad (1.48)$$

Note, gaugino exchange tends to increase the scalar mass, whereas corrections proportional to Yukawa couplings decrease the mass.

2) <u>Corrections to M</u>

At one-loop there are <u>no</u> divergent corrections to the operator $(\tilde\gamma\tilde\gamma)$. As a result we find the renormalized operator

$$M(\tilde\gamma\tilde\gamma) = M Z_3 (\tilde\gamma_r\tilde\gamma_r) = M_r (\tilde\gamma_r\tilde\gamma_r) \qquad (1.49)$$

or $M_r = M Z_3$.

If we recall that $e_r = Z_3^{1/2} e$ (see following eqn. (1.42)) , we find (at one-loop) that the expression

$$M_r / e_r^2 = M / e^2 \qquad (1.50)$$

is a renormalization group invariant. At two-loops, however, there is a divergent correction to the operator $(\tilde\gamma\tilde\gamma)$ (see fig. 6).

figure 6

We obtain a correction of the form

$$\delta M \sim \lambda^2 A e^2/(16\pi^2)^2 \ln(\Lambda/m) \quad . \qquad (1.51)$$

2. MINIMAL LOW ENERGY SUPERSYMMETRIC STANDARD MODEL

GAUGE HIERARCHY PROBLEM[11,12]

One would like to understand the origin of the small number $m_W/M_{LARGE} \lesssim 10^{-13}$ for $M_{LARGE} = M_{GUT}$ or M_{Planck}. The problem in understanding this small ratio is usually compounded by the fact that the ratio is extremely sensitive to radiative corrections , as we now show.

Let us assume that there exists a scale $M_{LARGE} \gg m_W$ and heavy states H with mass of order M_{LARGE} which couple to light states h with mass of order m_W. We assume a generic coupling constant g.

Consider first that the states h are fermions - f. They will typically receive radiative corrections to their mass of the form

$$\delta m_f \sim g^2/(16\pi^2)\, m_f \ln(M_{LARGE}/m). \qquad (1.52)$$

The result is only logarithmically dependent on M_{LARGE}, since a fermion mass is generally protected by chiral symmetries. Thus fermions are "naturally" light states.

Consider now that the states h are scalars - s. The radiative correction they receive is typically of the form

$$\delta m_s^2 \sim g^2/(16\pi^2)\; M_{LARGE}^2 \;. \qquad (1.53)$$

Scalars are typically sensitive to the physics at large energies, since there is generally no chiral symmetries protecting scalar masses.

Since in the standard model $m_W^2 \sim m^2_{Higgs}$ and $\delta m^2_{Higgs} \sim g^2/(16\pi^2)\; M_{LARGE}^2$ we expect that, without extreme fine tuning, $m_W/M_{LARGE} \sim 0.1$. This is the first gauge hierarchy problem. (We shall discuss the second gauge hierarchy problem when we discuss GUT's.)

In SUSY theories $\delta m_s^2 = \delta m_f^2$ (when SUSY is exact) and in a softly broken SUSY theory $\delta m_s^2 = \delta m_f^2 + g^2/(16\pi^2)\Lambda^2_{ss}$. Thus in a

SUSY theory, the Higgs scalar can be "naturally" light if $\Lambda_{ss} \lesssim 10\, m_W$. Of course, the question still remains, why is the ratio Λ_{ss}/M_{LARGE} so small? [SUSY breaking scenarios are discussed further in lecture 3.]

We shall now assume that $\Lambda_{ss} \lesssim 10\, m_W$ and discuss the ingredients of the minimal low energy SUSY standard model.

PARTICLE SPECTRUM

In table 1 we list the particle spectrum. The symmetry of the theory is taken to be SU(3)⊗SU(2)⊗U(1)⊗SUSY.

Table 1

ORDINARY PARTICLES		SUSY PARTNERS	
$l_i = \begin{bmatrix} \nu \\ e \end{bmatrix}_i^{-1}$	$\bar{e}_i^{+2}$	$\tilde{l}_i = \begin{bmatrix} \tilde{\nu} \\ \tilde{e} \end{bmatrix}_i$	$\bar{\tilde{e}}_i$
$q_i = \begin{bmatrix} u \\ d \end{bmatrix}_i^{+1/3}$	$\bar{u}_i^{-4/3}$, $\bar{d}_i^{+2/3}$	$\tilde{q}_i = \begin{bmatrix} \tilde{u} \\ \tilde{d} \end{bmatrix}_i$	$\bar{\tilde{u}}_i$, $\bar{\tilde{d}}_i$
$g\ ,\ \vec{W}\ ,\ B$		$\tilde{g}\ ,\ \vec{\tilde{W}}\ ,\ \tilde{B}$	
$h = \begin{bmatrix} h^+ \\ h^0 \end{bmatrix}^{+1}$	$\bar{h} = \begin{bmatrix} \bar{h}^- \\ \bar{h}^0 \end{bmatrix}^{-1}$	$\tilde{h} = \begin{bmatrix} \tilde{h}^+ \\ \tilde{h}^0 \end{bmatrix}$	$\bar{\tilde{h}} = \begin{bmatrix} \bar{\tilde{h}}^- \\ \bar{\tilde{h}}^0 \end{bmatrix}$

SUPERFIELDS

L_i	$\bar{E}_i$
Q_i	$\bar{U}_i$
	$\bar{D}_i$

V_3 , V_2 , V_1

H $\overline{H}$

Hypercharge assignments are given as superscripts, $Q = T_3 + Y/_2$ and $i = 1,2,3$ label generations. The superfields contain in an obvious way, we hope, all the states listed above.

The Lagrangian for the full low energy effective theory is as follows.

$$\begin{aligned}
\Omega = & \int d^2\Theta \left[\tfrac{1}{8g_3^2} Tr(W_3W_3) + \tfrac{1}{8g_2^2} Tr(W_2W_2) + \tfrac{1}{16g_1^2} W_1W_1 \right] \\
& + h.c. \\
& + \int d^4\Theta \Big[\quad Q_i^\dagger \exp(2g_3V_3 + 2g_2V_2 + \tfrac{1}{3}g_1V_1)\, Q_i \\
& \qquad + \overline{U}_i^\dagger \exp(-2g_3V_3^T - \tfrac{4}{3}g_1V_1)\, \overline{U}_i \\
& \qquad + \overline{D}_i^\dagger \exp(-2g_3V_3^T + \tfrac{2}{3}g_1V_1)\, \overline{D}_i \qquad\qquad (2.1) \\
& \qquad + L_i^\dagger \exp(\; 2g_2V_2 - g_1V_1)\, L_i \\
& \qquad + \overline{E}_i^\dagger \exp(\; 2g_1V_1)\, \overline{E}_i \\
& \qquad + H^\dagger \exp(\; 2g_2V_2 + g_1V_1)\, H \\
& \qquad + \overline{H}^\dagger \exp(\; -2g_2V_2^T - g_1V_1)\, \overline{H} \;\Big] \\
& + \left[\int d^2\Theta\; W + h.c. \right] + \delta\Omega_{SOFT}
\end{aligned}$$

where the superspace potential W is given by

$$W \equiv \lambda^u{}_{ij} H \overline{U}_i Q_j + \lambda^d{}_{ij} \overline{H}\,\overline{D}_i Q_j + \lambda^e{}_{ij} \overline{H}\,\overline{E}_i L_j + \mu \overline{H} H \qquad (2.2)$$

and the soft SUSY breaking term $\delta\Omega_{SOFT}$ is[13]

$$\begin{aligned}
\delta\Omega_{SOFT} = & M_3/_2(\tilde{g}\tilde{g}) + M_2/_2(\tilde{W}\tilde{W}) + M_1/_2(\tilde{B}\tilde{B}) \\
& + \mu^u{}_{ij} h \overline{u}_i \tilde{q}_j + \mu^d{}_{ij} \overline{h}\,\overline{d}_i \tilde{q}_j + \mu^e{}_{ij} \overline{h}\,\overline{e}_i \tilde{l}_j + B\mu \overline{h} h \\
& + \tilde{q}^\dagger m^2_{\tilde{q}} \tilde{q} + \overline{u}^\dagger m^2_{\overline{u}} \overline{u} + \overline{d}^\dagger m^2_{\overline{d}} \overline{d} \qquad\qquad (2.3) \\
& + \tilde{l}^\dagger m^2_{\tilde{l}} \tilde{l} + \overline{e}^\dagger m^2_{\overline{e}} \overline{e} + m^2_h |h|^2 + m^2_{\overline{h}} |\overline{h}|^2 .
\end{aligned}$$

($V_3 \equiv V_3{}^a T^a$, etc. and the charge conjugate generator $\overline{T}^a$ is given by $\overline{T}^a \equiv -T^{aT}$)

Note that this model necessarily contains two higgs doublets, h and $\bar{h}$. This is required in order to give mass to both up and down quarks in an anomally free theory. We cannot ,for example, introduce only the h field and use the scalar partner of the neutrino as the other higgs, even though it has identical quantum numbers, since this theory has anomalies. We are also not able to combine the two known massless particles , the neutrino and photon into one supermultiplet, since they have different electroweak quantum numbers.

SYMMETRIES OF $\mathfrak{L}$

Let us now discuss the global symmetries of $\mathfrak{L}$.

1) Baryon number B and Lepton number L_i , for each generation, is conserved. Note that we have explicitly left terms out of $\mathfrak{L}$ with dimension ≤ 4 which violate either B and/or L, such as $(L\,H)_F$, $(\overline{UDD})_F$, $(Q\bar{D}L)_F$, $(LL\bar{E})_F$.

2) R- parity, defined by the discrete symmetry $(-1)^{3(B-L)}(-1)^F$ (where F is fermion number and $L \equiv \sum_i L_i$), is conserved. All ordinary particles are R even, while all superpartners are R odd. As a result of R-parity the lightest superpartner [LSP] is stable[14,15].

3) In the limit that the parameter $\mu \to 0$, there is an exact Peccei-Quinn symmetry, $U(1)_{PQ}$, defined by the operations

$$H \to e^{i\alpha} H \ , \ \bar{H} \to e^{i\alpha} \bar{H} \ , \ F \to e^{-i\alpha/2} F \qquad (2.4)$$

where F means all quark and lepton superfields. Note when the higgs fields acquire vacuum expectation values then $U(1)_{PQ}$ is spontaneously broken and there exists a massless goldstone boson ,i.e. the standard axion. Since this axion is ruled out experimentally, we have taken $\mu \neq 0$.

4) In the limit $M_i \to 0$ and $\mu^u = \mu^d = \mu^e \to 0$, there exists a chiral symmetry which keeps gauginos massless to all orders in perturbation theory; a so-called global R invariance. It is defined

by the superfield transformations

$$\Phi'(x,\theta) = e^{iQ\alpha}\,\Phi(x, e^{-1/2\alpha}\theta) \qquad (2.5a)$$

$$V'(x,\theta,\bar\theta) = V(x, e^{-1/2\alpha}\theta, e^{1/2\alpha}\bar\theta)$$

or the component field transformations

$$\varphi'(x) = e^{iQ\alpha}\,\varphi(x),\ \ \psi'(x) = e^{i(Q-1/2)\alpha}\,\psi(x),\ \ F'(x) = e^{i(Q-1)\alpha}\,F(x)$$

$$A_\mu'(x) = A_\mu(x),\ \ \lambda'(x) = e^{1/2\alpha}\,\lambda(x),\ \ D'(x) = D(x) \qquad (2.5b)$$

with $Q = 2$ for H, $\bar{H}$ and $Q = 1$ for all other chiral fields.

ELECTROWEAK BREAKING

In order to discuss the spectrum of this model we must first consider the electoweak breaking $SU(2)\otimes U(1) \rightarrow U(1)_{EM}$. It has been shown by Inoue et. al.[16] that the higgs potential allows for electroweak breaking within a certain range of parameters. Consider the Higgs potential relevant for this issue. We have

$$V = m_1^2|h|^2 + m_2^2|\bar{h}|^2 - \mu B\bar{h}h + h.c.$$
$$+ g_2^2/_2(h^*\vec{\tau}/_2 h - \bar{h}^*\vec{\tau}^T/_2\bar{h})^2 + g_1^2/_2(\tfrac{1}{2}|h|^2 - \tfrac{1}{2}|\bar{h}|^2)^2 \qquad (2.5)$$

where we define $m_1^2 \equiv m_h^2 + \mu^2$ and $m_2^2 \equiv m_{\bar{h}}^2 + \mu^2$. Note, if $\langle h_0\rangle = \langle\bar{h}_0\rangle \neq 0$, then the quartic terms in V vanish. Thus we find that a) V is bounded from below only if $m_1^2 + m_2^2 > 2\mu B$ and b) the symmetric solution is unstable only if $\det m_h^2 < 0$ or equivalently, $m_1^2 m_2^2 < \mu^2 B^2$. The range of parameters which satisfy these constraints is described graphically in figure 7.

figure 7

The electroweak symmetry breaking solution is given by

$$\langle h \rangle = \begin{bmatrix} \\ \upsilon \end{bmatrix}, \quad \langle \bar{h} \rangle = \begin{bmatrix} 0 \\ \bar{\upsilon} \end{bmatrix} \tag{2.6}$$

with $\upsilon = v\cos\theta$, $\bar{\upsilon} = v\sin\theta$, $\sin 2\theta = \left[2\mu B/(m_1^2 + m_2^2)\right]$,

$$m_Z^2 = (g_1^2 + g_2^2)\, v^2/_2 \; = |(m_1^2 - m_2^2)/\cos\theta| - m_1^2 - m_2^2 \quad ,$$

$$m_W^2 = g_2^2\, v^2/_2$$

and $m_W = m_Z \cos\theta_W$. $[\ \sin^2\theta_W = g_1^2/(g_1^2 + g_2^2)\ ]$

We thus obtain the standard W and Z mass relations.

MASS SPECTRUM

GAUGINO MASSES

In general, the soft breaking masses M_i, $i=1,2,3$ are unequal , arbitrary parameters. (This situation changes, however, in the context of GUT's , a topic which we shall discuss in the next section.)

GLUINO MASS

The gluino mass is M_3.

CHARGED WINO MASS MATRIX

The relevant part of $\mathfrak{L}$ is

$$\mathfrak{L} \sim \sqrt{2}\, g_2 [\, -\bar{h}^{*i} (\tilde{W}_a (\tau_a^T/_2)^i{}_j \tilde{\bar{h}}^j) + \; h^*{}_i (\tilde{W}_a (\tau_a/_2)^j{}_i \tilde{h}^j)] + \text{h.c.}$$

$$\begin{aligned} = \; g_2 [\, -\bar{h}^{0*} (\tilde{W}^+ \tilde{\bar{h}}^-) + \; h^{0*} (\tilde{W}^- \tilde{h}^+) \; - \; \bar{h}^{-*} (\tilde{W}_3 \tilde{\bar{h}}^-)/\sqrt{2} \\ + \; h^{+*} (\tilde{W}_3 \tilde{h}^+)/\sqrt{2} \; - \; \bar{h}^{-*} (\tilde{W}^- \tilde{\bar{h}}^0) + \; h^{+*} (\tilde{W}^+ \tilde{h}^0) \\ + \; \bar{h}^{0*} (\tilde{W}_3 \tilde{\bar{h}}^0)/\sqrt{2} \; - \; h^{0*} (\tilde{W}_3 \tilde{h}^0)/\sqrt{2} \; + \text{h.c.}\,] \end{aligned} \tag{2.7}$$

where we have defined

$$\tilde{W}^{\pm} = (\tilde{W}_1 \mp i\, \tilde{W}_2)/\sqrt{2} \; , \quad \tau^{\pm} = (\tau_1 \pm i\, \tau_2)/2 \quad ,$$

$$\tilde{W}_a\, \tau_a/_2 = (\tilde{W}^+ \tau^+ + \tilde{W}^- \tau^-)/\sqrt{2} + \tilde{W}_3\, \tau_3/2 \quad .$$

The charged wino mass matrix is then given by

$$\begin{array}{ccc} & \tilde{W}^+ & \tilde{h}^+ \\ \tilde{W}^- & -M_2 & g_2 \upsilon \\ \tilde{h}^- & -g_2 \bar{\upsilon} & -\mu \end{array} \tag{2.8}$$

There are two Dirac fermions with mass of order m_W. In the limit $M_2, \mu \ll g_2 \upsilon \sim g_2 \bar{\upsilon}$, one state has mass less than m_W.

NEUTRALINOS

Let us define the states

$$\begin{array}{lll}
\tilde{\gamma} = (g_1 \tilde{W}_3 + g_2 \tilde{B})/\sqrt{g_1^2 + g_2^2} & \text{photino} & \\
\tilde{Z} = (-g_2 \tilde{W}_3 + g_1 \tilde{B})/\sqrt{g_1^2 + g_2^2} & \text{zino} & (2.9) \\
\tilde{S}^0{}_1 = (\bar{\upsilon} \tilde{h}^0 + \upsilon \tilde{\bar{h}}^0)/\sqrt{v} & \text{higgsino}_1 & \\
\tilde{S}^0{}_2 = (\upsilon \tilde{h}^0 - \bar{\upsilon} \tilde{\bar{h}}^0)/\sqrt{v} & \text{higgsino}_2 &
\end{array}$$

where $\overline{v}^2 = \upsilon^2 + \bar{\upsilon}^2$. The neutralino mass matrix is then given by

$$\begin{array}{c|cccc}
 & \tilde{S}^0{}_1 & \tilde{S}^0{}_2 & \tilde{Z} & \tilde{\gamma} \\
\hline
\tilde{S}^0{}_1 & 2\upsilon\bar{\upsilon}\mu/\overline{v}^2 & (\upsilon^2 - \bar{\upsilon}^2)\mu/\overline{v}^2 & 0 & 0 \\
\tilde{S}^0{}_2 & (\upsilon^2 - \bar{\upsilon}^2)\mu/\overline{v}^2 & -2\upsilon\bar{\upsilon}\mu/\overline{v}^2 & m_Z & 0 \\
\hline
\tilde{Z} & 0 & m_Z & \dfrac{(g_2^2 M_2 + g_1^2 M_1)}{(g_1^2 + g_2^2)} & \dfrac{g_1 g_2 (M_1 - M_2)}{(g_1^2 + g_2^2)} \\
\tilde{\gamma} & 0 & 0 & \dfrac{g_1 g_2 (M_1 - M_2)}{(g_1^2 + g_2^2)} & \dfrac{(g_1^2 M_2 + g_2^2 M_1)}{(g_1^2 + g_2^2)}
\end{array} \qquad (2.10)$$

In the limit $M_1, M_2, \mu \ll m_Z$ we have the approximate eigenstates

$\tilde{S}^0{}_1$ with mass $\sim 2\upsilon\bar{\upsilon}\mu/\overline{v}^2$

$\tilde{\gamma}$ with mass $\sim \sin^2\theta_W M_2 (1 + \cot^2\theta_W M_1/M_2)$

a pair of Majorana fermions , and

$\tilde{S}^0{}_2$, $\tilde{Z}$ a Dirac fermion with mass $\sim m_Z$.

HIGGS SCALARS

We can make some general statements about the higgs spectrum as a result of the fact that the quartic terms in the potential are due to supersymmetrized gauge interactions only[16,17]. There are five higgs', charged higgs' $\chi^{\pm}$, a pseudo-scalar χ^0 and two scalars φ^0 and η^0. Their masses are given by the expressions

$$m_{\chi^0}^2 = \mu B\,(v\backslash\bar{v} + \bar{v}\backslash v)$$

$$m_{\chi^\pm}^2 = m_{\chi^0}^2 + m_W^2 \qquad (2.11)$$

$$m^2_{\varphi^0(\eta^0)} = \tfrac{1}{2}\left(m_{\chi^0}^2+m_Z^2 +(-)\left[(m_{\chi^0}^2+m_Z^2)^2 - 4m_{\chi^0}^2 m_Z^2\left((v^2-\bar{v}^2)/v^2\right)^2\right]^{1/2}\right).$$

We note that

a) as $\mu \rightarrow 0$, $m_{\chi^0}^2$ vanishes. In this limit χ^0 becomes an axion, only receiving mass due to strong instanton corrections. In the limit $B \rightarrow 0$, $m_{\chi^0}^2$ also vanishes. In this limit there is an approximate R invariance which is explicitly broken by the gaugino mass parameters M_i. As a result χ^0 will obtain mass via radiative corrections.

b) χ^+ is heavier than the W bosons, and

c) one scalar is lighter than the Z^0 boson. Thus it can be produced in the process $Z^0 \rightarrow \eta^0 e^+ e^-$ at SLC or LEP.

QUARKS AND LEPTONS

The relevant terms in Ω for quark and lepton masses is

$$\Omega \sim \lambda^u_{ij}\, v\, \bar{u}^0_i u^0_j + \lambda^d_{ij}\bar{v}\, \bar{d}^0_i d^0_j + \lambda^e_{ij}\bar{v}\, \bar{e}^0_i e^0_j . \qquad (2.12)$$

We define the mass matrices

$$m^u_{ij} \equiv \lambda^u_{ij}\, v\,,\quad m^d_{ij} \equiv \lambda^d_{ij}\, \bar{v}\,,\quad m^e_{ij} \equiv \lambda^e_{ij}\, \bar{v}.$$

The mass matrices are in general <u>not diagonal</u> and the superscript 0 on the quarks and leptons denotes current eigenstates as opposed to mass eigenstates. The ν's are massless, just as in the standard model. We define the mass eigenstates by the following unitary transformations

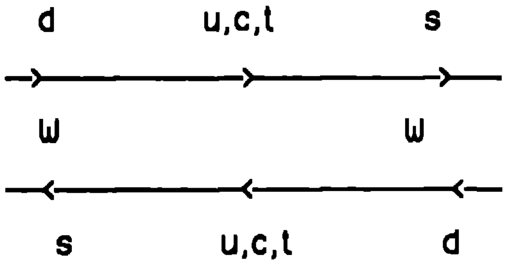
$$u^0_i = U^u_{ij}\, u_j \ ,\quad \bar{u}^0_i = \bar{u}_j\, V^{u\dagger}_{ji} \qquad (2.13a)$$

(with similar transformations for d and e) such that the diagonalized mass matrix is given by

$$m^u_{\text{diagonal}} = V_u^{\dagger}\, m^u\, U_u . \qquad (2.13b)$$

The Cabibbo-Kobayashi-Maskawa matrix $K = U_u^\dagger U_d$ is then defined in terms of the weak charged current interaction $W^+ u^{0\dagger}_i d^0_i \rightarrow W^+ u^\dagger K\, d$. As a result of this generation mixing we obtain the standard contribution to $\overline{K}^0 - K^0$ mixing as depicted in figure 8.

figure 8



$\overline{K}^0 - K^0$

SQUARKS AND SLEPTONS

The relevant part of $\mathfrak{L}$ for these mass terms is

$$
\begin{aligned}
\mathfrak{L} \sim \Big(&\lambda^u_{ij} H \overline{U}^0_i Q^0_j + \lambda^d_{ij} \underline{\overline{H}\,\overline{D}^0_i Q^0_j} + \lambda^e_{ij} \underline{\overline{H}\,\overline{E}^0_i L^0_j}\Big)_{F\text{-term}} \\
&+ \mu^u_{ij} \overline{u}^0_i \tilde{q}^0_j + \mu^d_{ij} \underline{\overline{h}\,\overline{d}^0_i \tilde{q}^0_j} + \mu^e_{ij} \underline{\overline{h}\,\overline{e}^0_i \tilde{l}^0_j} \\
&+ \tilde{q}^{0\dagger} m^2_{\tilde{q}} \tilde{q}^0 + \overline{u}^0 m^2_{\overline{u}} \overline{u}^{0\dagger} + \underline{\overline{d}^0 m^2_{\overline{d}} \overline{d}^{0\dagger}} \qquad (2.14) \\
&+ \tilde{l}^{0\dagger} m^2_{\tilde{l}} \tilde{l}^0 + \underline{\overline{e}^0 m^2_{\overline{e}} \overline{e}^{0\dagger}}\ .
\end{aligned}
$$

We shall study the terms which are underlined in more detail below. First we must once again distinguish between mass eigenstates and current eigenstates, this time for the squarks and sleptons. We shall define the transformed fields (without the superscript ^{0}) using the unitary transformations defined previously for quarks and leptons (eqn. 2.13). This goes part of the way toward defining the true mass eigenstates, but not necessarily all the way. In this way we will be able to study the new sources of generation mixing which can occur in SUSY theories. We define

$$
Q^0_i = U^u_{ij} Q'_j \qquad \text{with} \qquad Q'_i = \begin{bmatrix} U \\ K\,D \end{bmatrix}_i \qquad (2.15)
$$

$$
\overline{D}^0_i = \overline{D}_j V^{d\,\dagger}_{ji} \quad , \quad \overline{U}^0_i = \overline{U}_j V^{u\,\dagger}_{ji}
$$

$$
L^0_i = U^e_{ij} L_j \quad , \quad \overline{E}^0_i = \overline{E}_j V^{e\,\dagger}_{ji}
$$

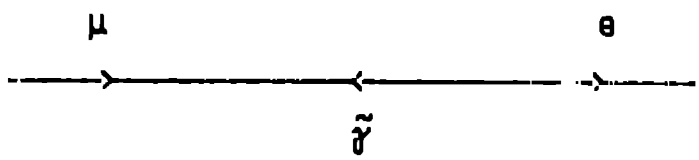
The underlined portion of equation (2.14) now becomes

$$\Omega \sim \left((\lambda^d_{diag.} K^\dagger)_{ij} \overline{H} \overline{D}_i Q'_j + (\lambda^e_{diag.})_{ij} \overline{H} \overline{E}_i L_j \right)_{F\text{-term}}$$
$$+ (V_d^\dagger \mu^d U_u)_{ij} \overline{h} \overline{d}_i \tilde{q}_j + (V_e^\dagger \mu^e U_e)_{ij} \overline{h} \overline{e}_i \tilde{l}_j \qquad (2.16)$$
$$+ \overline{d} (V_d^\dagger m^2_{\tilde{d}} V_d) \overline{d}^\dagger + \overline{e} (V_e^\dagger m^2_{\tilde{e}} V_e) \overline{e}^\dagger \; .$$

Note, that the squark and slepton mass matrices are not generally diagonalized in the same basis as the quarks and leptons. As a result we have new mechanisms for flavor symmetry breaking[18]. Consider the following two examples.

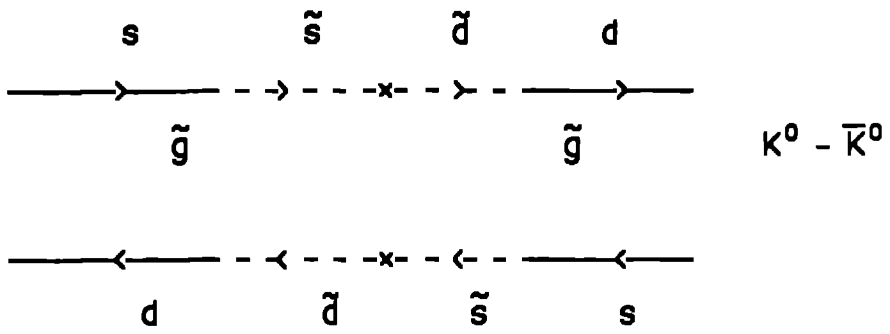
Example 1) In the standard model, lepton number is conserved for τ, μ and e separately. In the SUSY standard model only the total lepton number L is conserved in general. In figure 9 we show a graph contributing to the decay $\mu \rightarrow e + \gamma$. The decay rate is proportional to a μ-e squark mixing mass term $\delta m_{\mu e}{}^2$. Using the experimental limit for the branching ratio $BR(\mu \rightarrow e\gamma) \lesssim 10^{-11}$ we obtain the following limit on this mixing mass $\delta m_{\mu e}{}^2/\tilde{m}^2 \lesssim 10^{-3}$ (where $\tilde{m}^2$ is the average squark mass). Finally note that τ, μ and e lepton number symmetry is recovered in the limit that the parameters $\mu^e = m\, \lambda^e$ and $m_{\tilde{e}}{}^2$ and $m_{\tilde{\overline{e}}}{}^2$ are proportional to the unit matrix in flavor space.

figure 9



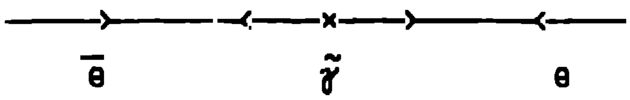
Example 2) There are additional contributions to $K^0 - \overline{K}^0$ mixing as given in figure 10. These contributions are proportional to the s-d squark mixing mass $\delta m_{sd}{}^2$. Assuming the SUSY contribution is no larger than the standard model result, we obtain the following limit on this mixing mass $\delta m_{sd}{}^2/\tilde{m}^2 \lesssim 10^{-3}$.

figure 10



The standard model has a chiral symmetry limit obtained by taking the quark and lepton masses to zero. In this limit quark and leptons will remain massless to all orders in perturbation theory. In the SUSY standard model chiral symmetry is in general not recovered in this limit. Consider the radiative contribution to the electron mass given in figure 11.

figure 11



We obtain the correction to the electron mass given by $\delta m_e \sim \alpha\, m_\mu (\delta m_{\mu e}{}^2/\tilde{m}^2)^2$. Thus even if m_e(bare) vanishes, we find $\delta m_e \neq 0$, unless $(V_e{}^\dagger \mu_e U_e)_{31} = (V_e{}^\dagger m_{\bar{e}}{}^2 V_e)_{31} = 0$ where the electron is taken to be in the third generation. Hence, the chiral symmetry limit (for the electron) in the SUSY standard model is obtained by taking the parameters $\mu^e = m\, \lambda^e$, $m_{\bar{e}}{}^2$ and $m_{\bar{e}}{}^2$. proportional to the unit matrix and setting electron mass to zero. The resulting symmetry is given by the global transformation $E_3{}' = e^{i\alpha} E_3$.

5. SUPERSYMMETRIC GUT'S

In this section we shall consider the consequences of supersymmetrizing the standard SU(5) GUT. Before we supersymmetrize, however, let us recall some of the successes and problems of standard SU(5).

Successes

1) provides an explanation of electric charge quantization
2) predicts the value of $\sin^2\theta_W$
3) predicts the value of the ratio m_b/m_τ

Problems

1) the simple predictions for other mass ratios ; i.e. $m_s/m_d = m_\mu/m_e$ are not satisfied, thus the simple higgs structure we shall present here, is clearly insufficient.
2) the simple prediction of the proton lifetime is too short.

It has been emphasized that these two problems are related and that a more realistic higgs structure could in principle solve both. We shall show that SUSY SU(5), with a simple higgs structure, can solve the problem of the proton lifetime while retaining the successes of standard SU(5) listed above. Unfortunately, the problem with fermion mass ratios remains.

STATES — SU(5)⊗SUSY

Simple SUSY SU(5) requires the superfields listed in table 2.

Table 2

Chiral Superfields

$$10_i \sim \begin{bmatrix} \bar{U} & Q \\ & \bar{E} \end{bmatrix}_i \qquad \bar{5}_i \sim \begin{bmatrix} \bar{D} \\ L \end{bmatrix}_i$$

$$H \sim \begin{bmatrix} H_3 \\ H_2 \end{bmatrix} \qquad \bar{H} \sim \begin{bmatrix} \bar{H}_3 \\ \bar{H}_2 \end{bmatrix}$$

24

Vector Superfield

$$V_5 = V_5^a T^a \; , \quad a = 1, \ldots , 24$$

The Lagrangian for this model is given below.

$$
\begin{aligned}
\Omega = \; & \frac{1}{8 g_5^2}\int d^2\theta \;\mathrm{Tr}(W_5 W_5) \\
& + \int d^4\theta \Big[\; 10_i^{\dagger} \exp(2 g_5 V_5(10))\, 10_i \\
& \qquad + \overline{5}_i^{\dagger} \exp(-2 g_5 V_5^{T})\, \overline{5}_i \\
& \qquad + 24^{\dagger} \exp(2 g_5 V_5(24))\, 24 \qquad\qquad (3.1) \\
& \qquad + H^{\dagger} \exp(2 g_5 V_5)\, H \\
& \qquad + \overline{H}^{\dagger} \exp(-2 g_5 V_5^{T})\, \overline{H} \; \Big] \\
& + \Big[\int d^2\theta \; W + \mathrm{h.c.}\Big] + \delta\Omega_{\mathrm{SOFT}}
\end{aligned}
$$

$$
\begin{aligned}
W \equiv \; & \lambda^{u}_{ij} H\, 10_i\, 10_j + \lambda^{d}_{ij} \overline{H}\, \overline{5}_i\, 10_j + M_1 \mathrm{Tr}(24^2) + \lambda_1 \mathrm{Tr}(24^3) \\
& + M_2 \overline{H} H + \lambda_2 \overline{H}\, 24\, H \\
\delta\Omega_{\mathrm{SOFT}} \equiv \; & M_5/2\, (\lambda_5^{a} \lambda_5^{a}) + \mu^{u}_{ij} H\, 10_i\, 10_j + \mu^{d}_{ij} \overline{H}\, \overline{5}_i\, 10_j \\
& + f(24, \overline{H}, H) + m_{10}^2 |10|^2 + m_5^2 |\overline{5}|^2 + m_H^2 |H|^2 \\
& + m_{\overline{H}}^2 |\overline{H}|^2 + m_{24}^2 |24|^2
\end{aligned}
$$

Note, there are two distinct scales in Ω. M_1 and M_2 are of order M_{GUT}; M_5, μ_{ij} and $\tilde{m}^2$ are assumed to be of order m_W.

SU(5) BREAKING

Let us first consider the SU(5) symmetry breaking. For this purpose it is convenient and a reasonable approximation to neglect $\delta\Omega_{\mathrm{SOFT}}$ and minimize the scalar potential. We find

$$
\langle 24 \rangle \sim \begin{bmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{bmatrix} V \qquad\qquad (3.2)
$$

is a possible SUSY ground state. In this ground state SU(5) is broken to SU(3)⊗SU(2)⊗U(1). In V_5, the states in V_3, V_2 and V_1 remain massless; the others obtain the mass M_{GUT}. All the states in 24 also obtain mass of order M_{GUT}.

SECOND HIERARCHY PROBLEM

In order to avoid rapid nucleon decay the color triplet Higgs must obtain masses of order 10^{11} Gev for scalars and 10^{17} Gev for their fermionic partners. The Higgs doublets, on the other hand, must have mass of order m_W. In this model we accomplish this mass splitting with the term

$$\overline{H} \left[\lambda_2 \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix} V + M_2 \right] H . \qquad (3.3)$$

H_3, $\overline{H}_3$ have mass $2\lambda_2 V + M_2 \sim M_{GUT}$ and we fine tune the parameters λ_2, M_2 such that the Higgs doublets H_2, $\overline{H}_2$ have mass $-3\lambda_2 V + M_2 \sim m_W$. This fine-tuning, though ugly, is ,nevertheless, "technically natural". We also require the additional fine-tuning that $f(24,H,\overline{H})$ is proportional to the term in the superspace potential containing H, $\overline{H}$ and 24 i.e. eq.(2.18).

RENORMALIZATION GROUP EQUATIONS

Below M_{GUT} the effective field theory is given by the states of Table 1 with the Lagrangian of equations (2.1-2.3). At M_{GUT} some of the parameters of the theory are fixed by symmetry relations. In particular, at M_{GUT} we have

$$M_1 = M_2 = M_3 = M_5 \quad \text{,gaugino masses are equal} \qquad (3.4)$$
$$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_5 \quad \text{,gauge couplings are equal}$$
$$m_b = m_\tau ,$$
$$m_s = m_\mu ,$$
$$m_d = m_e .$$

We now want to use the renormalization group equations to evaluate these parameters at the weak scale m_W. [For a review of the RGE's , see Srednicki, this school.] We first consider the gauge coupling constants. The RGE at one loop is given by

$$\frac{d}{dt} \alpha_n = -\frac{1}{2} b_n \alpha_n^2 \qquad (3.5)$$
$$t = \frac{1}{2\pi} \ln M/m_W .$$

The renormalization group parameters b_n for gauge group $SU(n)$ and representations R_n is given by

$$b_n = \frac{11}{3} C_2(SU(n)) - \frac{2}{3} T(R_n) N_f - \frac{1}{3} T(R_n) N_s \qquad (3.6)$$

where $C_2(G)$ is the quadratic casimir for the group G, $T(R_n)$ is defined by the expression $Tr(T^a T^b) = T(R_n)\delta^{ab}$ for the generators T^a in the representation R_n and T(fundamental) $= \frac{1}{2}$. $N_{f(s)}$ are the number of fermion (scalar) representations R_n. The formula (3.6) is general; for SUSY we can re-express (3.6) in terms of vector and chiral multiplets. In this case we have

$$b_n = 3\,C_2(SU(n)) - T(R_n)\,N_{CHIRAL} \qquad (3.7)$$

where the first term is the contribution of the vector multiplet and N_{CHIRAL} is the number of chiral multiplets in the representation R_n. The final result is given by

$$\begin{aligned} b_3 &= 9 - 2n_g \\ b_2 &= 6 - 2n_g - n_H \\ b_1 &= -2n_g - \tfrac{3}{5}n_H \end{aligned} \qquad (3.8)$$

where n_g is the number of generations and n_H is the number of Higgs pairs (H_2, $\overline{H}_2$). The result of running the gauge couplings α_n from M_{OUT} to m_W is depicted in figure 12.

figure 12

<u>M_{OUT}</u>

We find the value of M_{OUT} (for $\Lambda_{\overline{MS}} = 100$ Mev)[19]

$$M_{OUT}\,(\text{SUSY}) = 5 \times 10^{15}\ \text{Gev} \qquad (3.9)$$

$$M_{OUT}\,(\text{non-SUSY}) = 1.5 \times 10^{14}\ \text{Gev}.$$

Thus in SUSY SU(5), M_{OUT} is about 30 times larger than in non-SUSY SU(5).

<u>$SIN^2\Theta_W$</u>

We find the value of $\sin^2\Theta_W$[19]

$$\sin^2\theta_W(m_W) = \frac{b_3 - b_2 + \frac{5}{3}(b_2 - b_1)\,\alpha(m_W)/\alpha_3(m_W)}{b_3 - b_2 + \frac{5}{3}(b_3 - b_1)} \qquad (3.10)$$

Note that the contribution of n_g cancels in the differences of b_n's and the contribution of the gauge sector for SUSY as compared to non-SUSY cancels in the ratio. Thus , if we ignore the Higgs contribution, we find $\sin^2\theta_W(m_W)|_{SUSY\ GUT} = \sin^2\theta_W(m_W)|_{GUT}$. Including the Higgs we obtain

$$\sin^2\theta_W(m_W) = .238 \qquad \text{(SUSY)} \qquad (3.11)$$

$$\sin^2\theta_W(m_W) = .218 \qquad \text{(non-SUSY)} .$$

MASS RATIOS

At m_W the gaugino masses are in the ratio

$$M_3 : M_2 : M_1 = \alpha_3 : \alpha_2 : \alpha_1 \qquad (3.12)$$

as considered previously in equation (1.50). As a result we obtain the gluino-photino mass ratio

$$m_{\tilde{g}}/m_{\tilde{\gamma}} = \alpha_3/(\tfrac{8}{3}\alpha_2 \sin^2\theta_W) \qquad (3.13)$$

in the limit M_2 , $\mu \ll m_Z$.

For the quark-lepton mass ratio, m_b/m_τ, the result agrees with that of non-SUSY SU(5) (see Einhorn and Jones, ref.[19]).

PROTON DECAY

Dimension 4,5 and 6 operators can in principle contribute to nucleon decay in SUSY GUTs[20]. This is in contrast to non-SUSY theories where dimension 6 operators are the lowest dimension baryon violating operators consistent with $SU(3)\otimes SU(2)\otimes U(1)$.

GAUGE EXCHANGE

The dimension 6 contributions due to gauge exchange are naturally suppressed in SUSY GUTs as a consequence of the increased value of M_{GUT}, as discussed previosly (see eqn. (3.9)). [Recall that the proton lifetime $\tau_p \sim M_{GUT}^4/m_p^5$.] We find that the contribution to the proton lifetime due to gauge exchange is

$$\tau_p \sim 5 \times 10^{28\pm1} \text{ years} \qquad \text{non-SUSY SU(5)} \tag{3.14}$$

$$\tau_p \sim 4 \times 10^{34\pm1} \text{ years} \qquad \text{SUSY SU(5)}$$

(see Marciano and Senjanovic, ref.[19]).

NEW PROCESSES

As stated under our discussion of the symmetries of the effective low energy theory we have explicitly deleted any possible dimension 4 baryon and/or lepton violating operators. This is because, in order to be consistent with the observed nucleon lifetime ,of order 10^{31} years, such operators must be extremely suppressed. It is possible to forbid such operators by introducing new symmetries into the theory.* As an example, R-parity, introduced previously, does forbid all the dangerous baryon and/or lepton violating dimension 4 operators.

It has been shown that there are also dimension 5 baryon violating operators which are invariant under $SU(3)\otimes SU(2)\otimes U(1)\otimes SUSY$[20]. These are listed below.

$$\int d^2\Theta\, Q_i Q_j Q_k L_l \equiv A_{ijkl} \tag{3.15a}$$

$$\int d^2\Theta\, \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_l \equiv B_{ijkl}$$

$$\int d^2\Theta\, Q_i Q_j Q_k \overline{H}_2 \tag{3.15b}$$

$$\int d^4\Theta\, (Q_i Q_j \overline{D}_k^{\dagger} + \text{h.c.})$$

If we demand the symmetry R-parity to forbid dimension 4 operators then the operators in equation (3.15b) are also forbidden. We shall thus assume that only the operators of (3.15a) are present in the

*Note that even though it may be "technically natural" to delete dimension 4 operators in an effective low energy theory (meaning they will never be generated in any order in perturbation theory in the effective low energy theory), one must recognize that such operators can be generated at the tree level when one integrates out states with mass of order M_{GUT} in the process of defining the effective low energy theory. One would thus feel assured if there were a symmetry in the GUT theory which forbid this possibility.

effective low energy theory. Note, these operators are invariant under the symmetry B - L. One can now show that the operators A and B of (3.15a) satisfy the relation

$$A_{ijkl} = B_{ijkl} = 0 \quad \underline{\text{if}} \quad i = j = k = l. \qquad (3.16)$$

As a result (assuming these operators are the main contributors to nucleon decay) one concludes that the dominant decay modes will contain at least one second generation particle in the final state[21].

Using only dimensional analysis one might conclude that dimension 5 operators are dangerous and must also be forbidden by requiring a new symmetry. We shall show that this is not the case when we consider the origin of these operators Consider the graph of figure 13.

figure 13

The color triplet Higgs with mass of order M_{GUT} is exchanged. We obtain the effective dimension 5 operator

$$(1/M)\{D(K^T\lambda_u)U + U(\lambda_u K)D\}\{V(\lambda_d)D + E(\lambda_d K^\dagger)U\}$$
$$+ (1/M)\{\bar{E}(K^T\lambda_u)\bar{U} + \bar{U}(\lambda_u K)\bar{E}\}\{\bar{D}(\lambda_d K^\dagger)\bar{U}\} \qquad (3.17)$$

where K is the Cabibbo-Kobayashi-Maskawa matrix and λ_u and λ_d are diagonal Yukawa couplings. In order to calculate a nucleon decay rate one must then dress these dimension 5 operators at the weak scale to obtain more familiar dimension 6 four fermi operators. In figure 14 we give the graph which most likely gives the dominant contribution.

figure 14

$p \to k^+ \bar{\nu}_\mu$

$(n \to k^0 \bar{\nu}_\mu)$

The dimension 5 operator in this case is

$$\sim\left(m_c m_s \sin\theta_C/\nu\bar{\nu}M_{GUT}\right)\tilde{d}\,\tilde{c}\,s\,\nu_\mu \tag{3.18}$$

and the resulting dimension 6 operator is

$$\sim\left(\alpha_2\, m_c m_s \sin\theta_C{}^2/\nu\bar{\nu}\, M_{GUT}\tilde{m}\right) d\,u\,s\,\nu_\mu \tag{3.19}$$

where θ_C is the Cabibbo angle and $\tilde{m}$ is a typical superpartner mass. One sees that dimension 5 operators are sufficiently suppressed ,due to the small Yukawa couplings and one loop factors, to give acceptable contributions to nucleon decay[21,22]. In fact, with reasonable values of the parameters one expects to see an effect in the present nucleon decay experiments.

In figure 15 we present a graph which can be large and perhaps dominate for top quark masses of order 40 Gev[23].

figure 15

$p \rightarrow k^+\,\bar{\nu}_\mu$

The effective dimension 5 operator in this case is given by

$$\sim\left(m_t m_s \sin\theta_C K_{ts}/\nu\bar{\nu}\, M_{GUT}\right) s\,\tilde{t}\,\mu\,\tilde{u} \tag{3.20}$$

and the effective dimension 6 operator by

$$\sim\left(\alpha_3\, m_t m_s \sin\theta_C K_{ts}/\nu\bar{\nu}\, M_{GUT}\right)\left(\delta m^2_{\tilde{u}\tilde{t}}/\tilde{m}^2\right) s\,u\,\mu\,u. \tag{3.21}$$

$\delta m^2_{\tilde{u}\tilde{t}}$ is the $\tilde{u}$-$\tilde{t}$ mixing mass which in the standard model can be large as a result of renormalization group effects(see section 4). If the, as yet unmeasured, mixing angle K_{ts} is of order $\sin\theta_C{}^2$ and $m_t \geq 40$ Gev, then the process of figure 15 can dominate.

Finally, in figure 16 we present one more set of graphs which could in principle be significant in a particular range of parameters.

figure 16

$p \rightarrow k^0\,\mu^+$

The surprising fact is that these three graphs identically cancel each other if $m_{\tilde{u}} = m_{\tilde{s}} = m_{\tilde{d}}$[24]. However $(m_{\tilde{u}}^2 - m_{\tilde{d}}^2)$ is proportional to $M_W^2(v^2 - \bar{v}^2)/(v^2 + \bar{v}^2)$ and could in fact be large[25].

The preceding discussion illustrates some of the uncertainties inherent in any prediction for nucleon decay in SUSY GUTs[26]. Unlike the contribution of gauge exchange which is fully determined by the dynamics of unification, the new processes depend on both the unknown Higgs triplet mass and on the values of masses and mixing angles of the as yet unobserved superpartners.

4. SUPERSYMMETRY BREAKING

In this lecture we will consider mechanisms for spontaneous supersymmetry breaking in both rigid (or global) SUSY and in local SUSY (or supergravity). We shall conclude with a discussion of the minimal low energy supergravity model.

RIGID (OR GLOBAL) SUSY

We shall discuss three possible mechanisms for spontaneous SUSY breaking, going under the names O'Raifeartaigh mechanism, Fayet-Iliopoulos mechanism and dynamical mechanism. The first two mechanisms occur at the tree level. The last requires non-perturbative dynamics for its success.

1) O'Raifeartaigh mechanism

Consider the chiral superfields A, B, C and the lagrangian

$$\mathcal{L} = \int d^4\theta \, (A^*A + B^*B + C^*C) + \int d^2\theta \, W + h.c.; \qquad (4.1)$$

$$W = A(B^2 + M^2) + \mu C B.$$

Recall equation (1.34), the scalar potential is given by

$$V = |\partial W/\partial A|^2 + |\partial W/\partial B|^2 + |\partial W/\partial C|^2 . \qquad (4.2)$$

Thus supersymmetry requires

$$\partial W/\partial A = b^2 + M^2 = 0$$
$$\partial W/\partial B = 2ab + \mu c = 0 \qquad (4.3)$$
$$\partial W/\partial C = \mu b = 0$$

where a, b, c are the scalar components of A, B, C, respectively. It is easy to see that the three equations (4.3) cannot be satisfied simultaneously. Thus SUSY is spontaneously broken.

Let us now minimize the potential V. We have

$$V(a, b, c) = \mu^2|b|^2 + |b^2 + M^2|^2 + |2ab + \mu c|^2. \qquad (4.4)$$

Clearly the potential has a flat direction defined by

$$c_0/a_0 = -2b_0/\mu. \qquad (4.5)$$

(A subscript $_0$ denotes vacuum expectation value.) Thus the magnitude of c_0 is undetermined at the tree level. This phenomenon is typical in O'Raifeartaigh type models. The equations for b are as follows

$$\partial V/\partial b^* = \mu^2 b + 2(b^2 + M^2) b^* = 0 \qquad (4.6)$$
$$\partial V/\partial b = \mu^2 b^* + 2(b^{*2} + M^2) b = 0.$$

Recall a, b, c are complex. If M^2 is real then b^2 is real and the solution is

$$b_0 = i\sqrt{M^2 + {}^1/_2\mu^2}. \qquad (4.7)$$

The vacuum energy is given by

$$V(a_0, b_0, c_0) = \mu^2 (M^2 + \mu^2/_2) + \mu^4/_{16} > 0. \qquad (4.8)$$

Let us now discuss the spectrum of this model. We shall consider two limits defined by, a) $a_0 = c_0 = 0$ or b) $a_0, c_0 \gg \mu, M$

(a) $[a_0 = c_0 = 0]$ It is convenient to define

$$B = B' + b_0 \qquad (4.9)$$

$$G = \left(\langle \partial W/\partial A\rangle A + \langle \partial W/\partial C\rangle C\right) N^{-1/2}$$
$$L = \left(-\langle \partial W/\partial C\rangle A + \langle \partial W/\partial A\rangle C\right) N^{-1/2}$$
$$N = \left(\langle \partial W/\partial A\rangle^2 + \langle \partial W/\partial C\rangle^2\right)^{1/2} = V(a_0, b_0, c_0).$$

Note that

$$\langle G\rangle = \theta^2 N^{1/2}\ ,\ \langle L\rangle = 0\ ,\ \langle B'\rangle = 0\ . \qquad (4.10)$$

The lagrangian, written in terms of the new variables, is

$$\mathfrak{L} = \int d^4\theta\,(G^*G + L^*L + B'^*B') + \int d^2\theta\,\tilde{W} + \text{h.c.} \qquad (4.11)$$
$$W = N^{1/2} G + \left(2b_0\langle \partial W/\partial A\rangle + \mu\langle \partial W/\partial C\rangle\right) N^{-1/2} G B'$$
$$+ \left(-2b_0\langle \partial W/\partial C\rangle + \mu\langle \partial W/\partial A\rangle\right) N^{-1/2} L B'$$
$$+ \langle \partial W/\partial A\rangle N^{-1/2} G B'^2 - \langle \partial W/\partial C\rangle N^{-1/2} L B'^2.$$

Note that the coefficient of GB' vanishes identically. This exercise shows that

i) ψ_G is the massless Goldstino and

ii) $V(a_0, b_0, c_0) = |\partial W/\partial G|^2 = N$

which is consistent with the result of eqn.(1.23).

iii) The states contained in B' receive SUSY breaking mass contributions from the interaction $\langle G\rangle B'^2$. At the tree level the mass matrix, nevertheless, satisfies the relation

$$\mathrm{STr}\, M^2 = 0. \qquad (4.12)$$

This result is general and moreover it is a necessary condition for the absence of quadratic divergences in the theory at one loop (recall $\delta\mathfrak{L} \sim (\mathrm{STr}\, M^2)\Lambda^2$).

Note ,in a theory where $W \sim N^{1/2} G + L^2 + L^3$, SUSY is spontaneously broken <u>but</u> the Goldstino decouples and the spectrum of masses remains supersymmetric. In order to avoid this decoupling O'Raifeartaigh showed that one needs at least three chiral superfields as described in (4.1).

b) [$c_0 = -2b_0 a_0/\mu \sim M_{LARGE} \gg \mu,\ M \sim \Lambda_{ss}$] In this limit the superspace potential takes the form

$$W \sim \Lambda_{ss}{}^2 G + M_{LARGE} B'^2 + \Lambda_{ss} L B' + G B'^2 + L B'^2. \quad (4.13)$$

At the tree level B' has mass of order M_{LARGE} and L has mass of order Λ_{ss}^2/M_{LARGE}. Since however SUSY is broken at the scale $\Lambda_{ss} \gg \Lambda_{ss}^2/M_{LARGE}$, one might expect that L will receive SUSY breaking mass corrections in higher orders of perturbation theory of order Λ_{ss}. This guess is in fact wrong.

DECOUPLING OF SUSY BREAKING[27,28]

The graph contributing to a SUSY breaking mass correction for L ,at one-loop, is given in figure 17. The correction to the

figure 17

effective action is given by

$$\delta\Omega = \left(1/M^2_{LARGE}\right)\int d^4\theta\, G^\dagger G\, L^\dagger L \qquad (4.14)$$

which corresponds to a SUSY breaking mass for the scalar component of L given by

$$\delta m_L^2 \sim \left(\Lambda_{ss}^2/M_{LARGE}\right)^2. \qquad (4.15)$$

This result generalizes to all orders in perturbation theory and thus SUSY breaking effects decouple ,i.e. they are suppressed by the ratio Λ_{ss}/M_{LARGE}.

2) <u>Fayet-Iliopoulos mechanism</u>

Consider an SU(2)⊗U(1) gauge theory with one chiral superfield D, a doublet under SU(2) and charge 1 under U(1). The lagrangian for this theory is given by

$$\Omega = \int d^4\theta\,[\,D^\dagger(\,e^{2g_2 V_2 + 2g_1 V_1}\,)\,D\,] \qquad (4.16)$$
$$+ \int d^2\theta\,[\,\tfrac{1}{8g_2^2}\,Tr(W_2 W_2) + \tfrac{1}{16g_1^2}\,W_1 W_1\,] + g_1\xi D_1.$$

The scalar potential is given by

$$V = \tfrac{1}{2}\,(\,Tr(D_2^{\,2}) + D_1^{\,2}\,). \qquad (4.17)$$

Supersymmetry requires that

$$D_2^{\,i} = g_2\,(\,D^*\tau^i D\,)/2 = 0\;,\; i = 1,2,3 \qquad (4.18)$$
$$D_1 = g_1\,(\,D^* D + \xi\,) = 0.$$

Once again, these equations cannot be satisfied simultaneously and SUSY is spontaneously broken. In this case $\Lambda_{ss} \sim \xi$, and the Goldstino is one linear combination of the gauginos. Note that, in this example, the gauge currents are anomalous (see the triangle diagram of figure 18) and thus this model is inconsistent. It is very difficult in more realistic models to make use of this mechanism. The problem has been that the theory is either anomalous, supersymmetric or color is spontaneously broken.

figure 18

3) Dynamical SUSY breaking[12,29]

Consider an SU(n) gauge theory with N flavors of chiral multiplets Q_i , $\overline{Q}_i$, $i = 1,\ldots,N$. The lagrangian is given by

$$\mathfrak{L} = \int d^2\Theta \left[\frac{1}{8g^2} \mathrm{Tr}(WW) \right] + \mathrm{h.c.}$$
$$+\int d^4\Theta \left[Q_i^\dagger e^{2gV} Q_i + \overline{Q}_i^\dagger e^{-2gV^T} \overline{Q}_i \right]. \qquad (4.19)$$

We have assumed that the superspace potential is zero. Using the SUSY transformations (1.22) we obtain the relation

$$\{Q, (q_i\overline{q}_i + \tilde{q}_i\overline{q}_i)\} = \sqrt{2}(2q_i\overline{q}_i + F_i\overline{q}_i + \tilde{q}_i\overline{F}_i). \qquad (4.20)$$

However, using the equations of motion one finds that $F_i^\dagger = -\partial W/\partial Q_i = 0$ and $\overline{F}_i^\dagger = -\partial W/\partial \overline{Q}_i = 0$. Thus if $\langle q_i\overline{q}_i\rangle \neq 0$, then SUSY is spontaneously broken. The Goldstino would be the composite fermion created by the operator $q_i\overline{q}_i + \tilde{q}_i\overline{q}_i$. Recall , that in QCD a fermion condensate does indeed occur at a scale Λ_{QCD}. In this case, the pseudo-scalar octet, including pions, etc., are the goldstone bosons of a broken chiral symmetry.

Note, if we had introduced a mass term of the form $m\int d^2\Theta\, \overline{Q}_i Q_i$, then $F_i^\dagger = -m\overline{Q}_i$ and $\overline{F}_i^\dagger = -mQ_i$. In this case , the second and third terms on the right hand side of (4.20) no longer vanish identically and ,moreover, it can be shown, by using the

Witten index theorem, that SUSY cannot be spontaneously broken. Thus the entire RHS of (4.20) necessarily vanishes identically for any non-zero m. It is clear that if SUSY is spontaneously broken for $m=0$, then the $m=0$ theory is not obtainable as the limit of the $m \neq 0$ theory. We shall discuss the Witten index theorem in more detail shortly.

Gluino condensates can also spontaneously break SUSY. Consider the equal-time anti-commutator

$$\{\overline{Q}, q^{\dagger}_{i}\tilde{q}_{i}\} = \sqrt{2}\left(-\tilde{q}_{i}\,\partial W/\partial\tilde{q}_{i} + \frac{g}{8}\pi^{2}{}_{2}(\tilde{g}\tilde{g})\right) \qquad (4.21)$$

where the last term is the Konishi, Piguet, Sibold anomaly[30]. The anomaly is a consequence of the anomalous equation of motion

$$-\frac{1}{4}\overline{D}^{2}(Q^{\dagger}_{i}e^{2gV}Q_{i}) = -Q_{i}\,\partial W/\partial Q_{i} + \frac{g}{8}\pi^{2}{}_{2}\,(W^{\alpha}W_{\alpha}). \qquad (4.22)$$

Clearly, if $W=0$ and $\langle\tilde{g}\tilde{g}\rangle \neq 0$, then SUSY is spontaneously broken.

In the two preceding examples of dynamical symmetry breaking it has been assumed that the superspace potential W is identically zero. As a result of the non-renormalization theorem, $W=0$ at the tree level then it vanishes to all orders in perturbation theory. However, recently it has been shown that non-perturbative effects, such as instantons, can contribute to W. Thus great care must be taken when reaching any conclusion based on a vanishing W. Affleck et.al.[29] have used such non-perturbative corrections to W to obtain an explicit example of dynamical SUSY breaking in a "perturbative" regime.

WITTEN INDEX THEOREM

Supersymmetry implies the existence of an equal number of boson and fermion states with non-zero energy E. Consider the vacuum state $|0\rangle$. If the vacuum is supersymmetric then

$$H\,|0\rangle = Q_{\alpha}\,|0\rangle = 0. \qquad (4.23)$$

The energy levels of a supersymmetric theory are depicted in figure 19. Let us now define the Witten index Λ.

figure 19

$$\Delta \equiv Tr(-1)^F = Tr\left(e^{-\beta H}(-1)^F\right) \qquad (4.24)$$
$$= n_B^{E=0} - n_F^{E=0}$$

where F is fermion number and $n_B^{E=0}$ ($n_F^{E=0}$) is the number of zero energy boson(fermion) states. If $\Delta \neq 0$ then SUSY is unbroken ,i.e. a SUSY vacuum clearly exists. If $\Delta = 0$ then SUSY may or may not be broken. Witten has used this criteria to show that an SU(n) gauge theory with N massive flavors , Q_i , $\overline{Q}_i$, $i = 1,\ldots,N$ must be supersymmetric since $\Delta = n$. The limit $m \rightarrow 0$ may be non-uniform, as discussed previously.

LOCAL SUPERSYMMETRY ⇒ SUPERGRAVITY [SUGRA]

In this section title we indicate that local supersymmetry implies supergravity. We would like to motivate this assertion. Consider the transition from a global to a local symmetry for an abelian gauge theory. In particular consider the Lagrangian

$$\mathfrak{L} = i\,\psi^{\dagger} \not{\partial}\, \psi \qquad (4.25)$$

with the global phase invariance

$$\psi \rightarrow e^{i\alpha}\,\psi\ , \qquad (4.26)$$

with α, a space-time independent parameter. We now want to let $\alpha \rightarrow \alpha(x)$;i.e. α is now space-time dependent. The variation of $\mathfrak{L}$ under this transformation is given by

$$\delta\mathfrak{L} = -\partial_\mu\,\alpha(x)\,\psi^{\dagger}\gamma^\mu\,\psi \;\equiv\; -\partial_\mu\,\alpha(x)\,J^\mu(x) \qquad (4.27)$$

where J^μ is the conserved Noether current. In order to obtain a Lagrangian which is invariant under the local symmetry we must add to $\mathfrak{L}$ a term

$$\mathfrak{L}_g = J^\mu A_\mu \qquad (4.28)$$

such that the gauge field $A_\mu(x)$ transforms as

$$A_\mu \to A_\mu + \partial_\mu \alpha(x). \tag{4.29}$$

The resulting Lagrangian

$$\mathfrak{L} = i\,\psi^\dagger \gamma^\mu (\partial_\mu - i A_\mu)\,\psi \tag{4.30}$$

is gauge invariant.