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PARAMETRIC INSTABILITIES EXCITED BY LOCALIZED PUMPS NEAR THE LOWER-HYBRID FREQUENCY

BY

Y. Y. KUO AND LIU CHEN

PLASMA PHYSICS LABORATORY

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

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Parametric Instabilities Excited by Localized Pumps Near the Lower-Hybrid Frequency

Y. Y. Kuo and Liu Chen

Plasma Physics Laboratory, Princeton University,

Princeton, New Jersey 08540

ABSTRACT

Parametric instabilities excited in non-uniform plasmas by spatially localized pump fields oscillating near the local lower-hybrid frequency are analytically investigated. Corresponding threshold conditions, temporal growth rates, and spatial amplification factors are obtained for the oscillating-two-stream instability and the decay instabilities due to nonlinear electron and ion Landau dampings.

I. INTRODUCTION

Plasma heating using rf waves with frequency near the lower-hybrid frequency has attracted much theoretical and experimental interest. Currently, it is well known that nonlinear processes such as parametric instabilities play crucial roles in this heating scheme. While there has been extensive theoretical studies on the parametric instabilities in this frequency range, most of them are done with uniform plasmas and pumps of infinite spatial extent.[1] Yet, in realistic situations, the plasmas are always nonuniform and the pump waves are usually of finite spatial extent. Furthermore, studies on laser-plasma interactions have shown that both the plasma nonuniformities and spatial localization of pumps significantly modify the nature of parametric processes. It is, therefore, important to investigate these two effects on the lower-hybrid parametric instabilities in order to develop a better understanding of the heating processes.

Resonant and nonresonant (ion quasimode) decay instabilities in the WKB approximation were examined by Porkolab.[2] In the present work, we first study the oscillating-two-stream instability (OTSI) in the WKB approximation. We then investigate the nonresonant decay processes when the WKB approximation is invalid. We find that for sufficiently strong pumps the instability may become absolute (i.e., temporally growing) instead of being convective (i.e., spatially amplifying) as predicted with the WKB approximation.[2]

The basic set of equations and the theory of OTSI are presented in Sec. II. Section III contains the theory of nonresonant decay instabilities. Final conclusions and discussions are in Sec. IV.

II. OSCILLATING-TWO-STREAM INSTABILITY

We consider an inhomogeneous plasma having a density gradient in the x direction with scale length L. The pump is assumed to be spatially confined in the x direction and its frequency ω_o is close to the local lower-hybrid frequency. Since the waves considered here are electrostatic, the plasma dynamics can be described by the equation of motion, the continuity equation and Poisson's equation,

$$\frac{\partial}{\partial t}\vec{V}_s + (\vec{V}_s\cdot\vec{\nabla})\vec{V}_s + \Gamma_s\vec{V}_s = -\frac{\vec{\nabla}(\gamma_s n_s T_s)}{n_s m_s} + \frac{q_s}{m_s}\left(-\vec{\nabla}\phi + \frac{\vec{V}_s\times\vec{B}_o}{C}\right), \quad (1)$$

$$\frac{\partial n_s}{\partial t} + \vec{\nabla}\cdot(n_s\vec{V}_s) = 0 \quad , \quad (2)$$

and

$$\nabla^2\phi = -4\pi\sum_s n_s q_s \; , \qquad s = e,i \; , \quad (3)$$

where $\vec{V}_s$ denotes the velocity, n_s the density, Γ_s the damping rate, T_s the temperature, q_s the charge, m_s the mass, and γ_s the ratio of the specific heat of the s species. The external magnetic field $\vec{B}_o$ is in the z direction and ϕ is the electrostatic potential. Let the pump frequency satisfy the condition that $\omega_{ce} >> \omega_o >> \omega_{ci}$, where ω_{ce}, ω_{ci} are the electron and ion cyclotron frequencies, the ions thus respond as unmagnetized to both the pump field and all the high frequency modes which have frequencies

$\omega_h \sim \omega_o$. For the low-frequency modes, we assume that the frequencies ω are much smaller than ω_{ce} , but larger than ω_{ci} . The case for $\omega << \omega_{ci}$ will be presented later. The low-frequency modes are coupled to the high-frequency ones through the pump via the electron $(\nabla_\perp \cdot \nabla) V_\parallel$ term, where $V_\perp$ is dominantly the $\vec{E} \times \vec{B}$ drift velocity. On the other hand, the dominant nonlinear contribution to the high-frequency modes comes from the coupling between the pump and the low-frequency electron density perturbation, n^ℓ . The coupled equations, by taking $\phi^h \sim \exp(- i\omega_h t + ik_y y + ik_\parallel z)$ and $n^\ell \sim \exp(- i\omega t + ik_y y + ik_\parallel z)$ are found as

$$\left(\frac{\partial^2}{\partial t^2} + \Gamma_i \frac{\partial}{\partial t} - \gamma_i V_i^2 \nabla^2\right)\left(a\nabla_\perp^2 + b \frac{\partial^2}{\partial z^2}\right)\phi^h + \omega_{pi}^2 \left(1 + \frac{x}{L}\right) \nabla^2 \phi^h$$

$$= \left(\frac{\partial^2}{\partial t^2} + \Gamma_i \frac{\partial}{\partial t} - \gamma_i V_i^2 \nabla^2\right)\left\{\frac{d}{N_o} \nabla \cdot \left[n^\ell \left(\vec{E}_o + \frac{\vec{V}_{oe} \times \vec{B}_o}{C}\right)\right]\right\}$$

$$+ \frac{\omega_{pi}^2}{N_o} \vec{\nabla} \cdot (n^\ell \vec{E}_o) \quad , \tag{4}$$

$$\left(\frac{\partial^2}{\partial t^2} + \Gamma_2 \frac{\partial}{\partial t} - C_s^2 \nabla^2\right) n^\ell$$

$$= \frac{n_o e^2}{m_e m_i}\left[\left(- \frac{1}{\omega_{ce}^2} + \frac{m_e}{m_i} \frac{1}{\omega_o^2}\right) \frac{\partial}{\partial x} \nabla^2 + \frac{1}{i\omega_h \omega_{ce}} \frac{\partial}{\partial y} \nabla^2 \right] E_o \phi^h \quad , \tag{5}$$

where

$$d = \frac{\omega_{pe}^2}{-\omega_h^2 - i\omega_h \Gamma_e} , \tag{6}$$

$$a = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} \left(1 + \frac{x}{L}\right) , \tag{7}$$

and

$$b = 1 + d\left(1 + \frac{x}{L}\right) \simeq d\left(1 + \frac{x}{L}\right) . \tag{8}$$

Here, V_e , V_i are the electron and ion thermal velocities, N_o , ω_{pe} , ω_{pi} are the background plasma density and the plasma frequencies at $x = 0$, V_{oe} is the excursion velocity of electrons caused by the pump, $\Gamma_2 = [\Gamma_i + (k^2 m_e)/(k_{||}^2 m_i)\Gamma_e]$ is the damping decrement, and $C_s^2 = (\gamma_i T_i + \gamma_e T_e)/m_i$ is the ion sound velocity. In deriving Eqs. (4) and (5), we also assume $|k_\perp V_e/\omega_{ce}| << 1$, $k_\perp^2 >> k_{||}^2$, $\omega_{pe} \sim \omega_{ce}$, and $|\omega| << |k_{||}V_e| << \omega_h$.

In order to solve the coupled equations analytically, we proceed in the WKB approximation by letting

$$\phi^h \sim \hat{\phi}_\pm(x) \exp[- i(\omega \mp \omega_o)t + ik_x x + ik_y y + ik_{||}z] , \tag{9}$$

$$n^\ell \sim \hat{n}(x) \exp[- i\omega t + ik_x x + ik_y y + ik_{||}z] , \tag{10}$$

$$E_o \sim E(x) \exp(i\omega_o t) + c. c. . \tag{11}$$

where k_x is much larger than the quantities $1/L$, $(\partial\hat{\phi}/\partial x)/\hat{\phi}$, $(\partial\hat{n}/\partial x)/\hat{n}$, and $(\partial E/\partial x)/E$. Substituting the above expressions into Eqs. (4) and (5), we obtain

$$\left(\frac{d}{dx} + \beta_1 + \beta\right)\hat{\phi}_+ = \eta\hat{n}\,E \ , \tag{12}$$

$$\left(\frac{d}{dx} + \beta_1 - \beta\right)\hat{\phi}_- = -\,\eta\hat{n}\,E^* \ , \tag{13}$$

and

$$\hat{n} = \alpha(E\hat{\phi}_- - E^*\hat{\phi}_+) \ , \tag{14}$$

where

$$\beta_1 = \frac{+\frac{x}{L}(-k^2\omega_o^2 + \delta^2k^2) + \delta^2k^2\Theta}{2ik_x\Lambda^2} \ , \tag{15}$$

$$\beta = \frac{(2\omega + i\Gamma_1)}{2iV_{gx}} \ , \tag{16}$$

$$V_{gx} = -\frac{k_x\ \Lambda^2}{\omega_o k_\perp^2\Theta} \ , \tag{17}$$

$$\eta = \frac{-\,k_y\omega_o\omega_{pe}^2}{2ik_x\Lambda^2N_o\omega_{ce}} \ , \tag{18}$$

$$\alpha = \frac{-N_o e^2 k^2 k_y}{m_e m_i(-\omega^2 - i\omega\Gamma_2 + C_s^2 k^2)\omega_o \omega_{ce}} , \tag{19}$$

$$\Theta = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} , \tag{20}$$

$$\Lambda^2 = \gamma_i V_i^2 \left(1 + \frac{\omega_{pe}^2}{\omega_{ce}^2}\right) k_\perp^2 - \frac{k_\parallel^2}{k^2}\omega_{pe}^2 + \delta^2 , \tag{21}$$

δ^2 is the frequency mismatch defined by

$$\omega_o^2 = \omega_{LH}^2 \ (x = 0)\left(1 + \frac{k_\parallel^2 m_i}{k_\perp^2 m_e}\right) + \gamma_i V_i^2 k^2 + \delta^2 , \tag{22}$$

and Γ_1 is the linear damping rate of the high-frequency mode. The only explicit dependence on the scale length L of the coupled equations is through β_1 , which corresponds to wavenumber mismatch due to plasma inhomogeneities. It is obvious that, using the expression for $\hat{n}$ in (14), the differential Eqs. (12) and (13) are linear functions of $\hat{\phi}_\pm$, and the β_1 term can be easily eliminated by a transformation $\hat{\phi}_\pm \sim \exp(-\int \beta_1 \, dx)$. Therefore, the scale length of the background plasma in our WKB approximation does not play a direct role in determining the eigenfrequency and the region of localization for the instabilities. The importance of L may come in indirectly through the localization of the pump caused by the background plasma density gradient. We also want to point out that the mismatch δ^2 (which is understood to be much less than ω_o^2) can have any importance at all only if the condition $(m_i k_\parallel^2/m_e k^2) \simeq k_x^2 \lambda_{Di}^2$ is satisfied.

Let us write the pump field in the form of

$$E(x) = \tilde{E}\, f(x)$$

and make the transformation $\hat{\phi}_{\pm} = B_{\pm}\, e^{\int(-\beta_1 + i\lambda|f|^2 + f^*)dx}$, we obtain from Eq. (12) - (14) the second order differential equation

$$\left(\frac{d^2}{dx^2} + \lambda^2|f|^4 - \beta^2 - 2f^* \frac{\beta}{\Delta}\right)B_- = 0 \quad , \tag{23}$$

where

$$\lambda = i\eta\alpha|\tilde{E}|^2 = \frac{U_o^2 k_y^2 \lambda_{De}^2 k^2 \omega_{pi}^4}{2C_s^2 k_x \Lambda^2(-\,\omega^2 - i\omega\Gamma_2 + C_s^2 k^2)} \quad , \tag{24}$$

with $$U_o = \frac{C|\tilde{E}|}{B} \;.$$

So far we have derived a general formula for OTSI without specifying the spatial dependence of the pump wave. It is essential to know the function f(x) in order to calculate the turning points which in turn determines the growth rate and the spatial extent of localization for the instabilities. Because the formalism of Eq. (23) resembles a great deal to that for a Langmuir pump derived by Kuo et al.[3] except that β is replaced by $-\beta$, we are going to employ the same Lorentzien type of pump and quote the analytic solution from there directly to gain some insight into the general behavior of the threshold and the growth rate for OTSI in various pump strengths.

Let $f(x) = 1/(\frac{x}{\Delta} + i)$, where Δ is the width of the pump, which can be either $(L\lambda_{Di}^2)^{1/3}$ for the pump that obliquely incidents upon the resonant layer, or, in a crude approximation, the width of the resonance cone for a lower-hybrid pump. The instabilities are differentiated into two categories according to whether the turning points x_T for the instability are larger or smaller than Δ ; i.e., whether the instabilities are localized within or outside the pump width. And for each category we have calculated for the case: (i) $\omega << C_s k$, when the pump power is near threshold, and (ii) $\omega >> C_s k$, when the pump power is high that the growth rate is very large.

The results obtained are tabulated in Table I. We find that near the threshold, the waves confined within the pump width ($x_T < \Delta$) will grow as U_o^2 , while those outside ($x_T > \Delta$) will grow as $U_o^{4/3}$. The instabilities are stabilized not only by the usual linear damping but also by the convective loss due to the localization of the pump. Another feature of the instabilities in the presence of a finite pump is the real frequency shift ω_r for OTSI. Both ω_r and the convective loss vanish when $\Delta \to \infty$. Recall that in our calculation, the background plasma density scale length L does not play a direct role in evaluating the eigenvalue ω , therefore, our results should hold true also for the OTSI generated by a finite pump in a homogeneous plasma. The mismatch δ^2 contributes to the convective loss; i.e., the larger the value of δ^2 is, the higher the threshold value of the instability will be. However, if the pump power remains the same, there would be a threshold region for the pump frequency ω_o , so that the mismatch δ^2 could be minimized to destabilize the instabilities. The threshold pump power for the modes with $x_T > \Delta$ is slightly larger than that with $x_T < \Delta$.

As to the case of a high pump power that $\omega >> C_s k$, the waves will be localized within Δ and grow as $U_o^{2/3}$ if $(k_x \Lambda^2/\omega_{pi}\omega_o k^2 k\Delta\lambda_{Di}) < 1$; and be outside of Δ and grow as U_o^2 otherwise. Finally, as we had in the case for a Langmuir pump, the localization of the waves whose turning points are outside of Δ , is in the overdense region of the peak of the pump when $\omega << C_s k$, and shift toward the peak when $\omega >> C_s k$.

Before we go on to the nonresonant decay instabilities, we would like to present the calculation for OTSI when $\omega << \omega_{ci}$. Equation (4) is still true for the high frequency mode, while Eq. (5) for the low-frequency ones needs to be modified as

$$\left(\frac{\partial^2}{\partial t^2} + \Gamma_2 \frac{\partial}{\partial t} - c_s^2 \frac{\partial^2}{\partial z^2}\right) n^{\ell} = \frac{n_o e^2}{m_e m_i}\left[\left(- \frac{1}{\omega_{ce}^2} + \frac{m_e}{m_i}\frac{1}{\omega_o^2}\right)\frac{\partial^2}{\partial x}\frac{\partial^2}{\partial z^2} + \frac{1}{i\omega_h\omega_{ce}}\frac{\partial}{\partial y}\frac{\partial^2}{\partial z^2}\right] E_o \phi^h \quad , \tag{25}$$

which changes the expression λ of Eq. (24) to

$$\lambda = \frac{\omega_o^2 k_y^2 \lambda_{De}^2 k_{||}^2 \omega_{pi}^4}{2c_s^2 k_x \Lambda^2 \, (-\,\omega^2 - i\omega\Gamma_2 + c_s^2 k_{||}^2)} \tag{26}$$

and Γ_2 to $(\Gamma_i + m_e\Gamma_e/m_i)$. Then all of the previous calculations can be followed through to investigate the frequency ranges of (i) $\omega \ll c_s k_{||}$, and (ii) $\omega \gg c_s k_{||}$. The obtained results are tabulated in Table II.

III. NONRESONANT DECAY INSTABILITIES

In this section, we investigate the nonresonant decay instabilities due to nonlinear electron and ion Landau dampings; i.e., the induced scattering processes (also called the quasimode decays). For this decay, we ignore the upper sideband as being off-resonant; that is, according to Eq. (9) with ω_r , $\omega_o > 0$, we ignore $\hat{\phi}_-$. We then have from Eq. (4) the following equation for the lower sideband $\hat{\phi}_+$,

$$\left[\frac{\gamma_i v_i^2}{\omega_+^2}\left(1+\omega_{pe}^2/\omega_{ce}^2\right)\nabla_\perp^4+\left(1+\frac{\omega_{pe}^2}{\omega_{ce}^2}-\frac{\omega_{pi}^2}{\omega_+^2}\right)\nabla_\perp^2+k_\parallel^2\frac{\omega_{pe}^2}{\omega_+^2}\right]\hat{\phi}_+=-\frac{4\pi eck_yE}{\omega_+B_o}\hat{n} \quad . \tag{25}$$

Here, $\omega_+ \cong \omega - \omega_o + i\Gamma_1/2$ and $\nabla_\perp^2 = d^2/dx^2 - k_y^2$. To describe the low-frequency density response $\hat{n}$ to the high-frequency fields, we have to use the Vlasov equation for both electrons and ions in order to include the effect of nonlinear Landau damping. Since, as mentioned in the preceding section, the dominant nonlinear coupling comes from the parallel (to $\vec{B}_o$) ponderomotive force on electrons, $-m_e[(\vec{V}\cdot\vec{\nabla})V_\parallel]_e^\ell$, we can use this force along with the self-consistent electric field in the electron one-dimensional (in $\vec{B}_o$ direction) Vlasov equation to obtain the electron density response. As to the effectively unmagnetized ($\omega > \omega_{ci}$) ions, they only respond linearly to the self-consistent field. Substituting these density responses into Poisson's equation, we obtain a relation between the self-consistent field and the ponderomotive field produced by the high-frequency fields, which in turn gives the following relation between the low-frequency density perturbation $\hat{n}$ and the high-frequency fields

$$4\pi e\varepsilon(\vec{k}\ ,\ \omega\ ,\ x)\hat{n} = -\ k^2\chi_e\chi_i\ \frac{ck_y E^*}{B\omega_+}\ \hat{\phi}_+\ , \tag{26}$$

where

$$\chi_e = -\ \frac{\omega_{pe}^2(x)}{k^2}\int \frac{df_{oe}/dV_{\parallel}}{V_{\parallel} - \omega/k_{\parallel}}\ dV_{\parallel}\ , \tag{27}$$

$$\chi_i = -\ \frac{\omega_{pi}^2(x)}{k^2}\int \frac{df_{oi}/dV}{V - \omega/k}\ dV\ , \tag{28}$$

and

$$\varepsilon = 1 + \chi_i + \chi_e\ . \tag{29}$$

Note that $\vec{k}$ in general should be interpreted as an operator, $i\vec{k} = \vec{\nabla}$. The procedures used in deriving Eq. (26) is similar to those used by Drake et al.,[3] for unmagnetized plasmas. In the following, we look for solutions of the coupled equations, Eqs. (25) and (26), both with and without the WKB approximation. The WKB solutions, which previously have been studied by Porkolab,[2] are included here to have a complete description of the decay instabilities as well as to present a comparison with the solutions obtained without the WKB approximation.

First, we investigate the solutions with the WKB approximation. Let $\nabla_x = ik_x + d/dx$ and $|k_x| >> |d/dx|$. Equation (25) then reduces to Eq. (12), written in a more apparent form,

$$\left(\frac{d}{dx} + \frac{\Gamma_i/2 - i\omega}{V_{gx}} + \beta_1(x)\right)\hat{\phi}_+ = \eta\hat{n}E \ . \tag{30}$$

Here, V_{gx} and η are defined, respectively, in Eqs. (17) and (18). As to the low-frequency dynamics described by Eq. (26), since it is not a normal mode and the scale length of the pump field Δ is much less than the scale length of the inhomogeneity L , we can simply let $\nabla_x = ik_x$ and use the values at $x = 0$ for χ_e , χ_i and ε ; i.e.,

$$\hat{n} = \frac{k^2\chi_e^0\chi_i^0 ck_y E^*}{4\pi e\omega_o\varepsilon^0 B_o} \ . \tag{31}$$

Here, the superscript 0 denotes quantities evaluated at $x = 0$. Combining Eqs. (30) and (31), we obtain

$$\left(\frac{d}{dx} + F(x)\right)\hat{\phi}_+ = 0 \ , \tag{32}$$

where with

$$|E(x)|^2 = |\tilde{E}|^2 H(x) \ , \tag{33}$$

$$F(x) = \left[\Gamma_1/2 - i\omega - \lambda_o\omega_o H(x)\right]/V_{gx} + \beta_1(x) \ , \tag{34}$$

and

$$\lambda_o = -\frac{i}{2}\left|\frac{c\tilde{E}k_y}{B_o\omega_o}\right|^2\left(\frac{\chi_e^o\chi_i^o}{\varepsilon^o}\right)\left(1+\frac{\omega_{pe}^2}{\omega_{ce}^2}\right)^{-1} \quad .$$

This instability is convective. For a constant source I at $x = x_o$, the solution (assuming $V_{gx} > 0$) is

$$\hat{\phi}_+(x) = I\,\exp\left(-\int_{x_o}^{x} F(x')\,dx'\right) , \quad x > x_o \ . \tag{36}$$

For a typical H(x) with scale length Δ , the spatial amplification factor A is approximately given by

$$A \cong (\gamma_o - \Gamma_1/2)(\Delta/V_{gx}) \ . \tag{37}$$

Here, $\gamma_o = \omega_o \mathrm{Re}(\lambda_o)$ is the growth rate in uniform plasmas due to a dipole pump. Hence, a sufficiently large amplification ($A > 1$) occurs if

$$\gamma_o \gtrsim \frac{V_{gx}}{\Delta} + \frac{\Gamma_1}{2} \ . \tag{38}$$

Because the parametric coupling coefficient λ_o is only proportional to k_y , it is also interesting to examine the decay instabilities when $|k_x\Delta| \sim 1$ such that the WKB approximation becomes invalid. In this case, we assume $|k_y| \gg \left|\frac{d}{dx}\right|$ in order to have appreciable couplings; which is equivalent to assuming

$|k_y\Delta| >> 1$. Expanding Eq. (25) to order (d^2/dx^2) and combining with Eq. (26), we obtain with $z = k_y x$

$$\left(\frac{d^2}{dz^2} + \Omega + K_1 z + AH(z)\right)\hat{\phi}_+ = 0 \ , \tag{39}$$

where

$$\Omega = -\frac{2}{\omega_o}\left(1 + \frac{\omega_{pe}^2}{\omega_{ce}^2}\right)\left(\omega + i\frac{\Gamma_i}{2} - \frac{\delta^2}{2\omega_o}\right)\tilde{\Lambda}^2 \ , \tag{40}$$

$$K_1 = -\tilde{\Lambda}^2/k_y L \ , \tag{41}$$

$$A = \left|\frac{c\tilde{E}k_y}{B_o\omega_o}\right|^2 \left(\frac{\chi_e\chi_i}{\varepsilon}\right)^o \tilde{\Lambda}^2 \ , \tag{42}$$

and

$$\tilde{\Lambda}^2 = \omega_o^2\left(1 + \frac{\omega_{pe}^2}{\omega_{ce}^2}\right)^{-1}\left(k_y^2\gamma_i V_i^2 - \omega_{lh}^2\frac{k_{\parallel}^2 m_i}{k_y^2 m_e}\right)^{-1} \ . \tag{43}$$

To analyze further, we have to know H(z) . Let us assume H(z) is given as

$$H(z) = (1 - z^2/\tilde{\Delta}^2) \ . \tag{43}$$

Note that $\tilde{\Delta} = k_y\Delta$ and Δ is the characteristic width of pump localization. Equation (39) then can be reduced to a parabolic cylinder equation of the form

$$\left[\frac{d^2}{d\xi^2} - \left(\frac{1}{4}\,\xi^2 + a\right)\right]\hat{\phi}_+ = 0 \ , \tag{44}$$

where

$$\xi = (4A/\tilde{\Delta}^2)^{1/4}(z - z_o) \ , \tag{45}$$

$$z_o = K_1\tilde{\Delta}/2A \ , \tag{46}$$

and

$$a = - \left[\tilde{\Delta}^2(1 + \Omega/A) + z_o^2\right](A/4\tilde{\Delta}^2)^{1/2} \ . \tag{47}$$

We, thus, have spatially localized solutions which fall off away from the turning points like

$$\exp(- \xi^2/4) = \exp -\left[\sqrt{A}\; k_y(x - x_o)^2/2\Delta\right] \ , \tag{48}$$

if $a = - (n + \frac{1}{2})$ or

$$k_y\Delta A^{1/2}\left(1 + \frac{\Omega}{A} + \frac{x_o^2}{\Delta^2}\right) = 2n + 1 \ . \tag{49}$$

Equation (49) is the desired dispersion relation of the absolute instability. Note ImA > 0 and $x_o = z_o/k_y = K_1\Delta/2A$. In order to be consistent with H(z) given by Eq. (43), we require that the turning points z_t be within $\tilde{\Delta}$; i.e.,

$$|1 + \Omega/A| << 1 \ , \tag{50}$$

which indicates that the growth rate is approximately given by that of uniform plasma with a dipole pump

$$\omega_i = \frac{1}{2} \frac{\omega_o}{1 + \omega_{pe}^2/\omega_{ce}^2} \left|\frac{c\tilde{E}}{B_o\omega_o}\right|^2 \mathrm{Im}\left(\frac{\chi_e\chi_i}{\varepsilon}\right)^o - \frac{\Gamma_1}{2} \ . \tag{51}$$

In deriving Eq. (50), we have assumed that $|x_o| << \Delta$ or

$$\left|\frac{c\tilde{E}k_y}{B_o\omega_o}\right|^2 \left|\frac{\chi_e\chi_i}{\varepsilon}\right|^o |k_yL| >> 1 \ . \tag{52}$$

Furthermore, to be consistent with Eq. (49), the following condition also needs to be satisfied

$$|k_y\Delta\tilde{\Lambda}|^2 \left|\frac{c\tilde{E}k_y}{B_o\omega_o}\right|^2 \left|\frac{\chi_e\chi_i}{\varepsilon}\right|^o >> 1 \ . \tag{53}$$

Since $|\Delta/L| << 1$ and $\tilde{\Lambda}^2 \sim O(1)$, Eq. (53) is usually the more stringent condition. Because $|\chi_e\chi_i/\varepsilon| \sim O(1/k_y^2\lambda_{De}^2)$ and $\omega_o^2 \sim O(\omega_{pi}^2)$, Eq. (53) can also be written approximately as

$$\left|\frac{c\tilde{E}}{B_oC_s}\right|^2 |k_y\Delta|^2 >> 1 \ . \tag{54}$$

Currently, lower-hybrid heating experiments are usually running with $|c\tilde{E}/B_oC_s| \lesssim O(1)$ and $|k_y\Delta| >> 1$ (because k_y is only limited by λ_{Di} through ion Landau damping and $|\Delta| >> |\lambda_{Di}|$) , Eq. (54) is easily achievable. Finally, we remark that we have obtained similar results using a pump field with sharp boundaries.

IV. CONCLUSIONS AND DISCUSSIONS

We have calculated the threshold and the temporal growth rate for OTSI and nonresonant decay instabilities excited in an inhomogeneous plasma by a pump of finite spatial extent at a frequency close to the local lower-hybrid frequency. We have found that it is true for both an inhomogeneous and a homogeneous plasma that the localization of the pump causes firstly a convective loss in addition to the usual damping of the instabilities, and secondly a frequency shift for the OTSI from the pump frequency. Due to the mismatch of frequency, there is a threshold value for the frequency as well as the power of the pump field. We have also investigated the nonresonant decay instabilities due to nonlinear electron and ion Landau dampings. With WKB approximation, we recover the previous results that the instabilities are spatially amplifying. We demonstrate that without WKB approximation, however, for pump powers currently used in the lower-hybrid heating experiments, the instabilities can be temporally growing.

Finally, we would like to point out that our calculations are carried out under the assumption that the pump field is only localized in the x direction. If the pump wave does not propagate along but at an angle with respect to the $\vec{B}_o$ direction, the general formalism should be similar, and we expect that all the main features of OTSI and the nonresonant decay instabilities remain the same.

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Table I

Pump Power	Pump Width	Growth Rate	Frequency Shift	Turning Points
$\frac{U_o^2}{C_s^2} \gtrsim \frac{2k_\perp^2\omega_o^2\Theta}{k_y^2\omega_{pi}^2}\left(\frac{\sqrt{2}\lvert\Lambda^2\rvert k_x}{2\omega_o^2 k_\perp^2\Delta} - \frac{\Gamma_1}{2\omega_o}\right)$		$\omega_i \sim \left(\frac{U_o^2k_y^2}{2C_s^2k_\perp^2}\frac{\omega_{pi}^2}{\omega_o^2\Theta} - \frac{\sqrt{2}k_x\lvert\Lambda^2\rvert}{2k_\perp^2\Delta\omega_o^2\Theta} - \frac{\Gamma_1}{2\omega_o}\right)\omega_o$	$\omega_r \sim -\frac{k_x\lvert\Lambda^2\rvert}{k_\perp^2\Delta\omega_o^2}\,\omega_o$	$x_T < \Delta$
$\frac{U_o^2}{C_s^2} \gg \frac{2k^3\lambda_{Di}\omega_o\Theta}{k_y^2\omega_{pi}}$	$\frac{k_x\lvert\Lambda^2\rvert}{\omega_{pi}\omega_o k^3\Delta\lambda_{Di}} < \Delta$	$\omega_i \sim \left(\frac{U_o^2k_y^2\lambda_{De}^2\omega_{pi}^4}{2C_s^2\omega_o^4\Theta}\right)^{1/3}\omega_o$	$\omega_r \sim -\frac{k_x\lvert\Lambda^2\rvert}{k_\perp^2\Delta\omega_o^2}\,\omega_o$	$x_T < \Delta$
$\frac{U_o^2}{C_s^2} \gtrsim \frac{2k_\perp^2\omega_o^2\Theta}{k_y^2\omega_{pi}^2}\left(\frac{\lvert\Lambda^2\rvert k_x}{\omega_o^2k_\perp^2\Delta} - \frac{\Gamma_1}{2\omega_o}\right)$		$\omega_i \sim \left[\frac{4}{3}\left(\frac{U_o^4k_y^4k_x\omega_{pi}^4\lvert\Lambda^2\rvert}{4C_s^4k_\perp^6\Delta\omega_o^6}\right)^{1/3} - \frac{k_x\lvert\Lambda^2\rvert}{2k_\perp^2\omega_o^2\Delta\Theta} - \frac{\Gamma_1}{2\omega_o}\right]\omega_o$	$\omega_r \sim \frac{k_x\lvert\Lambda^2\rvert}{k_\perp^2\omega_o^2\Theta}\left[\frac{9}{4\Delta} + \frac{4}{3}\left(\frac{U_o^2k_y^2\omega_{pi}^2}{2C_s^2k_x^2\Lambda^2}\right)^{2/3}\frac{1}{\Delta^{1/3}}\right]\omega_o$	$x_T > \Delta$
$\frac{U_o^2}{C_s^2} \gg \frac{9k_x^2\Lambda^4}{k_y^2\lambda_{De}k_\perp k^2\Delta^2\omega_{pi}^3\omega_o\Theta}$	$\frac{k_x\lvert\Lambda^2\rvert}{\omega_{pi}\omega_o k^3\Delta\lambda_{Di}} > \Delta$	$\omega_i \sim \left(\frac{U_o^2k_y^2k^2k_\perp^2\lambda_{De}^2\Delta^2\omega_{pi}^4\Theta}{9C_s^2k_x^2\Lambda^4}\right)\omega_o$		$x_T > \Delta$

Threshold powers, growth rates, and frequency shifts for OTSI when the low frequency $\omega \gg \omega_{ci}$.

Table II

Pump Power	Pump Width	Growth Rate	Frequency Shift	Turning Points
$\frac{U_o^2}{C_s^2} \gtrsim \frac{2k_\perp^2\omega_o^2\Theta}{k_y^2\omega_{pi}^2}\left(\frac{\sqrt{2}k_x\lvert\Lambda^2\rvert}{2k_\perp^2\Delta\omega_o^2}+\frac{\Gamma_1}{2\omega_o}\right)$		$\omega_i \sim \left(\frac{U_o^2\omega_{pi}^2k_y^2}{2C_s^2\omega_o^2k_\perp^2\Theta} - \frac{\sqrt{2}k_x\lvert\Lambda^2\rvert}{2\omega_o^2k_\perp^2\Theta\Delta} - \frac{\Gamma_1}{2\omega_o}\right)\omega_o$	$\omega_r \sim -\frac{k_x\lvert\Lambda^2\rvert}{k_\perp^2\Delta\omega_o^2}\omega_o$	$x_T < \Delta$
$\frac{U_o^2}{C_s^2} \gg \frac{2k_\perp^2\omega_o k_\parallel\lambda_{De}\Theta}{k_y^2\omega_{pi}}$	$\frac{\lvert\Lambda^2\rvert k_x}{\omega_{pi}\omega_o k_\perp^2\Delta k_\parallel\lambda_{De}} < \Delta$	$\omega_i \sim \left(\frac{U_o^2k_y^2k_\parallel^2\lambda_{De}^2\omega_{pi}^4}{2C_s^2k_\perp^2\Theta\omega_o^4}\right)^{1/3}\omega_o$	$\omega_r \sim -\frac{k_x\lvert\Lambda^2\rvert}{k_\perp^2\Delta\omega_o^2}\omega_o$	$x_T < \Delta$
$\frac{U_o^2}{C_s^2} \gtrsim \frac{2k_\perp^2\omega_o^2\Theta}{k_y^2\omega_{pi}^2}\left(\frac{k_x\lvert\Lambda^2\rvert}{\omega_o^2k_\perp^2\Delta}+\frac{2\Gamma_1}{\omega_o}\right)$		$\omega_i \sim \left(\frac{U_c^2\omega_{pi}^2k_\parallel^2}{2C_s^2\omega_o^2k_\perp^2\Theta} - \frac{k_x\lvert\Lambda^2\rvert}{\omega_o^2k_\perp^2\Theta\Delta} - \frac{\Gamma_1}{2\omega_o}\right)\omega_o$	$\omega_r \sim \frac{k_x\lvert\Lambda^2\rvert}{k_\perp^2\omega_o^2\Theta}\left[\frac{9}{4\Delta}+\frac{4}{3\Delta^{1/3}}\left(\frac{U_o^2k_y^2\omega_{pi}^2}{2C_s^2k_x\lvert\Lambda^2\rvert}\right)^{2/3}\right]\omega_o$	$x_T > \Delta$
$\frac{U_o^2}{C_s^2} \gg \frac{9k_x^2\Lambda^4}{k_y^2k_\perp^2k_\parallel\lambda_{De}\Delta^2\omega_{pi}^3\omega_o\Theta}$	$\frac{\lvert\Lambda^2\rvert k_x}{\omega_{pi}\omega_o k_\perp^2\Delta k_\parallel\lambda_{De}} > \Delta$	$\omega_i \sim \left(\frac{U_c^2k_y^2k_\perp^2k_\parallel^2\lambda_{De}^2\Delta^2\omega_{pi}^4}{9C_s^2k_x^2\Lambda^4}\right)\omega_o$		$x_T > \Delta$

Threshold powers, growth rates, and frequency shifts for OTSI when the low frequency $\omega \ll \omega_{ci}$.