HW-28729

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XENON POISONING CALCULATIONS BASED ON TUBE POWER

by

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Summary

A method of calculating the steady state xenon poisoning has been developed using parameters derived from the power production of individual tubes in the pile. The power of individual tubes is now being recorded automatically at some of the production piles. The necessary parameter is derived from these data by the use of IBM computers. This method of calculating xenon poisoning eliminates the determination of the flattening in inhours, which is a parameter in the present method of determining the xenon poisoning for a flattened pile.

Introduction

The availability of data giving the power of individual tubes in the production piles offers the possibility of an improved method of calculating the poisoning of the pile due to xenon-135. The development of the xenon equation is given in "Review of the Xenon Problem" by P. F. Gast (HW-23729). In that document, the xenon reactivity effect was calculated for the unflattened pile only. Now it appears possible to extend the treatment to piles with any amount of flattening, using data recorded by the IBM temperature monitoring equipment. In previous treatments of this problem, the pile has been described by cylindrical geometry, with a smooth function describing the flux radially. Having a record of the power of individual tubes, plus having the equipment to derive useful information from these data removes the necessity of specifying a function to fit the flux radially. This should remove some of the uncertainty from the calculations.

Derivation

Using the results obtained in Appendix B of HW-23729, the change in the multiplication constant as a result of neutron absorption of xenon-135 is

$$\frac{dk}{k} = -\frac{f\eta\gamma}{\nu}(1+\delta)\frac{1}{1+\frac{3\pi}{8}\frac{w'}{w}\frac{\int z^2 r\,dr}{\int z^3 r\,dr}} \qquad (1)$$

where

k = multiplication constant of the lattice.
f = thermal utilization of the lattice = 0.870*
η = neutrons produced in thermal fission per neutron absorbed in the uranium = 1.32.

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* Clayton, E. D. , Private Communication

ν = neutrons per fission = 2.50.
δ = fast fissions per thermal fission = 0.057.
W = power of the central 1 cm. slab of the central tube.
Z = a function giving the power distribution in a radial direction. The form of Z will depend upon the flattening.
$w' = 9.265 \text{ KNX } (1+\delta) \frac{\sigma_f}{\sigma_x} = 2.03 \times 10^6 \frac{\sigma_f}{\sigma_x}$.

K = watt seconds/fission = 3.222×10^{-11}.
N = density of atoms of U-235 = 3.09×10^{20}
X = decay constant of Xe-135 = 2.091×10^{-5} sec.$^{-1}$.
σ_f = fission cross-section of U-235.
σ_x = absroption cross-section of Xe-135.
Y = total fission yield of Xe-135 and I-135 = 6.4%.

It is convenient to convert to the use of central tube power, for which it is necessary to assume a cosine distribution of flux from front to rear in the pile, as was done in the derivation of (1). There appears to be no way of avoiding such an assumption. Let p = power of the central tube in megawatts.

$$p = 10^{-6} \int W \cos \frac{\pi x}{L} \, dx = \frac{2WLx10^{-6}}{\pi} \int_0^{80^\circ} \cos y \, dy = \frac{1.97 \, WLx10^{-6}}{\pi} \qquad (2)$$

The integration is performed over the loaded zone of the pile, making the upper limit 80° in (2). Substituting for W and for some of the constants in (1), and converting to inhours (1% change in k = 385 inhours), we have

$$L_s = \frac{1205 \quad A \, p}{Ap + 0.7387 \, w'Lx10^{-6}} \quad , \qquad (3)$$

where

$$A = \frac{\int r Z^3(r) \, dr}{\int r \, Z^2(r) \, dr} \quad , \qquad (4)$$

and L = 795 cm., the extrapolated length of the pile. Since the temperature of the pile varies from point to point, the ratio σ_f / σ_x must be given an average value. The uncertainty in this quantity is given a detailed treatment in HW-23729.

Evaluation of the quantity A is accomplished by making use of tube fractions as described by R. O. Brugge in HW-26210. A quantity F_i represents the ratio of the power generation in the i-th tube to the average power generation per tube. Let P be the total power of the pile, N the number of power producing tubes, and p_i the tube power. Then

$$F_i = p_i \frac{N}{P} \quad . \tag{5}$$

We may obtain $\sum_{i=1}^{N} F_i$, $\sum_{i=1}^{N} F_i^2$, or $\sum_{i=1}^{N} F_i^3$ from the IBM calculations. The latter two are used to evaluate the quantity Ap as follows

$$Ap = \frac{\int r\, p^3\, Z^3\,(r)\, dr}{\int r\, p^2\, Z^2\,(r)\, dr} = \frac{\sum p^3{}_i}{\sum p^2{}_i} = \frac{P}{N} \frac{\sum F_i^3}{\sum F_i^2} \quad . \tag{6}$$

Evaluation of the quantity w in (3) is difficult because of the uncertainty in the ratio σ_f / σ_x which varies from point to point in the pile, due to the variations of temperature. Based on an experiment at 100-B in November of 1944, and a value of Y of 6.4%, the ratio σ_f / σ_x is given the value 1.627×10^{-4} by P. F. Gast in HW-23729. The value for w then becomes 330 watts and the xenon poisoning is

$$L_s = \frac{1205\ Ap}{Ap + 0.194} \tag{7}$$

for which the value of Ap is obtained from (6). Slightly different values for the constants were reported in the TAR for May, 1953, Applied Research, HW-28283. The equation reported there was normalized to give the same value for xenon poisoning at zero flattening as is given by the Xenon Tables. In order to perform the normalization, Z(r) was represented by a Bessel function and the integration in (4) performed.

[redacted]

Potential Xenon Poisoning

The iodine-135 in the pile is a potential poison, since it decays to xenon-135. This potential poisoning is calculated on the basis that all the iodine present is

suddenly converted into xenon. In HW-23729, the steady state concentration of iodine is given by

$$m_s = \frac{(1 - s)\, Y\, \phi\, N \sigma_f (1 + \delta)}{I} \qquad (8)$$

The simple dependency of m_s on the flux makes this problem easier than the preceding one. This problem will not be worked out completely here, since the method is demonstrated in the Appendices of HW-23729, where it is shown that the local reactivity effect is

$$\frac{d\,k}{k} = \frac{-f\eta}{\nu \sigma_f N} \frac{\int_u \phi\, x\, \sigma_x\, d\,v}{\int_u \phi\, dv} \qquad (9)$$

where x is the atomic concentration of the isotope, which we replace by (8), and ϕ is the flux in the metal. The flux ϕ is replaced by $aI_o(\mu r)$, giving

$$\frac{d\,k}{k} = \frac{-f\eta\,\sigma_x}{\nu I} (1-s)\, Y\, (1+\delta)\, a \frac{\int_o^R r\, I_o^{\,2}(\mu r)\, dr}{\int_o^R r\, I_o(\mu r)\, dr} \,. \qquad (10)$$

For R = 1.73 cm. and μ = 0.65 $cm.^{-1}$, the ratio of the two integrals becomes 1.1787. Also from HW-23729 we have $w = 10.91\, K\, N\, \sigma_f (1 + \delta)\, a$. (11)

Substituting for the integrals and for a in (10), the local reactivity effect becomes

$$\frac{d\,k}{k} = \frac{-f\eta\,\sigma_x}{\nu I} \frac{(1-s)\, Y\, 1.1787\, w}{10.91\, K\, N\, \sigma_f} \,. \qquad (12)$$

The local effect is averaged over the pile, using the square of the local flux or power as the weighting factor. If the power at any point in the pile is represented by

$$w = W \sin \frac{\pi x}{L} Z(r) \,, \qquad (13)$$

we have the following ratio to evaluate

$$\frac{\iint W^3 \sin^3 \frac{\pi x}{L} Z^3(r)\, r\, dr\, dx}{\iint W^2 \sin^2 \frac{\pi x}{L} Z^2(r)\, r\, dr\, dx}$$

Integrating over x, and converting to tube power by using (2) gives

$$1.703 \times 10^3\, p\, \frac{\int r\, Z^3(r)\, dr}{\int r\, Z^2(r)\, dr} = 1.703 \times 10^3\, Ap. \qquad (14)$$

For $N = 3.09 \times 10^{20}$ atoms/cc, (12) becomes

$$\frac{dk}{k} = 0.1128\, Ap. \qquad (15)$$

Converting to inhours, the potential poisoning of the iodine is

$$M_s = 4343\, Ap \qquad (16)$$

Conclusions

Equations have been developed giving the steady state values of the xenon poisoning and potential xenon poisoning for a pile in terms of tube powers. The quantities necessary for the calculations are available from computations now being made on a routine basis on IBM computers. If necessary, these equations may be normalized to results obtained from the Xenon Tables, HW-25565 for the case of no flattening.

With the equations developed here, the intermediate step of estimating the flattening of the pile in inhours is unnecessary. With the present system, evaluation of the quantity (Ap) is performed in the 700 Area, which may be inconvenient. This inconvenience may be small, since the quantity will vary slowly with changing power level or flattening. An alternative would be to set up a small computer to evaluate the summations in (6).

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The xenon tables in use at present are based on an approximation which, for low values of flattening, give an increase in the xenon reactivity effect for an increase in flattening(central tube power p remaining constant). For values of flattening in excess of about 200 inhours, the reactivity effect decreases, as was shown graphically in HW-28283. For a constant value of p, the quantity Ap in (6) increases with flattening, so that L_s in (7) increases steadily with flattening. This difference in behavior is shown graphically in Fig. 1, which was originally published in HW-28283.

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Figure 1

XENON POISONING
VS
INHOURS FLATTENING

W = Central Tube Power in M.W.