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SEQUENTIAL PROBABILITY RATIO TESTS FOR REACTOR SIGNAL VALIDATION AND SENSOR SURVEILLANCE APPLICATIONS

by

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Abstract

This paper examines the properties of sequential probability ratio tests (SPRT's) and the application of these tests to nuclear power reactor operation. Recently SPRT's have been applied to delayed-neutron (DN) signal data analysis using actual reactor data from the Experimental Breeder Reactor-II, which is operated by Argonne National Laboratory. The implementation of this research as part of an expert system is described. Mathematical properties of the SPRT are investigated, and theoretical results are validated with tests that use DN-signal data taken from the EBR-II in Idaho. Variations of the basic SPRT and applications to general signal validation are also explored.

1 Introduction

A *sequential probability ratio test* (SPRT) is a statistical hypothesis test which differs from the standard *fixed-sample test* in the way in which statistical observations are employed. In the fixed-sample test a given number of observations are used to select one hypothesis from two or more alternatives. The SPRT, however, examines one observation at a time and at some point makes a decision and selects an hypothesis, or, if the SPRT is not *closed*, it may continue to examine observations indefinitely without reaching a conclusion with some nonzero probability. SPRT's have been studied extensively in the statistical literature for quite some time beginning with Wald [1] and Wald and Wolfowitz [2]. Only more recently, however, have SPRT's found beneficial applications in the areas of safety and reliability of nuclear power reactor operation.

Given certain reasonable assumptions SPRT's have been shown to possess optimal properties versus corresponding fixed-sample tests [2,3,4]. For example, when choosing between two alternative hypotheses, the SPRT can make a correct decision faster (on average) than the fixed-sample test to which it corresponds. When this is the

case it is said that the SPRT is more *efficient* than the fixed-sample test. The validity of the above statement in a specific instance depends upon a number of statistical properties of the problem being studied, such as the form of the hypotheses, the underlying statistical distribution, the true value of various parameters and the specific kind of SPRT being used.

More recently SPRT's have been applied to pattern recognition problems [3]. A compound SPRT has been used to improve efficiency in two-class recognition problems with first-order Markov dependence among pattern classes [5], and a nonparametric SPRT has been formed to investigate Markov dependent data and problems which test more than two hypotheses [6].

It has been shown that the SPRT is generally more efficient than the fixed-sample size likelihood ratio test for the problem of detecting a constant signal in additive noise [7]. The SPRT has also been used to estimate system availability in systems which are continuously "on station", eg. early warning systems, patrol craft, etc. [8]

A project was initiated at the U.S. Nuclear Regulatory Commission in 1980 to attempt to transform the SPRT into a practical

tool for nuclear power plant data analyses [9]. This study endorses the value of a SPRT and encourages further testing. A truncated SPRT has been shown to possess large potential savings in average time spent monitoring employees for radiation exposure at the Rocky Flats Plant in Golden, Colorado [10]. A SPRT has also been suggested for rapid decision making and the ability to track signals emitted by radiation monitors used for nuclear safeguards and security [11].

The research presented in this paper primarily consists of the development of new SPRT signal-analysis tools and focuses on the integration of these tools into an AI based expert system to be used as an operator decision aid for liquid-metal cooled nuclear reactors (LMR's) that are licensed to operate with failed fuel [12]. The expert system is embodied in a device which monitors, processes and interprets information from nine groups of redundant plant sensors and displays to the reactor operator the diagnostic information needed to make proper decisions about technical-specification conformance during run-beyond-clad-breach (RBCB) operation.

The expert system is expected to enhance the safety, economics and licensibility of future-design LMR's. The system will enhance plant safety by making available to the operator online diagnosis and interpretation of a bewildering variety of interacting physical variables during exposed-fuel operation. Currently, full interpretation of these variables requires several days to weeks of detailed analysis by teams of specialists. The new system will provide the operator with very rapid (within one minute) identification of off-normal RBCB conditions, thereby enabling him to terminate or avoid events which might challenge safety or radiological performance guidelines. The system will enhance plant availability and economics by minimizing unnecessary reactor trips caused by events having no safety significance.

A detailed mathematical analysis has been performed on the applications of SPRT's to RBCB operation in relation to the aforementioned expert system. Current practice in all countries that have LMR programs (except possibly the Soviets) is to set conservative shutdown limits on the magnitude of delayed-neutron (DN) signals coming from breached fuel. By combining and evaluating informa-

tion from several groups of redundant plant sensors, the AI-based system will make it possible to significantly relax the conservatism in DN shutdown limits without compromising plant-safety assurance. Currently, the SPRT is being applied to the problem of providing rapid annunciation of discrepant signals or failed sensors with minimal “false alarm” probabilities. Since DN signals are functions of a number of interacting physical variables, a future goal is to apply the SPRT to questions about individual physical variables and combinations of physical variables influencing the DN signal. Examples include:

1. determining whether a change in the statistical quality of a DN signal (annunciated with a SPRT developed in the present paper) is a result of a change in source condition (breach condition or fission rate), or a change in primary pump performance characteristics and
2. using a modification of the SPRT developed here for rapid departure-from-background annunciation, which can pinpoint the onset of a new breach in the presence of background DN events

(from fissioning of "tramp" uranium) with greater accuracy and a lower "false-alarm" probability than simple threshold-limit tests currently in use for LMR's.

In most cases, the SPRT will provide an answer to each of these questions faster than all other statistical tests.

The expert system described above has been developed for incorporation into the conceptual design of Argonne National Laboratory's Integral Fast Reactor. Plans are underway to install a prototype apparatus at the EBR-II in Idaho. The device is also being incorporated into the conceptual design of General Electric's Power Reactor Inherently Safe Module.

2 Preliminaries

Consider the problem of discriminating between two *simple* hypotheses

$$H_0 : \theta = \theta_0, \quad H_1 : \theta = \theta_1, \quad \theta_0 \neq \theta_1 \quad (1)$$

In this case, we concentrate on the family $F_n(X_n; \theta), \theta = \theta_0, \theta_1$ (ie. the cumulative distribution function of the observations x_1, x_2, \dots, x_n given the parameter θ) containing only two distinct distributions [4].

In the usual sense, define

α = the probability that H_1 is chosen or *accepted* when

H_0 is true,

$1 - \alpha$ = the probability that H_0 is accepted when H_0 is true,

β = the probability that H_0 is accepted when H_1 is true,

$1 - \beta$ = the probability that H_1 is accepted when H_1 is true,

and n = the number of observations required to accept either

H_0 or H_1 .

α and β are called *error probabilities* and are usually determined

by the nature of the problem, and n is called the *sample number*.

There exist a number of statistical tests to approach this problem.

A 'good' test should accept H_0 when θ is close to θ_0 and *reject* H_0

(accept H_1) when θ is close to θ_1 with high probability. Define the

operating characteristic function (OC), $Q(\theta)$, as the probability that

H_0 is accepted as a function of θ , and the *power function*, $P(\theta)$, as

the probability that H_1 is accepted as a function of θ . As in [4], we

impose the validity criteria $P(n < \infty | \theta) = 1$ for $\theta = \theta_0, \theta_1$, and

$$Q(\theta_0) \geq 1 - \alpha, \quad Q(\theta_1) \leq \beta \quad (2)$$

Thus, the statistical test will always make a decision given a finite number of observations when $\theta = \theta_0$ or $\theta = \theta_1$. Also, the test will correctly accept H_0 when $\theta = \theta_0$ at least $100(1-\alpha)$ per cent of the time and will correctly accept H_1 when $\theta = \theta_1$ at least $100(1-\beta)$ per cent of the time. Note that if $P(n < \infty | \theta) = 1$ for all values of θ , the test is *closed*, i.e. it always makes a decision after some finite number of observations, and $Q(\theta) + P(\theta) = 1$ for all θ .

We now discuss two remaining questions in turn. Firstly, what statistical test shall we employ? A fixed-sample test, in which a constant n is selected so as to satisfy (2), is standard and simple. However, we shall choose a sequential probability ratio test for the reason discussed in the next paragraph. A *sequential probability ratio test* (SPRT) $S(b,a)$ for (1) is defined by [4]: Observe the sequence of observations $\{x_i\}$ ($i = 1, 2, \dots$) successively, and at stage $n \geq 1$

- (i) accept H_0 if $Z_n \leq b$,
- (ii) reject H_0 if $Z_n \geq a$,
- (iii) continue by observing x_{n+1} if $b < Z_n < a$,

where the *stopping bounds* (b,a) , $-\infty < b < a < \infty$, are two real

numbers, and

$$Z_n = \ln \frac{f_n(X_n; \theta_1)}{f_n(X_n; \theta_0)} \text{ for } n \geq 1$$

is the natural logarithm of the probability ratio at stage n. The inequality $b < Z_n < a$ is called the *critical inequality* of $S(b,a)$ at stage n. The sample number n at which a decision is made depends upon the observations themselves and the value of θ . In many cases, the *average sample number* (ASN) can be computed as a function of θ and is denoted by $E(n; \theta)$.

It can be shown [4] that if $S(b,a)$ is any SPRT which tests H_0 vs. H_1 in (1) in a one-parameter family with OC function $Q(\theta)$ and ASN function $E(n; \theta)$, if the x_i are independent identically distributed random variables and if certain other quite general assumptions hold, then for any other rival statistical test S whose OC function satisfies $Q_S(\theta) \geq Q(\theta)$ for all $\theta < \theta'$, $Q_S(\theta) \leq Q(\theta)$ for all $\theta > \theta'$, where θ' satisfies $E(z; \theta') = 0$ and

$$z = \ln \frac{f(x; \theta_1)}{f(x; \theta_0)},$$

we have $E_S(n; \theta) \geq E(n; \theta)$ for all $\theta \neq \theta'$. Note that $E(z; \theta)$ is the expected value of z and is, in general, a function of θ , since z is

a function of x , and x is a function of θ . θ' is the value of θ that makes $E(z; \theta)$ equal to zero. The proof that $E_S(n; \theta) \geq E(n; \theta)$ for all $\theta \neq \theta'$ is not given here, but is provided in detail in [4]. Hence, the SPRT makes a faster decision (on average) than any other test, which has equal or smaller error probabilities, provided the above assumptions are satisfied. Such a test is called a *uniformly most efficient* (UME) test. This is the major reason why the SPRT is a useful test and, in many cases, the best statistical test.

3 Main Results

We now turn to the second question of interest. What problem do we wish to solve? The problem of interest is that of determining whether DN detectors are functioning correctly. Consider the case in which there are two redundant detectors monitoring the same process. (See Figure 1.)

We form a discrete difference function $X_k = X_1(t_k) - X_2(t_k)$ by subtracting corresponding observations of X_1 and X_2 sampled at times t_k , $k = 1, 2, \dots$. Since both sensors are monitoring the same process, X_k should have mean 0. (See Figure 2.) We develop the

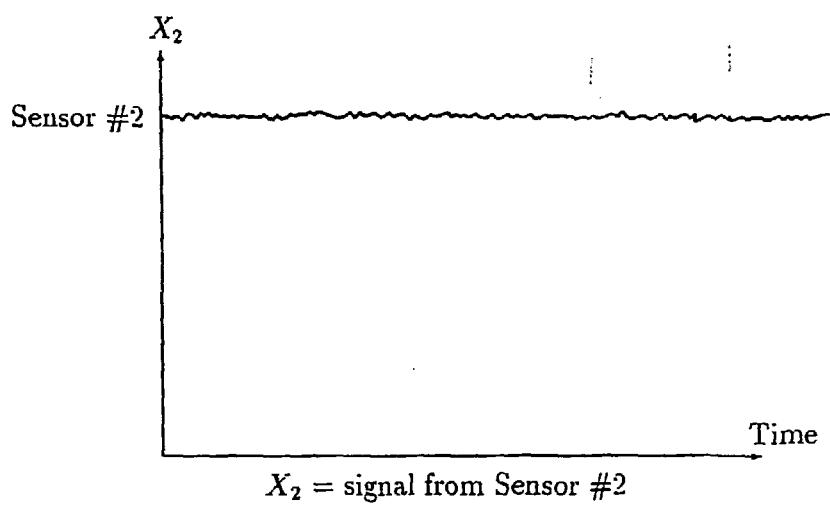
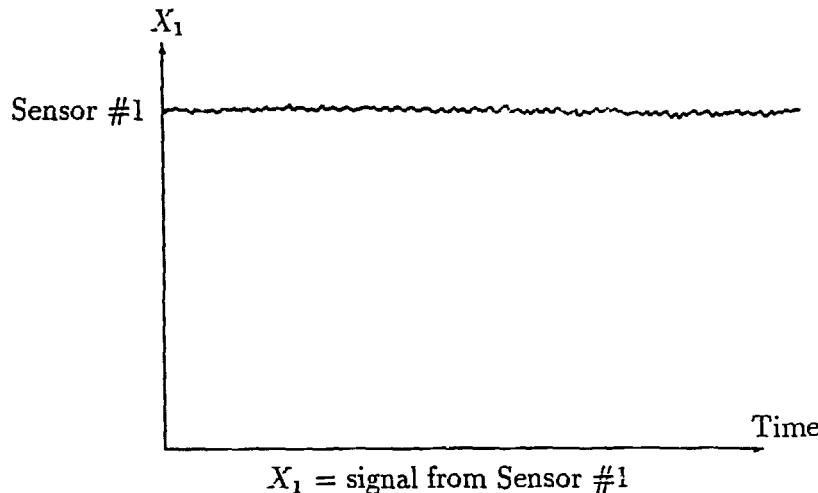


Figure 1. Two detectors monitoring the same process.

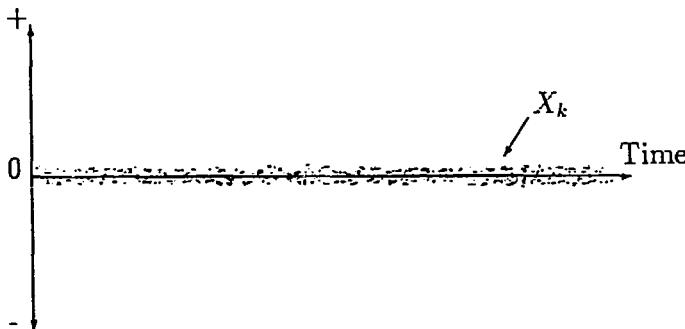


Figure 2. Graph of the difference function X_k .

following statistical hypotheses and use a SPRT to test them.

H_0 : X is Gaussian with mean $\mu_0 = 0$ and variance σ^2

H_1 : X is Gaussian with mean $\mu_1 \neq 0$ and variance σ^2

The value of σ^2 can be estimated from actual data. If the SPRT accepts H_1 , we declare sensor 1 or 2 failed. Note, however, that the SPRT alone does not indicate which sensor has failed.

If one sensor fails, it is possible for both μ and σ^2 to change. However, the SPRT as defined herein only tests for a change in the mean. Although the SPRT will continue to accept the null hypothesis, H_0 , if only the variance of the signal changes, the SPRT is sensitive to a change in the variance of the signal. Because the ASN is directly proportional to σ^2 , the variance of the difference function, as shown below, a change in the variance of one signal will

affect σ^2 and will subsequently cause a change in the ASN of the SPRT. Further study is currently underway to examine the effects of various kinds of failures on the properties of the SPRT.

Our test has been formulated as a test of Gaussian means. This leads to the question of whether the underlying DN signal distribution is Gaussian. This question has been explored using actual DN signals from experiments conducted at full power in EBR-II. Although a number of statistical tests of normality, including the sensitive Shapiro-Wilk test [13], indicate that the underlying distribution of X is not Gaussian, it can be seen (See Table 1) that the assumption that X is normally distributed does not significantly affect the performance of the SPRT for our application. That is, for the simulated SPRT applied to real DN-signal data the ASN was seldom more than 10% greater and was often significantly less than the theoretically predicted value of the ASN using the assumption that the DN-signal data were normally distributed. Hence, the mathematical theory of the SPRT applied to testing the mean of a Gaussian distribution provides accurate and conservative estimates for the ASN of the SPRT that has been applied to real DN-signal

data. This is fortunate, since much theoretical work has already been done in the area of SPRT's applied to normally distributed data.

The question of how close to Gaussian the signal has to be is very complicated. This issue is being studied carefully for DN signals, as well as for pump power and pump speed signals, and it has been shown that, in general, the SPRT becomes less accurate when $|\mu_1 - \mu_0| < \sigma$. For this reason, the authors are studying the applicability of various transformations to normality. SPRT's can be formulated for applications to signals contaminated by non-Gaussian noise, as long as the underlying signal distribution can be represented mathematically in some closed form or transformed to normality, but the analytical complexity may become far greater.

Signals containing periodic components pose no problem to SPRT analysis provided the components are in phase on the two instrument channels, as is the case when the periodicities originate in the physical process being observed by redundant sensors with identical time constants. In this case the periodic phenomena are filtered by

the differencing process.

Several properties of the SPRT are of interest. These are:

1. Reliability
2. Average sample number
3. Effect of the choice of μ_1
4. Robustness

The *reliability* or accuracy of the SPRT is determined by the error probabilities, α and β . Since these values are generally chosen based upon the nature of the problem, they can be selected by the SPRT user. As expected, their values do have an effect on other properties of the SPRT as shown below.

The authors have determined various other facts regarding the remaining SPRT properties. It can be shown [4] that the ASN is given by the following approximation when the underlying distribution is Gaussian:

$$E[n; \mu] \approx \frac{2\sigma^2 \left\{ \frac{A^h - 1}{A^h - B^h} \ln \frac{\beta}{1-\alpha} + \frac{1 - B^h}{A^h - B^h} \ln \frac{1-\beta}{\alpha} \right\}}{(\mu_1 - \mu_0)(2\mu - \mu_1 - \mu_0)}$$

where

$$h = \frac{\mu_1 + \mu_0 - 2\mu}{\mu_1 - \mu_0}, \quad A = \frac{1 - \beta}{\alpha}, \quad B = \frac{\beta}{1 - \alpha}$$

and \ln is the natural logarithm. Clearly, the ASN is a function of

μ , the true mean of the underlying distribution. As can be seen in Table 1, this approximation can be used to provide an accurate theoretical estimate of the ASN when the SPRT is applied to real DN-signal data. Note that the approximate ASN:

1. is directly proportional to σ^2
2. is inversely proportional to $(\mu_1 - \mu_0)^2$
3. is bounded, ie. has a maximum value
4. increases slowly as α and β decrease

It is not surprising that the ASN increases as σ^2 does. One would expect the SPRT to take longer to make a decision when the values observed vary to a greater degree. Conversely, it is easier for the SPRT to select an hypothesis as μ_1 and μ_0 grow farther apart. In fact, the SPRT takes longest to make a decision when $\mu = (\mu_1 + \mu_0)/2$. The ASN reaches a maximum at this point. Finally, a decrease of one order of magnitude in α and β only approximately doubles the ASN. This is what we mean by ‘increases slowly.’

The above properties of the ASN are encouraging. The SPRT also appears to be *robust* with respect to a number of important probability distributions. Hence, even if the underlying distribution

is not the distribution being used in theoretical calculations, those calculations can still often be used as an accurate estimate of the SPRT properties, as was seen to be the case in our research on the DN-signal data. What we desire is an underlying distribution, which, although it may not statistically be Gaussian, Poisson, etc., is 'close enough' for our theory to provide reasonable estimates. A distribution is close enough if

$$\frac{s - t}{t} \leq p$$

where s is the *simulated value*, t is the *theoretical value*, and $0 < p < 1$ is the allowable *percentage error* between the theoretical and actual OC, power and ASN. This percentage error is determined by the application. The theoretical value is that theoretical value calculated under the assumption of Gaussian data.

These considerations lead us to a discussion of the third property of the SPRT. What value should be chosen for μ_1 . In general, the authors suggest choosing the value for μ_1 based upon the sensitive malfunction value of μ , say μ' . If the value of μ_1 is chosen to be $4\mu'$, then the SPRT will begin signaling a problem (accepting H_1) relatively often when $\mu = \mu' = (\mu_0 + \mu_1)/4$. However, it will do so

quickly, since μ has not yet reached $(\mu_0 + \mu_1)/2$, the point at which the ASN is maximum. The value of μ' depends upon the situation being analyzed and upon the nature of sensor malfunction, ie. linear, quadratic, step, exponential, etc. The value of σ^2 also has an effect on the choice of μ_1 , since the ASN and robustness of the SPRT are functions of $(\mu_1 - \mu_0)/\sigma$. Additional work needs to be done in the area of how to choose μ_1 appropriately.

4 Example and Numerical Results

Table 1 illustrates the ASN, OC and power functions of the SPRT for theoretical (assuming an underlying normal distribution) and simulated (using actual DN-signal data from EBR-II) tests. The critical inequality is

$$\frac{\left(\ln \frac{\beta}{1-\alpha}\right) \sigma^2}{\mu_1 - \mu_0} + \frac{\mu_0 + \mu_1}{2} n < \sum_{i=1}^n x_i < \frac{\left(\ln \frac{1-\beta}{\alpha}\right) \sigma^2}{\mu_1 - \mu_0} + \frac{\mu_0 + \mu_1}{2} n$$

Other values are $\mu_0 = 0$, $\sigma^2 = 800.5138$, $\alpha = \beta = 0.001$, and $N=700$ data values. The three values of μ_1 were chosen to be approximately equal to σ ($\mu_1=28$), $\frac{1}{2}\sigma$ ($\mu_1=14$), and $\frac{1}{5}\sigma$ ($\mu_1=6$). It should be noted that, in practice, six sensor readings per second are common.

Thus an ASN value of 100 indicates that the SPRT takes approximately 17 seconds to select an hypothesis. This is much faster than methods currently used, which often take two to ten minutes. It can be seen that the theoretical values yield very accurate estimates for real DN-signal data. Errors are usually highest near $\mu = (\mu_0 + \mu_1)/2$, and they are conservative, ie. the theoretical ASN is higher than the simulated ASN. Corresponding fixed sample tests yield sample numbers of $n=39$ ($\mu_1 = 28$), $n=156$ ($\mu_1 = 14$), and $n=850$ ($\mu_1 = 6$), which are also greater than SPRT ASN's, even for simulations when μ is very close to $(\mu_0 + \mu_1)/2$. Discrepancies in the OC and power functions are also small, and indications are that the SPRT that is simulated on DN data is slightly more prone to signal a problem than is theoretically predicted. If this is a serious problem, α and β can be appropriately decreased with relatively small increases in the ASN, as indicated above.

μ_1	μ	Theoretical			Simulated		
		ASN	Q	P	ASN	Q	P
28	-5.000	10.39	.9999	.0001	11.34	1.000	0.000
	0.000	14.08	.999	.001	15.38	1.000	0.000
	5.000	21.43	.988	.012	20.76	0.939	0.061
	10.000	37.32	.878	.122	34.45	0.850	0.150
	14.000	48.69	.50	.50	42.50	0.50	0.50
	18.000	37.32	.122	.878	36.74	0.158	0.842
	23.000	21.43	.012	.988	23.20	0.033	0.967
	28.000	14.08	.001	.999	15.47	0.000	1.000
	33.000	10.39	.0001	.9999	11.78	0.000	1.000
14	-5.000	32.91	.99999	.00001	34.05	1.000	0.000
	0.000	56.31	.999	.001	57.67	1.000	0.000
	5.000	149.27	.878	.122	86.00	0.750	0.250
	7.000	194.85	.50	.50	93.17	0.50	0.50
	10.000	118.67	.049	.951	99.57	0.143	0.857
	14.000	56.31	.001	.999	58.55	0.000	1.000
	19.000	32.91	.00001	.99999	34.80	0.000	1.000
6	-3.000	153.58	.999999	.000001	123.00	1.000	0.000
	0.000	306.55	.999	.001	166.00	1.000	0.000
	3.000	1060.75	.50	.50	279.50	0.50	0.50
	6.000	306.55	.001	.999	300.00	0.000	1.000
	9.000	153.58	.000001	.999999	151.50	0.000	1.000

ASN, OC and power function of the SPRT.
Table I

5 Conclusions

The SPRT has been chosen as the method of selecting an hypothesis, because it has been mathematically proven to possess many properties desired by the statistician and has been shown to perform very well in the analysis of DN-signal data. Future goals of the work presented herein include:

- Mathematical investigation of optimum methods for selecting μ_1 , and a determination of the factors, which cause the maximum simulated ASN to be more significantly less than the maximum theoretical ASN as μ_1 approaches μ_0 .
- Further mathematical investigation of the relationship between various variables and properties of the SPRT, ie. α , β , ASN, OC, power and robustness. Extensive testing with newly available DN-signal data will be taken during breached-fuel operation under a variety of plant conditions. The 700 data values employed in the investigations to date were obtained during a brief steady-state exposed-fuel test conducted in EBR-II in May, 1988. (See Figure 3.) Plant modifications are currently underway (Spring, 1989) to install new high-sensitivity

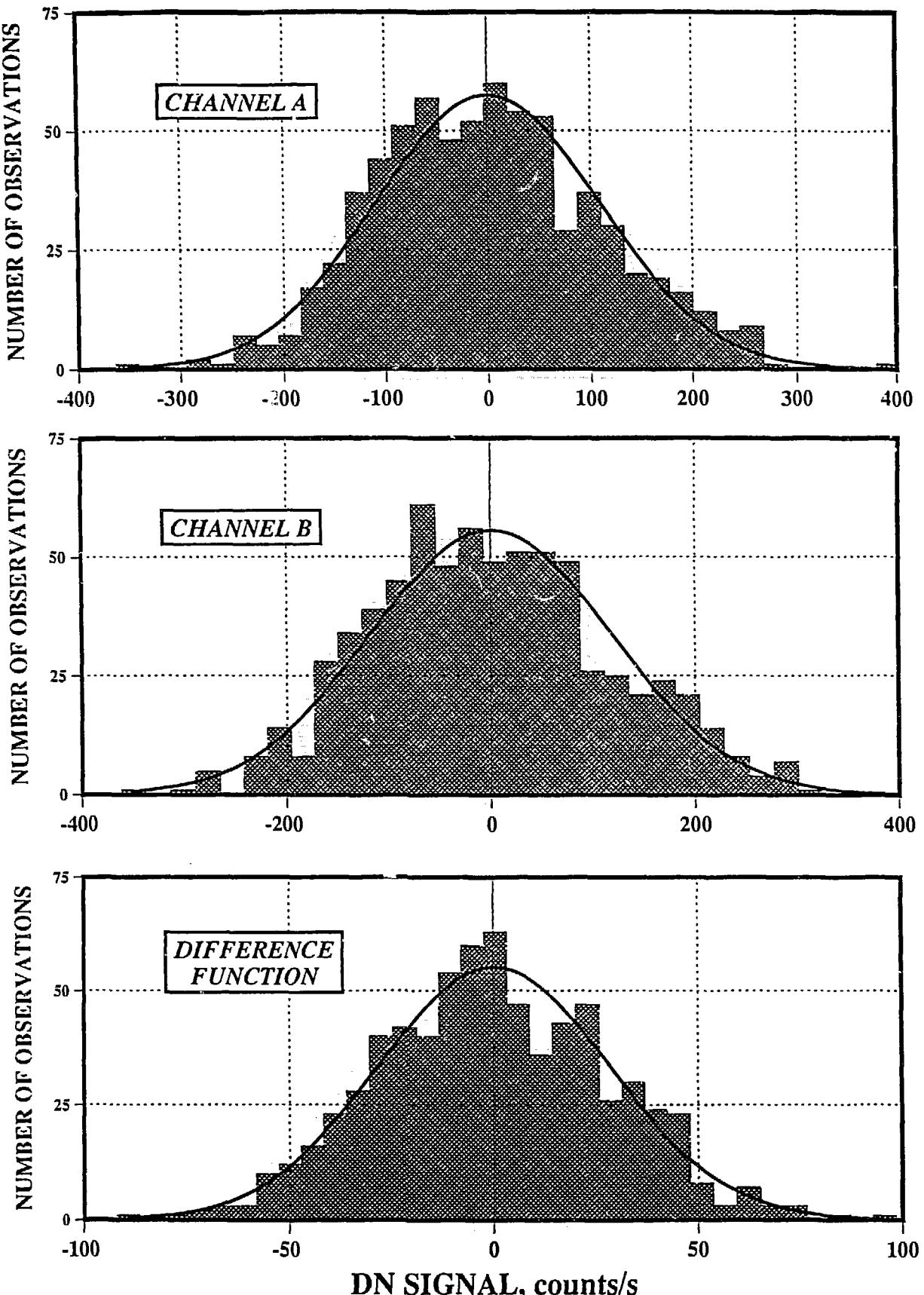


Figure 3. Frequency functions for X_1 , X_2 and X_k , computed from actual experimental DN signals.

DN detectors (DNDs). The new DND configuration will permit acquisition of DN-signal data from a series of breached-fuel tests that will be conducted under a variety of steady-state, transient-power and transient-flow conditions. This extensive experimental database will provide reliable values of the ASN, OC and power functions and enable the authors to fully explore the quality of the SPRT and discrepancies between theoretical and simulated values under all breached-fuel conditions anticipated during normal operation of an LMR.

- Research on the generalized truncated SPRT (GSPRT) to reduce the ASN [4]. This SPRT uses 'variable stopping boundaries' with the goal of enabling it to reach a decision faster when μ is close to $(\mu_0 + \mu_1)/2$. This advantage of the GSPRT has been shown to become more pronounced as the error probabilities become smaller [14]. The authors ultimately desire to combine the generalized truncated SPRT and the standard SPRT and to utilize the strong points of each to result in a faster decision. Research on the GSPRT is currently underway and results are encouraging.

- Research on clipped SPRT's and moving window SPRT's. These SPRT's do not allow the ASN to exceed some prespecified value. This is done by simply beginning the SPRT again (clipped SPRT) or by changing the function of observations in the critical inequality (moving window SPRT) in some mathematically convenient way.
- Application of the SPRT to more than two detectors.
- Encoding the final system in FORTRAN and implementing it as part of the expert system described in the introduction of this paper.
- Extension of the SPRT from analysis of DN-signal data, specifically, to general signal validation and sensor operability applications for all areas of nuclear reactor operation wherein a given physical variable is being monitored by multiple, redundant sensors (e.g. thermocouples, pressure transducers, flux monitors, flow meters, etc.).

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