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PARTICLE DIFFUSION IN TOROIDALLY SYMMETRIC SYSTEMS

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Particle Diffusion in Toroidally Symmetric Systems

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ABSTRACT

Since the two fluid equations of a plasma are expressions of momentum conservation, they must hold independent of the mean free path to system size ratio. Diffusion in a toroidally symmetric torus is calculated with the two fluid equations, using Pfirsch-Schlüter diffusion and neoclassical diffusion as examples.

I. INTRODUCTION

The plasma two fluid equations have only been used to calculate diffusion in the short mean free path or Pfirsch-Schlüter regime. However, the two fluid kinetic equations are basically statements of momentum conservation (or force balance) and should, with proper interpretation, apply independent of the ratio of mean free path to system size.

In this paper steady state toroidally symmetric diffusion will be calculated using two fluid equations. Particular emphasis is given to showing how neoclassical diffusion can be derived in this manner. The advantage of the method is a clarification of diverse concepts as bootstrap currents, intrinsic ambipolarity and the Ware drift. The linearized, time independent, two fluid equations for a plasma can be written for the ions and electrons respectively as

$$\vec{\nabla} P_i = en\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right) + \vec{f}_i - \vec{R}_{ei} \qquad (1)$$

$$\vec{\nabla} P_e = - en\left(E + \frac{\vec{u}}{c} \times \vec{B}\right) + \vec{f}_e + R_{ei} \qquad (2)$$

The various terms in these equations, which come from the first velocity moments of kinetic equations, require interpretation. The first term in each equation is the force produced by the pressure of the species. The pressure--or second moment of the distribution function--is in general a second rank tensor.

However, toroidally confined plasmas have an essentially scalar pressure due to their long confinement relative to relaxation times. Although a more general pressure tensor could be included in the theory, we will assume a scalar pressure.

The terms giving the forces on each species due electric and magnetic fields are quite general. The velocity of the ions, $\vec{v}$, and the electrons, $\vec{u}$, are defined so $m_+ n\vec{v}$ and $m_e n\vec{u}$ are the momentum density of the ions and the electrons and so that the continuity equations hold (the zeroth velocity moments of the kinetic equations)

$$\vec{\nabla}\cdot n\vec{v} = \vec{\nabla}\cdot n\vec{u} = S \ . \tag{3}$$

S is the source of new plasma to replace that lost by diffusion.

The forces $\vec{f}_i$ and $\vec{f}_e$ represent the viscosity of the two fluids. Actually, viscosity can be viewed as a manifestation of a tensor pressure. There are two reasons for considering the pressure a scalar and introducing a viscosity. First, the viscous force is small compared to the pressure gradient and will be considered in a different order in the solution. Second, at least in the examples that have been studied, the forces $\vec{f}_{i,e}$ are linearly dependent on the velocity of the species in question as one expects a viscosity to be. Strictly speaking, $\vec{f}_{i,e}$ represent any momentum transfer between groups of particles of the same species which is not included in the pressure gradient. In the usual fluid regime one finds the viscous force is dominated by parallel viscosity. That is, viscosity quickly establishes flow equilibrium within a magnetic surface while flow equilibrium across magnetic surfaces is established on such a long time scale that it usually can be neglected. This effect should be accentuated by longer mean free paths; so the cross field viscosity will be assumed negligible. The force $\vec{R}_{ei}$ is the momentum transfer between electrons and ions, often called the Braginskii force.[1] In this paper we will use the usual expression for the Braginskii force, $\vec{R}_{ei} = en\overleftrightarrow{\eta}\cdot\vec{j}$ where $\overleftrightarrow{\eta} = \eta_{\perp}(\overleftrightarrow{\delta}-\hat{b}\hat{b}) + \eta\,\hat{b}\hat{b}$ is the resistivity and $\vec{j}$ is the current $j = en(\vec{v} - \vec{u})$.

II. LOWER ORDER EQUILIBRIUM

The problem of diffusion in a torus is considered in two orders. In the lower order the forces $\vec{f}_{i,e}$ and $\vec{R}_{ei}$ are considered negligibly small. If the lower order two fluid equations are added, we obtain the usual ideal magnetohydrodynamic equation

$$\vec{\nabla}P = \frac{1}{c}\,\vec{j} \times \vec{B}\ , \quad P = P_e + P_i \tag{4}$$

The other equations are

$$\left(\vec{E} - \frac{1}{en}\ \vec{\nabla}P_i\right) + \frac{\vec{v}}{c} \times \vec{B} = 0 \tag{5}$$

$$\vec{\nabla}\cdot\vec{j} = 0\ , \quad \vec{\nabla}\cdot n\vec{v} = 0\ . \tag{6}$$

To present the results in a simple form, we will use the Knorr model[2] with the toroidal coordinate system given in Fig. 1

$$\vec{B} = B_\theta \hat{\theta} + B_\phi \hat{\phi} \quad , \; B_\theta = B_p(r) R_o / R$$

$$B_\phi = B_T(r) R_o / R \; , \; R = R_o - r \cos\theta \; . \tag{7}$$

We also define the inverse aspect ratio, $\varepsilon = r/R_o$, the ratio of field components $\Theta = B_\theta / B_\phi$, and the safety factor, $q = \varepsilon/\Theta$. Both ε and Θ will be considered small compared to unity. The unit vector along the magnetic field is $\hat{b} = \vec{B}/|\vec{B}|$.

Equation (4) implies $\hat{b} \cdot \vec{\nabla} P = 0$. Coupled with toroidal symmetry, this means the pressure is a function only of r and the current has no radial component. Zero current divergence implies

$$j_\theta = j_p(r) R_o / R \tag{8}$$

with j_p an arbitrary function of r . Equation (4) then requires

$$j_\phi = \frac{1}{\Theta} \left(j_p R_o / R - j_D R / R_o \right) \tag{9}$$

with

$$j_D(r) = \frac{c}{B_T} \frac{dp}{dr} \quad . \tag{10}$$

Equation (5) can be similarly analyzed to obtain the ion velocity distribution. In particular, one can show $\vec{E} - (\vec{\nabla} P_i)/en$

can have only a radial component and the ion velocity has only $\hat{\theta}$ and $\hat{\phi}$ components.

$$v_\theta = v_p(r) R_o/R \tag{11}$$

$$v_\phi = \frac{1}{\Theta}\left(v_p R_o/R - v_E R/R_o\right) \tag{12}$$

with v_p an arbitrary function of r and

$$v_E(r) = -\frac{c}{B_T}\left(E_r - \frac{1}{en}\, dP_i/dr\right) . \tag{13}$$

III. CONSISTENCY RELATIONS

The lower order equilibrium derived in Sec. II contains two arbitrary functions of r , v_p and j_p. These arbitrary functions can be evaluated by requiring the lower order equilibrium be consistent with the full two fluid equations, Eqs. (1) and (2).

Before deriving the consistency relations, an expression for the electric field must be obtained. In steady state $\vec{\nabla} \times \vec{E} = 0$. Coupled with toroidal symmetry and the fact that the electric field must be finite everywhere, this implies

$$\vec{E} = E_T \frac{R_o}{R} \hat{\phi} - \vec{\nabla}\Phi \tag{14}$$

Φ is the electrostatic potential and E_T , which must be a constant, drives the so-called Ohmic heating current.

We can now derive the consistency relations. The $\hat{b}$ component of the full electron equation is

$$\frac{enE_T}{(1+\Theta^2)^{1/2}} \frac{R_o}{R} + \frac{1}{(1+\Theta^2)^{1/2}} \frac{\Theta}{r} \left(\frac{\partial P_e}{\partial \theta} - en \frac{\partial \Phi}{\partial \theta} \right) = \hat{b}\cdot\vec{f}_e + \hat{b}\cdot\vec{R}_{ei} \ . \quad (15)$$

Since $\partial P_e/\partial\theta$, $\partial\Phi/\partial\theta$, and E_T are small--being of the same order as the dissipative terms--we can use the lower order equilibrium density, n , which is independent of θ , to solve to first order in the dissipation. This implies

$$\frac{enE_T}{(1+\Theta^2)^{1/2}} \int_o^{2\pi} \frac{R_o}{R} \frac{d\theta}{2\pi} = \int_o^{2\pi} \hat{b}\cdot\vec{f}_e \, d\theta/2\pi + \int_o^{2\pi} \hat{b}\cdot\vec{R}_{ei} \, d\theta/2\pi \quad (16)$$

A similar analysis of the equation derived by adding Eqs. (1) and (2) gives

$$\int_o^{2\pi} \hat{b}\cdot f_e \, d\theta/2\pi + \int_o^{2\pi} \hat{b}\cdot\vec{f}_i \, d\theta/2\pi = 0 \ . \quad (17)$$

Equations (16) and (17) determine the arbitrary functions of radius j_p and v_p .

To derive the consequences of Eqs.(16) and (17) the parallel viscosity must be evaluated. In the fluid regime the parallel viscosity is given by[3]

$$\vec{f} = \vec{\nabla}\cdot\left[\left(\hat{b}\hat{b} - \frac{1}{3}\overleftrightarrow{\delta}\right)F\right] \tag{18}$$

with $\overleftrightarrow{\delta}$ the identity tensor and

$$F = 2anT\tau\left[\hat{b}\cdot\vec{\nabla}(\hat{b}\cdot\vec{v}) - (\hat{b}\cdot\vec{\nabla}\hat{b})\cdot\vec{v} - \frac{K}{3}\vec{\nabla}\cdot\vec{v}\right] . \tag{19}$$

The constant, a , is 3/2 for ions and 9/8 for electrons[1] while τ is the collision time for the species in question. For the velocity distribution given in Eqs. (11) and (12), one can show[4]

$$\int_0^{2\pi} \hat{b}\cdot\vec{f}_i \, d\theta/2\pi = - a_i \frac{\Theta}{(1+\Theta^2)^{1/2}} \frac{n_i T_i \tau_i}{R_0^2} v_p . \tag{20}$$

The integral of $\hat{b}\cdot\vec{f}_e$ is likewise proportional to u_p , which is defined so $j_p = en(v_p - u_p)$. Using these results, Eq. (17) implies that

$$v_p = - \frac{a_e}{a_i} Z \frac{T_e}{T_i} \frac{\tau_e}{\tau_i} u_p \tag{21}$$

with Z being the ionic charge. For $T_e = T_i$, $v_p \simeq -Z(m_e/m_i)^{1/2}u_p$ and we can neglect the ion contribution to the poloidal current.

In the neoclassical regime, the momentum transfer between electron groups is dominated by the boundary layer between trapped and untrapped electrons. Galeev and Sagdeev[5] noted that this interaction could be calculated in the drift kinetic approximation in an analogous manner to the damping of an electrostatic wave as calculated by Zakharov and Karpman.[6] The electrons move with a velocity u_p/Θ through the oscillations of the magnetic field strength. Following Zakharov and Karpman, one can show the force the electrons feel due to their motion along the magnetic field undulations is approximately

$$\int_0^{2\pi} \hat{b}\cdot\vec{f}_e \, d\theta/2\pi \simeq - m_e n \frac{\varepsilon^{1/2}}{\tau_e} \frac{u_p}{\Theta} \quad . \tag{22}$$

Again, Eq. (17) implies $v_p \simeq - Z(m_e/m_i)^{1/2} u_p$ for $T_e \simeq T_i$ and we can again neglect the ion contribution to the poloidal current.

To carry out the analysis in a general manner we will define α so

$$\int_0^{2\pi} \hat{b}\cdot\vec{f}_e \, d\theta = - \alpha \frac{e^2 n^2 \eta_{\parallel}}{\Theta(1+\Theta^2)^{1/2}} u_p \tag{23}$$

with $\eta_{\parallel}$ the classical parallel resistivity. The classical parallel resistivity is $\eta_{\parallel} = m_e/(2e^2 n\tau_{ei})$ with τ_{ei} the time constant for electrons to transfer momentum to ions. (The parallel resistivity in the neoclassical regime[7] is equal to the classical value for small enough ε). The value of α in the fluid regime, α_F , and in the neoclassical regime, α_{NC} , are

$$\alpha_F = \frac{9}{4} \varepsilon^2 \frac{\tau_{ei}}{\tau_e}\left(\frac{\lambda_e}{qR_o}\right)^2 \tag{24}$$

$$\alpha_{NC} \simeq \frac{\tau_{ei}}{\tau_e} \varepsilon^{1/2}$$

with λ_e the electron mean free path, $\lambda_e = (T_e/m_e)^{1/2}\tau_e$. For the fluid picture to hold one must have $\lambda_e << qR_o$. In classical collision theory $\tau_{ei} = \tau_e$. However, it is of interest to retain the τ_{ei}/τ_e ratio in the equations. The parallel viscosity integral and α are plotted schematically in Figs. (2) and (3).

Eqs. (8) and (9) can be used to show

$$\int_o^{2\pi} \hat{b}\cdot\vec{R}_{ie}\, d\theta/2\pi = en\eta_\parallel \frac{1}{\Theta(1+\Theta^2)^{1/2}}\left[\frac{1+\Theta^2}{(1-\varepsilon^2)^{1/2}} j_P - j_D\right] \tag{25}$$

We can now evaluate Eq. (16) to the required order in ε and Θ . This gives

$$j_P = \frac{1}{\alpha+1+\Theta^2+\frac{1}{2}\varepsilon^2}\left[j_D + \Theta E_T/\eta_\parallel\right] \tag{26}$$

We have assumed $-enu_p = j_P$ for $u_P >> v_P$. The parallel current can also be evaluated

$$j_{\parallel} \equiv \int_0^{2\pi} \hat{b}\cdot\vec{j}\; d\theta/2\pi = -\frac{\alpha}{1+\alpha}\,\frac{j_D}{\Theta} + \frac{E_T}{(1+\alpha)\eta_{\parallel}} \tag{27}$$

The first term in this expression is the famous bootstrap current j_{BS} .

$$j_{BS} = -\frac{\alpha}{1+\alpha}\,\frac{j_D}{\Theta} = -\frac{\alpha}{1+\alpha}\,\frac{c}{B_P}\,\frac{dP}{dr} \tag{28}$$

The bootstrap current is negligible in the fluid regime and assumes its well known value[8] for $\alpha = \sqrt{\varepsilon}$ in the neoclassical regime. The second term is the so-called Ohmic heating current with the resistivity enhanced by a factor $(1+\alpha)$. This effect is also well known in the neoclassical regime.

IV. DIFFUSION

To calculate diffusion in a toroidally symmetric system, we use the $\hat{\phi}$ components of the two fluid equations. To illustrate a point, we will consider the ion equation first, including the inertial term $m_+ n\partial v/\partial t$. The $\hat{\phi}$ component of the ion equation is

$$m_+ n\partial v_\phi/\partial t = enE_T\,\frac{R_o}{R} + \frac{en}{c}\,v_r\,B_\theta + \hat{\phi}\cdot\vec{f}_i - \hat{\phi}\cdot\vec{R}_{ie} \;. \tag{29}$$

To calculate diffusion we want the average ion flux across the magnetic surfaces $\langle nv_r\rangle$. This is

$$\langle nv_r\rangle = \int_0^{2\pi} \left(\frac{R}{R_o}\right) nv_r\; d\theta/2\pi \;. \tag{30}$$

Multiplying Eq. (29) by $(R/R_o)^2$ and integrating over θ , we find

$$\frac{\partial}{\partial t}\frac{1}{R_o}\int_o^{2\pi} \frac{R}{R_o}\,(Rm_+ nv_\phi)\; d\theta/2\pi = enE_T \int_o^{2\pi} \frac{R}{R_o}\; d\theta/2\pi$$

$$+ \frac{e}{c}\,\langle nv_r\rangle\; B_p + \int_o^{2\pi} \left(\frac{R}{R_o}\right)^2 \hat{\phi}\cdot\vec{f}_i\; d\theta/2\pi - \int_o^{2\pi} \left(\frac{R}{R_o}\right)^2 \hat{\phi}\cdot R_{ei}\; d\theta/2\pi \;. \tag{31}$$

The intergral on the left side of Eq. (31) is the average value of the toroidally directed angular momentum within a magnetic surface. This means the integral over $\hat{\phi}\cdot\vec{f}_i$ in Eq. (31) represents the rate toroidally directed angular momentum is destroyed by the viscosity acting within a magnetic surface. Due to the toroidal symmetry, this must vanish, i.e.,

$$\int_o^{2\pi} \left(\frac{R}{R_o}\right)^2 \hat{\phi}\cdot\vec{f}_i\; d\theta/2\pi = 0 \;. \tag{32}$$

This can be explicitly shown for the classical parallel viscosity given in Eqs. (18) and (19).[4] Equation (31) implies in the time independent case that

$$\langle nv_r\rangle = \frac{c}{eB_p}\int_o^{2\pi} \left(\frac{R}{R_o}\right)^2 \hat{\phi}\cdot\vec{R}_{ei}\; d\theta/2\pi - nc\,\frac{E_T}{B_p} \tag{33}$$

A similar analysis for the electrons gives

$$\langle nu_r \rangle = \langle nv_r \rangle \ . \tag{34}$$

Equation (34) is equivalent to $\langle j_r \rangle = 0$. That is, there is no net current across the magnetic surfaces if the viscous interaction between magnetic surfaces is neglected and there is no other mechanism for damping toroidally directed angular momentum. The property $\langle j_r \rangle = 0$ independent of the radial electric field is called intrinsic ambipolarity. The intrinsic ambipolarity of neoclassical diffusion was controversial for several years.[5,9,10,11,12]

To evaluate the diffusion rate in both the classical and the neoclassical regimes, we assume $\vec{R}_{ei} = en\vec{\eta}\cdot\vec{j}$ with $\overleftrightarrow{\eta} = \vec{\eta}_\perp(\overleftrightarrow{\delta}-\hat{b}\hat{b}) + \eta_\parallel\hat{b}\hat{b}$. With this approximate form for the Braginskii force the integral in Eq. (33) can be carried out using the lower order equilibrium solution Eqs. (8) and (9). Using Eq. (26) and the definition of i_D , Eq. (10) one finds

$$\langle nv_r \rangle = -\frac{\alpha}{1+\alpha}\ \eta_\parallel \frac{nc^2}{B_p^2}\frac{dp}{dr} - \left(\eta_\perp + \frac{2q^2}{1+\alpha}\eta_\parallel\right)\frac{nc^2}{B_T^2}\frac{dp}{dr} - \frac{\alpha}{1+\alpha}\,nc\,\frac{E_T}{B_p} \tag{35}$$

In the fluid regime $\alpha \ll \varepsilon^2$ and Eq. (35) implies

$$\langle nv_r \rangle = -\left(\eta_\perp + 2q^2\eta_\parallel\right)\frac{nc^2}{B_T^2}\frac{dp}{dr} \tag{36}$$

the well-known Pfirsch-Schlüter diffusion rate.[13]

In the neoclassical regime, $\alpha \simeq \tau_{ei}/\tau_e \, \varepsilon^{1/2}$; so

$$\langle nv_r \rangle = - \frac{\tau_{ei}}{\tau_e} \varepsilon^{1/2} \frac{n\eta c^2}{B_p^2} \frac{dP}{dr} - \frac{\tau_{ei}}{\tau_e} \varepsilon^{1/2} nc \frac{E_T}{B_p} \tag{37}$$

the usual neoclassical result [5,12] with the addition of the Ware drift[14]. Equation (37) has one peculiar feature, the τ_{ei} in front cancels the τ_{ei} in the resistivity. The fact $\langle nv_r \rangle$ depends on τ_e rather than τ_{ei} was responsible for much of the confusion about the intrinsic ambipolarity of neoclassical diffusion.

V. DISCUSSION

The two fluid equations have been shown powerful enough to calculate neoclassical diffusion. The difference between the Pfirsch-Schülter and the neoclassical regimes is the importance of the parallel viscosity. The parallel viscosity tends to establish rigid body rotation within a magnetic surface. Due to the assumed toroidal symmetry, it can damp poloidal but not toroidal rotation. As electrons diffuse radially, the force $\vec{u} \times \vec{B}$ accelerates the electrons perpendicular to the magnetic field. This acceleration maintains the diamagnetic current against the dissipative effects of resistivity and viscosity. The viscosity, damping only poloidal rotation, exerts a net force along the magnetic field lines, which must be balanced by the electron-ion interaction (the parallel resistivity.) This balance gives the bootstrap current.

The bootstrap current is the net current in a magnetic surface, driven by the pressure gradient and parallel to the magnetic field lines. Once the bootstrap current is obtained the correct neoclassical diffusion rate, to lowest order in the inverse aspect ratio, can be obtained by exactly the same method that Pfirsch-Schülter diffusion is obtained from the toroidal current in the fluid regime.

Since tokamaks experimentally do not behave as one would expect with just classical collisions, it is of interest to ask what happens in the presence of microinstability. The electrostatic waves associated with microinstabilities in tokamaks can have two qualitatively different effects. First, they can enhance the transfer of energy and momentum between various groups of particles within a magnetic surface. Second, they can transfer energy and momentum from a group of particles in a magnetic surface to a distant point in the plasma. A good review of the transfer of energy and momentum by waves has been given by Bers.[15] If the transfer distance described by second effect is comparable to the size of the plasma, one can not easily think of the process as diffusion and within the context of this paper little can be said. However, if the effect of instability is just to enhance inter-particle momentum transfer within a surface, the instability can be represented by enhanced collision frequencies. In our notation, an enhanced isotropization rate for the electrons would be represented by a shortened τ_e . An enhanced rate of momentum transfer between electrons and ions would shorten τ_{ei} and increase the resistivity. Since there is both a parallel and a perpendicular resistivity, there are both a parallel and a perpendicular momentum transfer times, $\tau_{ei\parallel}$ and $\tau_{ei\perp}$. Classically, $\tau_e = \tau_{ei\parallel} = \tau_{ei\perp}$, but in the presence of microinstability these ratios can take any value.

An interesting question is what happens to the bootstrap current in the presence of microinstability. The importance of the bootstrap current is determined by the dimensionless parameter α, introduced in Eq. (23), which is basically the ratio of the parallel electron viscosity and the parallel resistivity. The bootstrap current gives the dominate diffusion for $\alpha > \varepsilon^2$, and it determines the magnetohydrodynamic equilibrium (β limitations, topology of surfaces, etc.) for $\alpha > \varepsilon$ with ε the inverse aspect ratio. If the bootstrap current is to determine the equilibrium one requires $\lambda_{ei\parallel}\lambda_e > (qR_o)^2/\varepsilon$ with $\lambda_{ei\parallel} = (T_e/m_e)^{1/2}\tau_{e\parallel}$, $\lambda_e = (T_e/m_e)^{1/2}\tau_e$, and qR_o the connection length. The collision times τ_e and $\tau_{ei\parallel}$ are of course the effective collision times in the presence of instabilities. If the mean free paths are long enough $\lambda_e > qR_o/\varepsilon^{3/2} < \lambda_{ei\parallel}$, the neoclassical expression for α can be used $\alpha \simeq (\tau_{ei\parallel}/\tau_e)\varepsilon^{1/2}$. When $\tau_{ei\parallel} >> \tau_e/\varepsilon^{1/2}$, $\alpha >> 1$. The large α case is quite interesting, for in this limit the equations we have derived are identical to those for a tokamak without a toroidal field. In other words in the large α case, the toroidal field serves only to give magnetohydrodynamic stability and does not affect the equilibrium or diffusion. In this case the parallel viscosity is so strong the electrons do not rotate at all poloidally. The value of α to be expected in large tokamaks is not known. Present theories of instability induced diffusion make heavy use of the $\gamma/k_\perp^2$ estimate of the diffusion rate.[16] These theories do not attempt to calculate the parameters necessary to evaluate α and hence the bootstrap current. The instabilities of greatest interest, the trapped particle modes, have $k_\parallel \ll k_\perp$ and hence strongly enhance $\eta_\perp$ but not $\eta_\parallel$. Despite the large effect these modes have on the transport they may have little effect on the bootstrap current.

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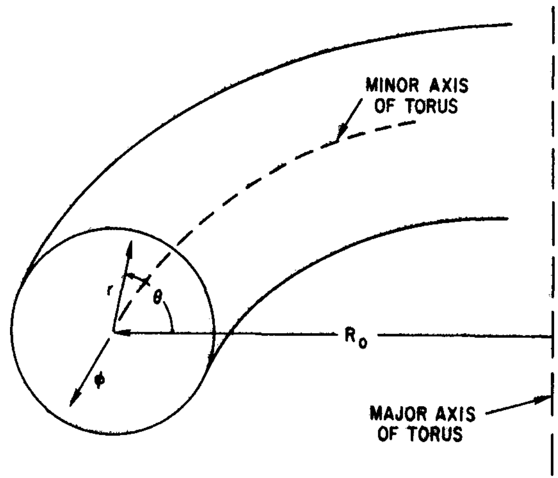

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Fig. 1. The Toroidal Coordinate System. In this system

$$R = R_o - r\cos\theta \quad , \quad \vec{\nabla}F = \hat{r}\frac{\partial}{\partial r}F + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial\theta}F + \hat{\theta}\frac{1}{R}\frac{\partial}{\partial\phi}F$$

$$\vec{\nabla}\cdot\vec{B} = \frac{1}{rR}\frac{\partial}{\partial r}(rRB_r) + \frac{1}{rR}\frac{\partial}{\partial\theta}(RB_\theta) + \frac{1}{R}\frac{\partial}{\partial\phi}B_\phi$$

$$\vec{\nabla}\times\vec{B} = \frac{1}{R}\left[\frac{1}{r}\frac{\partial}{\partial\theta}(RB_\phi) - \frac{\partial}{\partial\phi}B_\theta\right]\hat{r} + \frac{1}{R}\left[\frac{\partial}{\partial\phi}B_r - \frac{\partial}{\partial r}RB_\phi\right]\hat{\theta}$$

$$+ \frac{1}{r}\left[\frac{\partial}{\partial r}rB_\theta - \frac{\partial}{\partial\theta}B_r\right]\hat{\phi}$$

$$d\hat{r} = \hat{\theta}d\theta - \hat{\phi}\cos\theta d\phi \quad , \quad d\hat{\theta} = -\hat{r}d\theta + \hat{\phi}\sin\theta\, d\phi$$

$$d\hat{\phi} = (\cos\theta\ \hat{r} - \sin\theta\ \hat{\theta})d\phi$$

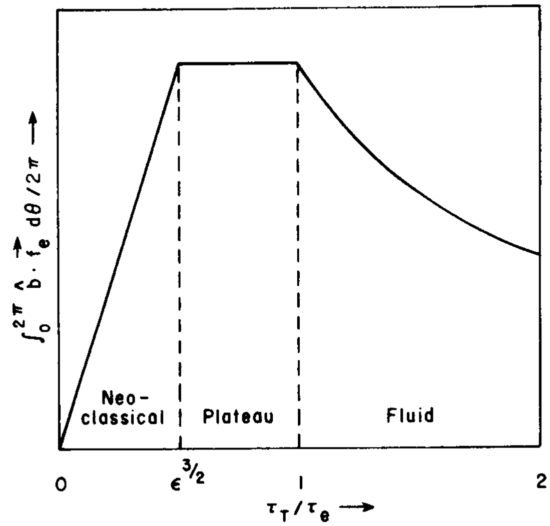

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Fig. 2. The Average Parallel Viscosity along the Field Lines. The average of the parallel viscous force along the magnetic field lines is given versus the electron collision time τ_e . The electron transit time, $\tau_T = qR_o/v_e$, is the time it takes an electron to travel the connection length of the field lines.

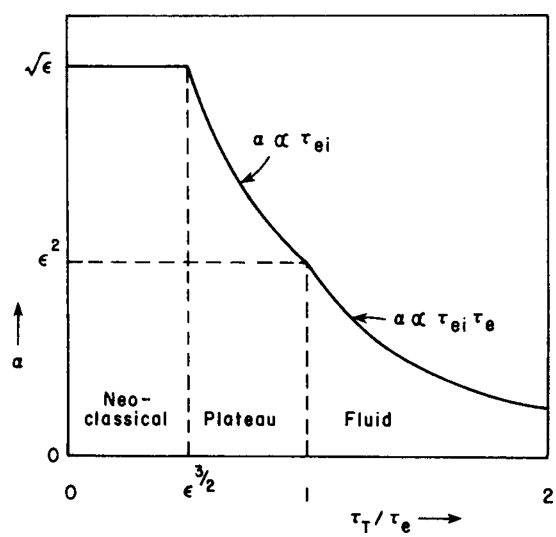

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Fig. 3. The Bootstrap Current Parameter α . The importance of the bootstrap current is determined by α . It is non-negligible for $\alpha \geq \varepsilon^2$. The electron transit time is $\tau_T = qR_o/v_e$ and the electron collision time is τ_e .