Inv. Talk - International Symposium on "Three-Body Force in the Three-Nucleon System"
The George Washington University, Washington, DC, 24-26 April 1986

CONF-
CONF-8604194--8
DE87 001578

RELATIVISTIC PARTICLE QUANTUM DYNAMICS AND THREE-BODY FORCES IN THE THREE-NUCLEON SYSTEM

F. Coester
Argonne National Laboratory, Argonne, IL 60439-4843

INTRODUCTION

The most important data on short-range effects are produced by electromagnetic probes, and the central questions relating the three-nucleon system to fundamental theory concern the role which explicit quark degrees of freedom should play in the three-nucleon dynamics. Three-body forces do not necessarily play a pivotal role in these problems. Requirements of relativistic invariance, however, can be expected to have an important quantitative impact on all short-range properties of the three-nucleon system.

The relativistic dynamics of any quantum system implies the existence of a unitary representation $U(a,\Lambda)$ of translations and Lorentz transformations (Poincaré group).[1,2] The generators P^μ of the translations have the physical significance of energy and momentum. For any system the unitary representation of the Poincaré group also specifies the dynamics. For Lagrangian field theories the Lagrangian which specifies the dynamics also determines the corresponding unitary representation of the translations and Lorentz transformations.[3,4]

The covariance requirement for currents

$$U(\Lambda) j^\mu(x)\, U^{-1}(\Lambda) = \Lambda^\mu{}_\nu j^\nu(\Lambda^{-1}x) \tag{1}$$

as well as the continuity equation

$$[P_\mu, j^\mu(x)] = 0 \tag{2}$$

imply consistency conditions which relate the strong-interaction dynamics to the electromagnetic and weak currents.

One of the most important qualitative features of relativistic quantum dynamics stems from the fact that the group structure demands that some transformations, other than the time evolutions, depend on the dynamics.[2]

It is possible to choose the representation of a kinematic subgroup to be independent of the dynamics. The choice leads to different "forms of dynamics"[2] which are unitarily equivalent as far as the observable consquences are concerned.[5] In the familiar "instant form" the kinematic subgroup (translations and rotations) leaves the hyperplane $t \equiv x^0 = 0$ invariant, and the Lorentz boosts are dynamical transformations. In the "front-form" dynamics[4,6] the kinematic subgroup leaves the light front $\tau \equiv x^0 + x^3 = 0$ invariant, and the rotations about any transverse axis are dynamical transformations.

In a Fock-space representation of a Lagrangian field theory, restriction to a finite number of particles always destroys the relativistic invariance. However relativistic dynamics exists also for finite numbers of particles, which need not be elementary.[6,7] Both the three-nucleon system and quark models consisting of a finite number of light quarks are examples. I believe it is important to emphasize the fact that the need for a relativistic dynamics does not in itself imply the necessity of describing the system in terms of its ultimate elementary constituents. Questions concerning the relevant subnucleon degrees of freedom can and should be separated from questions concerning the relativistic invariance.

The construction of relativistic quantum particle dynamics is based on the following observations.[7] The generators of the infinitesimal dynamical transformations can be obtained as functions of the kinematic generators, the invariant mass operator of the interacting system and additional operators which <u>may</u> be obtained from the noninteracting system. In the instant form these additional operators are the components of the Newton-Wigner position operator,[8] while in the front form they are the transverse components of the spin.

The front form is particularly convenient because the kinematic subgroup includes the Lorentz transformations which are important in the calculation of electromagentic form factors and inelastic structure functions. In the front form, relativistic quantum particle dynamics and the Fock-space representation of quantum field theory are more closely related than in the instant form.

RELATIVISTIC QUANTUM DYNAMICS

The generators of infinitesimal Poincaré transformations are the four-momentum $\{P^0, \vec{P}\}$, the angular momentum $\vec{J}$, and the Lorentz boosts $\vec{K}$. The generators of the front-form kinematic subgroup are $P^+ \equiv P^0 + P^3$, the transverse component of the momentum $\vec{P}_T$, the longitudinal components of the angular momentum J_3, the boosts K_3, and

$$\vec{E} \equiv \vec{K}_T + \vec{n} \times \vec{J}_T \quad ; \quad \vec{n} \equiv \{0,0,1\} \ . \tag{3}$$

The dynamic generators are $P^- \equiv P^0 - P^3$ and the transverse components of the angular momentum $\vec{J}_T$. The mass operator M is, of course, related to the four-momentum by

$$M^2 = P^+P^- - \vec{P}_T^2 \ . \tag{4}$$

A bound state must be an eigenfunction of the spin operator $\vec{I}^2$ as well as the mass operator M. The spin is related to the Pauli-Lubanski[9] vector $\{W^0,\vec{W}\} \equiv \{\vec{P}\cdot\vec{J},P^0\vec{J}+\vec{P}\times\vec{K}\}$ by

$$I_3 = W^+/P^+ \quad ; \quad M\vec{I}_T = \vec{W}_T - \vec{P}_T \ . \tag{5}$$

Conversely, if the mass M and the transverse spin $\vec{I}_T$ are known, then the dynamic generators P^- and $\vec{J}_T$ are determined by

$$P^- = [M^2 + \vec{P}_T^2]/P^+ \ , \tag{6}$$

and

$$\vec{J}_T = \frac{M}{P^+}\vec{I}_T + \frac{P^+-P^-}{2P^+}\vec{n}\times\vec{E} + \frac{\vec{P}_T}{P^+}I_3 + \frac{\vec{n}\times\vec{P}_T}{P^+}K_3 \ . \tag{7}$$

Projection of a meson field theory onto the two- or three-nucleon sector of the Fock space produces an expression for M^2 which is invariant under the kinematic subgroup, but the projections of the spin components do not commute with M^2 and do not satisfy the correct commutation relations. Poincaré invariant dynamical models can be constructed by assuring that M^2 commutes with the total spin of the free particles.

The main advantages of the front-form dynamics are:

1. The Hamiltonian $H \equiv P^- = [M^2+\vec{P}_T^2]/P^+$ is a linear function of M^2, while the instant form involves the square root relation $H \equiv P^0 = \sqrt{M^2+\vec{P}^2}$.

2. Since the spectrum of P^+ is non-negative the Fock vacuum and the physical vacuum of a field theory are the same if admixtures of $p^+=0$ states of massless particles and ultraviolet divergencies at $p^+=0$ are cut off.

3. The inital and final states $|p^+,-1/2Q,0\rangle$ and $|p^+,1/2Q,0\rangle$ of

elastic scattering are related by a kinematic Lorentz transformation, and form factors can be extracted from the matrix elements of a single component of the current, $j^+(0)$, which is invariant under the kinematic group,

$$[\vec{E}, j^+(0)] = 0 \quad ; \quad [J_3, j^+(0)] = 0 \quad ; \quad [K_3, j^+(0)] = i j^+(0) \ . \tag{8}$$

The main <u>disadvantage</u> is the complicated relation between the spin and the transverse angular momentum shown in Eq. (7), and the complicated relations between the spins of subsystems and the total spin. I believe the advantages outweigh the disadvantages.

TWO-BODY SYSTEMS

Sates $|\psi\rangle$ of a single nucleon are represented by square integrable functions $\psi(p,\mu)$, where $\mu = \pm 1/2$ is the longitudinal component of the spin and $p \equiv \{p^+, \vec{p}_T\}$. States $|\psi\rangle$ of a two-nucleon system are represented by square integrable functions $\psi(p_1, \mu_1, p_2, \mu_2)$. All the kinematic generators are additive in the two nucleons. Appropriate internal variables are

$$\xi = p_1^+/P^+ \quad \text{and} \quad \vec{k}_T = \vec{p}_{1T} - \xi \vec{P}_T \ , \tag{9}$$

where $P = p_1 + p_2$. (I am using ξ instead of the usual x for the momentum fraction in order to avoid possible confusion with space-time points.)

The mass operator M is given by

$$M^2 = \frac{m^2 + \vec{k}_T^2}{\xi(1-\xi)} + 4mV_{12} = M_0^2 + 4mV_{12} \ , \tag{10}$$

and the Hamiltonian is

$$H = H_0 + 4mV_{12}/P^+ \ , \tag{11}$$

where the operator V_{12} is the nucleon-nucleon potential. Lorentz invariance requires that V_{12} commute with P and be independent of P. Furthermore the dynamics so formulated is Poincaré invariant if, and only if, M^2 commutes with the spin $\vec{I}$. A bound-state wave function $\psi(\xi, \vec{k}_T, \mu_1, \mu_2)$ must be an eigenfunction of M^2 and $\vec{I}^2$. The invariance of M can be assured in the following manner. <u>Define</u> the longitudinal component of the internal momentum $\vec{k}$ as a function of ξ and $\vec{k}_T$ by

$$\vec{k}\cdot\vec{n} = \frac{1}{2}\ \{M_0\xi - \frac{m^2+\vec{k}_T^2}{M_0\xi}\}\ . \tag{12}$$

The spin of the noninteracting two-nucleon system can then be expressed in the form[10]

$$\vec{I} = i\nabla_k\times\vec{k} + R(\xi,\vec{k}_T,m,M_0)\vec{s}_1 + R(1-\xi,-\vec{k}_T,m,M_0)\vec{s}_2\ , \tag{13}$$

where R denotes a Melosh rotation,[11]

$$R(\xi,\vec{k}_T,m,M_0) = \frac{m+M_0\xi - i\vec{\sigma}\cdot(\vec{n}\times\vec{k}_T)}{\sqrt{(m+M_0\xi)^2+\vec{k}_T^2}}\ . \tag{14}$$

Expressed as a function of the vector $\vec{k}$ the mass operator M is given by

$$M^2 = 4(k^2+m^2+mV_{12})\ , \tag{15}$$

where V_{12} must commute with the spin (13). Thus the dynamical equations for the internal coordinates have the same form as in the nonrelativistic case. The relations of $\vec{k}$ and P to the individual nucleon momentua p_1 and p_2 differ, of course, from the nonrelativistic relations. This difference becomes manifest in three-nucleon systems as well as in form factors of the deuteron. For slow nucleons we have the nonrelativistic approximation

$$\vec{p}_1+\vec{p}_2 = \{P^+-2m,\vec{P}_T\}\quad ;\quad \frac{1}{2}(\vec{p}_1-\vec{p}_2) = \{m(2\xi-1),\vec{k}_T\}\ . \tag{16}$$

Larger relativistic effects can be expected in the bound states of light quarks. As an illustration, consider a toy pion as a bound state of a spinless quark and antiquark. The complete representation of the bound state is given by

$$(P,\xi,\vec{k}_T|\Psi|P_i) = \delta(P-P_i)\ \psi(\xi,\vec{k}_T) \tag{17}$$

(Note that no special reference frame is involved in this description of the bound state.) The well-known expression for the charge form factor[12]

$$F_\pi(Q^2) = \int d\xi\int d^2k_T\psi(\xi,\vec{k}_T+(1-\xi)\vec{Q})\psi(\xi,\vec{k}_T) \tag{18}$$

follows immediately. The fact that the pion has spin zero imposes a nontrivial constraint on the wave function,[13]

$$\psi(\xi,\vec{k}_T) = [\xi(1-\xi)]^{-1/2}\,\chi\{[m^2+\vec{k}_T^2]/[\xi(1-\xi)]\} \,. \tag{19}$$

It follows that for large Q and zero quark mass $F_\pi(Q^2)$ is proportional to $1/Q^2$, while the nonrelativistic form obtains for heavy quarks.[13,14] By expressing ξ as a function of k_3 and $\vec{k}_T$ one obtains by a Fourier transform a spatial distribtuion of the quarks. For light quarks the rms radius of this distribution can differ substantially from the standard "charge radius" extracted from the charge form factor $F_\pi(Q^2)$.[14]

The two-body dynamics formulated here implies an obvious description of two nucleons in the presence of a noninteracting spectator. The transition to a fully interacting three-nucleon system involves new problems which I will address in the next Section.

THREE-NUCLEON SYSTEMS

As in the description of nonrelativistic systems the convenient choice of internal variables distinguishes one of the three particles. Let

$$P = p_1 + p_2 + p_3 \,, \tag{20}$$

$$\xi_{12} = p_1^+/(p_1^+ + p_2^+) \quad ; \quad \xi_3 = p_3^+/P^+ \,, \tag{21}$$

$$\vec{k}_T = \vec{p}_{1T} - \xi_{12}(\vec{P}_T - \vec{p}_{3T}) \quad ; \quad \vec{q}_T = \vec{p}_{3T} - \xi_3\vec{P}_T \,. \tag{22}$$

The mass operator M_{12} of the interacting 12 subsystem is given by Eqs. (10) or (15). All the Poincaré generators are additive in two-body cluster and the spectator. The mass and spin operators are unambiguously defined as functions of those generators. However, the spin operator so defined depends on the interaction V_{12}, and the operator $M^2_{12,3}$,

$$M^2_{12,3} = \frac{M^2_{12}+\vec{q}_T^{\,2}}{1-\xi_3} + \frac{m^2+\vec{q}_T^{\,2}}{\xi_3} = M_0^2 + \frac{4mV_{12}}{1-\xi_3} \,, \tag{23}$$

does not commute with the spin $\vec{I}_0$ of the noninteracting three-nucleon system, which commutes with M_0^2,

$$M_0^2 = \frac{m^2+\vec{k}_T^2}{\xi_{12}(1-\xi_{12})(1-\xi_3)} + \frac{m^2}{\xi_3} + \frac{\vec{q}_T^2}{\xi_3(1-\xi_3)} . \tag{24}$$

The noninteracting spin operator is[10]

$$\vec{I}_0 = i\nabla_q \times \vec{q} + R(1-\xi_3, -\vec{q}_T, M_{012}, M_0)\vec{I}_{12} + R(\xi_3, \vec{q}_T, m, M_0)\vec{s}_3 , \tag{25}$$

where the longitudinal component of the vector $\vec{q}$ is defined by

$$\vec{q}\cdot\vec{n} = \frac{1}{2}\{M_0\xi_3 - \frac{m^2+\vec{q}_T^2}{M_0\xi_3}\} . \tag{26}$$

The interaction-dependent spin operator $\vec{I}_{12,3}$ that commutes with $M_{12,3}$ can be obtained from (25) and (26) by replacing M_0 and M_{012} by $M_{12,3}$ and M_{12} respectively.

The Hamiltonian $H_{12,3} \equiv P^-_{12,3}$,

$$P^-_{12,3} = \sum_i p_i^- + 4mV_{12}/(p_1^+ + p_2^+) \tag{27}$$

has all the required invariance properties, but the addition of two or three two-body interactions destroys the invariance unless an appropriate three-body interaction is added. The expected result for the fully interacting three-ncuelon system is

$$P^- = \sum_i p_i^- + \sum_{i<j} 4mV_{ij}/(p_i^+ + p_j^+) + 6mV_{123}/P^+ \tag{28}$$

and

$$M^2 = M^2_{12,3} + M^2_{31,2} + M^2_{23,1} = 2M_0^2 + 6mV_{123} \tag{29}$$

The task at hand is to show that a three-body potential V_{123} which establishes the invariance of the three-body dynamics exists, can be constructed explicitly and is small. This is accomplished by constructing an operator $\bar{M}_{12,3}$ that commutes with $\vec{I}_0$ and describes the same two-body dynamics as $M_{12,3}$. The two mass operators $\bar{M}_{12,3}$ and $M_{12,3}$ are related by Sokolov's[15] unitary "packing" transformation $B_{12,3}$.

The operator V_{12} in Eq. (23) has the matrix representation

$$(k', \ell', S|\hat{V}_{12}|S, \ell, k) \times (\vec{q}_T', \xi', \mu_{12}', \mu_3'|1|\mu_3, \mu_{12}, \xi, \vec{q}_T) ; \tag{30}$$

The operator $\bar{V}_{12}$ designed to commute with $\vec{I}_0$ can be defined by the matrix representation

$$(k',\ell',S|\hat{V}_{12}|S,\ell,k) \times (q',L',I',\mu'|1|\mu,I,L,q) \; . \tag{31}$$

Manifestly the dynamics of the two-body subsystem is completely specified by the operator $\hat{V}_{12}$ which is the same in (30) and (31). The subsystem mass operators

$$M_{12}^2 = M_{012}^2 + 4mV_{12} \quad \text{and} \quad \bar{M}_{12}^2 = M_{012}^2 + 4m\bar{V}_{12} \tag{32}$$

yield the same two-body bound-state energies and the same scattering observables. The same is true for $M_{12,3}$ defined by Eq. (23) and $\bar{M}_{12,3}$ defined by

$$\bar{M}_{12,3} = \quad \bar{M}_{12}^2+q^2 + \quad m^2+q^2 \; . \tag{33}$$

They commute respectively with $\vec{I}_{12,3}$ and $\vec{I}_0$. Therefore, there exists a unitary transformation $B_{12,3}$ which transforms $\vec{I}_{12,3}$ into $\vec{I}_0$ and $M_{12,3}$ into $\bar{M}_{12,3}$. It follows that

$$B_{12,3}\xi B_{12,3}^{\dagger} - \xi = [\vec{n}\cdot\vec{q} + \quad m^2+q^2][M_{12,3}^{-1} - M_0^{-1}] \tag{34}$$

and

$$B_{12,3}^{\dagger}\vec{n}\cdot\vec{q}\; B_{12,3} - \vec{n}\cdot\vec{q} =$$
$$\frac{1}{2}\,\{(M_{12,3}-M_0)\xi - \frac{m^2+\vec{q}_T^{\,2}}{\xi}\,[M_{12,3}^{-1} - M_0^{-1}]\} \; . \tag{35}$$

Equation (34) or (35) provide the basis for an approximation. The effect of $B_{12,3}$ is small, i.e. $B_{12,3}\approx 1+ i\beta_{12,3}$ with $\beta_{12,3}$ of the order $\|V_{12}M_0^{-1}\|$.

Since $\bar{M}_{12,3}$ commutes with the spin $\vec{I}_0$ the mass operator

$$\bar{M}^2 = \bar{M}_{12,3}^2 + \bar{M}_{31,2}^2 + \bar{M}_{23,1}^2 - 2M_0^2 + 6m\bar{V}_{123} \tag{36}$$

is invariant for any three-body interaction $\bar{V}_{123}$ that commutes with the spin $\vec{I}_0$. The choice of $\bar{V}_{123}$ is subject to the same arbitrariness and restrictions as the nonrelativistic three-body potential.[16] A three-nucleon dynamics based on $\bar{M}$ and $\vec{I}_0$ satisfies all the invariance requirements, but the dynamic generators do not become additive if one of the particles is at a large distance. This property is essential for the construction of a consistent many-body theory.[7,15] It can be achieved by a unitary transformation B that transforms (36) into (29), and $\vec{I}_0$ into $\vec{I}$.

$$B\bar{M}B^{\dagger} = M \quad ; \quad B\vec{I}_0B^{\dagger} = \vec{I} \ . \tag{37}$$

An appropriately defined product of $B_{12,3}$, $B_{23,1}$ and $B_{31,2}$ will serve that purpose. The approximate form

$$B \approx 1 + i(\beta_{12,3} + \beta_{23,1} + \beta_{31,2}) \tag{38}$$

should be adequate for the three-nucleon system.

The three-nucleon dynamics so constructed satisfies all the Poincaré invariance requirements and has the correct cluster separability properties. The transformation (38) introduces three-body interactions in (29) even if $\bar{V}_{123}$ vanishes. Since the β's in Eq. (38) are small, of the order $\|V_{12}M_0^{-1}\|$, the effects of the three-nucleon forces required by Poincaré invariance can be expected to be relatively small.

SUMMARY

Questions concerning the effects of relativistic invariance can and should be separated from questions concerning the relevant subnucleon degrees of freedom that should be treated explicitly.

It is possible to formulate Poincaré invariant quark models with a finite number of quarks. In such models hadron states have definite spin, a feature which is absent in light-front perturbative treatments of QCD. Substantial differences from nonrelativistic quark models can occur for very light quarks.

It is possible to formulate a Poincaré invariant three-nucleon dynamics which has the same qualitative features as the nonrelativistic dynamics, including semiphenomenological two- and three-body forces. The invariance requirements do not constrain the allowable two-body forces and impose only a weak constraint on acceptable three-body forces.

Work supported by the U.S. Department of Energy, Nuclear Physics Division, under contract W-31-109-ENG-38.

REFERENCES

1. E. P. Wigner, Ann. Math. 40, 149 (1939).
2. P. A. M. Dirac, Rev. Mod. Phys. 21 392 (1949).
3. J. Schwinger, Phys. Rev. 127, 324 (1962).
4. Shau-Jin Chang et al., Phys. Rev. D 7, 1133 (1973).
5. S. N. Sokolov, Teor. Mat. Fiz. 24, 236 (1975).
6. H. Leutwyler and J. Stern, Ann. Phys. (N.Y.) 112, 94 (1978); B. L. G. Bakker, L. A. Kondratyuk and M. V. Terent'ev, Nucl. Phys. B158, 497 (1979); L. A. Kondratyuk and M. V. Terent'ev, Yad. Fiz. 31, 1087 (1980); F. M. Lev, Fortschr. Physik 31, 75 (1983).
7. F. Coester and W. N. Polyzou, Phys. Rev. D 26, 1348 (1982); and references quoted therein.
8. T. D. Newton and E. P. Wigner, Rev. Mod. Phys. 21, 400 (1949); M. H. L. Pryce, Proc. R. Soc. London A 195, 62 (1949), definition (e).
9. J. K. Lubanski, Physica 9, 310 (1942); V. Bargmann and E. P. Wigner, Proc. Nat. Acad. 34, 214 (1948).
10. V. R. Berestetski and M. V. Terent'ev, Yad. Fiz. 24, 1044 (1976).
11. H. Melosh, Phys. Rev. D 9, 1095 (1974); see also F. M. Lev, in Ref. 6.
12. S. J. Brodsky, et al., Banff Summer Institute on Particles and Fields (1981); A. Z. Capri and A. N. Kamal, eds. Plenum Press NY (1983). See Eq. (3.2) on p. 155.
13. M. V. Terent'ev, Yad. Fiz. 24, 207 (1976).
14. P. L. Chung, private communcation.
15. S. N. Sokolov, Dokl. Akad. Nauk USSR 233, 575 (1977) [Sov. Phys. Dokl. 22, 198 (1977)]; Teor. Mat. Fiz. 36, 193 (1978).
16. V. R. Pandharipande, these proceedings.