THE ADAPTIVE LINE ENHANCER APPLIED TO CHIRP DETECTION

Glenn R. Elliott
Samuel D. Stearns, Fellow, IEEE

Sandia National Laboratories
Albuquerque, NM 87185-5800

SAND--90-0231C

DE90 006772

Abstract - **This paper discusses the ability of an adaptive line enhancer (ALE) driven by the least-mean-squares (LMS) algorithm to track the frequency of a chirping signal in broadband noise. The dynamic behavior of the weights is described and a weight tracking error bound is derived in terms of the chirp rate. Frequency tracking and weight behavior are illustrated in examples.**

1. INTRODUCTION

Sandia Labs is currently studying the detection of a chirping sinusoid and the tracking of its frequency. Our subject of this paper is the chirp resulting from passing an impulse through a dispersive medium. The basic dispersion formula is

$$t = \frac{a}{f^2} \tag{1}$$

where t is time, f is frequency, and a is the dispersion constant.

A typical impulse response is shown in Fig. 1. Because of

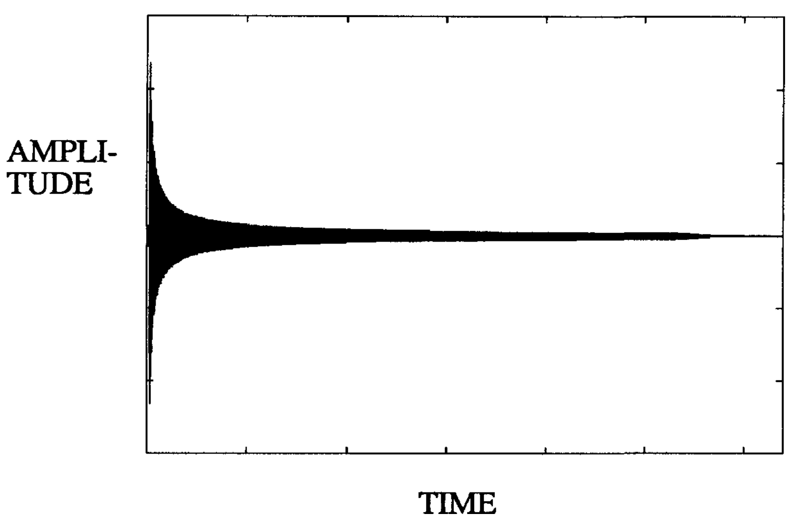

Fig. 1 Typical Impulse Response

the dispersion represented in Eq. 1, the high frequencies arrive first, followed by the lower frequencies.

We have studied several different approaches for detecting and tracking the frequency of a chirping signal in the presence of noise. The adaptive line enhancer (ALE) is used in one of

these approaches. We have investigated the dynamic tracking characteristics under varying conditions of noise and chirp rate. This paper discusses the way the individual weights in the ALE change in response to the chirping signal in the presence of noise.

2. THE ADAPTIVE LINE ENHANCER

The adaptive line enhancer (ALE) is usually in the form of the linear predictor shown in Fig. 2, with the predictor weights

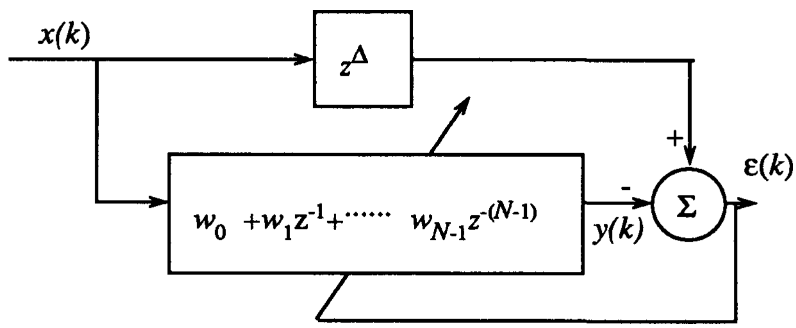

Fig. 2. Adaptive line enhancer in the form of a linear predictor.

$[w_n]$ driven by a normalized version of the well-known least-mean-squares (LMS) algorithm [1,5]. The n^{th} weight, w_n, is adjusted at the k^{th} sample time as follows:

$$w_n(k+1) = w_n(k) + \frac{2u}{NP}\epsilon(k)x(k-n); \qquad 0 < u < 1 \qquad (2)$$

The LMS convergence parameter is normalized by N, the filter size in Fig. 2, and by P, the input signal power, P=E[x_k^2]. (The latter may also be adjusted during the adaptive process [6].)

A stationary narrowband signal (spectral line) in the presence of wideband noise is "enhanced" by the ALE if the delay (Δ) is long enough to decorrelate the noise. Then, in order to minimize the mean-squared error (MSE), E[ε_K^2], the LMS algorithm will adjust $[w_n]$ such that the filter, $W(z)$, has a narrowband transfer function at the frequency of the line. Then the narrowband signal is predicted, the noise is rejected as well as possible by the filter, the MSE is minimized, and the frequency of the line is the peak of the inverse predictor gain, i.e.,.

$$\text{line frequency} = \max\left[z^{-\Delta} - W(z)\right]^{-1} \quad \text{with } z = e^{j2\pi f} \quad (3)$$

Here f is the frequency in Hz-s. In this paper we are primarily concerned with the ability of the ALE to track the moving line of a chirping signal.

The performance of the ALE in Fig. 2 has been studied extensively, mostly for one or more stationary sinusoids in white Gaussian noise. For this case Griffiths [1] has shown that the convergence factor u in Eq. 2 should be in the range shown and that certain constraints on the input correlation matrix (R_x) must be met for convergence. Treichler [2] has also characterized the convergence in terms of the eigenvalues of R_x, and Rickard and Zeidler [3] have provided a quantitative analysis of the ability of the ALE to enhance a narrowband signal. Regarding the choice of the delay (Δ), Yoganandam et. al. [4] have shown that the ALE performance is significantly improved by choosing an optimal value, Δ_{opt}, for a given situation.

3. TRACKING CONSIDERATIONS

In the present case the ALE is supposed to converge or lock on to a nonstationary, chirping narrowband signal which appears suddenly in broadband background noise, and to track the frequency of the signal through time. We are particularly interested in the tracking capabilities and performance of the ALE. Suppose that the ALE is successfully tracking a sinusoid at frequency f (Hz-s) in broadband noise. With a low signal-to-noise ratio (the condition in which we are most interested), the magnitude of the filter transfer function given by $|W(e^{j2\pi f})|$ will ideally be a unit impulse at frequency f. The filter weights, $[w_n]$, will therefore be ideally sinusoidal, in the form

$$w_n = \frac{2}{N}\sin(2\pi n f + \alpha); \quad 0 < n < N-1 \tag{4}$$

If we assume that f is being tracked and that both f and $[w_n]$ are functions of f, the time step, then the weights and their rates of change are given by

$$\begin{aligned} w(k) &= \frac{2}{N}\sin[2\pi n f(k) + \alpha]; \\ \dot{w}(k) &= \frac{4\pi n \dot{f}(k)}{N}\cos[2\pi n f(k) + \alpha]; \end{aligned} \tag{5}$$

From this result we see that the weights themselves change sinusoidally with time (k) and that the rate of change is bounded by

$$|\dot{w}_n(k)| \leq \frac{4\pi n}{N}\left|\dot{f}(k)\right| \qquad \text{Hz}-\text{s/sample} \tag{6}$$

Thus, for a given chirp rate, the higher-order weights must change faster. This result is fortuitous for high-signal-to-noise cases where the lower-order weights tend to dominate (only 2 weights are required theoretically in the noise-free case). For low-SNR cases, a further upper bound on the weight rate of change required for tracking is obtained using the final weight (assuming N is large, we use $n=N$ in Eq. 6.):

$$|\dot{w}_{N-1}| \leq 4\pi\left|\dot{f}(k)\right| \qquad \text{Hz}-\text{s/sample} \tag{7}$$

In Eqs. 6 and 7 we have essentially established bounds on the required tracking capability of the ALE. On the other hand, the convergence of any gradient-search algorithm like the LMS algorithm in Eq. 2 can be approximated [5] by

$$\dot{w}(k) \equiv w(k+1) - w(k) = 2u\left[w_{opt} - w(k)\right] \tag{8}$$

Here the weight tracking error is the difference between the optimal and current weight values, i.e., w_{opt} - $w(k)$. The relationship in Eq. 8 is really good only for a single weight or in the case where R_x has equal eigenvalues; however, the latter is approximately true in low-SNR cases. Since the weights themselves are in general proportional to $2/N$ as seen in Eq. 4, we define a normalized tracking error for weight w_n as follows:

$$E_n(k) = \frac{N}{2}|w_{opt} - w_n(k)| \tag{9}$$

Combining Eqs. 6, 8, and 9, we have

$$E_n(k) \leq \frac{n\pi\left|\dot{f}(k)\right|}{u} \tag{10}$$

During tracking, the range from 0 to 0.5 Hz-s is divided into N intervals. We therefore express the chirp rate in terms of $r(k)$, the number of frequency intervals/sample, and obtain

$$\mathrm{E}_n(k) \leq \frac{n\pi|r(k)|}{Nu} \tag{11}$$

Using this result, we can at least approximately predict the success of an ALE tracking the frequency of a chirping signal. The filter size (N) must be large enough to allow adequate frequency discrimination. For adequate performance of the LMS algorithm in terms of weight noise [5], the convergence parameter (u) must generally be much smaller than the limit in Eq. 2. Since the weight variance due to noise is proportional to u [5], the actual choice of u must be based on the SNR for which the ALE is designed. In our present study we have SNR's around -10 dB, and typical values of N and u are N=32 and u=0.01. The implication in Eq. 11 is that on the order of $\pi/(Nu)$ to π/u, or 10 to 300 samples, may be required for adjusting the weights to switch the ALE response successfully from one frequency increment to the next. In experiments we have found that the bound in Eq. 11 is reasonable.

Regarding the choice of Δ, the forward delay in Fig. 2, the optimal choice will decorrelate the noise but not the signal at the summing junction. With steady-state signals, Δ can be chosen on the basis of noise characteristics alone, but when the signal is chirping, any value of Δ will tend to decorrelate the signal at the summing junction in Fig. 2. For example, suppose the signal component of $x(k)$ is chirping at a constant rate c Hz-s/sample. Then, in order to cancel the signal at the summing junction, the weights must "tumble" [4] at approximately $c\Delta$ Hz-s, the difference between the frequency of the signal in $x(k)$ and the frequency of the signal in $x(k+\Delta)$. On the other hand, in Eq. 5 with the constant chirp rate assumed here, the time-varying sine argument becomes $2\pi nck$, so neglecting tumbling, the n^{th} weight must oscillate at the frequency cn in order to track the signal. To summarize,

Constant chirp rate c Hz-s/sample:

(maximum) tracking frequency of weight $w_N = cN$

additional tumbling frequency of any weight $= c\Delta$ (12)

From this result we can conjecture that if Δ is a significant fraction of N, the tracking capabilities of the LMS-driven weights will be affected.

The tracking characteristics discussed in this section are demonstrated in the following examples.

4. TRACKING EXAMPLES

In this section we demonstrate the results above using the type of signal described in section 1. First, a signal is illustrated with low noise (average SNR≅0dB) in Fig. 3. The waveform

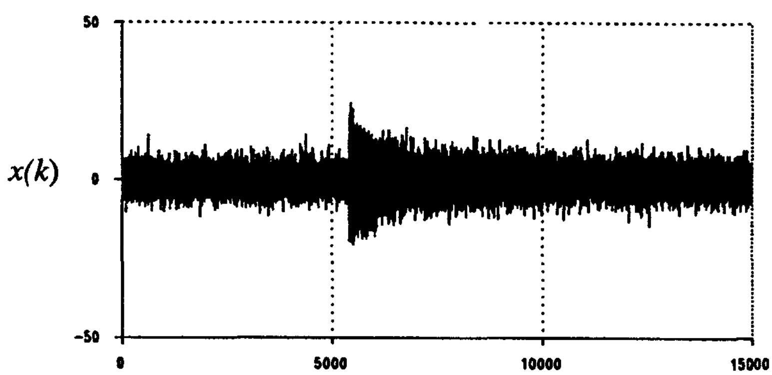

Fig. 3. Signal with frequency described in Eq 13, k_0=5000 and SNR=0 dB.

begins at k_0=5000 samples and has the frequency in accordance with Eq. 1

$$f(k) = \frac{10}{\sqrt{k - k_0}} \quad \text{Hz-s} \tag{13}$$

The additive noise is Gaussian and has uniform power density from 0.05 to 0.45 Hz-s. The signal first appears at k=5400 samples where, in accordance with Eq. 13, $f(k)$ comes within the range (0,0.5) Hz-s. From Eq. 13, the chirp rate amplitude is

$$\left|\dot{f}(k)\right| = 5(k - k_0)^{-3/2} \quad \text{Hz}-\text{s/sample} \tag{14}$$

This rate has a maximum of 6.25E-4 Hz-s/sample at k=5400 where the signal first appears. Using u=0.01, Eq. 10 gives a reasonable tracking error for the low-order weights. Thus we would expect the LMS algorithm to begin tracking the signal near its onset, and of course the amplitude in Eq. 14 decreases rapidly with k, so the tracking should improve with k.

A tracking example for the waveform in Fig. 3 is shown in Fig. 4. In the upper plot, the ALE is seen to improve the SNR

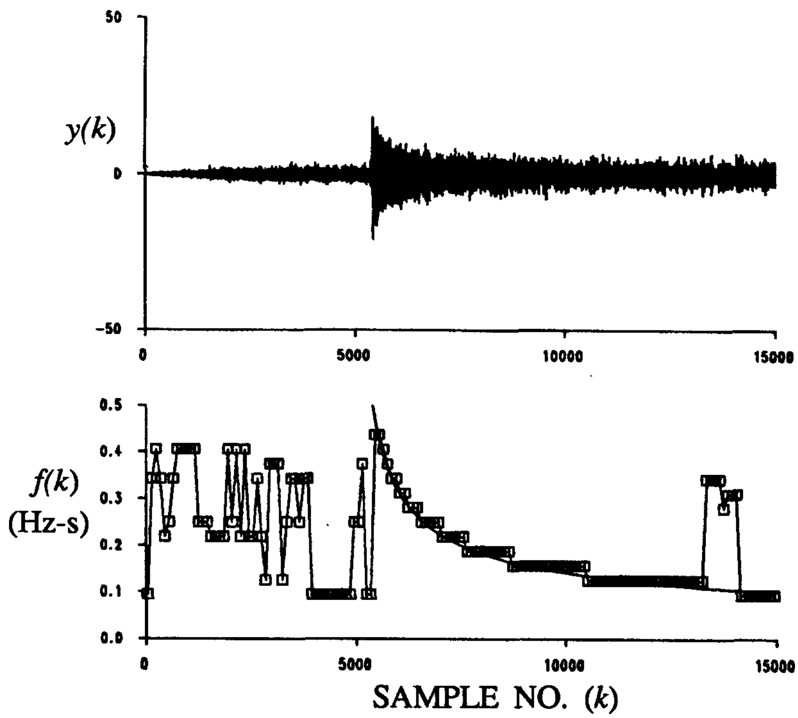

Fig. 4. ALE tracking under low noise conditions. Input waveform $x(k)$ as in Fig 3 with k_0=5000, Δ=1, N=32, u=0.01, and avg. SNR=0 dB.

of the waveform in Fig. 3, and the frequency estimation in Eq. 3 (boxes) in the lower plot matches the actual frequency (smooth curve) until the instantaneous SNR, which decreases during the signal, decreases to a low value. For this example we used N=32 weights, Δ=1, and u=0.01. The input power, P in Eq. 2 was tracked using a single-pole smoothing filter described in [6] with forgetting factor α=0.001.

, Plots of the weights $w_8(k)$ and $w_{16}(k)$ vs k, which exhibit the relationship in Eq. 5, are shown in Fig. 5. The amplitude of

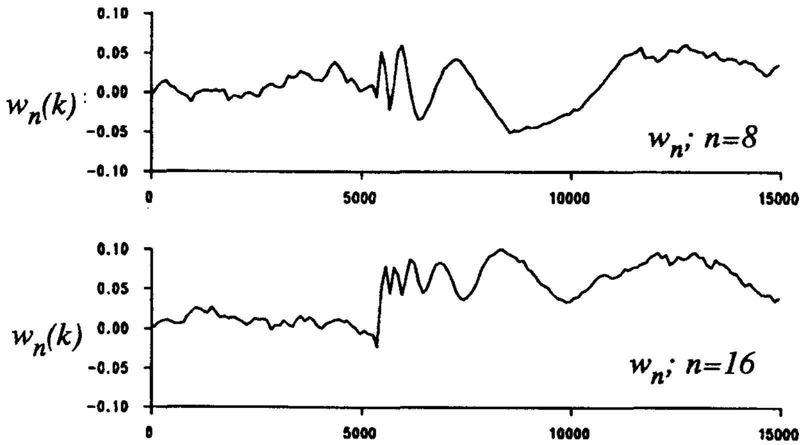

Fig. 5. Plots of weights w_8 and w_{16}, illustrating the behavior predicted in Eq. 5

the variation of w_8 is close to the value 0.06 predicted by Eq. 5. The amplitude of w_{16} is less because, with a high SNR, not all of the weights are required for signal cancellation. The frequency of weight variation is found by taking the derivative of the phase $\theta_n(k)$, then substituting Eq. 13 for the present example:

$$\begin{aligned} \text{phase } \theta_n(k) &= 2\pi n f(k) + \alpha \\ \text{frequency } f_w(k) &= \frac{1}{2\pi}\left|\dot{\theta}_n(k)\right| = n\left|\dot{f}(k)\right| \quad \text{Hz}-\text{s} \qquad (15) \\ &= 5n(k-k_0)^{3/2} \quad \text{Hz}-\text{s} \end{aligned}$$

For example, at k=7500, this result gives $f_w(k)$=(4.E-5)n and hence periods of 3125 and 1562 samples for weights w_8 and w_{16} respectively, which agree with the plots in Fig. 5.

The degradation in tracking caused by a larger value of Δ, which in turn causes the weight tumbling expressed in Eq. 12, is shown in Fig. 6 where Δ is increased from 1 to 5 samples. The

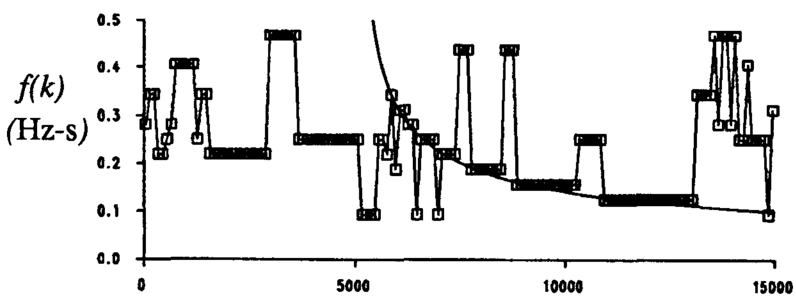

Fig. 6. Loss of tracking from Fig. 4 caused by increasing Δ from 1 to 5.

Gaussian noise was made white (with the same SNR) for this case so that no other correlated components were present in the waveform. As predicted, the tracking capability is affected by making Δ a significant fraction of N.

A waveform similar to Fig. 3 but with SNR decreased to -10 dB is plotted in Fig. 7 and tracked using the same ALE

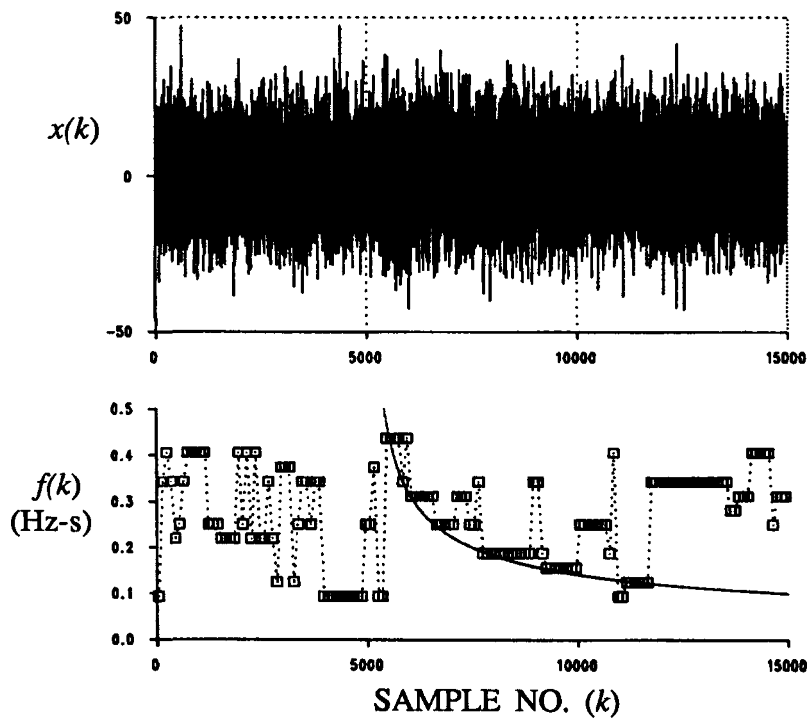

Fig. 7. Tracking as in Fig. 4 but with avg. SNR=-10 dB.

used for Fig. 4. Two effects can be noted in the frequency track in the lower plot. First, the tracking error, $E_n(k)$, in Eq. 10 has not changed and, as expected, tracking is still accurate except where the weights are misadjusted due to the increased noise in the weight solutions [5,6] due to the lower SNR. Note, however, that lowering the convergence parameter u in order to lower the misadjustment would in this case produce a proportional increase in $E_n(k)$, and produce an adverse effect on tracking. Secondly, as we would expect, the overall frequency track is noisier due to the increased misadjustment over the case in Fig. 4.

5. CONCLUSIONS

In this paper we have examined the ability of the ALE using the LMS algorithm to track the frequency of a chirping signal in broadband noise. We have found that the tracking error bound in Eq. 11 and other conclusions arising from an analysis of the expected behavior of the ALE weights are helpful in predicting the tracking performance.

6. REFERENCES

1. L. J. Griffiths, "Rapid Measurement of Digital Instantaneous Frequency", IEEE Trans. ASSP-23, No. 2, April 1975, p. 207.
2. J. R. Treichler, "Transient and Convergent Behavior of the Adaptive Line Enhancer", IEEE Trans. ASSP-27, No. 1, Feb. 1979, p. 53.
3. J. T. Rickard and J. R. Zeidler, "Second-Order Output Statistics of the Adaptive Line Enhancer", Ibid., p. 31.
4. Y. Yoganandam, V. U. Reddy, and T. Kailath, "Performance Analysis of the Adaptive Line Enhancer for Sinusoidal Signals in Broad-band Noise", IEEE Trans. ASSP-36, No. 11, Nov. 1988, p. 1749
5. B. Widrow and S. D. Stearns, Adaptive Signal Processing, Prentice-Hall, 1985, Chapters 4,6,12.
6. J. S. Lim and A. V. Oppenheim, Advanced Topics in Signal Processing, Prentice-Hall, 1988, Chapter 5.