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Global Properties of High Pressure Flux Conserving Tokamak Equilibria

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CONTENTS

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GLOBAL PROPERTIES OF HIGH PRESSURE FLUX CONSERVING TOKAMAK EQUILIBRIA*

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ABSTRACT

An approximate analytic theory is developed to calculate the poloidal beta β_I and the diamagnetic parameter μ_I for a sequence of axisymmetric toroidal MHD equilibria confining high pressure plasmas $[\beta \sim \mathcal{O}(a/R)]$ under the constraint of flux conservation. To satisfy the equilibrium equations, the plasma current must be increased with pressure as $p^{1/3}$ and previously calculated equilibrium limits on poloidal beta are avoided.

I. THE RELATIONSHIP OF PLASMA HEATING AND MHD EQUILIBRIUM

Successful auxiliary heating of thermonuclear plasmas to ignition requires a characteristic heating time τ_h shorter than the energy containment time τ_E. Using high power neutral particle injection, Tokamak experiments with

$$\tau_A < \tau_h \leq \tau_E \qquad (1)$$

where τ_A is the Alfven-time, have been conducted (ORMAK) and a number of future devices (PLT, ORMAK Upgrade, TFTR) will satisfy this criterion. Experimental evidence so far (e.g., ATC, Tuman) indicates

$$\tau_E < \tau_{pol} \tag{2a}$$

where τ_{pol}, the poloidal skin penetration time, is given by

$$\frac{1}{\tau_{pol}} \equiv \frac{1}{\psi}\frac{\partial\psi}{\partial t} \stackrel{\sim}{\sim} \frac{\eta}{4\pi a^2} \tag{2b}$$

with ψ the poloidal magnetic flux, η the resistivity and a the minor plasma radius. (The experimentally effective η tends to be $\lesssim$ 5 times the Spitzer value.) Using the classical value (2b) and Pfirsch-Schlüter heat conduction scaling for τ_E, one obtains

$$\tau_E/\tau_{pol} = \left(2\sqrt{\frac{m_e}{m_i}}\,/q^2\beta\right) << 1 \quad ,$$

in theoretical accordance with (2a) [q and β are defined below]. The combination of conditions (1) and (2a) necessitates the theoretical study of a series of neighboring MHD equilibria under the constraint of poloidal flux conservation.

The toroidal magnetic flux $\phi \equiv 2\pi\int dr\; r\, B_\phi$ diffuses on a time scale slower than τ_{pol} by a factor $\beta \equiv 8\pi\, \overline{p}/B_\phi^{\,2}$. To show this we define $F(\psi) \equiv R\, B_\phi$, and recalling that quite generally

$$\frac{\psi_0}{F}\frac{dF}{d\psi} \sim O(\beta) \quad ,$$

one obtains

$$\frac{1}{\Phi}\frac{d\Phi}{dt} \approx \frac{1}{F}\frac{dF}{dt} \approx \frac{\beta}{\psi}\frac{d\psi}{dt} \quad .$$

Thus certainly

$$\left(\frac{1}{\Phi}\frac{\partial\Phi}{\partial t}\right)^{-1} >> \tau_h$$

and since the safety factor q is given by

$$q(\psi) = \frac{1}{2\pi}\frac{d\Phi}{d\psi}$$

one concludes that $q(\psi)$ will be an invariant for these equilibria. In our own analysis we will take $q(\psi)$ to be determined by its value in the low beta initial state and examine the evolution of the plasma equilibrium as the pressure is raised.

The condition $\tau_h < \tau_E$ implies an adiabatic equation of state, augmented in this case by a heat and particle source term due to neutral injection. However, the present problem differs from the well-known "adiabatic compressor" problem[1,2,3] in that the major radius R remains essentially constant and flux conservation is realized by imposing

$$\frac{d\psi_0}{dt} = 0$$

where ψ_0 is the poloidal flux at the fixed plasma boundary. To keep the analysis simple and analytically tractable we will drop the coupling between the adiabatic equation of state and the equilibrium equation and assume henceforth that the average plasma pressure $p(\psi)$ shall be a free parameter. This assumption is realistic in experimental devices with powerful auxiliary heating. In principle, therefore, we will solve the MHD equilibrium problem with $p(\psi)$ and $q(\psi)$ given.

II. MHD EQUILIBRIUM RELATIONSHIPS

The principal macroscopic parameters characterizing the Tokamak equilibrium are[4]

$$\beta_I = 2 \int p dV/(I^2 \cdot 2\pi R_c) \qquad (3a)$$

$$\mu_I = 2 \int dV \frac{1}{8\pi} (B_{\phi_0}^2 - B_\phi^2)/(I^2 \cdot 2\pi R_c) \qquad (3b)$$

$$\ell_i = 2 \int dV \frac{1}{8\pi} B_p^2/(I^2 \cdot 2\pi R_c) \qquad (3c)$$

where $I = I(\psi_0)$ is the total current inside the circular flux surface boundary $\psi = \psi_0$ with major radius R_c. B_ϕ, B_p are the toroidal/poloidal magnetic fields and B_{ϕ_0} is the vacuum toroidal magnetic field. β_I measures the poloidal beta, μ_I the plasma diamagnetism and ℓ_i the internal inductivity (inductance/cm gaussian) of the plasma column, a geometric factor of O(1) determined by the shape of the current profile.

For a complete solution of the equilibrium problem one must solve the Grad-Shafranov Equation

$$\Delta^* \psi/R^2 = - 4\pi p'(\psi) - F\, F'(\psi)/R^2 \qquad (4a)$$

where F has to be expressed through $q(\psi)$ using

$$q(\psi) = \frac{1}{2\pi} \frac{d\phi}{d\psi} = FV'(\psi) \langle R^{-2} \rangle / 4\pi^2 \quad . \qquad (4b)$$

Here,

$$V'(\psi) \langle R^{-2} \rangle / 2\pi = \oint_\psi \frac{d\ell}{B_p} R^{-2} \quad , \qquad (4c)$$

where $R B_p = |\nabla\psi|$ closes this set of equations. (These equations are conveniently derived in Callen and Dory.[5])

In principle, the calculation of this flux average can be facilitated by an expansion of the flux function

$$\psi = S(\rho^2,\theta) = S_{circ.}(\rho^2) + S_{ellip.}(\rho^2,\theta) + S_{triang.}(\rho^2,\theta) + \cdots \quad (5)$$

where (ρ,θ) are suitable polar coordinates inside the circular flux shell and the RHS is developed in the deviation from a circular flux surface cross section, in powers of the small aspect ratio $\varepsilon = a/R_c$. A hierarchy of equations for $S_{circ.}, S_{ellip.}, S_{triang.} \cdots$ can be generated from Eq. (4a) by a multipole expansion.[6]

In practice, if one is content with a knowledge of the global parameters β_I and μ_I of Eqs. (3), the problem can be simplified substantially by using two fundamental equations, viz. the integral form of the virial theorem and the equilibrium equation.

When evaluated on the outer flux surface of an axisymmetric toroidal plasma, these equations assume the form

$$\int dV \; [3p + \frac{1}{8\pi} (B_p^2 + B_\phi^2 - B_{\phi_0}^2)] = \int \frac{1}{8\pi} B_p^2 \; \underline{n} \cdot \underline{r} \; d \; S_n$$

$$2\pi \int d \; S_\phi [p + \frac{1}{8\pi} (B_p^2 + B_{\phi_0}^2 - B_\phi^2)] = \int \frac{1}{8\pi} B_p^2 \; \underline{n} \cdot \underline{e}_x \; d \; S_n$$

where $d\; S_n$ is a flux surface element, $d\; S_\phi$ a cross sectional element, and $\underline{r}$ and $\underline{e}_x$ are depicted in Fig. 1.

III. ANALYTIC METHOD FOR OBTAINING ARBITRARY BETA MHD EQUILIBRIA

As shown by Shafranov,[4] for a circular flux shell of minor radius a, these two relationships reduce to

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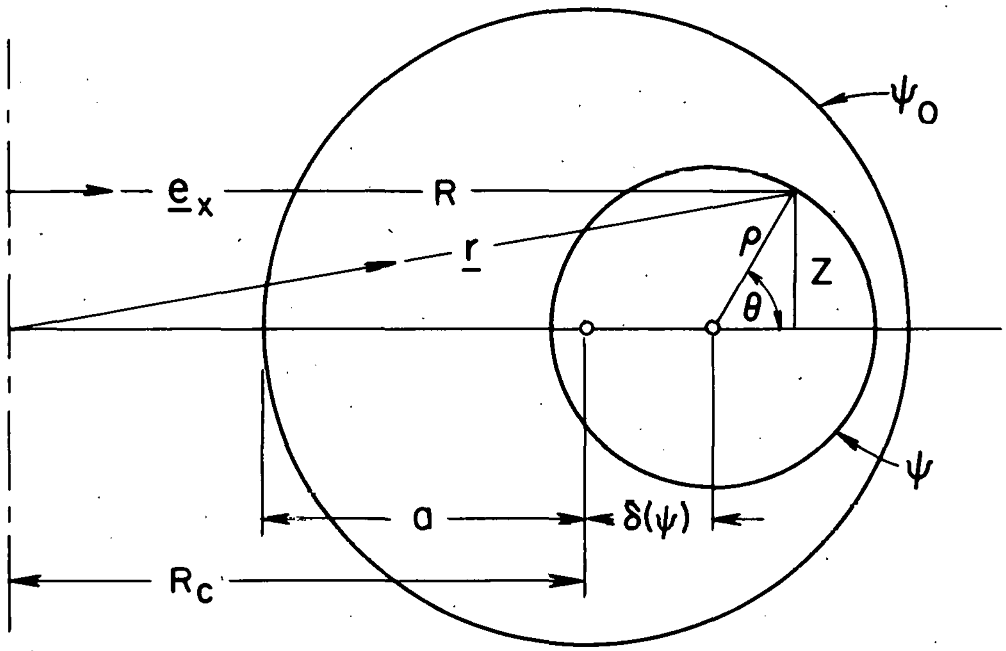

Fig. 1. Circular flux surface model geometry.

$$3\beta_I + \ell_i - \mu_I = 2(S_1 + S_2) \tag{6a}$$

$$\beta_I + \ell_i + \mu_I = 2S_2 \tag{6b}$$

whence

$$\beta_I = S_2 + \frac{S_1}{2} - \frac{\ell_i}{2} \tag{7a}$$

$$\mu_I = S_2 - \frac{S_1}{2} - \frac{\ell_i}{2} \tag{7b}$$

Here

$$2\pi\, I^2 R_c\, S_1 = \oint_{\psi_0} \frac{1}{8\pi} B_p^2\, a\, d\, S_n \tag{8a}$$

$$2\pi\, I^2\, S_2 = \oint_{\psi_0} \frac{1}{8\pi} B_p^2 \cos\theta\, d\, S_n \tag{8b}$$

where

$$d\, S_n = 2\pi R_c\, (1 + \varepsilon \cos\theta) a\, d\, \theta$$

and we have neglected higher order terms in a/R_c. In these equations the total plasma current is given by

$$I(\psi_0) = V'(\psi)\, \langle B_p^2 \rangle_{\psi_0} / 8\pi^2 \tag{8c}$$

Since we are treating the average pressure $\bar{p}$ as a variable controlled by auxiliary heating, Eq. (7a) should be looked upon as a relationship connecting the variable $\bar{p}$ with the surface integrals $\langle B_p^2 \rangle_{\psi_0}$, S_1 and S_2. Once these are determined, Eq. (7b) yields μ_I as a function of $\bar{p}$ and ℓ_i.

Since the problem centers around the calculation of the surface averages (8a,b,c) at the circular plasma boundary $\psi = \psi_0 = $ const, it is justifiable to drop the higher order noncircular terms in (5) and adopt the following circular flux surface approximation, describing a set of nested toroidal flux surfaces with circular cross section

$$\psi = S(\rho^2), \; \rho^2 = (R - R_\psi)^2 + z^2, \; R_\psi = R_c + \delta(\psi) \qquad (9)$$

where R_ψ extends to center of the circular flux tube $\psi(\rho^2)$, shifted from the geometric center R_c by an amount $\delta(\psi)$, as shown in Fig. 1.

This model equilibrium contains only two arbitrary functions $S(\rho^2)$ and $\delta(\psi)$. We shall determine the function S from flux conservation and the parameter $\delta(\psi)$ or, more precisely, $\delta'(\psi) \equiv \partial\delta/\partial\psi$ from Eq. (7a) after the surface integrals are performed. Had we defined a more complicated model flux function with additional free parameters such as ellipticity, we would have required additional moments of the plasma force balance equations to determine them.[6]

It follows from Eq. (9) that in our simple model

$$R B_p = \left|\nabla\psi\right| = 2\rho\dot{S}/D \quad ,$$

$$B_p = 2\tilde{\varepsilon}\dot{S}/(1 + \tilde{\varepsilon} \cos\theta)D \qquad (10a)$$

where $D = 1 + d \cos\theta$, $d = 2\rho\dot{S}\,\delta'(\psi)$, and

$$\dot{S} = \frac{d\psi}{d\rho^2} \; , \; \tilde{\varepsilon} = \frac{\rho}{R_\psi} \; , \; R = R_\psi(1 + \tilde{\varepsilon} \cos\theta) \qquad (10b)$$

The volume inside a flux surface ψ = const is $V(\psi) = 2\pi^2\rho^2R_\psi$. In this flux model, the flux average becomes

$$\frac{V'\langle X\rangle}{(2\pi)^2} = \frac{R_\psi}{2\dot{S}} \int \frac{d\theta}{2\pi} D(1 + \tilde{\varepsilon} \cos\theta)X$$

The integrations (8c), (4c) are elementary and yield

$$I(\psi) = \rho\, \dot{S}\, \tilde{\varepsilon}\, (1 + \frac{\tilde{\varepsilon}d}{2} + \cdots)/\sqrt{1-d^2}\ ; \qquad (11a)$$

$$V'\, \langle R^{-2} \rangle / 4\pi^2 = \frac{\tilde{\varepsilon}}{2\rho \dot{S}} \left(1 - \frac{1}{2}\, \tilde{\varepsilon} d + \cdots \right) \qquad (11b)$$

The dots indicate higher order terms in ε.

As the flux surfaces shift outward under the increased plasma pressure in the high beta regime, one expects that $|d| \to O(1)$ at $\psi = \psi_0$. Concomitantly, the poloidal field (10a) has a non-expandable dependence on $\cos\theta$ in marked contrast to the widely used low beta model[7]

$$B_p = B_{p0}\, (1 + \varepsilon\Lambda \cos\theta) \quad .$$

As long as flux is conserved, the singularity can only be approached asymptotically as the pressure is raised. As will be shown below, it is the correct treatment of the dependence of B_p, which eliminates any equilibrium limit on the poloidal beta. The remaining surface integrals are

$$\frac{a}{8\pi} \oint_{\psi_0} B_p^2 \; dS_n = a^2\, 2\pi\, R_c\, (\tilde{\varepsilon}\dot{S})^2 \left(\frac{1 + \tilde{\varepsilon}d}{(1-d^2)^{3/2}} \right)$$

$$\frac{1}{8\pi} \oint_{\psi_0} B_p^2 \cos\theta \; dS_n = a\, 2\pi\, R_c\, (\tilde{\varepsilon}\dot{S})^2 \left(- \frac{d + \tilde{\varepsilon}}{(1-d^2)^{3/2}} + \frac{\tilde{\varepsilon}}{d^2} \cdot \frac{1 - \sqrt{1-d^2}}{\sqrt{1-d^2}} \right)$$

In summary,

$$S_1 \cdot I^2 = (a\tilde{\varepsilon}\dot{S})^2 \frac{1 + \tilde{\varepsilon}d}{(1-d^2)^{3/2}} \tag{12a}$$

$$S_2 \cdot I^2 = \frac{R_c}{a} (a\tilde{\varepsilon}\dot{S})^2 \left\{ - \frac{d + \tilde{\varepsilon}}{(1-d^2)^{3/2}} + \frac{\tilde{\varepsilon}}{d^2} \frac{1 - \sqrt{1-d^2}}{\sqrt{1-d^2}} \right\} \tag{12b}$$

and from (11a)

$$I^2 = (\rho \dot{S} \tilde{\varepsilon})^2 \frac{\left(1 + \frac{\tilde{\varepsilon}d}{2}\right)^2}{1-d^2} \tag{12c}$$

The $(1-d^2)^{-n}$ terms will be shown below to dominate the high beta equilibrium properties. For example, combining (12a) and (12c),

$$S_1 \simeq (1-d^2)^{-1/2} >> 1 \cdots \tag{12d}$$

in the high beta flux conserving equilibrium vs. $S_1 \sim O(1)$ in Ref. 4.

Defining a pressure variable normalized by the initial low beta toroidal current

$$\overline{\beta}_I = 2\int dVp/2\pi \; R_c \; I_i^2$$

we find for β_I defined in (3a), using (12c)

$$\frac{\beta_I}{\overline{\beta}_I} = \frac{\dot{S}_i^2}{\dot{S}^2} \frac{\left(1 + \frac{\varepsilon d_i}{2}\right)^2}{\left(1 + \frac{\varepsilon d}{2}\right)^2} \cdot \frac{1-d^2}{1-d_i^2} \tag{13}$$

where the subscript i stands for the initial value and $\varepsilon = a/R_c$. All quantities on the RHS are evaluated at $\psi = \psi_0$.

IV. DETERMINATION OF PARAMETERS SPECIFYING THE HIGH BETA EQUILIBRIUM

Having expressed I, β_I, S_1 and S_2 in terms of $S(\rho^2)$ and $d(\psi)$, we must determine these latter two quantities for the flux conserving equilibrium.

From Eqs. (7a,b) and (12a,b) it is clear that large increases in β can be produced if $d \to 1$. One expects a small decrease in ℓ_i as beta is increased $(d \to 1)$ since the denominator of Eq. 3c becomes large as beta is increased while the numerator which depends only on an integral over $(1-d^2)$ remains finite. Qualitatively this follows since increasing beta in a flux conserving system tends to drive a current increasing toward the surface [Eq. (11a)]. Thus, the external magnetic field is increased more than the internal field. Numerical calculations[8] confirm that ℓ_i does not have a strong dependence on beta and in what follows we shall approximate it by its low beta value.

The functional form of $S(\rho^2)$ is specified by the invariance of $q(\psi)$ in a flux conserving system. From Eqs. (4b) and (11b), $q(\psi)$ can be written

$$q(\psi) = \frac{F}{2\dot{S}\ R_c} \left\{1 - \frac{\tilde{\varepsilon} d}{2}\right\} \qquad (14)$$

Now, in general, F is a constant plus an order β term. Thus, order unity variations in q can only be produced by changes in $\dot{S}$. Conversely, since q is an invariant on each flux surface as β is increased in a flux conserving tokamak, $\dot{S}$ must also be an invariant to zero order in beta. Finally, for simplicity, we will specialize to the case of q constant on all flux surfaces which implies that $\dot{S}$ is a constant to zero order in β, i.e.,

$$S = \psi_0 \rho^2/a^2 + O(\beta) \tag{15}$$

We will neglect the change in the functional form of S as the pressure is increased.

The above arguments leading to the specification of the function S are valid near the surface where the circular shape of the flux surfaces is guaranteed by the boundary condition. As shown by numerical calculation,[8] when the beta is raised in a flux conserving tokamak, strong deformation of the initially circular flux surfaces occurs in the plasma interior and Eq. (15) has a more complicated form. Fortunately, for the present work, we require a knowledge of S only near the surface where Eq. (15) is sufficiently accurate.

V. PROPERTIES OF FLUX CONSERVING HIGH BETA EQUILIBRIA

We are thus justified to approximate ℓ_i and $\dot{S}$ by their initial low beta values. Moreover, if we retain the shearfree model $q = q_0$, $\dot{S} \simeq \psi_0/a^2$ can be taken for all values of β. Then Eqs. (7a,b) can serve to determine β_I and μ_I as follows. Approximating (13) by

$$\beta_I = \overline{\beta}_I \; (1-d^2) \tag{16}$$

since $d_i \sim \varepsilon$, β_I becomes uniquely dependent on d^2. Using Eqs. (12), (7a) assumes the form

$$\frac{\ell_i}{2} + \overline{\beta}_I(1-d^2) = \frac{(1-d^2)}{(1+\frac{\varepsilon d}{2})^2}\left\{\frac{[-\frac{1}{\varepsilon}(d+\varepsilon) + \frac{1}{2}(1+\varepsilon d)]}{(1-d^2)^{3/2}} + \frac{1}{d^2}\,\frac{1-\sqrt{1-d^2}}{\sqrt{1-d^2}}\right\} \tag{17}$$

which yields the desired relation between the unknown parameter d and the pressure variable $\overline{\beta}_I$. In the high beta limit this relation reduces to

$$\overline{\beta}_I \simeq (-\frac{d}{\varepsilon})(1-d^2)^{-3/2} \tag{18}$$

Solving for d, the inversion of (18) for high beta is

$$-d \simeq [1 - \frac{1}{3} \cdot (\varepsilon\overline{\beta}_I)^{-2/3}]^{3/2} \tag{19}$$

showing that $d^2 \to 1$ for $\overline{\beta}_I \to \infty$ as discussed above. The low beta limit follows trivially from (16), viz. $\beta_I \simeq \overline{\beta}_I$.

To calculate β_I as a function of the pressure variable $\overline{\beta}_I$ in the high beta limit, we use Eqs. (16) and (19) to obtain

$$\beta_I/\overline{\beta}_I \simeq 1-d^2 = 1 - [1 - \frac{1}{3} \cdot (\varepsilon\overline{\beta}_I)^{-2/3}]^3$$

In the regime $\varepsilon\overline{\beta}_I > 1$ this can be expanded to yield

$$\beta_I/\overline{\beta}_I \simeq (\varepsilon\overline{\beta}_I)^{-2/3} \tag{20a}$$

or

$$\beta_I = \varepsilon^{-2/3} (\overline{\beta}_I)^{1/3} \tag{20b}$$

The pressure dependence of the plasma current follows immediately. From (12c) and $\dot{S} \simeq \psi_0/a^2$

$$I(\psi_0) = \begin{cases} \dfrac{\psi_0}{R_c} & \text{low } \beta \quad (21a) \\ \dfrac{\psi_0}{R_c}(1-d^2)^{-1/2} \simeq \dfrac{\psi_0}{R_c}(\varepsilon\overline{\beta}_I)^{1/3} & \text{high } \beta \quad (21b) \end{cases}$$

From (7b), one obtains for the diamagnetic parameter

$$\mu_I + \frac{\ell_i}{2} = -(1 + \frac{\varepsilon d}{2})^{-2} [\frac{1}{\varepsilon}(d + \varepsilon) + \frac{1}{2}(1 + \varepsilon d)]/\sqrt{1-d^2}$$

$$+ d^{-2} (1 + \frac{\varepsilon d}{2})^{-2} [\sqrt{1-d^2} - (1-d^2)] \tag{22a}$$

In the high beta regime ($-\,d/\varepsilon > 1$, $d^2 \rightarrow 1$)

$$\mu_I + \frac{\ell_i}{2} \rightarrow -\frac{d}{\varepsilon}\,(1-d^2)^{-1/2} = (1-d^2)\bar{\beta}_I > 0 \tag{22b}$$

where the dependence of d on pressure is given in (19). Comparing this with (16) shows $\mu_I \rightarrow \beta_I$ in asymptotic agreement[9] with Eq. (40) of Ref. 4. Thus, from (22b), $(1/8\pi)\int dV(B^2_{\phi_0} - B^2_\phi) > 0$, showing the high beta diamagnetism since B_{ϕ_0} is the toroidal field for zero plasma pressure.

An upper bound for μ_I exists as $\int dV\, B^2_\phi/B^2_{\phi_0} \rightarrow 0$, or equivalently, $\beta \rightarrow 1$. At $\beta = 1$, or $\bar{\beta}_I = q^2/\varepsilon^2$, from Eq. (19)

$$d^2 = 1 - \varepsilon^{2/3} \cdot q^{-4/3} \tag{23a}$$

and from Eq. (20b)

$$\beta_I = q^{2/3} \cdot \varepsilon^{-4/3} \tag{23b}$$

The exact behavior of β_I, μ_I and the plasma current $I(\psi_0)$ as a function of the pressure variable $\bar{\beta}_I$ is shown in Fig. 2. Since $\bar{\beta}_I \equiv q^2\beta/\varepsilon^2$ and the maximum value of β is one, the maximum value of $\bar{\beta}_I$ in the figure is dependent on the choice of q.

Setting μ_I in (22a) equal to zero, the change from para-to diamagnetism occurs for $-d = 5/4\,\varepsilon$. Inserting this value for d in (17) results in $\bar{\beta}_I \simeq 1$ as expected. In the low β limit [$d \sim O(\varepsilon)$], Eqs. (17) and (22a) combine to yield $\mu_I = \beta_I - S_1$, where from (12d), $S_1 \simeq 1$, in agreement with Eq. (41) of Ref. 4.

VI. CONCLUSIONS OF AXISYMMETRIC HIGH BETA EQUILIBRIUM ANALYSIS

1. Increasing the average pressure $\int dVp/V$ (by means of auxiliary heating) on a flux conserving time scale, the equilibrium equations of

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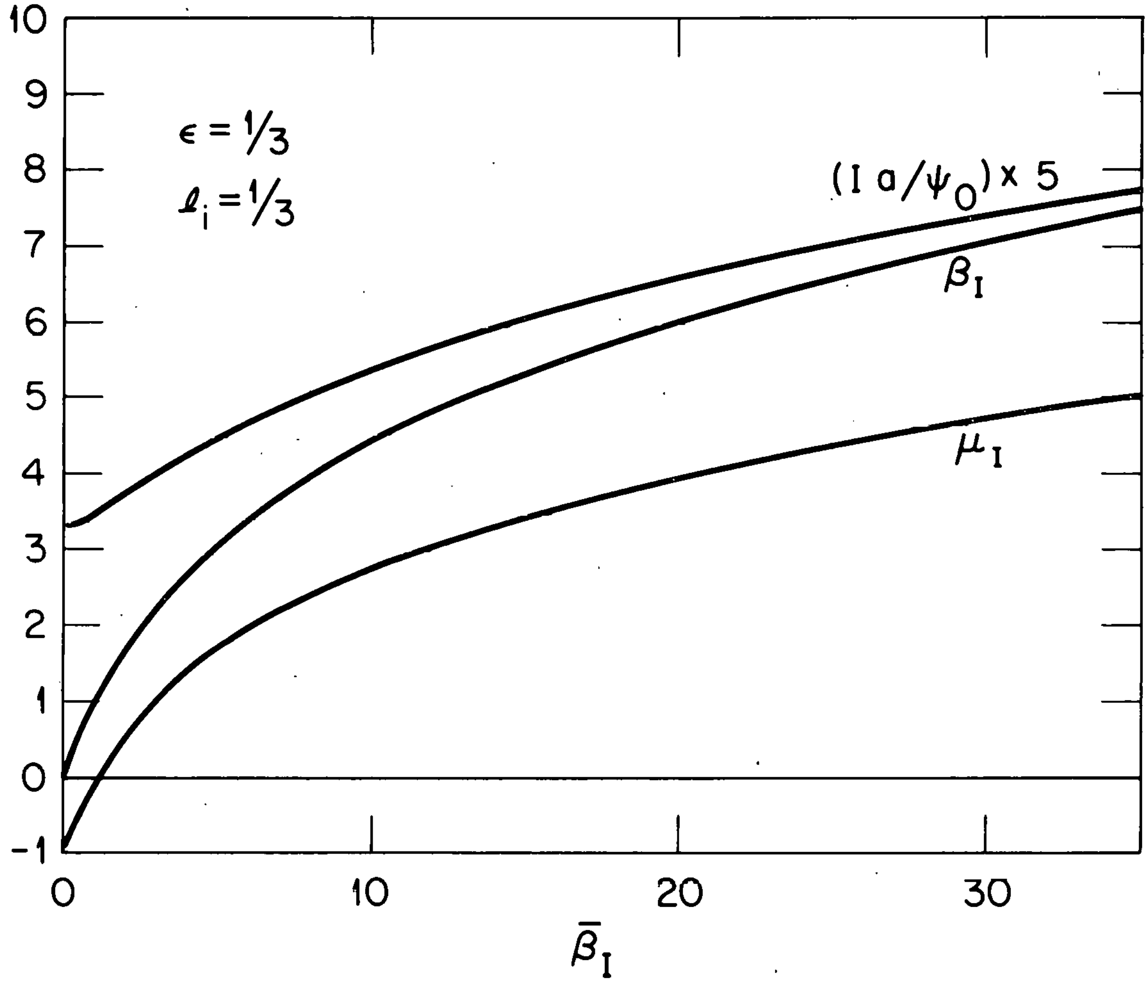

Fig. 2 The behavior of β_I, μ_I and I are shown as a function of $\bar{\beta}_I$ for $\varepsilon = 1/3$, $\ell_i = 1/2$.

an axisymmetric toroidal plasma permit a continuous transition from low to arbitrarily high values of beta (Eqs. 16 and 17).

2. In a flux conserving equilibrium, the plasma current must be allowed to increase with pressure as $(\varepsilon\bar{\beta}_I)^{1/3}$ [Eq. (21b)].

3. Consequently, the poloidal beta grows slower than linearly with pressure, (Eq. 20b), and the often used scaling relation $\beta_I = q^2\beta/\varepsilon^2$ does not apply for the flux conserving tokamak. It is replaced by (20b) asymptotically.

4. The flux conserving equilibria considered in this paper do not permit formation of a second magnetic axis as can be seen from Eq. (10a):

$$B_p(\psi = \psi_0,\ \theta = \pi) = \frac{2\varepsilon(d\psi/d\rho^2)\ \psi_0}{(1-d)(1-\varepsilon)}$$

Since from (17) and(23a), $0 < -d < 1$, B_p can therefore not vanish. This confirms the physical notion that the topology of flux surfaces cannot change if flux is conserved. Thus, there is no equilibrium limit such as implied by the condition $\beta_I \leq \varepsilon^{-1}$ obtained for non-flux conserving equilibria at constant plasma current.[10]

5. In the high beta limit, the diamagnetic parameter μ_I approaches β_I, implying confinement by the toroidal diamagnetic well. There are no equilibrium limits short of the ultimate limit at $\beta = 1$ where $\beta_I = q^{2/3}\varepsilon^{-4/3}$ (Eq. 23b).

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9. Note, however, that owing to Eq. (12d), Eq. (41) of Ref. 4 is not valid for high beta flux conserving equilibria for the reason given after Eq. (11b).
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