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$D^{o*} \to D^{o}\gamma$ AND OTHER RADIATIVE DECAYS OF VECTOR MESONS[†]

by

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Abstract

Using SU(4) as a spectrum generating group the radiative decay rates of the charmed vector mesons and of $J(\psi)$ are calculated. With the known decay rates of the old mesons $\Gamma(\omega \to \pi\gamma)$, $\Gamma(\phi \to \eta\gamma)$, $\Gamma(\rho \to \pi\gamma)$, $\Gamma(K^{o*} \to K^{o}\gamma)$ as input one obtains $\Gamma(K^{+*} \to K^{+}\gamma) = 2.6$ keV, $\Gamma(\omega \to \eta\gamma) = 220$ eV, $\Gamma(\rho \to \eta\gamma) = 4.8$ keV, $\Gamma(\psi \to \chi\gamma) = 1.6$ keV, $\Gamma(D^{o*} \to D^{o}\gamma) = 350$ eV and $\Gamma(D^{+*} \to D^{+}\gamma) = 22$ eV.

In a preceding communication we have discussed the radiative decay of the $J(\psi)$ in an approach in which SU(4) is considered as a spectrum generating group.[1] This spectrum generating group is a non-perturbative approach to broken SU(4),[2] similar to that in which SU(4) is considered as the dynamical stability group of the velocity operator[3] $P_\mu M^{-1}$.

As a consequence of this assumption the amplitude contains in addition to the SU(4) Clebsch-Gordon coefficients, a symmetry breaking factor (suppression factor) Φ, which is a function of the masses involved. The precise form of Φ as a function of the vector and pseudoscalar meson masses m_V and m_P appearing in the radiative decays $V \to P\gamma$ depends upon the assumption about the SU(4) property of the "current" operators V_μ^a.[4] This assumption should be chosen such that in the limit, when the spectrum generating group $SU(4)_E$ goes into the SU(4) symmetry group, the V_μ^a become SU(4) tensor operators. As there are many possible generalizations away from this limit, in reference 1 we determined the precise functional form of the suppression factor Φ phenomenologically from the known radiative decay rates of the old vector mesons $\Gamma(\omega \to \pi\gamma)$, $\Gamma(\phi \to \eta\gamma)$, $\Gamma(\rho \to \pi\gamma)$, $\Gamma(K^{0*} \to K^0\gamma)$. The three functions which fitted these decay rates are[5]

(1) $$\Phi(m_V, m_P) = \frac{m_V^P + m_P^P}{m_V m_P} \qquad \text{for } P = 1/2,\ 1,\ 3/2$$

The decay rate for the process $V \to P\gamma$ is given by

(2) $$\Gamma(V \to P\gamma) = |g_{VP}|^2 \frac{1}{24}\alpha m_V^3 (1 - (m_P/m_V)^2)^3$$

with

(3) $$g_{VP} = g \cdot \langle P|V^{el}|V\rangle \cdot \Phi(m_V, m_P)$$

where α is the fine structure constant, g is an overall constant which could have been absorbed into the reduced matrix elements of V_μ^a and $\langle P|V^{el}|V\rangle$ is the $SU(4)_E$ matrix element of the SU(4) part of the electromagnetic current. In the symmetry limit $\Phi = 1$, (2) becomes the usual expression for magnetic dipole transitions.

At the time reference 1 was written, the masses of the charmed mesons were unknown and also the exact expression for the electromagnetic current operator was not experimentally justified. With the discovery of the D meson[6] both these deficiencies have been overcome, and from correspondence with the charge in SU(4),

$$Q = I_3 + \tfrac{1}{2}Y + \tfrac{2}{3}\chi + \tfrac{1}{2}B, \text{ where charm} = \tfrac{3}{4}B + \chi$$

the electromagnetic current operator V_μ^{el} in SU(4) is--in the phase convention that we shall use here--given by[7]

(4) $$V_\mu^{el} = V_\mu^{\rho^0} + \frac{1}{\sqrt{3}} V_\mu^{\eta} - \sqrt{\frac{2}{3}} V_\mu^{\xi} + V_\mu^{\eta'}$$

The SU(4) scalar term $V_c^{\wedge}$ in (4) whose matrix element between baryon states is proportional to the baryonic charge B and whose matrix element between meson states is zero, is essential in this calculation. Between meson vectors with opposite charge parity, like the pseudoscalar vector mesons, it is different from zero, $\langle P|V^{c}|V\rangle \neq 0$, and it is the occurrence of this term which makes it possible to fit the experimental ratio of $\Gamma(\omega \to \pi\gamma)/\Gamma(\rho \to \pi\gamma)$ which otherwise could not be explained by any form of the symmetry breaking factor $f(m_i, m_f)$. The old SU(3) or naive quark model assumption expressed by $\langle$old pseudoscalar$|-\sqrt{\frac{2}{3}}V^{8}+V^{0}|$old vector meson$\rangle = 0$, can also *not fit this ratio of the decay rates.*

There are four reduced matrix elements for all $\langle P|V^{el}|V\rangle$. It can be seen that the F-type reduced matrix element is zero as a consequence of the transformation property under charge conjugation.

We will further assume, in order to keep the number of parameters as small as possible, that the vector and pseudoscalar mesons belong to a pure 63-plet of $SU(8) \supset SU(4) \times SU_{spin}(2)$ and that the particle vectors are given by the basis vectors which are connected to the subgroup chain

$$SU(8) \supset SU(6) \times SU_{S_+}(2) \supset SU_W(4) \times SU_{S_Y}(2) \times SU_{S_+}(2)$$

where SU(6) is the Gürsey-Radicati SU(6), S_+ is the charmed

spin, $SU_W(4)$ is the Wigner SU(4) (and distinct from the $SU(4)_f$ or symmetry SU(4) described above) and S_Y is the hypercharged spin. In this approximation the vector mesons belong to an ideally mixed 16-plet of $SU(4)_f$ and the pseudoscalar mesons belong to a pure 15-plet of $SU(4)_f$; η-η' mixing, deviation from ideal mixing and isospin mixing are ignored. Deviation from ideal mixing should not be considered separately without considering isospin mixing (ρ^0-ω or π^0-η mixing) because they are of the same magnitude and perhaps of the same origin. However, including all of these mixings by arbitrary mixing angles would introduce too many parameters to result in anything useful.

The narrow width of $J(\psi)$ is usually explained as a consequence of the fact that its decays into old particles are first forbidden transitions, i.e., they are forbidden if one assumes ideal mixing and the Zweig rule. If one considers the approximation in which the first forbidden transitions are zero, then one obtains a relation between the D type reduced matrix elements $D = \langle P\,15\,\|V^{(15)}\|\,15\,V\rangle$ and the reduced matrix element $A = \langle P\,15\,\|V^{15}\|\,1\,V\rangle$.[8]

All the matrix elements $\langle P|V^{el}|V\rangle$ can then be expressed in terms of the two parameters (reduced matrix elements) $A = \frac{4}{\sqrt{3}}D$ and $S = \langle P\,\bar{3}\,\|\,V\rangle$ and SU(4) Clebsch Gordan coefficients[9].

$$
\begin{aligned}
\langle \pi^{0} | V^{el} | \omega \rangle &= d \\
\langle \eta | V^{el} | \phi \rangle &= \sqrt{\tfrac{2}{3}}\,(d + S) \\
\langle \pi^{\pm} | V^{el} | \rho^{\pm} \rangle &= S \\
\langle K^{+} | V^{el} | K^{+*} \rangle &= S \\
\langle K^{0} | V^{el} | K^{0*} \rangle &= d + S \\
\langle \eta | V^{el} | \rho^{0} \rangle &= -\frac{1}{\sqrt{3}}\, d \\
\langle \eta | V^{el} | \omega \rangle &= \frac{\sqrt{3}}{[illegible]}\,(-d + S) \\
\langle D^{0} | V^{el} | D^{0*} \rangle &= -d + S \\
\langle D^{+} | V^{el} | D^{+*} \rangle &= S \\
\langle F^{+} | V^{el} | F^{+*} \rangle &= S \\
\langle \pi^{0} | V^{el} | \phi \rangle = \langle \pi^{0} | V^{el} | \psi \rangle &= \langle \eta | V^{el} | \psi \rangle \frac{1}{\sqrt{3}} = 0
\end{aligned}
\tag{5}
$$

The two free parameters cannot be calculated from any further theoretical assumption and have to be determined from the experimental data of the old mesons $\Gamma(\omega \to \pi\gamma)$, $\Gamma(\phi \to \eta\gamma)$,

$\Gamma(\rho \to \pi\gamma)$, $\Gamma(K^{0*} \to K^{0}\gamma)$.

We use the experimental values of $\Gamma(\omega \to \pi\gamma)$ and $\Gamma(\phi \to \eta\gamma)$ to determine d and S, respectively, and obtain $|S| \approx (1/5)\,|d|$.[10] The experimental values of $\Gamma(\rho \to \pi\gamma)$ and $\Gamma(K^{0*} \to K^{0}\gamma)$ then determine the relative phase of d and S, and restrict the possible choices for P. We find that only P = 1/2, 1, 3/2 and $S = +(1/5)d$ give acceptable results[5] (note that the old SU(3) or naive quark model assumption requires $S = -(1/3)d$).

The decay rates calculated in this way are shown in Table 1, in columns 3, 4 and 5. All quantities are in units of keV. The experimental values that have been used are shown in column 2. Our predictions are in the last [illegible] lines of the table, in columns 3, 4 and 5. Those in column 4, for P = 1, are considered the best because the old rates are fitted best in that case. In particular, $\Gamma(D^{0*} \to D^{0}\gamma) = (0.35 \pm .04)$ keV. The errors shown were estimated from the experimental errors of the input, $\Gamma(\eta \to \gamma\gamma)$ and $\Gamma(\omega \to \pi\gamma)$. Changes in the predictions due to using a χ^2 fit to determine d and S, or using 751 MeV for the ρ mass,[11] lie within these error estimates.

We conclude with a remark concerning the deviation from ideal mixing: One can easily see that a small deviation from ideal mixing has an effect of only a few percent upon the radiative decay rates calculated above. However, this small deviation has a large enough effect to explain the value of the first-forbidden transitions. As we have already mentioned above it does not make sense to include the deviation from ideal mixing without also taking the isospin mixing into account. Thus we should not expect to obtain a prediction if we arbitrarily take one formula for the non-ideal $|\psi^{\text{non-ideal}}\rangle$ out of the literature, e.g.,[12]

$$|\psi^{\text{non-ideal}}\rangle = -0.9995|\psi\rangle - 0.0101|\phi\rangle - 0.0130|\omega\rangle$$

$$|\eta^{\text{non-ideal}}\rangle = 0.9999|\eta\rangle + 0.0115|\psi\rangle$$

With this admixture of ϕ and ω in $\psi^{\text{non-ideal}}$ and of ψ in $\eta^{\text{non-ideal}}$ one obtains with the values in column 2 of the table $\langle\eta^{\text{non-ideal}}|V^{el}|\psi^{\text{non-ideal}}\rangle = -0.0031$ and therewith $\Gamma(\psi^{\text{non-ideal}} \to \eta^{\text{non-ideal}}\gamma) = 0.13$ keV. This is already above the experimental value[13] of $\Gamma^{\exp}(\psi \to \eta\gamma) = 0.069$ keV. Ignoring the ϕ admixture in η one even obtains $\Gamma(\psi^{\text{non-ideal}} \to \eta\gamma) = 0.94$ keV. This, however, also demonstrates that the small $\rho^0 - \omega$ or $\eta - \pi^0$ mixing cannot be ignored.

Table 1 Calculated values and error estimates for the decay rates, in keV.

Decay	Experiment	P = 1/2	P = 1	P = 3/2
$\omega \to \pi\gamma$	870 ± 61[14]	870 ± 61	870 ± 61	870 ± 61
$\phi \to \eta\gamma$	74 ± 15[15]	51 ± 4	76 ± 6	98 ± 7
$\rho \to \pi\gamma$	35 ± 10[16]	35 ± 10	35 ± 10	35 ± 10
$K^{0*} \to K^0\gamma$	75 ± 35[17]	66 ± 5	87 ± 7	98 ± 8
$K^{+*} \to K^+\gamma$	< 80[15]	1.9 ± 0.6	2.6 ± 0.7	3.0 ± 0.8
$\omega \to \eta\gamma$	< 50[14]	0.18 ± 0.05	0.22 ± 0.06	0.25 ± 0.07
$\rho \to \eta\gamma$	< 152[18]	3.9 ± 0.3	4.8 ± 0.3	5.0 ± 0.4
$\psi \to \eta\gamma$	< 2[19]	0.30 ± 0.03	1.6 ± 0.2	7.5 ± 0.8
$D^{0*} \to D^0\gamma$	...	0.10 ± 0.01	0.35 ± 0.04	1.0 ± 0.1
$D^{+*} \to D^+\gamma$	...	0.006 ± 0.002	0.022 ± 0.007	0.07 ± 0.02
$F^{+*} \to F^+\gamma$	...	0.006 ± 0.002	0.022 ± 0.007	0.07 ± 0.02

References and Footnotes

1) A. Bohm, M. Hossain, R. A. Kagali, P. Moylan, R. B. Teese, P. M. Valanju and H. Alhendi, Phys. Rev. D14, Sept. (1976).

2) A non-perturbative approach to a "broken symmetry" has been advanced for a long time; see e.g. S. Oneda, in Proceedings of the Coral Gables Conference on Fundamental Interactions, 1973, and A. Bohm, Phys. Rev. 158, 1408 (1967). For SU(4), with its large mass differences, this kind of approach seems to become essential.

3) H. van Dam and L. C. Biedenharn, Phys. Rev. D14, 405 (1976).

4) An explanation of the exact meaning of the "current" operators V_μ^a is given in A. Bohm, Phys. Rev. D13, 2110 (1976).

5) We have also tested many other expressions for $\phi(m_V,m_P)$ which follow from simple assumptions of the transformation property of the V_μ^a under $SU(4)_E$:

$$(m_V,m_P) = (m_V^P + m_P^P)(m_V m_P)^Q; \quad P,Q = 0, \pm 1/2, \pm 1, \pm 3/2, \pm 2$$

We found that of these only (1) fitted the decay rates of the old mesons.

6) G. Goldhaber, et al., in Proceedings of the SLAC Summer Institute (1976).

7) Due to a misunderstanding of the conventions none of the three cases considered for X in reference 1 is identical to (4). Therefore our prediction here for $\psi \to \chi\gamma$ differs from the predictions there. The fit to the old vector mesons, however, is independent of the value for X.

8) The Zweig rule in this formulation turns out to be the relation $(D/\sqrt{6} + A) = 0$ between the otherwise arbitrary reduced matrix elements D and A of the vector current 15-plet. This results--with ideal mixing--in $\langle\pi^0|V^{el}|\phi\rangle = -\langle\pi^0|V^{el}|\omega\rangle = -\langle\eta|V^{el}|\psi\rangle \cdot \frac{1}{\sqrt{3}} = 0$ and deviations from this relation are usually considered as deviations from the ideal mixing assumption for ϕ and ω.

9) For the SU(4) Clebsch-Gordon coefficients we used the tables by V. Rabl, G. Campbell, Jr., K. C. Wali, J. Math. Phys. 16, 2494 (1975), and Y. Miyata, S. Iwai and K. Kudoh, Tokyo Institute of Technology preprint TIT/HEP 21 (August, 1975), and adjusted for our phase convention. To obtain consistency in the phase factors is the most troublesome problem in these calculations.

10) A more elaborate fit which does not depend this strongly on the experimental value of $\Gamma(\rho^- \to \pi^-\gamma)$ which has been determined in only one experiment, B. Gobbi, et al., Phys. Rev. Lett. 42B, 511 (1972), is under investigation.

11) H. Hallock, S. Oneda and Milton D. Slaughter, University of Maryland preprint 76-294 (June, 1976), and references therein.

12) S. Oneda and I. Takasugi, "SU(4)-Algebraic Approach to the New Resonances," in Proceedings of the Joint International Symposium on Mathematical Physics, Mexico City (1976).

13) P. S. Schmüser, in Proceedings of the SLAC Summer Institute (1976).

14) Particle Data Group, Rev. Mod. Phys. 48, No. 2, Part II (April, 1976).

15) For $\Gamma(\psi \to \eta\gamma)$ we use the average of the Orsay results: D. Benaksas, et al., Phys. Lett. 42B, 511 (1972); C. Bemporad, in Proceedings of the 1975 International *Symposium on Lepton and Photon Interactions at High Energies*, Stanford, California, edited by W. T. Kirk (SLAC, Stanford, 1976), p. 113.

16) B. Gobbi, et al., Phys. Rev. Lett. 33, 1450 (1974).

17) W. C. Carithers, et al., Phys. Rev. Lett. 35, 349 (1975).

18) C. Bemporad, et al., Nuc. Phys. B51, 1 (1973).

19) This value was reported by Batke, et al., at the Tbilisi Conference (July, 1976). The decay $\psi \to \gamma\gamma$ has been reported by the DASP collaboration: W. Braunschweig, et al., Phys. Lett. 57, 407 (1975), and the DESY-Heidelberg collaboration: J. Heintz, et al., in Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California, edited by W. T. Kirk (SLAC, Stanford, 1976) p. 97.