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Configuration Space Faddeev Calculations

Progress Report

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G. L. PAYNE, W. H. KLINK, and W. N. POLYZOU
Department of Physics and Astronomy
The University of Iowa
Iowa City, Iowa 52242-1479

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ABSTRACT

The detailed study of few-body systems provides one of the most precise tools for studying the dynamics of nuclei and nucleons. Our research program consists of a careful theoretical study of few-body systems and methods for modeling these systems. During the past year we have completed several aspects of this program.

I. PERSONNEL

Principal Investigators

Gerald L. Payne, Professor of Physics
Department of Physics and Astronomy
The University of Iowa
Iowa City, Iowa 52242-1479
Telephone: (319) 335-1854

William H. Klink, Professor of Physics
Department of Physics and Astronomy
The University of Iowa
Iowa City, Iowa 52242-1479
Telephone: (319) 335-1757

Wayne N. Polyzou, Professor of Physics
Department of Physics and Astronomy
The University of Iowa
Iowa City, Iowa 52242-1479
Telephone: (319) 335-1856

Graduate Student Research Assistants

Hsien-Chih Jean
Philip Moore
Michael Rogers

Time Devoted to Contract

Gerald L. Payne	40%
William H. Klink	40%
Wayne N. Polyzou	40%

On the following pages is a summary of the theoretical research which has been carried out under Contract No. DE-FG02-86ER40286 at the Department of Physics and Astronomy of The University of Iowa. The requirements of the contract have been fulfilled.

II. SUMMARY OF RESEARCH COMPLETED

The Relativistic Three Quark Problem

H. C. Jean, G. L. Payne, and W. N. Polyzou

We have numerically solved the relativistic three-body problem for systems of confined constituent quarks. This was done by solving the Faddeev equations for the eigenfunctions of the mass operator for a system of three constituent quarks. To the best of our knowledge, this is the first fully relativistic Faddeev calculation for systems of three light constituent quarks.

The model studied is described below. Baryons are modeled as bound systems of three constituent quarks of mass m_i. The model Hilbert space is the tensor product of three single-quark Hilbert spaces. The interactions consist of a Coulomb plus linear confining interaction between the individual constituent quarks. With a suitable interaction strength, the sum of three pairwise linear confining interactions is known to be a good approximation to a stringlike three-body interaction.

The three-body mass operator has the form

$$M = M_0 + \sum_{i=1}^{3} V_i$$

$$V_i = \sqrt{q_i^2 + (m_{0jk} + v_{jk})^2} - \sqrt{q_i^2 + m_{0jk}^2}$$

where v_{jk} is the basic Coulomb plus linear interaction.

The computations are more complicated than the corresponding trinucleon computations because the angular momentum recoupling coefficients are momentum dependent and because the square root operators are best treated in momentum space while the confining interactions are most naturally treated in configuration space.

The confining interactions were treated directly in momentum space using a method due to Eyre and Vary [1] while the square roots were treated using a method of Glöckle and Offermann [2]. The angular momentum recoupling was done by generalizing the method introduced by Balian and Brézin [3] to the relativistic case which considerably simplifies the numerical computation.

The calculation was performed in a mixed basis consisting of momentum eigenstates for free particles and two-body mass eigenstates. The input required an accurate determination of a large number of two-body wave functions.

The most important result of this research is the momentum space wave function for a nucleon consisting of three light constituent quarks. Although spectral properties can be accurately determined by variational calculations, our experience is that variational calculations are not as reliable for the computation of wave functions. Accurate wave functions are needed for the precise determination of nucleon form factors at high momentum transfer. The calculations were the subject of H. C. Jean's Ph.D. thesis.

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[1] D. Eyre and J. Vary, Phys. Rev. D **34** (1986) 3467.

[2] W. Glöckle and R. Offermann, Phys. Rev. **16** (1977) 2039.

[3] R. Balian and E. Brézin, Nuovo Cimento **61** (1969) 403.

The Relativistic Balian-Brézin Method

H. C. JEAN, G. L. PAYNE, AND W. N. POLYZOU

We have shown, using explicit numerical calculations, that the method proposed by Balian and Brézin [1] can be used to simplify computations in relativistic three-body calculations in the same manner that it is used in the nonrelativistic case. The simplification in the relativistic case is comparable to that of the nonrelativistic case and can be applied in any of Dirac's forms [2] of the dynamics.

In performing three-body computations it is convenient to work in bases where the basic two-body interactions have the simplest matrix elements. Recoupling coefficients are used to transform between bases that are natural for each interacting pair. In the nonrelativistic case the recoupling coefficients are related to Racah coefficients for the rotation group, while in the relativistic case they are related to Racah coefficients for the Poincaré group.

In the nonrelativistic case the coefficients needed for the change of basis are expressed in terms of an integral of an angle-dependent function over four angles. By exploiting the rotational symmetry, it can be shown that the integrand is a function of one combination of the angles, reducing the integral to a one-dimensional integral multiplied by a phase space factor. The integrand is invariant under a three-parameter set of transformations that change the orientation of the rest system. This invariance can be exploited to evaluate the integrand using a convenient choice of orientation of the coordinate system. This method, first suggested by Balian and Brézin, has been utilized in all of our recent three-nucleon computations.

We have shown that a similar method can be successfully applied in the relativistic case. The relativistic change of basis formulas are more complicated because they involve momentum dependent Wigner and/or Melosh rotations. The change of basis matrix elements also involve integrals over four angles, three of which can be reduced to a single phase space factor by using the Poincaré symmetry. Since the integrand is invariant its computation is simplified by exploiting the invariance to evaluate it in a zero total momentum frame with a convenient orientation of the coordinate system. This method was used in the relativistic three quark computations.

[1] R. Balian and E. Brézin, Nuovo Cimento **61** (1969) 403.

[2] P.A.M. Dirac, Rev. Mod. Phys. **21** (1949) 392.

Spin in Light Front Quantum Mechanical Models

W. N. Polyzou

Sufficient conditions are given for a relativistic light-front formulation of quantum mechanics with an interacting spin operator to be scattering equivalent to a relativistic light-front formulation of quantum mechanics with a noninteracting spin operator.

There exist systematic methods for constructing interacting representations of the Poincaré Lie algebra with the kinematic subgroup of the light front when the spin is noninteracting [1]. Local quantum field theories quantized on the light front have a representation of the Poincaré group with an interacting spin. Because of this, it is difficult to construct approximations of the Poincaré generators that preserve the commutation relations. In the case of a noninteracting spin, approximations that preserve the relativistic invariance can be obtained by approximating the mass operator by an operator that commutes with the spin and the kinematic subgroup of the light front. It is thus natural to ask which representations with interacting spin are related to a representation with a noninteracting spin by unitary transformation that preserves the scattering cross sections?

The existence of such transformations provides the justification for using models with a kinematic spin, such as those that have been used to successfully treat deuteron form factors, and provides formal techniques for constructing invariant approximations when the spin is not interacting.

We have found conditions for the existence of such a unitary transformation in the case of multichannel scattering with a fixed number of particles.

[1] B. D. Keister and W. N. Polyzou, in *Advances in Nuclear Physics*, Vol. 20, edited by J. W. Negele and E. Vogt (Plenum Publishing Co., 1991).

Proton-Deuteron Scattering and Reactions

J. L. FRIAR AND G. L. PAYNE

We have completed a review paper on the present status of the use of the configuration-space Faddeev equations to solve three-body problems when long-range Coulomb interactions are present in the Hamiltonian. This review will be a chapter in a book on the Coulomb problem edited by Frank Levin and David Micha. The review provides a detailed discussion on the numerical solution of both the two-body and three-body problems with Coulomb interactions. A careful derivation of the appropriate boundary conditions for both cases is presented, along with a discussion of the role of polarization potentials in the calculations. A new derivation of the boundary conditions for the three-body case is given, and the result agrees with the boundary conditions obtained by Merkuriev [1] which were derived using Green's functions. The use of variational calculations to test the accuracy of the numerical solutions for the three-body Faddeev equations is reviewed. We discuss the ability of the present calculation to reproduce the experimental observations and we discuss the need for additional work on unsolved problems.

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[1] S. P. Merkuriev, Yad. Fiz. **24** (1976) 289; Sov. J. Nucl. Phys. **24** (1976) 150.

Point Form Relativistic Quantum Mechanics of Constituents

W. H. Klink

One of the ways of formulating relativistic quantum mechanics—called the "point form" by Dirac [1]—is to define the four-momentum operator as the mass operator times a four-velocity operator. All of the relativistic dynamics is contained in the mass operator, and in particular it is possible to write the mass operator as the sum of a free-mass operator and interacting-mass operator, as is done nonrelativistically. In the point form an n-body wave function can be written in terms of the overall four velocity of the system, along with internal momentum and spin variables. Under Lorentz transformations these internal variables transform in the same way as their nonrelativistic counterparts, so that it is possible to couple orbital and spin angular momenta together in the usual way, even though the resulting total angular momentum is an eigenvalue of the square of the Pauli-Lubanski operator [2].

Using these features of the point form, it is possible to take a nonrelativistic potential used for baryon spectroscopy and make it fully relativistic. In particular for relativistic harmonic oscillator potentials the eigenfunctions and eigenvalues can be determined exactly, as is the case nonrelativistically.

But for these relativistic harmonic oscillator wave functions to be useful they must also incorporate the correct $SU(6)$ spin-flavor and $SU(3)$ color symmetries. In Ref. [3] we have formulated a relativistic $SU(6)$ model that incorporates all the correct internal and space-time symmetries and should be useful in hadronic spectroscopy [4].

[1] P.A.M. Dirac, Rev. Mod. Phys. **21** (1949) 392.

[2] W. H. Klink, "Relativistic Spin in Point Form Relativistic Quantum Mechanics" (submitted for publication).

[3] W. H. Klink, "Point Form Relativistic Quantum Mechanics and Relativistic $SU(6)$" (submitted for publication).

[4] A. Hey and R. Kelly, Phys. Rep. **96** (1983) 72, and references cited therein.

Solutions of Quantum Mechanical Anharmonic Oscillators

W. H. KLINK

Certain sextic and higher-order anharmonic oscillators have known ground-state solutions [1], but nothing is known about their excited state solutions. For quartic anharmonic oscillators not even ground-state solutions are known. By rewriting the Schrödinger equation as a Ricatti equation and using certain properties of the Ricatti equation, we have found classes of solutions of quartic anharmonic oscillators for ground and excited states, as well as extended the class of known solutions for sextic and higher-order anharmonic oscillators [2]. These solutions can also be transformed to give eigenfunctions of particles in nonconstant electric and magnetic fields.

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[1] G. P. Flessis, Phys. Lett. **81A** (1981) 17; J. Phys. **A14** (1981) L209.

[2] W. H. Klink, "Nonpolynomial Solutions of Quantum Mechanical Anharmonic Oscillators" (submitted for publication).

Shift Operators and the $U(N)$ Multiplicity Problem

W. H. KLINK, TUONG TON-THAT, AND RANDALL G. WILLS*

In many of the algebraic formulations of collective nuclear models [1], a resolution of the multiplicity problem is needed in order to compute nuclear wave functions. In the past several years we have found a way of dealing with the multiplicity problem [2], wherein the representation of a subgroup may appear more than once in the representation of a group.

In this paper a computationally effective method for decomposing r-fold tensor products of irreducible representations of $U(N)$ in a basis independent fashion is given. The multiplicity arising from the tensor decomposition is resolved by the eigenvalues of the invariant operators chosen from the universal enveloping algebra generated by the infinitesimal operators of the dual representation. Shift operators which commute with the $U(N)$ invariant operators, but not with the dual invariant operators, are introduced to compute the eigenvectors and eigenvalues of the dual invariant operators algebraically.

*Department of Mathematics, Southeastern Louisiana University, Hammond, Louisiana 70402.

[1] See for example, G. Rosensteel and D. J. Rowe, Ann. Phys. **126** (1980) 343.

[2] For a review, see W. H. Klink and T. Ton-That, Ulam Quarterly **1** (1992) 41, and references cited therein.

III. PAPERS PUBLISHED, ACCEPTED OR SUBMITTED

Papers Published
(1 November 1991 – 31 October 1992)

1. W. H. KLINK
Clebsch-Gordan and Racah Coefficients of the Poincaré Group
Ann. Phys. **213**, 31–53, 1992

2. W. H. KLINK
$SU(3)$ Clebsch-Gordan Coefficients with Definite Permutation Symmetry
Ann. Phys. **213**, 54–73, 1992

3. W. H. KLINK and T. TON-THAT
Invariant Theory of the Block Diagonal Subgroup of $GL(n, \mathbf{C})$ and Generalized Casimir Operators
J. Alg. **145**, 187–203, 1992

4. W. GLÖCKLE and G. L. PAYNE
Boundary Conditions for Three-Body Scattering in Configuration Space
Phys. Rev. C **45**, 974–984, 1992

5. W. H. KLINK and T. TON-THAT
Duality in Representation Theory
Ulam Quarterly **1**, 41, 1992

Papers Accepted or Submitted
(as of 31 October 1992)

1. W. N. POLYZOU
   A Theorem on Light Front Quantum Mechanics
   J. Math. Phys. [to be published]

2. W. H. KLINK
   Relativistic Quantum Mechanics on Fock Space
   II Wigner Symposium
   [to be published]

3. A. S. LAHAMER and G. L. PAYNE
   Numerical Techniques for the Two-Body Schrödinger Equation
   J. Comput. Phys. [submitted]

4. J. L. FRIAR and G. L. PAYNE
   Proton-Deuteron Scattering and Reactions
   The Coulomb Problem, edited by Frank Levin and David Micha
   [submitted]

5. W. H. KLINK
   Point Form Relativistic Quantum Mechanics and Relativistic $SU(6)$
   [submitted]

6. W. H. KLINK
   Relativistic Spin in Point Form Relativistic Quantum Mechanics
   [submitted]

7. W. H. KLINK, T. TON-THAT, and R. WILL
   Shift Operators and the $U(N)$ Multiplicity Problem
   [submitted]

8. W. H. KLINK
   Nonpolynomial Solutions of Quantum Mechanical Anharmonic Oscillators
   [submitted]

Meetings Attended

G. L. Payne

Gordon Conference on Photonuclear Reactions, Tilton, NH,
August 1992

W. H. Klink

Workshop on Harmonic Oscillators, College Park, MD,
March 1992

Colloquium on Group Theoretical Methods in Physics,
Salamanca, Spain, 28 June–4 July 1992

W. N. Polyzou

Spring Meeting of the American Physical Society,
Washington D.C., April 1992

Workshop on Q.C.D. on the Light-Front, Dallas, TX
24 May - 31 May 1992

Baryons '92, Yale University, New Haven, CT
1 June - 6 June 1992

Midwest Theory Get Together, Argonne, IL, 27–28 September 1991

H. C. Jean

Spring Meeting of the American Physical Society,
Washington D.C., April 1992

Invited Talks

G. L. Payne

"Three-Body Continuum," Gordon Conference on Photonuclear Reactions, August 1992

W. H. Klink

"Point Form Relativistic Quantum Mechanics and Production Reactions" University of Pittsburgh, February 1992

"Relativistic Harmonic Oscillators and Relativistic $SU(6)$" Harmonic Oscillators Workshop, College Park, MD, March 1992

"Relativistic Spin in Point Form Relativistic Quantum Mechanics" Salamanca, Spain, July 1992

W. N. Polyzou

"Relativistic Quantum Mechanics of Particles" The George Washington University, Washington, D.C., January 1992

Visitors

Walter Glöckle, Ruhr Universität, Bochum, West Germany

Rocco Sciavilla, Argonne National Laboratory

Craig Roberts, Argonne National Laboratory

Hsien-Chih Jean, Florida State University

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