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Bunch Lengthening in SPEAR and Extrapolation to PEP

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A recent theory of bunch lengthening suggests that it is caused primarily by the combination of many high-frequency resonators which add up to produce an effective broad-band resistive impedance. The ultimate source is thought to be such small chamber discontinuities as vacuum flanges. Since it is the resistive component of the impedance that is responsible, the bunch lengthening should be directly correlated with the heat dissipated in these ring elements.

From the theory given by Messerschmid and Month[1,2] the equilibrium bunch length can be obtained from

$$G(Z_R, n_0, a, \theta_{rms}) = g \quad .$$

The quantity g is a scaling parameter which depends on the machine and beam parameters but not on the bunch length;

$$g = \frac{h\, V \cos\, \phi_s\, k^2\, (I)}{I} \quad ,$$

with

- h the harmonic number of the rf system,
- V the peak voltage,
- I the dc current,
- ϕ_s the rf stable phase angle, and
- $k(I)$ a function of current representing the change of synchrotron frequency with current (I in units, mA).

We use for k(I):[3]

$$k(I) = \left(\frac{1}{1 + 25 \cdot I^2}\right)^{1/2} \qquad \text{(SPEAR I)}$$

$$k(I) = 1 \qquad \text{(SPEAR II and PEP)}$$

The function G depends on the character of the coupling impedance and in particular on the central mode number, n_0, related to the frequency by

$$f_{peak} = n_0 f_0 \quad ,$$

where f_0 is the ring revolution frequency, and the field decay rate parameter,

$$a = \frac{n_0}{2Q} \quad ,$$

with Q the quality factor; a is related to the width of the impedance by

$$(\Delta f)_{fwhm} = 2 a f_0 \quad ,$$

and Z_R is the peak of the resistive part of the impedance; i.e., at frequency f_{peak}. The bunch length σ is given by

$$\sigma = R \theta_{rms} \quad ,$$

where R is the overall machine radius.

Specifically, the function G is

$$G = \frac{Z_{eff}(a, \theta_{rms})}{n_0 \theta_{rms}^2 \left[1 + 4/\pi \, n_0 \theta_{rms}\right]} \quad .$$

Because the impedance source is a combination of many resonant objects in the ring, we can expect the "sum" to be dominated by a resistive component. Thus, neglecting the reactive term and writing a Lorentzian shape for the impedance function, we have

$$Z_{eff}(a,\theta_{rms}) = Z_R \sum_{n=-\infty}^{\infty} \frac{a^2}{a^2 + (n - n_0)^2} e^{-(n-n_0)^2\theta_{rms}^2} .$$

The power loss to these source elements is due to the same impedance function

$$P = \tfrac{1}{2} I^2 R_{hm} (Z_R, a, n_0, \epsilon_{rms}) ,$$

where the higher-mode resistance R_{hm} is given by[4]

$$R_{hm} = Z_R \sum_{n=-\infty}^{\infty} \frac{a^2}{a^2 + (n - n_0)^2} e^{-n^2\theta_{rms}^2} .$$

We can fit the bunch lengths observed[3,5] in SPEAR I and II with the following impedance parameters:

$$n_0 = 4000 \qquad f_{peak} = 5.1 \text{ GHz}$$
$$a = 700 \ (Q = 2.9) \qquad (\Delta f)_{fwhm} = 1.8 \text{ GHz}$$
$$Z_R = 2 \times 10^5 \ \Omega \qquad Z_R/n_0 = 50 \ \Omega .$$

The theory and data point comparisons are given in Fig. 1 for SPEAR I and Fig. 2 for SPEAR II. Remember that both sets of data are fitted with the same impedance. Now, as has been pointed out, this impedance also determines the higher-mode resistance, which is plotted in Fig. 3 and compared with a few measured[6] points in SPEAR II.

To apply the theory to PEP,[7] we must guess at the impedance. Assuming that the vacuum chamber in PEP will be of similar design to that in SPEAR, we simply scale Z_R with circumference.

Because PEP is to have three bunches, care must be exercised in using the above formulae. In Table I are listed some PEP parameters, particularly those which must be modified to take the three-fold bunch periodicity into account.

Table I: PEP Parameters

Parameter	Value	Value for Bunch Lengthening Analysis	
V (peak voltage)	44	44	MV
I (dc current)	100	100	mA
h (harmonic number)	2589	863	
$\bar{R}$ (average radius)	345.0	115.0	m
E (energy)	15	15	GeV
u_0 (energy loss/turn)	26.4	26.4	MeV
$\cos\phi_s$	0.80	0.80	
ν_s	6.63×10^{-2}	6.63×10^{-2}	
σ_{nat}	2.4	2.4	cm

The impedance parameters must be scaled from the SPEAR to the PEP circumference. In Table II these scaled parameters and their values are given for use in the above formulae which were derived from a one-bunch analysis. We use

$$\frac{R_{PEP}}{R_{SPEAR}} = 9.2 \quad .$$

Table II: Scaled Impedance Parameters for PEP

Parameter		Value	Value for Bunch Lengthening Analysis
n_0	(mode number)	12000×3	12000
a	(field decay rate parameter)	2100×3	2100
Q	(quality factor)	2.9	2.9
f_{peak}	(peak frequency)	5.0	5.0 GHz
$(\Delta f)_{fwhm}$	(frequency width)	1.74	1.74 GHz
Z_R	(peak resistive impedance)	1.84	1.84 MΩ
θ_{rms}	(bunch length)	σ/R_{PEP}	$3\sigma/R_{PEP}$

In Fig. 4 we plot the bunch length function $G(Z_R, a, n_0, \theta_{rms})$ for the impedance characterized in Table II, and in Fig. 5 we plot the higher-mode resistance for the same impedance function. For the PEP parameters we find $g = 304 \times 10^9$. From Fig. 4 it can be seen that the equilibrium bunch length $\theta_{rms} = 0.89$ mrad, or $\sigma = 10.2$ cm -- more than four times the natural bunch length $\sigma_{nat} = 2.4$ cm. We find from Fig. 5 that the higher-mode resistance for the 10.2-cm bunch length is 110 MΩ.

References

1. E. Messerschmid and M. Month, "Theory for the Fast Blowup of Particle Bunches in Accelerators", BNL Report 21891 (1976). (Submitted to Nucl. Inst. Meth.)

2. E. Messerschmid and M. Month, "Anomalous Length of Electron Bunches as an Instability Threshold", to be issued as a BNL Report, 1976.

3. M.A. Allen, G.E. Fischer, M. Matera, A.P. Sabersky and P.B. Wilson, Proc. IXth Int. Conf. on High Energy Accel., Stanford University, May 1974, p. 352. (SPEAR I data)

4. See, for example, E. Keil, CERN Report CERN/ISR-TH/74-15 (1974).

5. SPEAR Group, IEEE Trans. Nucl. Sci. NS-22, No. 3, June 1975, p. 1366. (SPEAR II data)

6. M.A. Allen, J.M. Paterson, J.R. Rees and P.B. Wilson, IEEE Trans. Nucl. Sci. NS-22, No. 3, June 1975, p. 1838.

7. PEP Conceptual Design Report, LBL-4288/SLAC-189, February 1976.

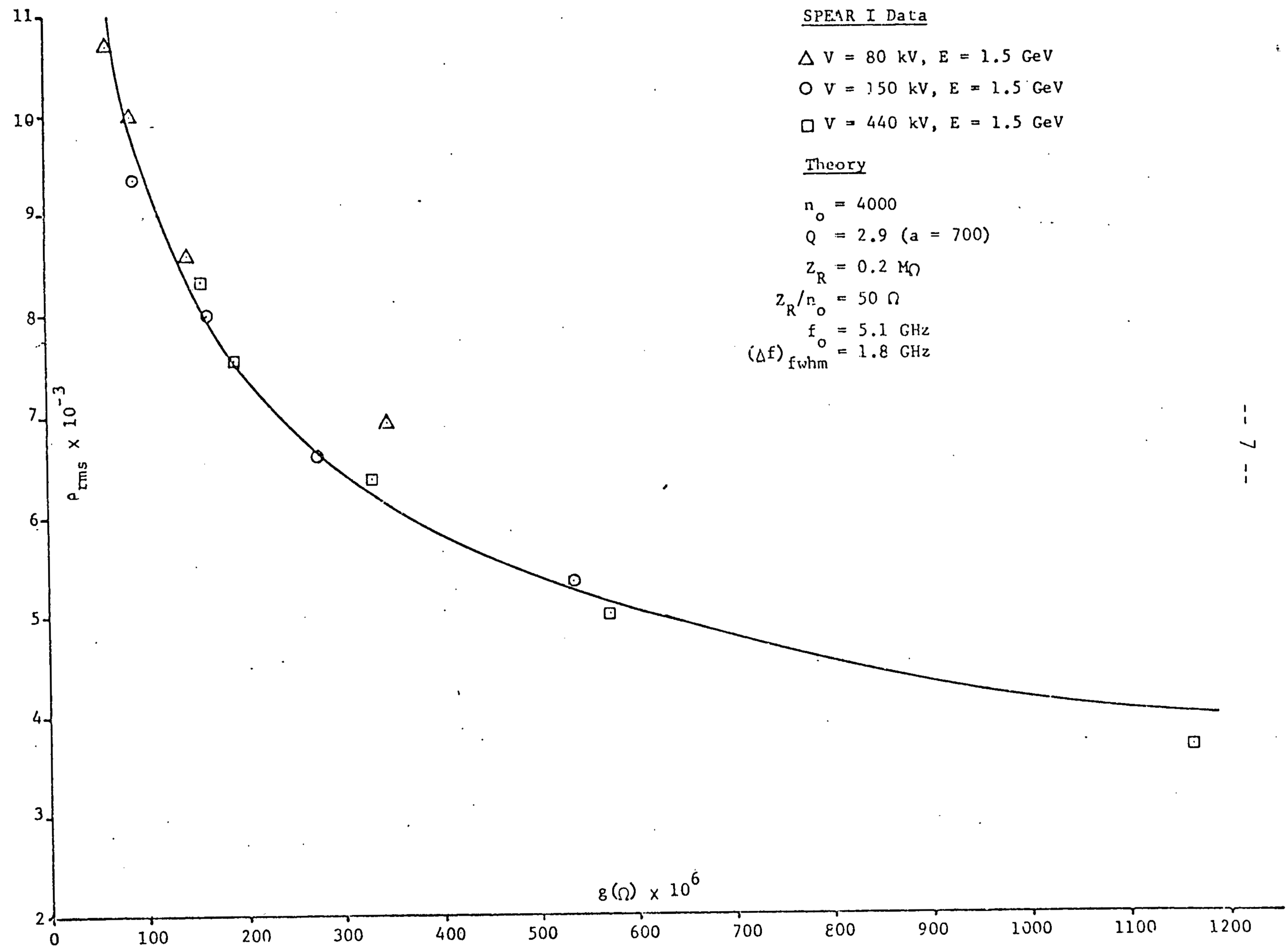

Fig. 1 -- Bunch Length -- SPEAR I

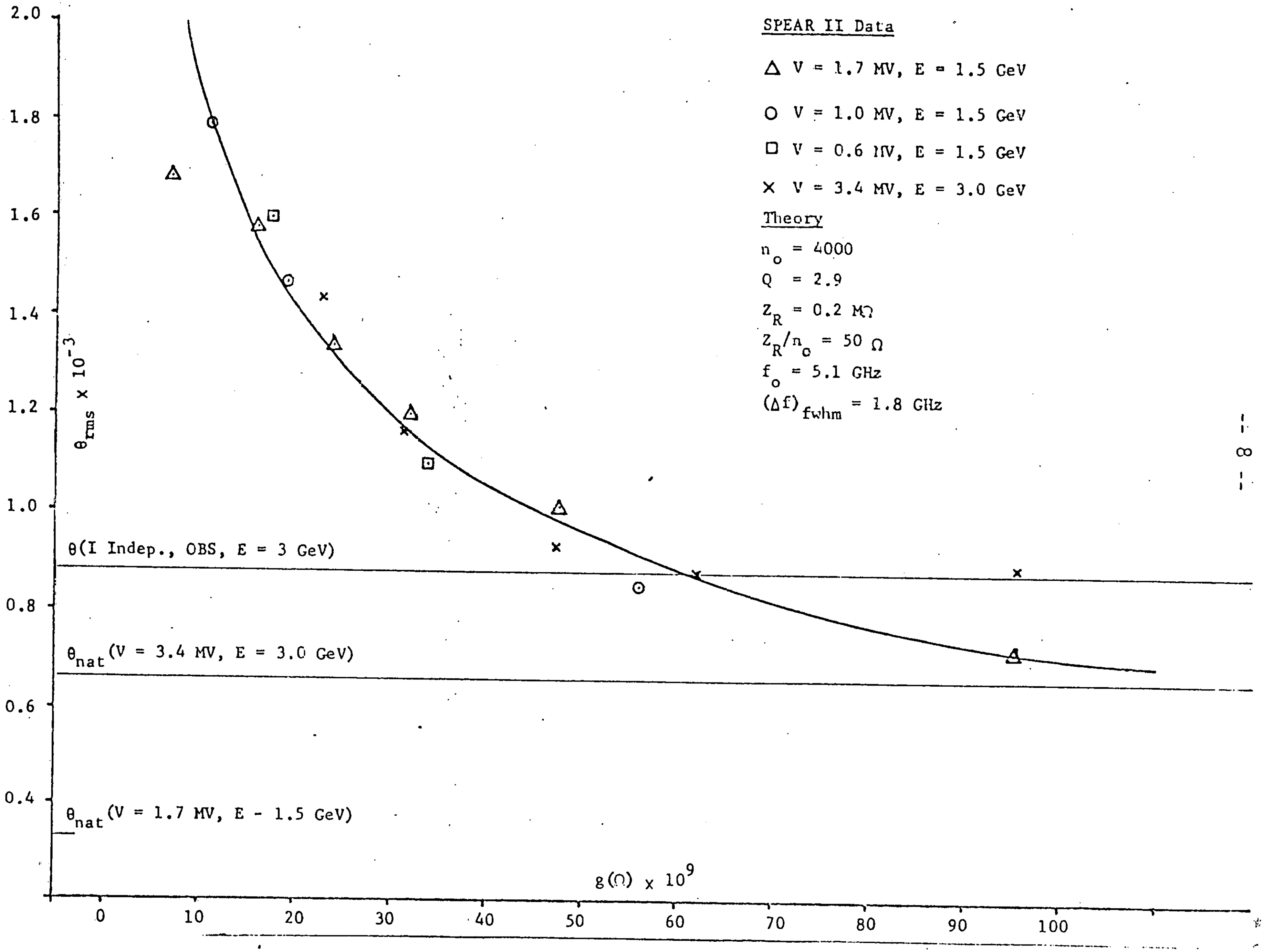

Fig. 2 -- Bunch Length -- SPEAR II

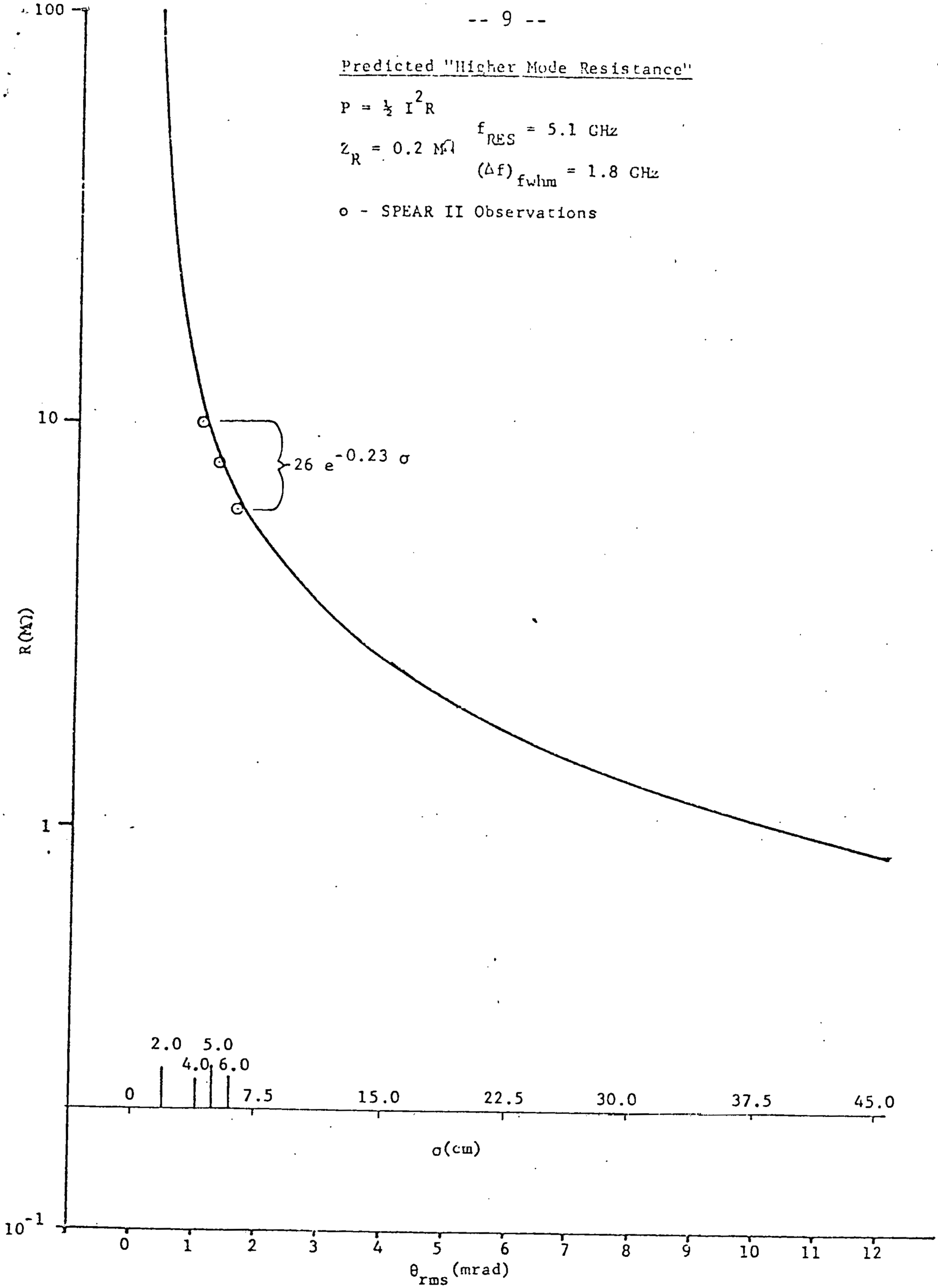

Fig. 3 -- Higher-mode Resistance -- SPEAR

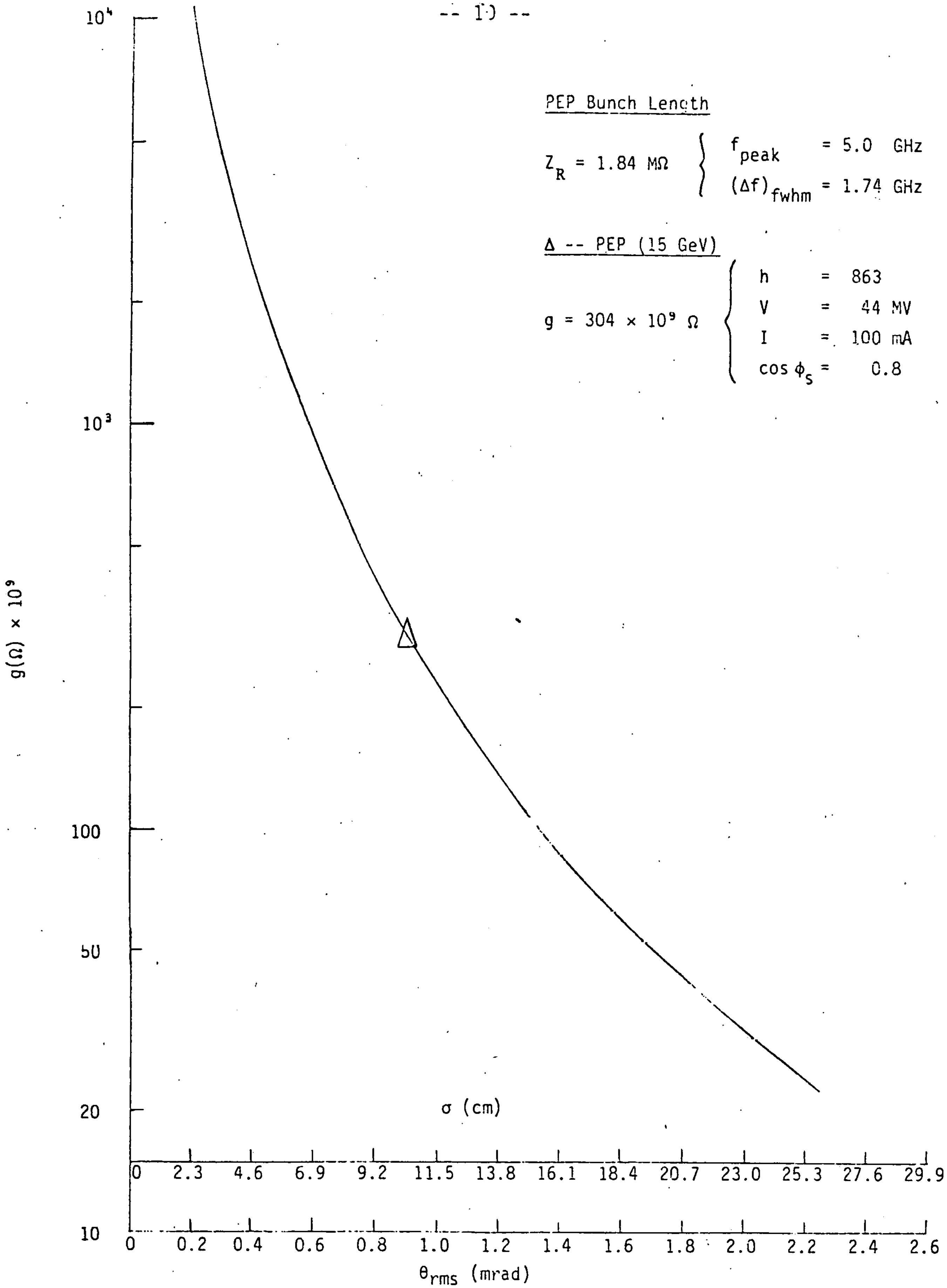

Fig. 4 -- Bunch Length -- PEP

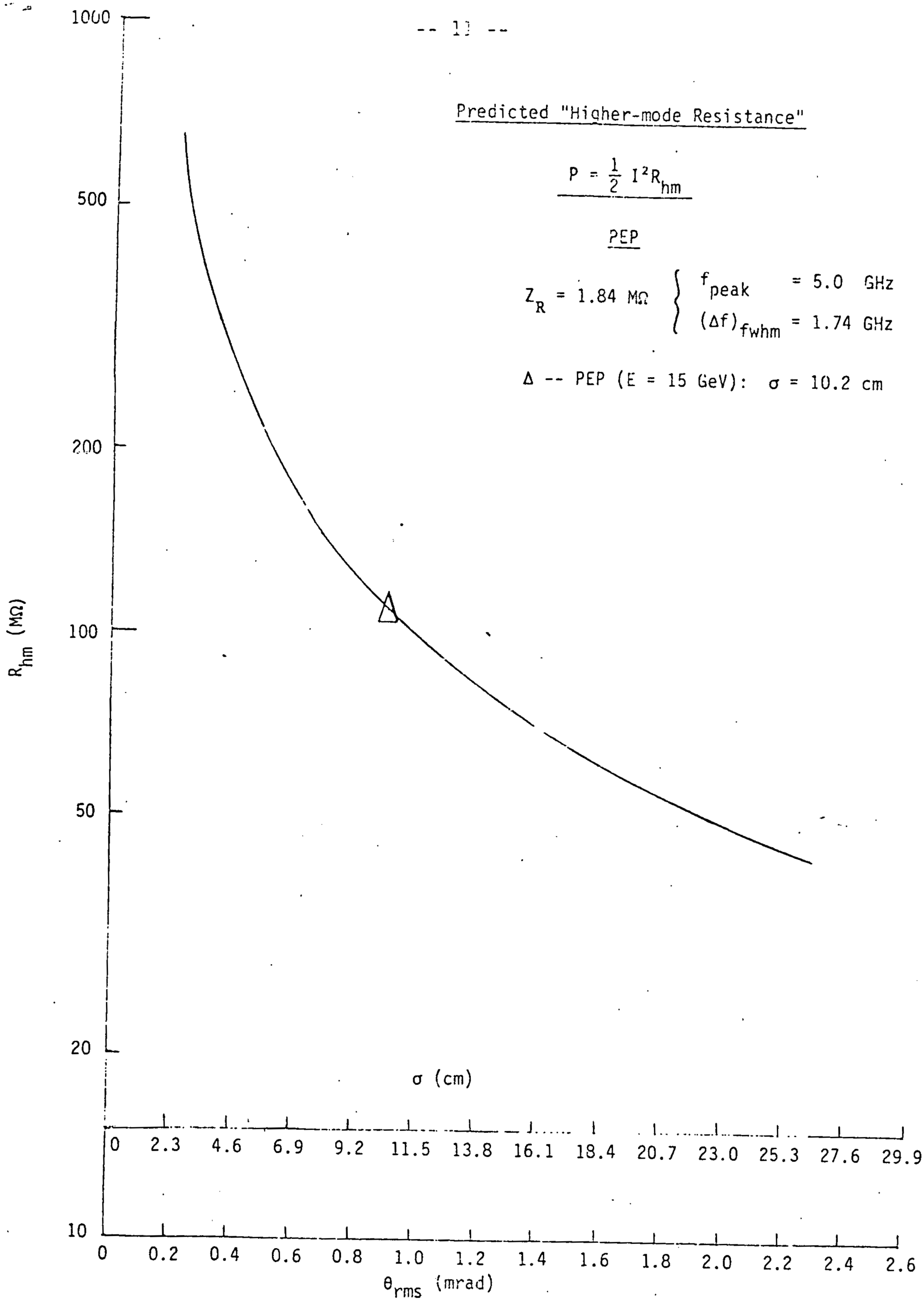

Fig. 5 -- Higher-mode Resistance -- PEP