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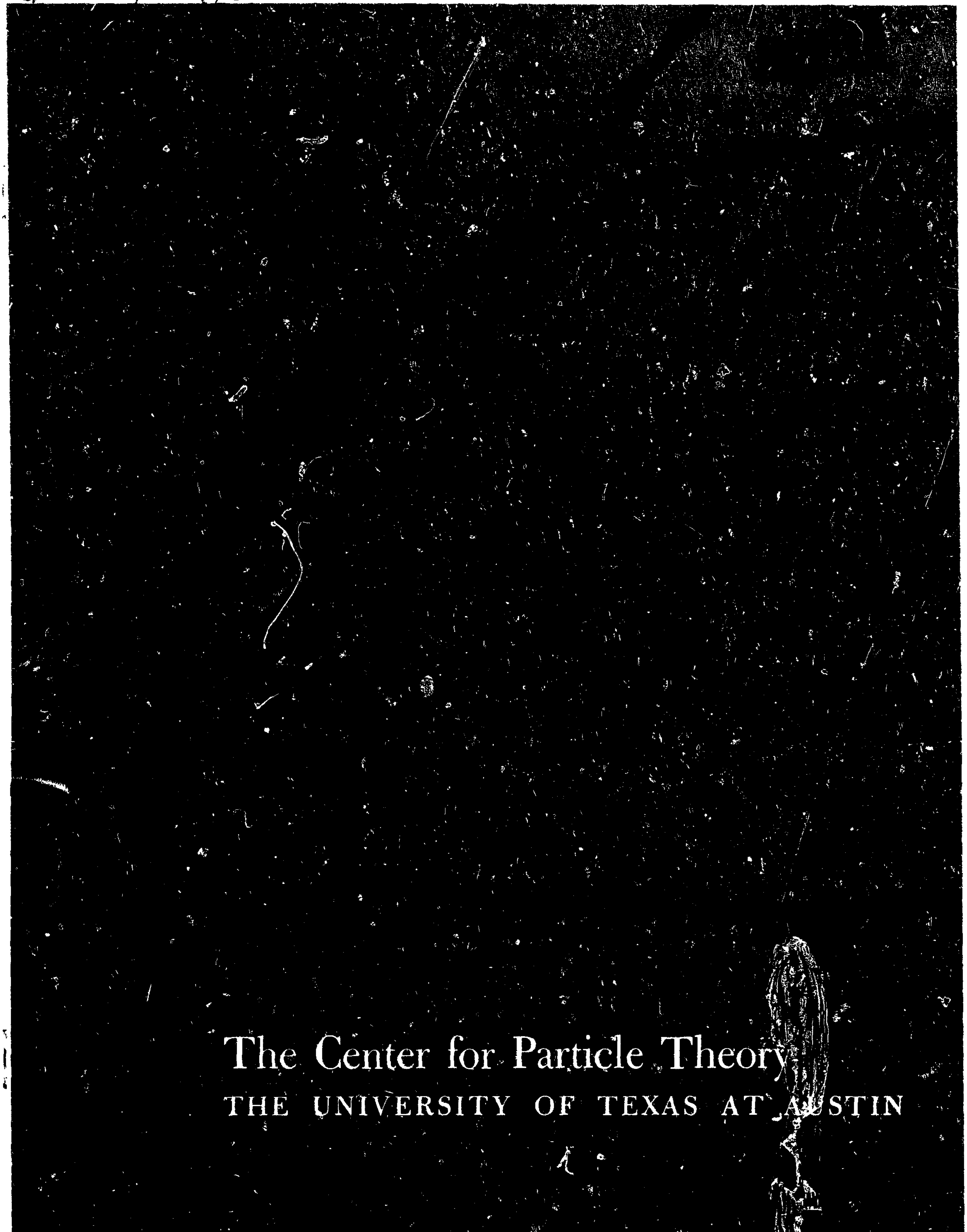

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The Role of Higg's Mesons in the

Violation of Muon Number

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*Work supported in part by the Energy Research and Development Administration.

†Work supported in part by the National Science Foundation

Abstract

The branching ratio for the process $\mu \to e\gamma$ is calculated by assuming that the couplings of both the scalar (Higg's) mesons and the gauge mesons to heavy leptons violate muon number conservation. When the Higg's mesons are somewhat lighter than the gauge mesons their contribution to $\mu \to e\gamma$ is the more important one.

The possibility exists that muon number is not strictly conserved. For example, violation of muon number conservation could manifest itself through the decay $\mu \to e\gamma$ with a branching ratio for this process of the order of 10^{-8} or less.[1] Such violation can be incorporated into a gauge theory of the weak interaction by mixing the e and μ in a manner analogous to the way the GIM mechanism mixes the d and s quarks.[2] In addition, if the violation is to be anywhere near the upper limit allowed by experiment, heavy leptons must be included in the theory.

Consider an $SU(2) \otimes U(1)$ gauge theory with the following multiplets of particles

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix}_L , \quad \begin{pmatrix} e^- \\ L_e^o \end{pmatrix}_R , \quad (L_e^o)_L , \qquad (1)$$

where the doublets have weak hypercharge Y = -1 and the singlet has Y = 0 ($Q = -T_3 + Y/2$). Identical multiplets exist for the muon. Now couple these multiplets to the gauge mesons and then mix the electron type particles with the muon type by performing the transformations[2]

$$\begin{pmatrix} e \\ \mu \end{pmatrix}_{L,R} \rightarrow V_{L_1R} \begin{pmatrix} e \\ \mu \end{pmatrix}_{L,R}$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}_{L} \rightarrow V_{L} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}_{L} \qquad (2)$$

$$\begin{pmatrix} L_e^o \\ L_\mu^o \end{pmatrix}_{L,R} \rightarrow U_{L_1R} \begin{pmatrix} L_e^o \\ L_\mu^o \end{pmatrix}_{L,R}$$

with

$$U_R^{-1} V_R = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}$$

The usual $\bar{e}\gamma^\alpha \frac{1+\gamma_5}{2}\nu_e W_\alpha^-$ and $\bar{\mu}\gamma^\alpha\frac{1+\gamma_5}{2}\nu_\mu W_\alpha^-$ couplings are unchanged but we have an additional interaction of the form

$$\begin{aligned} \mathcal{L}_{WL} = \frac{g}{\sqrt{2}} W_\alpha^- \bar{e}\gamma^\alpha \frac{1-\gamma_5}{2} [L_e^o \cos\phi - L_\mu^o \sin\phi] \\ + \frac{g}{\sqrt{2}} W_\alpha^- \bar{\mu}\gamma^\alpha \frac{1-\gamma_5}{2} [L_e^o \sin\phi + L_\mu^o \cos\phi] \end{aligned} \qquad (3)$$

$\mathcal{L}_{WL}$ will clearly allow the process $\mu \rightarrow e\gamma$ if the mass of L_e^o, M_e, is different than the mass of L_μ^o, M_μ.[3] (The neutral current interactions are unchanged by (2) because they are simply multiplied by UU^{-1} or VV^{-1}.) More interesting, however, is what the transformations (2) do to the scalar (Higgs) meson sector of the theory.

The simpiest way to couple the leptons to the Higg's particles is to take one complex doublet, (ϕ^-, ϕ^o) to couple the singlet in (2) to the right handed doublet and a complex singlet χ^o to couple the two doublets. The lepton masses are then generated by $\langle\phi^o\rangle_o = a$ and $\langle\chi^o\rangle_o = b$. In this case, the ϕ^- is gauged away and there is no scalar lepton coupling that can contribute to $\mu \to e\gamma$. If we add one more scalar multiplet, however, say an isotopic triplet (ρ^-, ρ^o, ρ^+) with $\langle\rho^o\rangle_o = c$ then the ϕ^- will remain coupled to the leptons and an interaction of the form

$$f_1 (\overline{L_e^o})_L (\phi^{-*}, \phi^{o*}) \begin{pmatrix} e^- \\ L_e^o \end{pmatrix}_R$$

$$+ f_2 (\overline{L_\mu^o})_L (\phi^{-*}, \phi^{o*}) \begin{pmatrix} \mu^- \\ L_\mu^o \end{pmatrix}_R \qquad (4)$$

will, under the transformation (2), not only generate masses for the heavy leptons through

$$U_L^{-1} \begin{pmatrix} f_1 & \\ & f_2 \end{pmatrix} U_R = \frac{1}{a} \begin{pmatrix} M_e & \\ & M_\mu \end{pmatrix} , \qquad (5)$$

but will also give a coupling of ϕ^- to the leptons that is proportional to the heavy lepton mass,

$$\phi^{-*} \frac{1}{a} \Big\{ M_e \overline{L_e^o} \frac{1-\gamma_5}{2} (e \cos\phi + \mu \sin\phi)$$

$$+ M_\mu \overline{L_\mu^o} \frac{1-\gamma_5}{2} (-e \sin\phi + \mu \cos\phi) \Big\} \qquad (6)$$

This interaction can also contribute to $\mu \to e\gamma$ and may give a much larger contribution than that of (3). The mass of the vector meson gets contributions from both a^2 and c^2 with the result that

$$\frac{1}{a^2} \geq \sqrt{2}\, G \tag{7}$$

so the coupling constant in (6) may also be relatively large but in any event is bounded from below.

The result for $\mu \to e\gamma$, using both (3) and (6), is[4]

$$\frac{\Gamma_{\mu\to e\gamma}}{\Gamma_{\mu\to e\nu\bar{\nu}}} = \frac{3}{64}\,\frac{\alpha}{\pi}\,\sin^2\phi\,\cos^2\phi\,\frac{(M_e^2 - M_\mu^2)^2}{m_W^4}\left(1 + \frac{1}{6}\,n\varepsilon\,\frac{m_W^2}{m_\phi^2}\right)^2 \tag{8}$$

where m_W is the vector meson mass and m_ϕ is the mass of the scalar ϕ^-. In writing (8) we have neglected higher order terms in $(M_{e^-}^2 - M_\mu^2)/m_W^2$.

The second term in the bracket of (8) is the contribution from the scalar mesons as in (6). As we have discussed, this term may be absent ($n = 0$). If there are enough scalars in the theory so that it is present then, by (7),

$$n \equiv \frac{1}{\sqrt{2}\, G a^2} \geq 1 \tag{9}$$

ε is a known factor which takes into account higher order terms in $(M_e^2 - M_\mu^2)/m_\phi^2$,

$$\varepsilon = \left[\frac{\alpha_e A(\alpha_e) - \alpha_\mu A(\alpha_\mu)}{\alpha_e - \alpha_\mu}\right]^2 \qquad (10)$$

with $\alpha_e = \frac{M_e^2}{m_\phi^2}$, $\alpha_\mu = \frac{M_\mu^2}{m_\phi^2}$, and

$$A(\alpha) = \frac{1}{1-\alpha} - \frac{3\alpha}{(1-\alpha)^2} - \frac{6\alpha^2}{(1-\alpha)^3} - \frac{6\alpha^2}{(1-\alpha)^4}\ln\alpha\ . \qquad (11)$$

ε is shown in Fig. 1 for various values of α_e and α_μ.

Since m_ϕ may be much less than m_W, the scalar contribution may well dominate the decay rate. The lower bound on the Higgs particle masses is only ~5 GeV[6] while the gauge meson must have a mass of ~50 GeV. The ε factor suppresses the scalar contribution somewhat while n can enhance it.

If $M_e^2 - M_\mu^2$ is taken as 1 GeV^2 and m_W is taken to be 50 GeV, then the coefficient of the bracket in (8) is less than 1.8×10^{-11}. This coefficient could, of course, be much less than this number, in fact, if the scalar contribution dominates, then it is probably required to be much less.

Another process which would show the violation of muon number and which may be easier to detect is $\mu \to ee\bar{e}$. In our theory this could proceed by attaching an electron positron pair to the photon of the $\mu e\gamma$ vertex. It may also occur by the exchange of two (scalar and/or gauge) mesons but here the scalar exchange is not enhanced even for low Higg's masses. Thus one expects that

$$\Gamma_{\mu\to ee\bar{e}} \lesssim 10^{-2}\ \Gamma_{\mu\to e}$$

In conclusion we note that the scale of muon number nonconservation in simple models such as the above increases with the heavy lepton mass difference and decreases with gauge meson masses. It is enhanced by Higg's mesons,[7] if they can contribute, roughly by $(m_W/m_\phi)^4$ and thus, in a rich enough theory, the Higg's meson contribution should dominate.

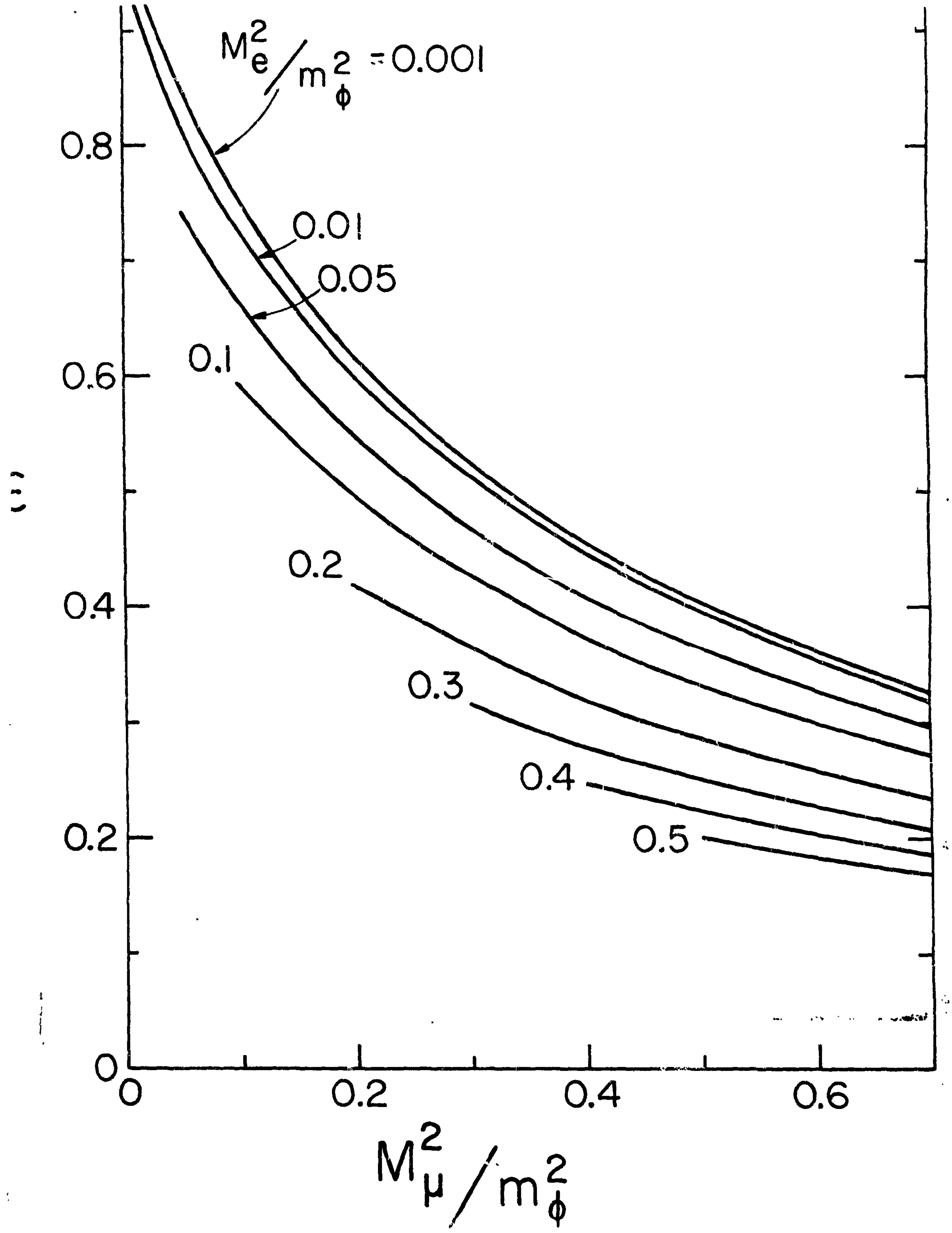
It is a pleasure to acknowledge a number of very helpful conversations with Professor Royce L. P. Zia and also a stimulating discussion with M. A. Bég, H. Schnitzer and G. Segrè.

References

1. The experimental upper bound on the branching ratio for $\mu \to e\gamma$ is 2×10^{-8}. See S. Parker, H. L. Anderson, and C. Rey, Phys. Rev 133B, 768 (1964); S. Frankel et al., Nuovo Cimento 27, 894 (1963).
2. For an illustrative example of such mixing see S. Weinberg, Phys. Rev. Letters 37, 657 (1976).
3. T. P. Cheng and L.-F. Li, Carnegie-Mellon preprint.
4. The vector meson part of this result has also been derived by Cheng and Li in Ref. 3. Their preprint was brought to our attention by Professor Zia while our work was in progress. Our answer is smaller than that of Ref. 3 by an overall factor of 1/4.
5. This calculation was performed in the U-gauge so there are no contributions from any other particles such as unphysical scalars.
6. S. Weinberg, Phys. Rev. Letters 36, 294 (1976); A. D. Linde, to be published.
7. A gauge model for $\mu e\gamma$ without heavy leptons but rich in Higg's mesons has been proposed by J. D. Bjorken and S. Weinberg, SLAC preprint.

Figure Caption

Figure 1 The factor ε, defined in Eq. (10), is shown for values of M_e^2/m_ϕ^2 and M_μ^2/m_ϕ^2.


$M_e^2/m_\phi^2 = 0.001$
0.01
0.05
0.1
0.2
0.3
0.4
0.5
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
M_μ^2/m_ϕ^2