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THE INSTALLATION OF HORIZONTAL SEISMOMETERS IN THE LLL SEISMIC NET AND THEIR CALIBRATION

Marvin D. Denny

January 25, 1977

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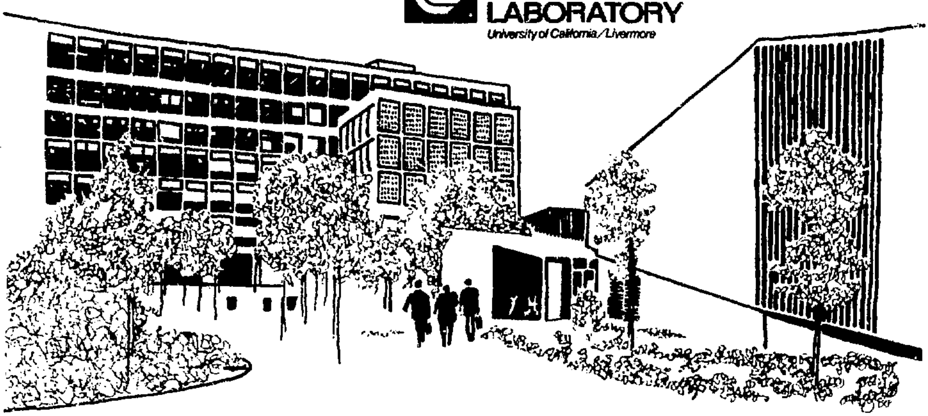

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Abstract

We have upgraded the Lawrence Livermore Laboratory seismic net by installing two horizontal seismometers at each of the four LLL stations. These seismometers record radial and transverse ground motion from underground nuclear explosions at the Nevada Test Site and complement the vertical components which were installed several years ago. Each station now monitors three orthogonal components of ground velocity over a broad frequency band.

Introduction

During 1976, the Lawrence Livermore Laboratory seismic net was upgraded by the installation of two horizontal seismometers at each of the four LLL stations. The horizontal seismometers record radial and transverse ground motion from underground nuclear explosions at the Nevada Test Site (NTS). They complement the vertical components installed several years ago. Each station now monitors three orthogonal components of ground velocity over a broad frequency band. This report summarizes the work that went into the upgrading of the seismic net.

Seismometer General Description

At each station, the two new horizontal seismometers complement the existing vertical seismometer, thereby giving us a matched set of three-component instrumentation. Both the horizontal and the older vertical seismometers were manufactured by Sprengnether Instruments of St. Louis, Missouri, and have the following common features:

1. Mass position monitor
2. Mass alignment motor
3. Calibration coil/magnet assembly
4. Two large coil/magnet assemblies

Features 1 and 2 are used to detect and remove drift of the seismometer mass while feature 3 is used chiefly to confirm that all system components are functioning properly. The seismic data are provided by the relative motion of one of the two large coil/magnet assemblies while the other magnet is used to damp the seismometer. Since the data are generated by the movement of a coil in a magnet, they are proportional to velocity. While the coil/magnet assemblies have nominally the same electrical properties, their location with respect to the hinge is somewhat different on the horizontal from where they are on the vertical. Hence, when the system is calibrated by means of the calibration coil, a different constant is required for the horizontal from that required for the verticals. More details are given in the Appendices.

Telemetry and Control System

While the basic telemetry system remained unchanged in concept from the one originally used with the vertical components,* much of the hardware both at the central and the remote stations had to be changed. In Fig. 1, we show the telemetry system including data acquisition and command facilities, and in Fig. 2, we show the command panel in detail. Of the items shown in Fig. 1, the new ones in the central station are the control panel, status display, and status decommutators, while at the remote stations the new ones are the amplifiers, the calibration, status monitor, and VCO chassis.

The data are recorded continuously, as in the past, but now we have only two channels from the vertical, instead of four, and one channel for each horizontal seismometer. The fifth VCO brings back status information, as before, but now all status information is sent sequentially in digital form. In the old system the operator could read only mass position or gain at any one time. Now both are displayed simultaneously. In addition, if a calibration is being run, the current in use is displayed.

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*For a description of the original single-component system, see D. D. Judd, *Determination of Transfer Function of Seismometer/Telemetry System Through Use of Pseudo-Random Binary Sequences*, Lawrence Livermore Laboratory Rept. UCRL-50490 (1969).

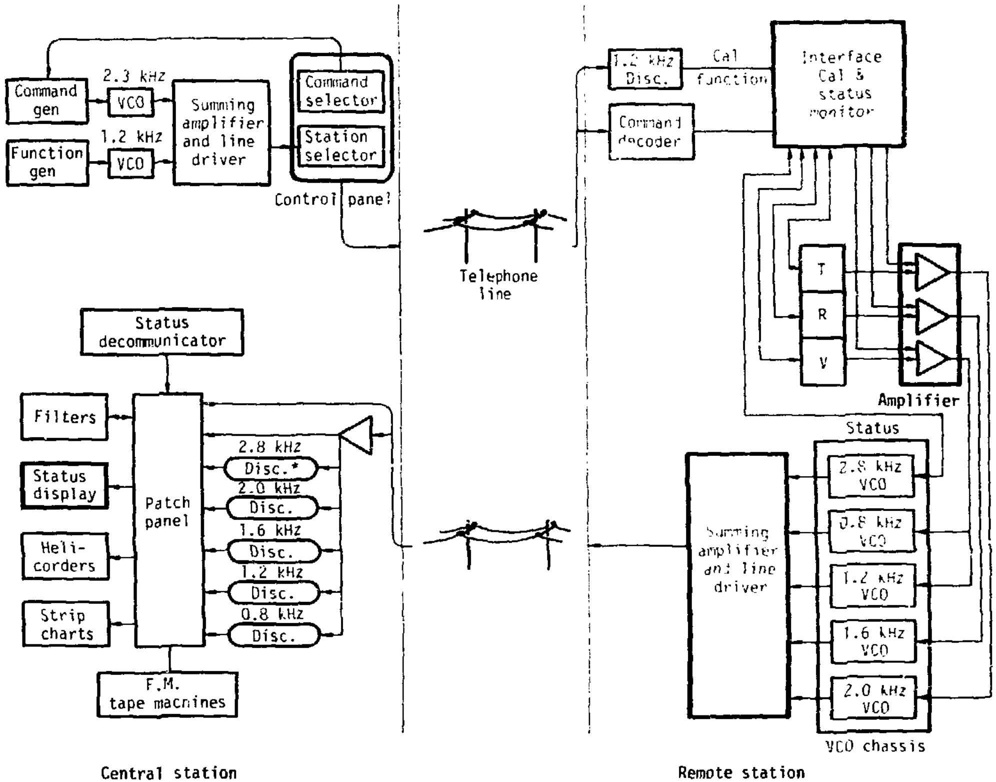


Fig. 1. Telemetry system. Items in heavy boxes are new to the telemetry system with the exception that while the VCO chassis is new, the VCO's are not. *Disc. is discriminator.

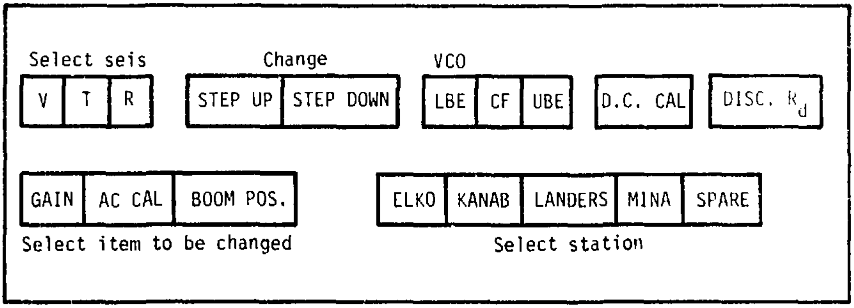


Fig. 2. Control panel layout.

Three 14-t.. FM tape machines are in use. The slow tape, which runs at 0.06 ips (inches per second), continuously records one channel from each of the eight horizontal seismometers and the low gain channel from each of the four vertical seismometers. In addition, it records IRIG time code "C" on the two remaining tracks. At shot time, two machines are run at 7.75 ips. On one are both channels of vertical data from each of the stations plus IRIG time codes "B" and "C." On the other are all the horizontal channels, the low gain verticals, and the time codes. Also, at shot time, four six-channel Brush strip charts are run to record all data from each station.

The operation of the control panel (see Fig. 2) is self explanatory, with four exceptions. One is that to attempt to calibrate all three seismometers at any one station, while possible, will lead to failure of the current driver. Another is that the "disconnect damping resistance" button disconnects all seismometers at the stations selected. Furthermore, while it is possible to put all the seismometers into free period at the same time, the results will be erroneous. See, for example, the free period test of the Kanab R&T seismometer on 9/20/76 shown in Appendix D (Fig. D-1). What happens is that the calibration coils are latched together in parallel thus inducing additional damping. The last is that "UBE, CF, LBE" commands apply to all channels at the stations selected. Other than that, the procedure is to select a station or stations; select a seismometer; select either GAIN, AC CAL, BOOM POS., or DC CAL; and then to adjust one of these operations with the

STEP UP or STEP DOWN commands. For example, the sequence of commands to put a seismometer into a free period is:

1. Select station
2. Select seismometer
3. D.C. CAL (initiation)
4. STEP UP (this command is repeated until the desired boom displacement is reached).
5. Disc. R_d
6. D.C. CAL (termination)

This example also illustrates another point: that all commands except STEP UP and STEP DOWN are continuously sent once the button is pushed in and the light comes in until it is pushed a second time.

Emplacement

As stated earlier the seismometers were located so as to record radial and tangential ground motion from NTS explosions. A centrally located point in Rainier Mesa at NTS more or less midway between Pahute Mesa and Yucca Valley was arbitrarily chosen for the focus. This point is 37.2°N, 116.1°W. The location of the stations with respect to this point is given in Table 1, and the orientation of the seismometers is shown in Fig. 3. Positive ground motion is away from NTS for the radial component and counter clockwise for the tangential component. The stations' coordinates were determined by a survey in 1969 at which time two monuments at each station were set. These monuments made it possible to use a surveyor's transit to scribe on the existing concrete pads radial and tangential lines with respect to NTS. Once the alignment was determined, specially made lucite enclosures for the seismometers were bolted to the concrete using a stud gun. As moisture is a major problem at Elko and Kanab, the following measures were taken. The heads of the bolts were covered with RTV silastic sealant so that the bolts would not serve as cold points on which moisture could condense. The enclosures were fabricated with three 3 in. × 3 in. square holes in the bottom. Glass plates were then placed in these holes and glued with epoxy to

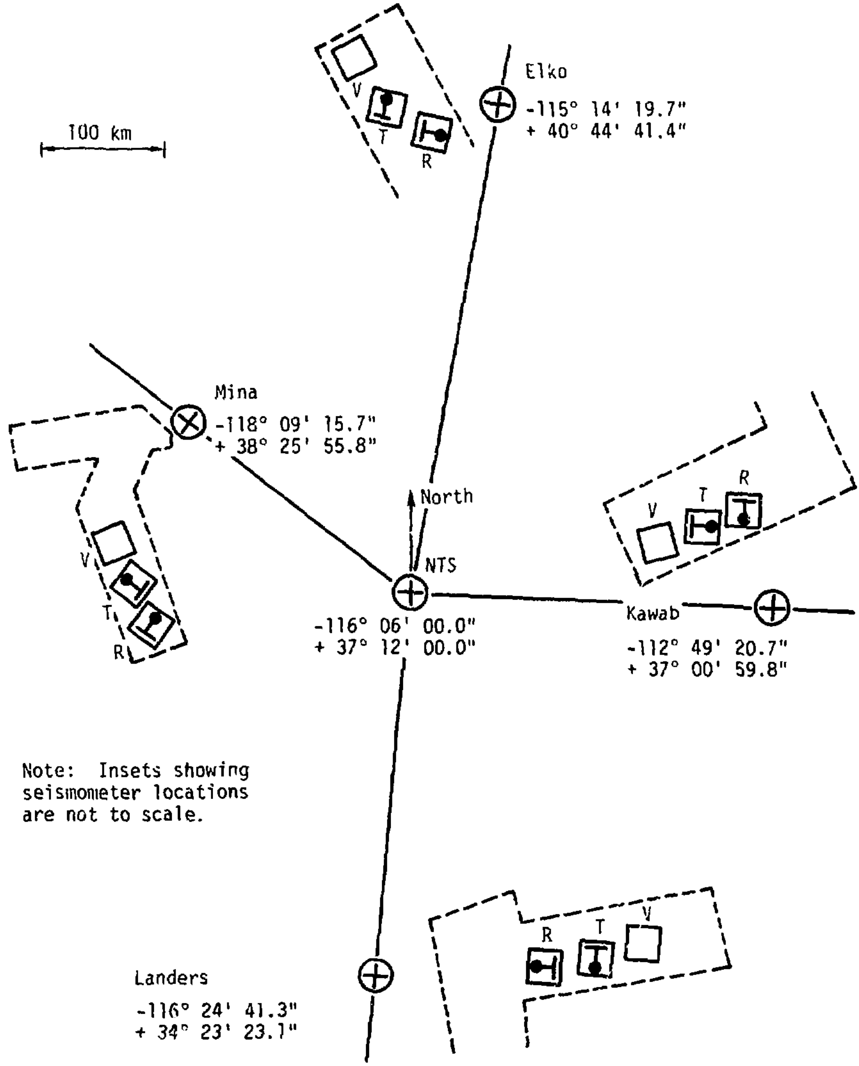
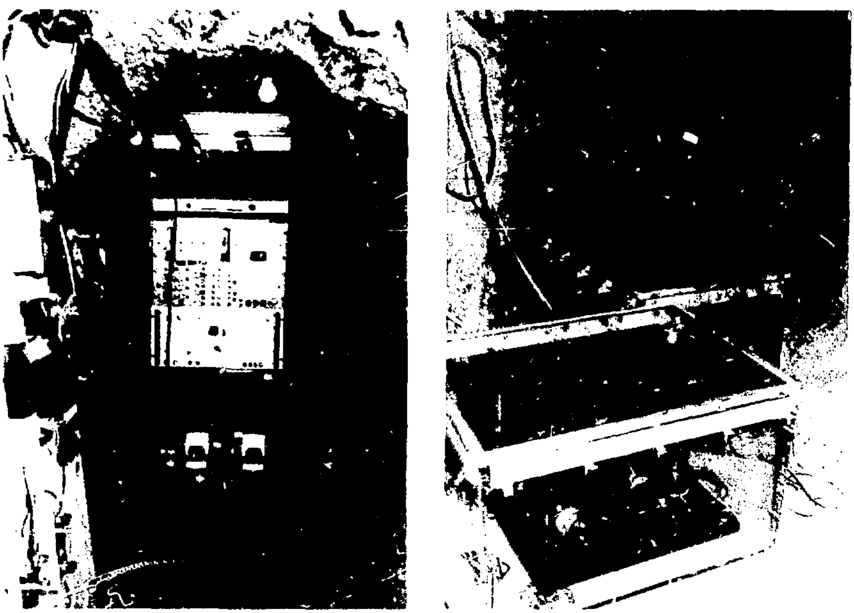


Fig. 3. Relative station locations with respect to Nevada Test Site. The insets at each station show approximate locations within the vaults of the seismometers.

Table 1. Station locations with respect to 37.2° N, 116.3°W.

	Distance (km)	Azimuth (degrees)	Back azimuth (degrees)
Elko	3.603° 400.54	10.4699	191.01
Kanab	2.626° 292.03	93.01	274.98
Landers	2.816° 313.09	185.25	5.07
Mina	2.040° 226.78	307.71	126.46



(a) (b)

Fig. 4. (a) Telemetry and associated electronics. (b) The seismometers. The nearest is the radial, then the tangential, and the farthest is the vertical.

the concrete. The gap between the glass and the lucite was then also sealed with RTV silastic sealant. Two of the glass plates were drilled and countersunk so that the front feet of the seismometer were fixed in place. The third plate was not so that the period adjustment foot would be free. The holes for the cables were sealed with silastic and the lid was placed on a rubber gasket and secured with turn buckles. Hence every reasonable effort was made to seal the seismometer in a moisture proof container. As a final effort to keep the moisture from condensing on the seismometer, each of the enclosures were outfitted with four 1-W bulbs place near the top of the enclosure, and several bags of desiccant were placed in each enclosure. Fig. 4 shows the installation at Mina of both the seismometers and the electronics with the exception of the amplifier.

System Parameters

The system parameters of interest in this report are those that control the seismometer damping and those that govern the calibration. The damping term, in general, is complex; however, for the range of frequencies over which the seismic net operates, the complex term is negligible as will be seen. The damping term is given by, neglecting inductance,

$$\eta = \left[\frac{\ell_s^2}{M\ell_o\ell_m}\left(\frac{K^2}{R_c + R_d}\right)\right]\frac{1}{2\omega_n} + h, \quad * \tag{1}$$

where ℓ_s is the distance from the hinge to the center of the signal coil used for damping

ℓ_o is the distance from the hinge to the center of oscillation

ℓ_m is the distance from the hinge to the center of mass

M is the mass in kilograms or $\frac{N\text{-}sec^2}{m}$

K is the generator constant of the moving coil and magnet in N/A or V-sec/m

*While textbook derivations of the equation of motion abound, most make simplifying assumptions which eliminate the geometry of construction of the seismometer. Therefore, a derivation including the relevant geometry is given in Appendix A.

R_c is the internal resistance of the damping coil

R_d is the external resistance needed for a given value of η

ω_n is the undamped (open circuit) natural frequency of the seismometer in rad/sec

h is the natural (open circuit) inherent mechanical damping of the seismometer.

The system sensitivity can be given in terms of magnification, velocity sensitivity, or acceleration sensitivity. Since the transducer that produces the data is proportional to velocity, velocity sensitivity is the logical choice. The seismometer can be independently calibrated in two ways. Using the known generator constant, K_s, for the signal coil, one finds the velocity sensitivity is

$$V.S._{sc} = K_s \left(\frac{\ell_s}{\ell_o}\right) \left[\prod_{i=1}^{N} X_i\right] \bar{T}\ (\omega), \qquad (2)$$

where ℓ_s and ℓ_o are as defined above,

X_i is the gain or scaling constant for the electrical components,

$\bar{T}\ (\omega)$ is the transfer function for the seismometer.

The alternate method uses the motor constant, G_c, for the calibration coil which is driven by a selected current, I, at an angular frequency ω.

$$V.S._{CC} = \left[\frac{\ell_m}{\ell_c}\ \frac{M}{G_c}\right] \omega\ \frac{A(\omega)}{I(\omega)}\ , \qquad (3)$$

where ℓ_c is the distance from the hinge to the center of the calibration coil,

G_c is the motor constant in N/A of the calibration coil,

$A(\omega)$ is the strip chart amplitude resulting from $I(\omega)$.

The mechanical parameters, as taken from the manufacturer's specification and the telemetry transmission and recorder parameters, are listed in Tables 2 and 3, respectively. Both sets of parameters are independent of seismometers. Table 4 gives the electro-mechanical parameters of the coil-magnet assembies in addition to damping information. The motor constants were measured in the laboratory using a method in which a known

Table 2. Mechanical parameters.

Mechanical parameters		Horizontal	Vertical
Distance from hinge to center of signal coil,	ℓ_s	14.03 in.	13.87 in.
Distance from hinge to center of oscillation,	ℓ_o	14.07 in.	14.08 in.
Distance from hinge to center of calibration coil,	ℓ_c	6.76 in.	9.53 in.
Distance from hinge to center of mass,	ℓ_m	13.2 in.	12.66 in.
Mass of inertial assembly,	M	11.13 kg	10.95 kg

Table 3. Data transmission parameters.

Amplifier gain	X_1 = 0, 1., 2., 4., ... 16384. In steps of 2
VCO scale factor	X_2 = 100 Hz/5 V for horizontals and low gain verticals
	= 100 Hz/0.2 V for high gain verticals
Discriminator scale factor	X_3 = 1.25 V/100 Hz
Brush strip-chart scale factor	X_4 = 20 mm/1.25 V

Table 4. Electro-mechanical parameters.

Station	Inst.	Serial No.	Date installed	Signal Coils R_g (Ω) Rt	R_g (Ω) Lt	L (H) Rt	L (H) Lt	Cal. Coil R_{cc} (Ω)	L_{cc} (H)	h	Results of force-balance and frequency response tests[a] K_s (N/A) Rt	K_s (N/A) Lt	G_c (N/A)	$\frac{R_c + R_d}{T_n}$ (Ω/sec)	R_d (Ω)	T_n^b (sec)	η^c
Landers	R[c]	4986	1-23-76	560	560	1.2	1.3	58	0.02	0.027	101.5	103.30	5.060	114.8	R 4100	40.0	0.612 ± 0.008
											94.8 ± 0.6	96.[illegible] ± 0.6	4.97 ± 0.06	100.7 ± 1.4	R 3170 ± 50	39.2	.707 ± .01
	T	4991		560	560	1.2	1.2	58	.02	.027	101.1	101.5	5.330	113.9	R 4060	40.7	.607 ± 0.05
											93.1 ± 1.1	93.4 ± 1.1	4.94 ± 0.04	96.6 ± 2	R 3270 ± 90	39.6	.707 ± 0.01
	V	3621	8-9-68	485	474	1.2	1.15	65	.013	.02	94.4	92.97	5.42	97.08	1340	20.9	.80
Mina	R	4987	3-9-76	572	571	1.02	1.1	63	.014	.026	92.5	93.1	4.6	95.88	L 3245	39.8	.713 ± 0.12
											92.7 ± 1.0	93.3 ± 1.0	5.12 ± 0.1	96.73 ± 2	L 3240 ± 80	41.5	.707 ± .01
	T	4992		565	565	1.06	1.05	61	.013	.026	97.0	95.4	5.1	104.83	R 3690	40.6	.711 ± .03
											97.3 ± 1.9	94.6 ± 1.9	5.15 ± 0.12	105.4 ± 3.8	R 3720 ± 150	40.6	.707 ± .01
	V	3269	12-7-67	405	405	0.94	0.94	65	.015	.03	81.5	81.8	5.6	64.91	930	19.3	.79
Kanab	R	4993	5-10-76	580	584	1.3	1.3	64.	.023	.029	96.1	96.5	5.2	103.97	L 3610	40.34	.787 ± .02
											95.1 ± 1.1	95.6 ± 1.1	5.2 ± 0.08	102.2 ± 2.4	L 3560 ± 100	40.56	.707 ± .01
	T	4988		580	578	1.4	1.3	64	.022	.029	95.8	96.5	5.2	104.6	L 3600	39.96	.686 ± .01
											94.9 ± 0.9	94.7 ± 0.9	5.57 ± 0.07	99.89 ± 1.8	L 3420 ± 70	39.98	.707 ± .01
	V	3612	10-18-68	464	463	1.09	1.55	65	.013	.031	92.6	92.7	5.04	87.75	1290	20.0	.80
Elko	R	4989	8-9-76	581	580	1.3	1.3	64	.023	.024	94.0	93.4	5.2	94.94	L 3170	39.5	.668 ± .02
											93.9 ± 1.0	92.0 ± 1.0	5.32 ± 0.1	94.0 ± 2	L 3220 ± 90	40.4	.707 ± .01
	T	4994		578	572	1.3	1.3	64	.023	.024	94.20	95.4	5.01	92.28	L 3110	39.9	.666 ± .01
											92.9 ± 0.8	91.6 ± 0.8	5.15 ± 0.05	93.1 ± 1.6	L 3180 ± 60	40.4	.707 ± 0.01
	V	3677	9-9-69	459	459	1.1	1.11	61	0.013	0.049	93.3	94.5	5.0	97.09	2540	30.9	0.77

[a] For each horizontal seismometer the top line has the force-balance test results while the second line has the results from the frequency response test. The values from the frequency response tests are the final values.

[b] The free period changed with time as the seismometer settled in.

[c] The top line for each horizontal seismometer is the value of η determined by the frequency response test. These values were the result of using the force-balance test data to calculate the damping resistance corresponding to η = 0.707.

force is applied to the seismometer. While this test is easily done on the verticals by merely setting a weight on the boom, it is more difficult to perform on the horizontals. In order to apply a known force to the horizontals, a jig was made which placed a vertical post at a known distance from the boom's center position. A string of known length was then attached to the boom and the vertical post, and a small weight was suspended from the string. A current is then applied to one of the coils, the others being open, and is increased until the seismometer is driven back to its original position. Several problems were encountered with this arrangement. Some of the tests turned out well and others did not, but we could never assess at the test time the quality of the results. Nevertheless, the motor constants determined in this way were used to calculate the initial, or test, values of the damping resistance, R_{dT}, for $\eta = 0.707$.

Following the installation and checkout of all the station electronics, we determined the frequency response of each seismometer using frequencies of 0.01 Hz through 5.0 Hz. From the results of this test, the actual damping, or test value η_T, was found using the value Eq. (3) takes at large frequencies, compared to the free period frequency ω_n as follows:

$$\eta_T = \frac{1}{2} \frac{(V.S.)_\omega}{(V.S.)_{\omega_n}} ,$$

where $\omega \gg \omega_n$. (4)

The correct value was then found from

$$R_c + R_d = \frac{(R_c + R_{dT})}{0.707 - h} \times (\eta_T - h) , \quad (5)$$

where R_{dT} is the value of the damping resistance determined from the force balance test and used during the

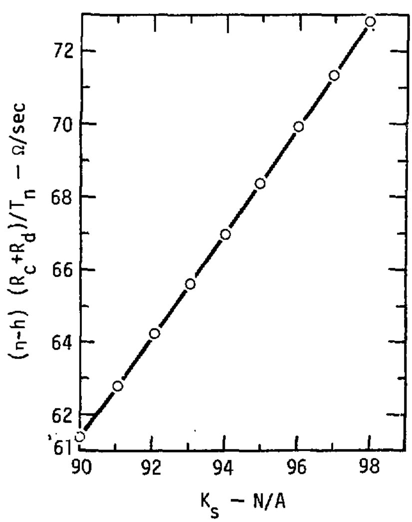


Fig. 5. Relationship of system damping, free period, and damping resistance.

frequency response test. The motor constant for the damping coil could then be deduced from Eq. (1), which is plotted in Fig. 5. The uncertainties shown in Table 4 result from the uncertainty in the measurements of (V.S.) at large frequencies and at ω_n. The motor constant for the signal coil was then found from a simple test in which the relative outputs of both coils are measured. Finally, the motor constant of the calibration coil was deduced by equating Eqs. (2) and (3). A complete record of the frequency response tests and calculations just described are given in Appendices B through E.

Magnification

This section is included because seismologists have a tendency to compare systems only in terms of magnification. Therefore, for reference, the system magnification on the Brush strip-chart recorders for a nominal vertical and horizontal seismometer at the routine gain setting of 2048 is

$$M = K_s \left(\frac{\ell_s}{\ell_o}\right)\left[\prod_{i=1}^{N} X_i\right] \omega|\bar{T}(\omega)|$$

$$M_{vertical} = \left(93\frac{\text{V-sec}}{\text{m}}\right)\left(\frac{13.87}{14.08}\right)(2048)\left(\frac{100\ \text{Hz}}{5\ \text{v}}\right)\left(\frac{1.25\ \text{V}}{100\ \text{Hz}}\right)\left(\frac{20\ \text{mm}}{1.25\ \text{V}}\right)\omega|\bar{T}(\omega)|$$

$$= (750.5\ \text{sec})\ \omega|\bar{T}(\omega)|$$

$$M_{horizontal} = \left(93\frac{\text{V-sec}}{\text{m}}\right)\left(\frac{14.03}{14.07}\right)(2048)\left(\frac{100\ \text{Hz}}{5\ \text{V}}\right)\left(\frac{1.25\ \text{V}}{100\ \text{Hz}}\right)\left(\frac{20\ \text{mm}}{1.25\ \text{V}}\right)\omega|\bar{T}(\omega)|$$

$$= (759.7\ \text{sec})\ \omega|\bar{T}(\omega)|$$

These equations are plotted in Fig. 6 for free periods of 40, 30, and 20 sec. The maximum magnification is seen to be approximately 80K at 18 Hz.

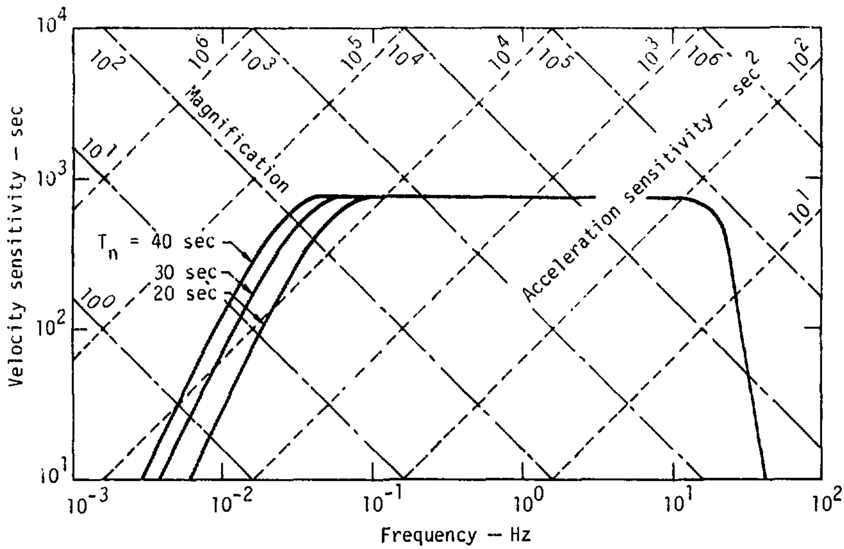


Fig. 6. Nominal system response.

Acknowledgements

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Appendix A.
Seismometer Basic Equations

There are two equations which govern the behavior of the seismometer. One is the second-order equation of motion, which can be written down by summing the torques about the hinge, and the other is the first-order circuit equation. Figure A-1 shows the forces acting on the seismometer. The summation of torques, assuming the seismometer is being driven by the calibration coil, is given by

$$\left(M\ell_m^2 + I_m\right)\ddot{\theta} + \ell_s \left(K_1 i_1 + K_2 i_2\right) + d\ell_m^2\dot{\theta} + \ell_m^2 k\theta = \ell_c G_c I_c \,, \qquad \text{(A-1)}$$

where θ is the angular deflection of the boom and the other parameters are defined in Fig. A-1. Figure A-2 shows an electrical representation of the coil and magnet. The circuit equation for the coil and magnet assembly is

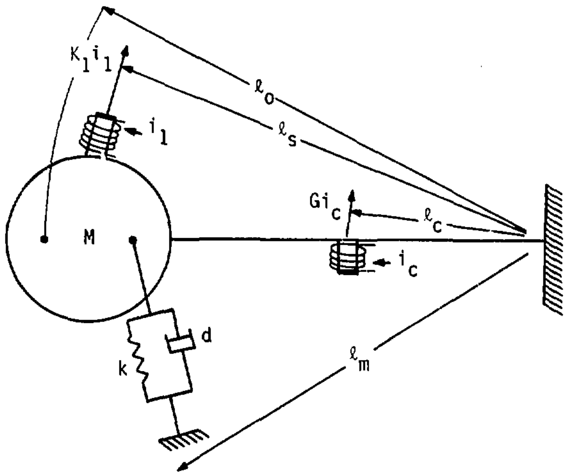
$$K\ell_s \dot{\theta} = iR_c + \frac{di}{dt} L_c + Zi \,. \qquad \text{(A-2)}$$

In practice, one of the signal coils is used to damp the seismometer with a resistive load while the other drives a high-impedance amplifier. Since the damping resistance is on the order of a kilo-ohm and the amplifier impedance is on the order of a mega-ohm, only the damping current need be retained in Eq. (A-1) and Z is just R_d the external damping load. Combining the Fourier transforms of Eqs. (A-1) and (A-2) results in

$$-\omega^2\bar{\theta} + \frac{\ell_s^2}{M\ell_o\ell_m}\left(\frac{K_d^2}{R_c + R_d + j\omega L_c}\right) j\omega\bar{\theta} + 2h\omega_n j\omega\bar{\theta} + \omega_n^2\bar{\theta} = \frac{\ell_c}{\ell_o\ell_m}\,\frac{G_c\,\bar{i}_c}{M} \,, \qquad \text{(A-3)}$$

where K_d is the generator constant of the coil used for damping,

$\omega_n^2 = \frac{\ell_m}{\ell_o}\frac{k}{M}$, $2h\omega_n = \frac{\ell_m}{\ell_o}\frac{d}{M}$, $\ell_o = \ell_m\left(1 + \frac{I_m}{\ell_m^2 M}\right)$, ω_n is the open circuit or natural frequency, and h is the open circuit damping factor. The contribution of the damping term of the inductance, ωL, at ω_n is about 0.2 Ω which is insignificant when compared to $R_c + R_d$ which is approximately 3700 Ω. Hence we can safely neglect the inductance, and then Eq. (A-3) reduces to



ℓ_c = distance to center of calibration coil

ℓ_m = distance to center of mass

ℓ_o = distance to center of oscillation

ℓ_s = distance to center of signal coils

K_n = generator constant for signal coil n = 1,2

I_m = moment of inertia about center of mass

d = mechanical damping at center of mass

k = effective spring constant at center of mass

M = mass

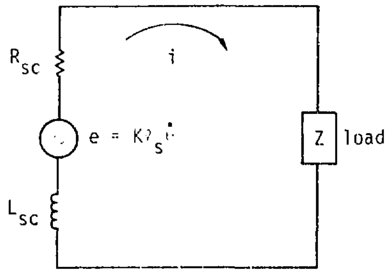
G = motor constant for cal coil

Note: second signal coil is not shown

Fig. A-1. Forces acting on the seismometer.

$$\theta\left(-\omega^2 + 2\eta\omega_n j\omega + \omega_n^2\right) = \frac{\ell_c}{\ell_o\ell_m}\;\frac{G_c\bar{i}_c}{M} ,$$

where $$\eta = \frac{\ell_s^2}{M\ell_o\ell_m}\;\frac{K_d^2}{(R_c + R_d)\;2\omega_n} + h \; . \qquad \text{(A-4)}$$



R_{sc} = electrical resistance of signal coil

L_{sc} = electrical inductance of R_{sc} signal coil

Fig.A-2. Electrical representation of coil and magnet.

When the driving force on the seismometer is the earth then the inertial term is dependent on the absolute motion so that Eq. (A-1) becomes

$$\ell_m M\ddot{y} + I\ddot{\theta} + \ell_s(K_1 i_1 + K_2 i_2) + d\ell_m^2\dot{\theta} + \ell_m^2 k\theta = 0, \quad \text{(A-5)}$$

where $y = z + \ell_m\theta$, z is the absolute earth motion, and y is the absolute mass motion.

As with Eq. (A-1), Eq. (A-5) becomes upon transformation (neglecting inductance)

$$\bar{\theta}\left(-\omega^2 + 2\eta\omega_n j\omega + \omega_n^2\right) = \frac{\omega^2\bar{z}}{\ell_o} \quad . \qquad \text{(A-6)}$$

From Eqs. (A-4) and (A-6) the equivalence between the calibration current and ground displacement is seen to be

$$\frac{\ell_c}{\ell_m} \; \frac{G_c}{M} \bar{i}_c = \omega^2\bar{z} \quad . \qquad \text{(A-7)}$$

From Eq. (A-6) the seismometer transfer function, a dimensionless ratio of mass motion to base motion, can be written down

$$\bar{T}(\omega) = \frac{\ell_o\bar{\theta}}{\bar{z}} = \frac{\omega^2}{\omega_n^2 - \omega^2 + 2\eta\omega_n j\omega} \quad . \qquad \text{(A-8)}$$

The transfer function approaches unity for $\omega >> \omega_n$ while for $\omega << \omega_n$ it approaches $(\omega/\omega_n)^2$.

The system may be calibrated for displacement, velocity, or acceleration, as follows:

$$\text{Magnification, } M = \frac{\text{seismometer output}}{\text{ground displacement}} = \left|\frac{\text{output}}{\bar{z}}\right|$$

$$\text{Velocity sensitivity, V.S.} = \frac{\text{seismometer output}}{\text{ground velocity}} = \left|\frac{\text{output}}{j\omega\bar{z}}\right|$$

$$\text{Acceleration sensitivity, A. S.} = \frac{\text{seismometer output}}{\text{ground acceleration}} = \left|\frac{\text{output}}{-\omega^2\bar{z}}\right|$$

Output can be given in volts, but when the signal is displayed on a recorder of some kind then it is given in mm and the calibration units are:

	Volts	mm
M	mV/mm	dimensionless
V.S.	$\frac{\text{mV}\cdot\text{sec}}{\text{mm}}$	sec
A.S.	$\frac{\text{mV}\cdot\text{sec}^2}{\text{mm}}$	sec^2

Since the data for the LLL seismic net is provided by a velocity transducer, calibration is routinely given in terms of velocity sensitivity. The seismometer output in volts is given by

$$V_{out} = K_s \ell_s \dot{\theta} ,$$

whose Fourier transform is

$$\bar{V}_{out} = K_s \ell_s \; j\omega\bar{\theta}.$$

Therefore, the velocity sensitivity at the seismometer is by definition and Eq. (A-8):

$$\text{V.S.} = \left|\frac{K_s \ell_s j\omega\bar{\theta}}{j\omega\bar{z}}\right| = \frac{K_s \ell_s}{\ell_o} \left|\bar{T}(\omega)\right| \text{ in } \frac{\text{mV}\cdot\text{sec}}{\text{mm}} . \qquad \text{(A-9)}$$

Equation (A-9) shows that the velocity sensitivity is flat for $\omega >> \omega_n$. To determine the velocity sensitivity on a given strip chart, we simply multiply Eq. (A-9) by the various scale factors in the data transmission system and in the recorder as follows:

$$V.S. = K_s \frac{\ell_s}{\ell_o} \left|\bar{T}(\omega)\right| \left[\prod_{i=1}^{n} X_i \right] . \tag{A-10}$$

In the LLL system, N = 4 and X_1 is amplifier gain, X_2 is VCO sensitivity, X_3 is discriminator sensitivity, and X_4 is the Brush strip-chart sensitivity.

Alternatively, the system can be calibrated using a known current $\bar{I}_c(\omega)$ to drive the seismometer. In this case, the output is the measured amplitude $A(\omega)$ on the strip chart so that by definition and Eq. (A-7):

$$V.S. = \left|\frac{A(\omega)}{j\omega\bar{z}}\right| = \frac{A\ (\omega)}{\frac{\ell_c}{\ell_m} \frac{G}{M} \frac{i_c(\omega)}{\omega}}$$

or

$$V.S. = \frac{\ell m}{\ell_c} \frac{M\ \omega\ A(\omega)}{G_c i_c(\omega)} . \tag{A-11}$$

From Eq. (A-11), a calibration constant for this method is defined by

$$C.C. = \frac{\ell_m}{\ell_c} \frac{M}{G_c} \text{ in } \frac{uA \cdot sec^2}{mm} .$$

For the nominal calibration coil value of 5.0 N/A, the calibration constant for the horizontals is

$$C.C._{horizontal} = 4.35 \frac{mA \cdot sec^2}{mm} ,$$

while for the verticals it is

$$C.C._{vertical} = 2.91 \frac{mA \cdot sec^2}{mm} .$$

From the above, it can be seen that for the same velocity sensitivity (V.S.), a given calibration current will result in smaller amplitudes from the horizontal seismometers than from verticals.

Appendix B.
Landers Station Calibration

FORCE BALANCE TEST

STATION Landers INSTRUMENT Radial SN #4986

AEC LLL 216800

DATE 1-23-76

Coil Resistances

Lt Coil 560 Ω

Rt Coil 560 Ω

Cal Coil 58 Ω

Coil Inductances

Lt Coil 1.3 H

Rt Coil 1.19 H

Cal Coil 20 mH

Natural Damping

$$h_o^2 = 1 \Big/ \left[\left(\frac{\pi(j - i)}{\ell_n A_j/A_i} \right)^2 + 1 \right] \qquad j > 1$$

or for $h_o << 1$

$$h_o \approx - \frac{\ell_n (A_j/A_i)}{(j - i)\pi}$$

from plot (Fig. B-1) $h_o = 0.027$

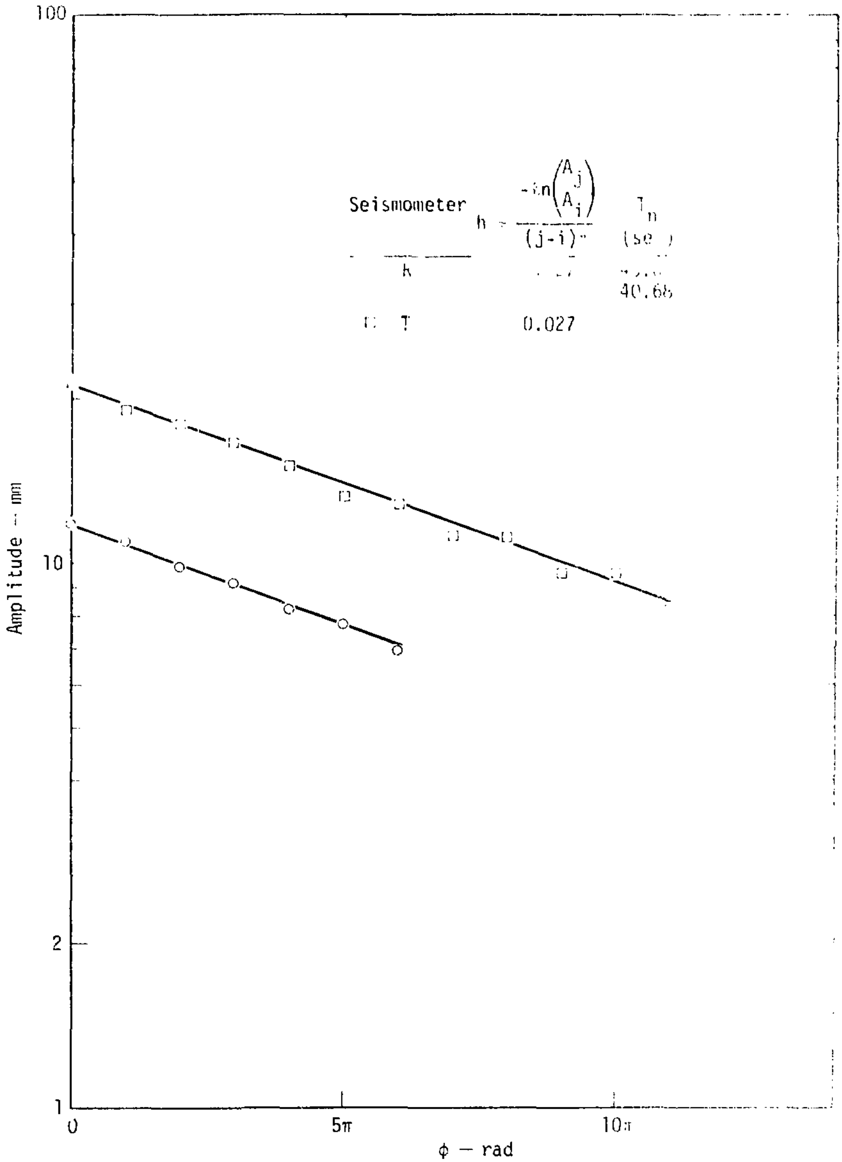


Fig. B-1. Landers station. Natural decay of amplitude while in free period.

Generator Constants

$$K = 9.8 \left(\frac{\ell_w}{\ell_c}\right) \frac{F_{wt}}{I}$$

ℓ_w = 17.0 in.

ℓ_c = 14.03 in. for signal coil

6.76 in. for cal coil

$$F_{wt} = \frac{6}{4.5}\left(\frac{M}{2}\right) = \frac{2}{3}\,m$$

Coil	m (g)	I (mA)	K(N/A)
Rt	100	7.8	101.5
Lt	100	7.67	103.3
Cal	10	32.5	5.06

Damping Resistance on Right Coil

$$(R_d + R_c)/T_n = \frac{\ell_s^2}{\ell_o \ell_m} \frac{K^2}{4\pi M(0.707 - h_o)}$$

$$= 0.00758 \frac{(101.5)^2}{0.707 - 0.027}$$

$$= 114.8\ \Omega/\text{sec}$$

FORCE BALANCE TEST

STATION Landers INSTRUMENT Tangential SN #4991

AEC LLL 216805

DATE 1-23-76

Coil Resistances

Lt Coil 560 Ω

Rt Coil 560 Ω

Cal Coil 58 Ω

Coil Inductances

Lt Coil 1.19 H

Rt Coil 1.19 H

Cal Coil 20 mH

Natural Damping

$$h_o^2 = 1 \quad \left[\left(\frac{\pi(j - i)}{\ell_n \; A_j/A_i} \right)^2 + 1 \right] \qquad j > 1$$

or for $h_o << 1$

$$h_o \approx - \frac{\ell_n \; (A_j/A_i)}{(j - i)\pi}$$

from plot (Fig. B-1) $h_o = 0.027$

Generator Constants

$$K = 9.8 \left(\frac{\ell_w}{\ell_c}\right)\frac{F_{wt}}{I}$$

ℓ_w = 17.0 in.

ℓ_c = 14.03 in. for signal coil

6.76 in. for cal coil

$$F_{wt} = \frac{6}{4.5}\left(\frac{M}{2}\right) = \frac{2}{3}\,m$$

Coil	m (g)	I (mA)	K(N/A)
Rt	100	7.83	101.1
Lt	100	7.8	101.5
Cal	10	30.8	5.33

Damping Resistance on Right Coil

$$(R_d + R_c)/T_n = \frac{\ell_s^2}{\ell_o \ell_m} \frac{K^2}{4\pi M(0.707 - h_o)}$$

$$= 0.00758 \frac{(101.1)^2}{0.707 - 0.027}$$

$$= 113.9\ \Omega/\text{sec}$$

FREQUENCY RESPONSE TEST

STATION Landers INSTRUMENT Radial SN #4986

DATE 1-28-76

System Scale Factor, $F_{ss} = X_1 X_2 X_3 X_4$

$$= (32)\left(\frac{100\ \text{Hz}}{200\ \text{mV}}\right)\left(\frac{1.25\ \text{V}}{100\ \text{Hz}}\right)\left(\frac{20\ \text{mm}}{1.25\ \text{V}}\right) = 3.2\ \frac{\text{mm}}{\text{mV}}$$

$R_{dT} = 4100\ \Omega$ $R_c = 560\ \Omega$ $h = 0.027$ $T_n = 40.0$ sec

f(Hz)	I(μA)	A(mm)	$A/F_{ss}I\left(\frac{\mu V}{\mu A}\right)$	$fA/F_{ss}I\left(\frac{Hz \cdot \mu A}{\mu A}\right)$ [a]	
0.01	200	37.1	57.97	0.58	
.02	60	20.5	106.77	2.14	
.025		22	114.58	2.8645	
.03		22	114.58	3.44	
.04		17.3	90.1	3.60	
.05		14.0	72.92	3.65	
.06		11.4	64.42	3.63	
.07		9.8	51.04	3.57	
.08		8.6	44.79	3.58	
.09		7.7	40.10	3.61	
.09	200	25.4	39.69	3.57	
.1		23	35.94	3.59	
.12		19	29.69	3.56	
.15		15.1	23.59	3.54	
.20		11.3	17.66	3.53	
.3	600	22.5	11.72	3.52	RMS= 3.503 ± 0.023 (for .3 to 1.0)
.4		16.8	8.75	3.50	
.5		13.5	7.03	3.52	
.7		9.5	4.95	3.46	
0.7	2000	32.2	5.03	3.52	
1.0		22.4	3.50	3.50	
2.0		11.0	1.72	3.44	
3.0		7.2	1.13	3.38	
4.0		5.2	0.81	3.25	

[a]These data are plotted in Fig. B-2.

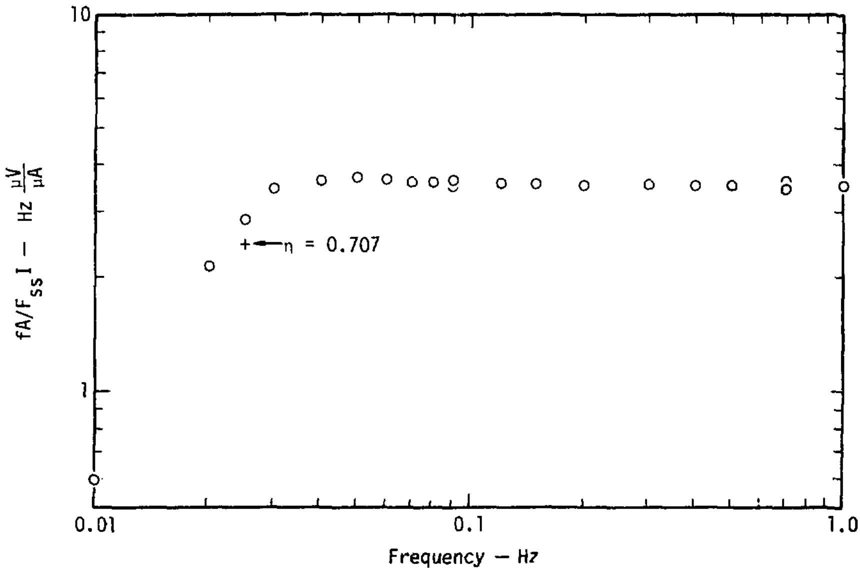


Fig. B-2. Landers radial velocity response with trial R_{dT} = 4100 Ω.

$\underline{\eta_T}$

Asymptotic value of $(fA/F_{ss}I)_{\omega >> \omega_n} = 3.503 \pm 0.023$

from plot (Fig. B-2) at f_n $(fA/F_{ss}I)_{\omega_n} = 2.8645 \pm 0.02$

$$\eta_T = \frac{1}{2} \frac{(fA/F_{ss}I)_{\omega >> \omega_n}}{(fA/F_{ss}I)_{\omega_n}} = \frac{3.503 \pm 0.023}{2(2.8645 \pm 0.02)} = 0.6115 \pm 0.008$$

$\underline{(R_d + R_c)/T_n}$

$$(R_c + R_d)/T_n = \frac{(\eta_T - h)}{(0.707 - h)} \frac{(R_c + R_{dT})}{T_n}$$

$$= \frac{(0.6115 \pm 0.008 - 0.027)}{(0.68)} \frac{(4660)}{40.0} = 100.7 \pm 1.4\ \Omega/\text{sec}$$

$\underline{K_d}$

$$K_d^2 = 4\pi M \left(\frac{\ell_o \ell_m}{\ell_s^2} \right) (\eta - h) \frac{(R_c + R_d)}{T_n}$$

from plot (Fig. 5) $K_d = 94.8 \pm 0.6$ N/A

<u>Ratio of Coils</u>

$$\frac{K_s}{K_d} = \frac{32.3}{31.7} = 1.019$$

$$K_s = 96.6 \pm 0.6 \text{ N/A}$$

Calibration Coil Constant

$$G_c = \frac{\ell_m \ell_o}{\ell_c \ell_s} \frac{M}{K_s} \frac{2\pi}{|\bar{T}_{(\omega)}|} \left(\frac{fA}{F_{ss} I}\right)_{\omega>>\omega_n}$$

$$(fA/F_{ss} I)_{\omega>>\omega_n} = 3.503 \pm 0.023$$

$$G_c = \frac{(13.2)}{6.76} \left(\frac{14.07}{14.03}\right) \frac{11.13}{(966 \pm 0.6)} 2\pi \quad (3.503 \pm 0.023)$$

$$= 4.966 \pm 0.064 \text{ N/A}$$

Calibration Constant

$$C.C. = \frac{\ell_m}{\ell_o} \frac{M}{G_c}$$

$$= \frac{13.2}{6.76} \left(\frac{11.13}{4.966 \pm 0.064}\right)$$

$$= 4.38 \pm 0.06 \frac{\text{mA}\cdot\text{sec}^2}{\text{mm}}$$

FREQUENCY RESPONSE TEST

STATION Landers INSTRUMENT Tangential SN #4991

DATE 1-28-76

System Scale Factor, $F_{ss} = X_1 X_2 X_3 X_4$

$$= (32)\left(\frac{100\ Hz}{200\ mV}\right)\left(\frac{1.25\ V}{100\ Hz}\right)\left(\frac{20\ mm}{1.25}\right) = 3.2\ \frac{mm}{mV}$$

$R_{dT} = 4050\ \Omega$ $R_c = 560\ \Omega$ $h = 0.027$ $T_n = 40.68$ sec

f(Hz)	I(μA)	A(mm)	$A/F_{ss}I\left(\frac{\mu V}{\mu A}\right)$	$fA/F_{ss}I\left(\frac{Hz \cdot \mu A}{\mu A}\right)$[a]		$f^2A/F_{ss}I\left(\frac{Hz^2 \cdot \mu V}{\mu A}\right)$[b]
0.01	200	36.9	57.66	0.577		0.00577
.02	60	20.5	106.8	2.135		.04271
.025	↓	21.8	113.56	2.839		.07096
.03	↓	20.8	108.33	3.25		.09749
.04	↓	16.9	88.021	3.521		.14083
.05	↓	13.5	70.313	3.5156		.17578
.06	↓	11.1	57.81	3.4688		.20813
.07	200	31.8	49.69	3.4781		.24347
.08	↓	27.6	43.13	3.4500		.2760
.09	↓	24.6	38.438	3.4594		.31134
.10	↓	22.0	34.375	3.4375		.34375
.12	↓	18.0	28.125	3.3750		.40500
.15	↓	14.4	22.50	3.375		.50625
.20	600	32.5	16.927	3.3854		0.67708
.30	↓	21.5	11.198	3.3594		1.0078
.4	↓	16.2	8.4375	3.375	RMS =	1.3500
.5	↓	12.9	6.7188	3.3594	3.372 ±	1.6797
.7	↓	9.3	4.8438	3.3906	0.015	2.3734
0.7	2000	31.0	4.8438	3.3906		2.3734
1.0		21.5	3.3594	3.3594		3.3594
2.0		10.6	1.6563	3.3125		6.6750
2.5		8.5	1.3281	3.3203		8.3008

[a]These data are plotted in Fig. B-3.

[b]These data are plotted in Fig. B-4.

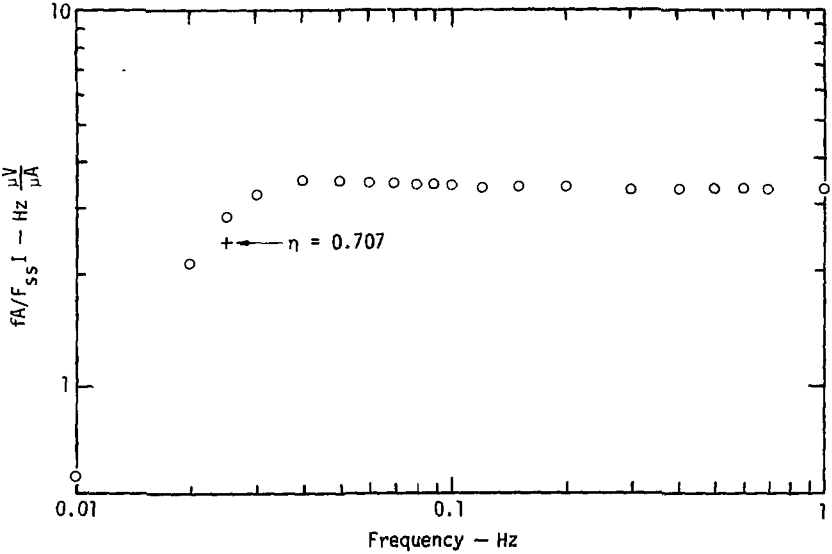


Fig. B-3. Landers tangential velocity response with trial R_{dT} = 4050 Ω.

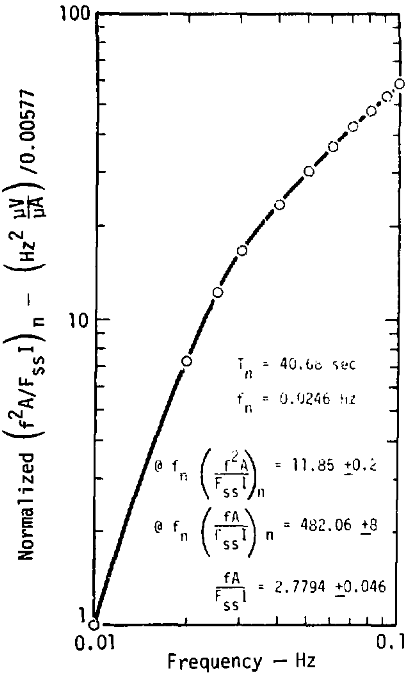


Fig. B-4. Landers tangential displacement response with trial R_{dT} = 4050 Ω.

$\underline{\eta_T}$

Asymptotic value of $(fA/F_{ss}I)_{\omega>>\omega_n} = 3.372 \pm 0.015$

from plot (Fig. B-4) @ f_n $(fA/F_{ss}I)_{\omega_n} = 2.779 \pm 0.046$

$$\eta_T = \frac{1}{2}\,\frac{(fA/F_{ss}I)_{\omega>>\omega_n}}{(fA/F_{ss}I)_{\omega_n}} = \frac{3.372 \pm 0.015}{2(2.779 \pm 0.046)} = 0.6067 \pm 0.013$$

$\underline{(R_d + R_c)/T_n}$

$$(R_c + R_d)/T_n = \frac{(\eta_T - h)}{(0.707 - h)} \quad \frac{(R_c + R_{dT})}{T_n}$$

$$= \frac{(0.6067 \pm 0.013 - 0.027)}{(0.68)} \quad \frac{(4610)}{40.68} = 96.6 \pm 2.0\ \Omega/\text{sec}$$

$\underline{K_d}$

$$K_d^2 = 4\pi M \left(\frac{\ell_o \ell_m}{\ell_s^2}\right)(\eta - h) \quad \frac{(R_c + R_d)}{T_n}$$

from plot (Fig. 5) $K_d = 93.1 \pm 1.1$ N/A

<u>Ratio of Coils</u>

$$\frac{K_s}{K_d} = \frac{31.0}{30.9} = 1.0032$$

$$K_s = 93.4 \pm 1.1 \text{ N/A}$$

Calibration Coil Constant

$$G_c = \frac{\ell_m \ell_o}{\ell_c \ell_s} \frac{M}{K_s} \frac{2\pi}{|\bar{T}_{(\omega)}|} \left(\frac{fA}{F_{ss} I}\right)_{\omega >> \omega_n}$$

$$(fA/F_{ss} I)_{\omega >> \omega_n} = 3.372 \pm 0.015$$

$$G_c = \frac{(13.2)}{6.76}\left(\frac{14.07}{14.03}\right) \frac{11.13}{(93.4 \pm 1.1)} \quad 2\pi \ (3.372 \pm 0.015)$$

$$= 4.94 \pm 0.09 \text{ N/A}$$

Calibration Constant

$$C.C. = \frac{\ell_m}{\ell_o} \frac{M}{G_c}$$

$$= \frac{13.2}{6.76} \quad \frac{11.13}{4.94 \pm 0.09}$$

$$= 4.40 \pm 0.08 \ \frac{\text{mA}\cdot\text{sec}^2}{\text{mm}}$$

Appendix C.
Mina Station Calibrations

FORCE BALANCE TEST

STATION Mina INSTRUMENT Radial SN #4987

AEC LLL 216801

DATE 3-9-76

Coil Resistances

Lt Coil 571 Ω

Rt Coil 572 Ω

Cal Coil 63 Ω

Coil Inductances

Lt Coil 1.109 H

Rt Coil 1.019 H

Cal Coil 14 mH

Natural Damping

$$h_o^2 = 1 \Big/ \left[\left(\frac{\pi(j - i)}{\ell_n\, A_j/A_i} \right)^2 + 1 \right] \qquad j > 1$$

or for $h_o << 1$

$$h_o \approx - \frac{\ell_n\, (A_j/A_i)}{(j - i)\pi}$$

from plot (Fig. C-1) $h_o = 0.0258$

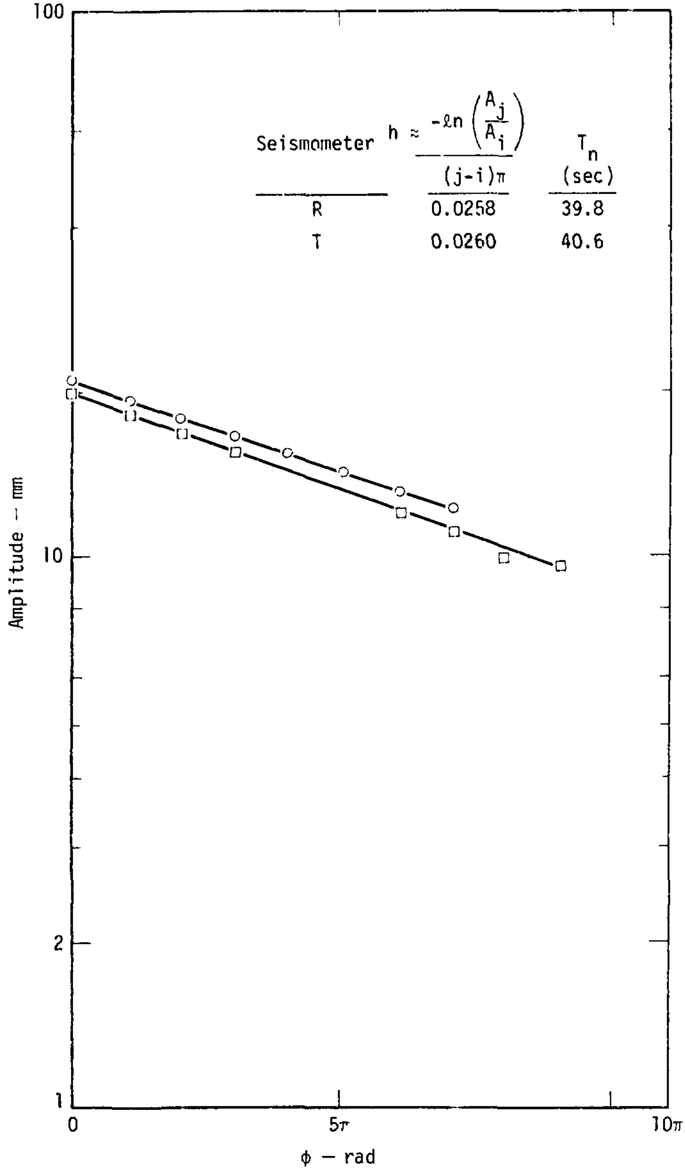


Fig. C-1. Mina station. Natural decay of amplitude while in free period.

Generator Constants

$$K = 9.8\left(\frac{\ell_w}{\ell_c}\right)\frac{F_{wt}}{I}$$

ℓ_w = 17.0 in.

ℓ_c = 14.03 in. for signal coil
6.76 in. for cal coil

$$F_{wt} = \frac{6}{4.5}\left(\frac{M}{2}\right) = \frac{2}{3}\, m$$

Coil	m (g)	I (mA)	K(N/A)
Rt	50	4.28	92.48
Lt	50	4.25	93.13
Cal	5	17.89	4.59

Damping Resistance on Left Coil

$$(R_d + R_c)/T_n = \frac{\ell_s^2}{\ell_o \ell_m}\,\frac{K^2}{4\pi M(0.707 - h_o)}$$

$$= 0.00758\ \frac{(93.13)^2}{0.707 - 0.0258}$$

$$= 95.88\ \Omega/\text{sec}$$

FORCE BALANCE TEST

STATION Mina INSTRUMENT Tangential SN #4992

AEC LLL 216806

DATE 3-9-76

Coil Resistances

Lt Coil 565 Ω

Rt Coil 565 Ω

Cal Coil 61 Ω

Coil Inductances

Lt Coil 1.048 H

Rt Coil 1.063 H

Cal Coil 13 mH

Natural Damping

$$h_o^2 = 1 \Big/ \left[\left(\frac{\pi(j - i)}{\ell_n A_j/A_i} \right)^2 + 1 \right] \qquad j > i$$

or for $h_o << 1$

$$h_o \approx - \frac{\ell_n (A_j/A_i)}{(j - i)\pi}$$

from plot (Fig. C-1) $h_o = 0.0260$

Generator Constants

$$K = 9.8\left(\frac{\ell_w}{\ell_c}\right)\frac{F_{wt}}{I}$$

ℓ_w = 17.0 in.

ℓ_c = 14.03 in. for signal coil
6.76 in. for cal coil

$$F_{wt} = \frac{6}{4.5}\left(\frac{M}{2}\right) = \frac{2}{3}\,m$$

Coil	m (g)	I (mA)	K(N/A)
Rt	50	4.08	97.01
Lt	50	4.15	95.38
Cal	5	16.1	5.09

Damping Resistance on Right Coil

$$(R_d + R_c)/T_n = \frac{\ell_s^2}{\ell_o \ell_m}\,\frac{K^2}{4\pi M(0.707 - h_o)}$$

$$= 0.00758\ \frac{(97.01)^2}{0.707 - 0.026}$$

$$= 104.83\ \Omega/\text{sec}$$

FREQUENCY RESPONSE TEST

STATION Mina   INSTRUMENT Radial   SN #4987

DATE 3-11-76

System Scale Factor, $F_{ss} = X_1 X_2 X_3 X_4$

$$= (256)\left(\frac{100\ \text{Hz}}{5\ \text{V}}\right)\left(\frac{1.25\ \text{V}}{100\ \text{Hz}}\right)\left(\frac{20\ \text{mm}}{1.25\ \text{V}}\right) = 1.024\ \frac{\text{mm}}{\text{mV}}$$

$R_{dT} = 3245\ \Omega$ $R_c = 571\ \Omega$ $h = 0.0258$ $T_n = 39.8$ sec

f(Hz)	I(μA)	A(mm)	$A/F_{ss}I\left(\frac{\mu V}{\mu A}\right)$	$fA/F_{ss}I\left(\frac{Hz\cdot\mu A}{\mu A}\right)$[a]		$f^2A/F_{ss}I\left(\frac{Hz^2\cdot\mu V}{\mu A}\right)$
0.01	600	33.4	54.362	0.54362		0.0054362
.015	200	15.9	77.637	1.16455		.017468
.02	↓	19	92.7734	1.85547		.0371094
.025		19.9	97.168	2.4292		.06073
.03		19.7	96.1914	2.88574		.0865723
.04		16.7	81.543	3.26172		.130469
.05		13.8	67.3828	3.36914		.168457
.06		11.7	57.1289	3.42773		.205664
.06	600	35.1	57.1289	3.42773		.205664
.07	↓	30.3	49.3164	3.45215		.24165
.08		26.6	43.2943	3.46354		.27708
.09		23.7	38.5742	3.47168		.312451
.1		21.2	34.5052	3.45052		.345052
.15		14.3	23.2747	3.49121		.[illegible]3682
.2		10.7	17.4154	3.48307		.696615
.2	2000	35.1	17.1387	3.42773		0.685547
.3	↓	23.6	11.5234	3.45703		1.03711
.4		17.7	8.64258	3.45703		1.38281
.5		14.1	6.88477	3.44238	RMS =	1.72119
.7		10.1	4.93164	3.45215	3.465 ±	2.41656
0.7	6000	30.8	5.01302	3.50911	0.026	2.45638
1.0		21.6	3.51563	3.51563		3.51563
2.0		10.7	1.74154	3.48307		6.96615
3.0		7.1	1.15560	3.46680		10.4004
5.0		4.2	0.683594	3.41797		17.0898

[a] These data are plotted in Fig. C-2.

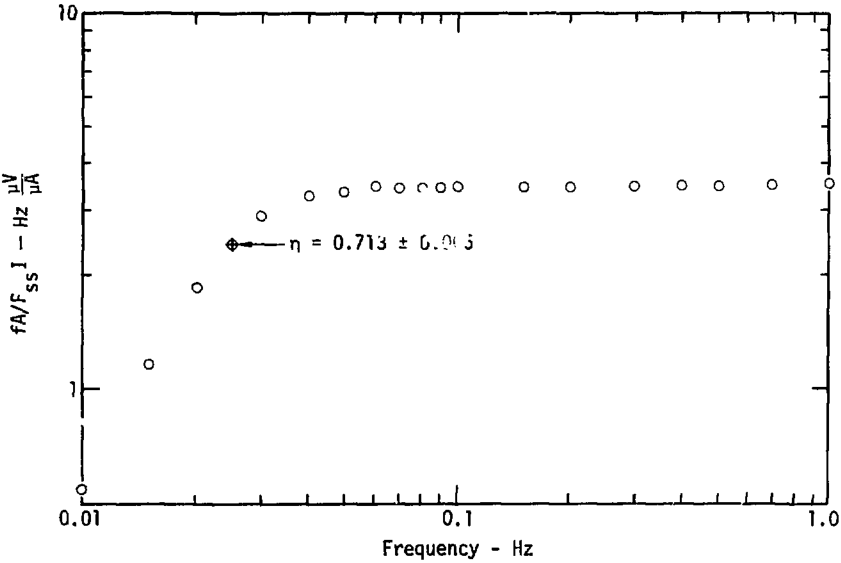


Fig. C-2. Mina radial velocity response with trial R_{dT} = 3245 Ω. T_n = 39.8 sec.

$\underline{\eta_T}$

Asymptotic value of $(fA/F_{ss}I)_{\omega>>\omega_n}$ = 3.465 ± 0.026

from plot (Fig. C-2) @ f_n $(fA/F_{ss}I)_{\omega_n}$ = 2.4292 ± 0.03

$$\eta_T = \frac{1}{2}\frac{(fA/F_{ss}I)_{\omega>>\omega_n}}{(fA/F_{ss}I)_{\omega_n}} = \frac{3.465 \pm 0.026}{2(2.4292 \pm 0.03)} = 0.713 \pm 0.015$$

$\underline{(R_d + R_c)/T_n}$

$$(R_c + R_d)/T_n = \frac{(\eta_T - h)}{(0.707 - h)} \quad \frac{(R_c + R_{dT})}{T_n}$$

$$= \frac{(0.713 \pm 0.015 - 0.0258)}{(0.707 - 0.0258)} \quad \frac{(3816)}{39.8} = 96.73 \pm 2.0$$

$\underline{K_d}$

$$K_d^2 = 4\pi M \left(\frac{\ell_o \ell_m}{\ell_s^2}\right)(\eta - h) \quad \frac{(R_c + R_d)}{T_n}$$

from plot (Fig. 5) K_d = 93.25 ± 1.0 N/A

Ratio of Coils

$$\frac{K_s}{K_d} = \frac{30.8}{31.0} = 0.9935$$

$$K_s = 92.65 \pm 1.0 \text{ N/A}$$

Calibration Coil Constant

$$G_c = \frac{\ell_m \ell_o}{\ell_c \ell_s} \frac{M}{K_s} \frac{2\pi}{|\bar{T}_{(\omega)}|} \left(\frac{fA}{F_{ss} I}\right)_{\omega >> \omega_n}$$

$$(fA/F_{ss}I)_{\omega >> \omega_n} = 3.465 \pm 0.026$$

$$G_c = \frac{(13.2)}{6.76} \left(\frac{14.07}{14.03}\right) \frac{11.13}{(92.65 \pm 1.0)} \; 2\pi \quad (3.465 \pm 0.026)$$

$$= 5.12 \pm 0.1 \text{ N/A}$$

Calibration Constant

$$\text{C.C.} = \frac{\ell_m}{\ell_o} \frac{M}{G_c}$$

$$= \frac{13.2}{6.76} \left(\frac{11.13}{5.12 \pm 0.1}\right)$$

$$= 4.245 \pm 0.084 \; \frac{\text{mA} \cdot \text{sec}^2}{\text{mm}}$$

FREQUENCY RESPONSE TEST

STATION Mina INSTRUMENT Tangential SN #4992

DATE 3-11-76

System Scale Factor, $F_{ss} = X_1\ X_2\ X_3\ X_4$

$$= (256)\left(\frac{100\ \text{Hz}}{5\ \text{V}}\right)\left(\frac{1.25\ \text{V}}{100\ \text{Hz}}\right)\left(\frac{20\ \text{mm}}{1.25\ \text{V}}\right) = 1.024\ \frac{\text{mm}}{\text{mV}}$$

R_{dT} =3690 Ω R_c = 565 Ω h = 0.026 T_n = 40.6 sec

f(Hz)	I(μA)	A(mm)	$A/F_{ss}I\left(\frac{\mu V}{\mu A}\right)$	$fA/F_{ss}I\left(\frac{Hz \cdot \mu A}{\mu A}\right)$ [a]		$f^2A/F_{ss}I\left(\frac{Hz^2 \cdot \mu V}{\mu A}\right)$ [b]
0.01	600	35.9	58.43	0.5843		0.0058431
.015	200	17.1	83.496	1.2524		.018787
.02		20.1	98.145	1.9629		.039258
.025		21.0	102.54	2.5635		.064087
.03		21.3	104.400	3.1201		.093604
.04		17.3	84.473	3.3789		.13516
.05		14.5	70.801	3.5400		.17700
.06		12.2	59.570	3.5742		.21445
.06	600	36.1	58.757	3.5254		.2115
.07		31.0	50.456	3.5319		.24723
.08		27.2	44.271	3.5412		
.09		24.3	39.551	3.5596		
.10		22.0	35.807	3.5807		.35807
.15		14.6	23.763	3.5645		.53467
.2		11.2	18.229	3.6458		.72917
.2	2000	36.2	17.676	3.5352		0.70703
.3		24.2	11.816	3.5449		1.0635
.4		18.2	8.8867	3.5547	RMS =	1.4219
.5		14.5	7.0801	3.5400	3.561 ±	1.7700
.7		10.5	5.1270	3.5889	0.046	2.5122
0.7	6000	31.7	5.1595	3.6117		2.5282
1.0		22.3	3.6296	3.6296		3.6292
2.0		11.1	1.8066	3.6133		7.7766
3.0	20,000	24.2	1.1816	315449		
5.0		14.1	0.68848	3.4424		

[a]These data are plotted in Fig. C-3.

[b]These data are plotted in Fig. C-4.

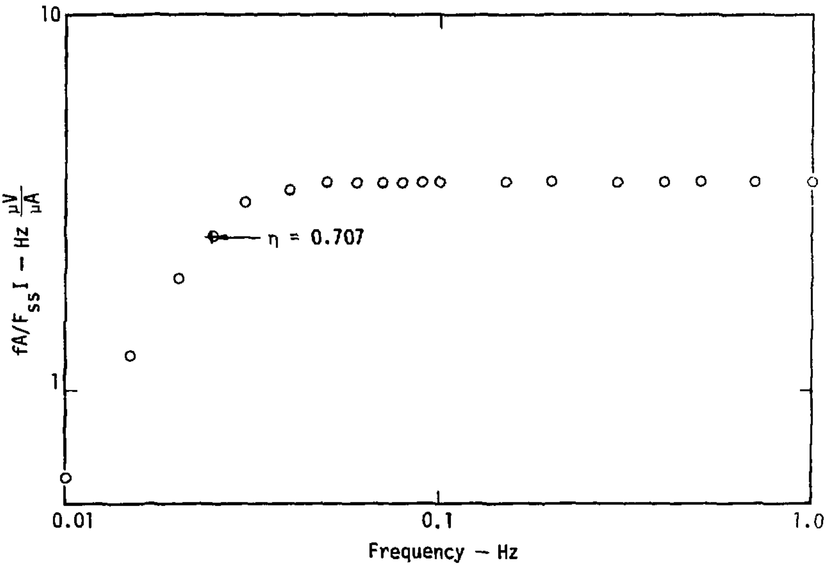


Fig. C-3. Mina tangential velocity response with trial R_{dT} = 3690 Ω. T_n = 40.6 sec.

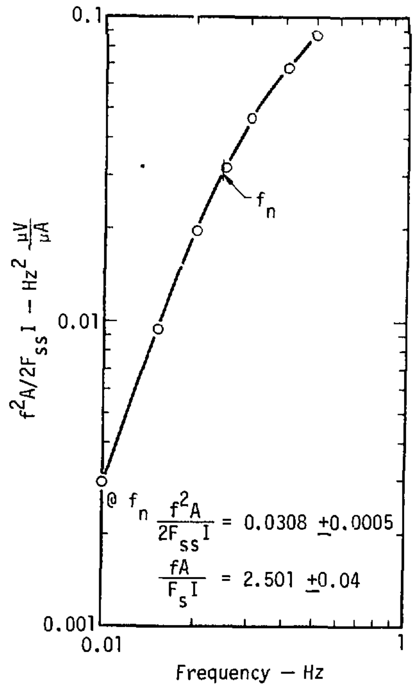


Fig. C-4. Mina tangential displacement response with trial R_{dT} = 3696 Ω.

$\underline{\eta_T}$

Asymptotic value of $(fA/F_{ss}I)_{\omega>>\omega_n} = 3.561 \pm 0.046$

from plot (Fig. C-4) @ f_n $(fA/F_{ss}I)_{\omega_n} = 2.501 \pm 0.04$

$$\eta_T = \frac{1}{2}\,\frac{(fA/F_{ss}I)_{\omega>>\omega_n}}{(fA/F_{ss}I)_{\omega_n}} = \frac{3.561 \pm 0.046}{2(2.5 \pm 0.04)} = 0.711 \pm 0.025$$

$\underline{(R_d + R_c)/T_n}$

$$(R_c + R_d)/T_n = \frac{(\eta_T - h)}{(0.707 - h)} \quad \frac{(R_c + R_{dT})}{T_n}$$

$$= \frac{(0.711 \pm 0.025 - 0.026)}{(0.681)} \left(\frac{4255}{40.6}\right) = 105.4 \pm 3.8\ \Omega/\text{sec}$$

$\underline{K_d}$

$$K_d^2 = 4\pi M \left(\frac{\ell_o \ell_m}{\ell_s^2}\right) (\eta - h) \quad \frac{(R_c + R_d)}{T_n}$$

from plot (Fig. 5) $K_d = 97.3 \pm 1.9$ N/A

<u>Ratio of Coils</u>

$$\frac{K_s}{K_d} = \frac{35.2}{36.2} = 0.972$$

$$K_s = 94.6 \pm 1.9 \text{ N/A}$$

Calibration Coil Constant

$$G_c = \frac{\ell_m \ell_o}{\ell_c \ell_s} \frac{M}{K_s} \frac{2\pi}{|\bar{T}_{(\omega)}|} \left(\frac{fA}{F_{ss} I}\right)_{\omega >> \omega_n}$$

$$(fA/F_{ss}I)_{\omega >> \omega_n} = 3.561 \pm 0.046$$

$$G_c = \frac{(13.2)}{6.76} \left(\frac{14.07}{14.03}\right) \frac{11.13}{(94.6 \pm 1.9)} \quad 2\pi \quad (3.561 \pm 0.046$$

$$= 5.15 \pm 0.17 \text{ N/A}$$

Calibration Constant

$$\text{C.C.} = \frac{\ell_m}{\ell_o} \frac{M}{G_c}$$

$$= \frac{13.2}{6.76} \left(\frac{11.13}{5.15 \pm 0.17}\right)$$

$$= 4.220 \pm 0.14 \frac{\text{mA} \cdot \text{sec}^2}{\text{mm}}$$

Appendix D.
Kanab Station Calibration

FORCE BALANCE TEST

STATION Kanab — INSTRUMENT Radial — SN #4993

AEC LLL 216807

Date 4-18-76

Coil Resistances

Lt Coil 584 Ω

Rt Coil 580 Ω

Cal Coil 64.2 Ω

Coil Inductances

Lt Coil 1.3 H

Rt Coil 1.3 H

Cal Coil 23 mH

Natural Damping

$$h_o^2 = 1 \Big/ \left[\left(\frac{\pi(j - i)}{\ell_n \; A_j/A_i} \right)^2 + 1 \right] \qquad j > 1$$

or for $h_o \ll 1$

$$h_o \approx - \frac{\ell_n \; (A_j/A_i)}{(j - i)\pi}$$

from plot (Fig. D-1) $h_o = 0.0285$

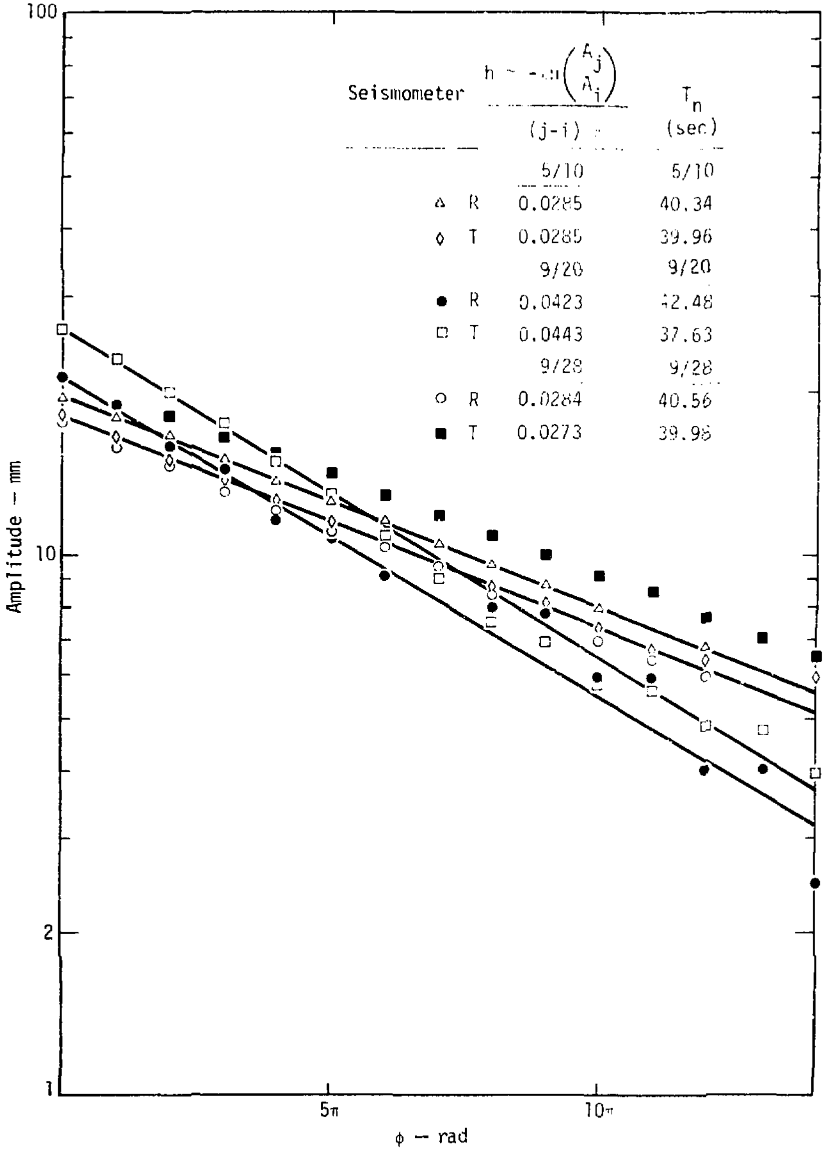


Fig. D-1. Kanab station. Natural decay in amplitude while in free period.

Generator Constants

$$K = 9.8\left(\frac{\ell_w}{\ell_c}\right)\frac{F_{wt}}{I}$$

ℓ_w = 17.0 in.

ℓ_c = 14.03 in. for signal coil

6.76 in. for cal coil

$$F_{wt} = \frac{6}{4.5}\left(\frac{M}{2}\right) = \frac{2}{3}\,m$$

Coil	m (g)	I (mA)	K(N/A)
Rt	50	4.12	96.07
Lt	50	4.1	96.54
Cal	5	15.8	5.2

Damping Resistance on Left Coil

$$(R_d + R_c)/T_n = \frac{\ell_s^2}{\ell_o \ell_m}\,\frac{K^2}{4\pi M(0.707 - h_o)}$$

$$= 0.00758 \quad \frac{(96.54)^2}{0.707 - 0.0285}$$

$$= 103.97\ \Omega/\text{sec}$$

FORCE BALANCE TEST

STATION Kanab | INSTRUMENT Tangential | SN #4988

AEC LLL 216802

DATE 4-18-76

Coil Resistances

Lt Coil 578 Ω

Rt Coil 580 Ω

Cal Coil 63.9 Ω

Coil Inductances

Lt Coil 1.3 H

Rt Coil 1.4 H

Cal Coil 22 mH

Natural Damping

$$h_o^2 = 1 \Big/ \left[\left(\frac{\pi(j - i)}{\ell_n A_j/A_i}\right)^2 + 1\right] \qquad j > 1$$

or for $h_o \ll 1$

$$h_o \approx -\frac{\ell_n (A_j/A_i)}{(j - i)\pi}$$

from plot $h_o = 0.0285$

Generator Constants

$$K = 9.8\left(\frac{\ell_w}{\ell_c}\right)\frac{F_{wt}}{I}$$

ℓ_w = 17.0 in.

ℓ_c = 14.03 in. for signal coil
6.76 in. for cal coil

$$F_{wt} = \frac{6}{4.5}\left(\frac{M}{2}\right) = \frac{2}{3}\,m$$

Coil	m (g)	I (mA)	K(N/A)
Rt	50	4.13	95.85
Lt	50	4.1	96.54
Cal	5	15.75	5.22

Damping Resistance on Left Coil

$$(R_d + R_c)/T_n = \frac{\ell_s^2}{\ell_o \ell_m}\,\frac{K^2}{4\pi M(0.707 - h_o)}$$

$$= 0.00758\,\frac{(96.54^2)}{0.707 - 0.0258}$$

$$= 104.6\ \Omega/\text{sec}$$

FREQUENCY RESPONSE TEST

STATION Kanab INSTRUMENT Radial SN #4993

AEC LLL 216807

DATE 9-2-76

System Scale Factor, $F_{ss} = X_1 \; X_2 \; X_3 \; X_4$

$$= (X_1)\left(\frac{100\ \text{Hz}}{5}\right)\left(\frac{1.25\ \text{V}}{100\ \text{Hz}}\right)\left(\frac{20\ \text{mm}}{1.25\ \text{V}}\right) = 4X_1 \times 10^{-3}\ \frac{\text{mm}}{\text{mV}}$$

$R_{dT} = 3300\ \Omega$ $R_c = 584\ \Omega$ $h = 0.0258$ $T_n = 42.48$ sec

X_1	f(Hz)	I(μA)	A(mm)	$A/F_{ss}I\left(\frac{\mu V}{\mu A}\right)$	$fA/F_{ss}I\left(\frac{Hz \cdot \mu A}{\mu A}\right)$ [a]	$f^2A/F_{ss}I\left(\frac{Hz^2 \cdot \mu V}{\mu A}\right)$ [b]
128	0.01	600	18.6	60.55	0.6055	0.006055
	.015		25.5	83.008	1.2451	.01868
	.020		29.2	95.052	1.901	.03802
	.024		30.0	97.656	2.344	.05626
	.025		30.0	97.656	2.441	.06103
	.026		29.8	97.005	2.522	.06557
	.030		28.8	93.75	2.813	.08439
	.04		24.7	80.40	3.216	.1286
	.05		20.7	67.38	3.369	0.1685
	.06		18.0	58.594	3.5156	
	.07		15.6	50.781	3.5547	
	.08		13.7	44.596	3.5677	
	.09		12.2	39.714	3.5742	
	.10		11.0	35.807	3.5807	
	.10	2000	34.6	33.789	3.3789	
	.15		23.1	22.559	3.3838	
	.20		17.5	17.09	3.418	
	.30		11.6	11.328	3.398	
	.40		8.8	8.5938	3.4375	
	.40	6000	21.1	6.8685	2.7474	
	.50		16.7	5.4362	2.7181	
512	.50	2000	28.0	6.8359	3.418	
	0.60		23.3	5.6885	3.413	

[a]These data are plotted in Fig. D-2.

[b]These data are plotted in Fig. D-3.

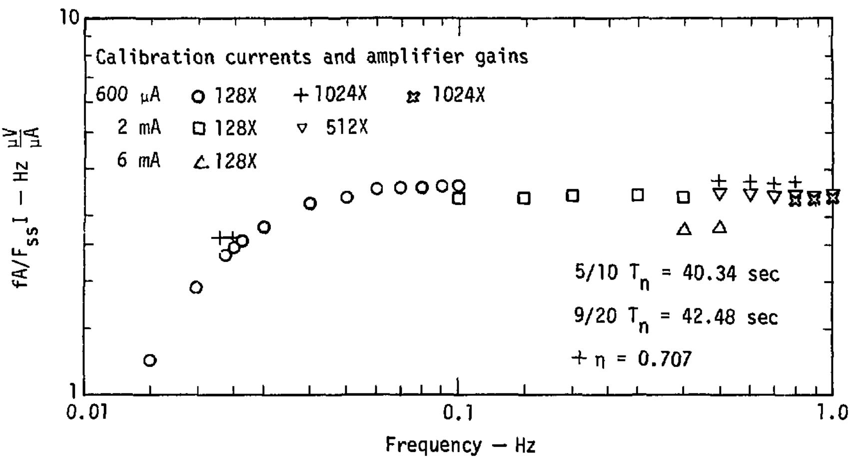


Fig. D-2. Kanab radial velocity response with trial R_{dT} = 3300 Ω.

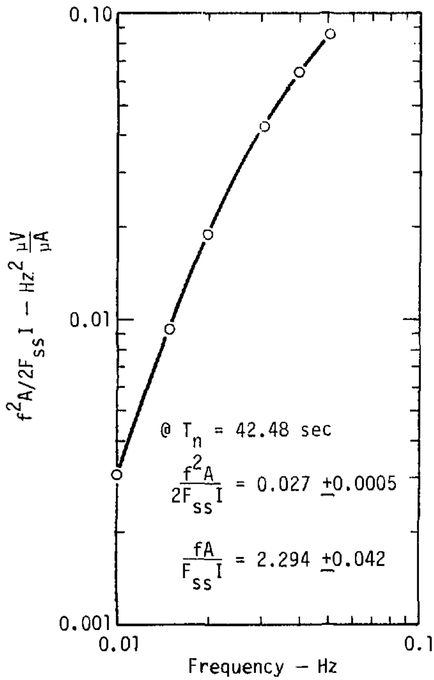


Fig. D-3. Kanab radial displacement response with trial R_{dT} = 3300 Ω.

X_1[a]	f(Hz)	I(μA)	A(mm)	$A/F_{ss}I$	$fA/F_{ss}I$	
512	0.7	2000	20.0	4.8828	3.418	
	0.8	↓	17.5	4.2725	3.418	
	0.9		15.5	3.7842	3.406	
	1.0		14.0	3.418	3.418	
	1.5		13.3	3.247	4.87	
	2.0		6.8	1.6602	3.3203	
1024	0.5	600	17.8	7.2428	3.6214	
	.6	↓	14.8	6.0221	3.6133	
	.7		12.6	5.1270	3.5889	RMS =
	.8		11.1	4.5166	3.6133	3.609 ±
	.8	2000	34.8	4.2480	3.3984	0.014
	0.9		31.2	3.8086	3.4277	
	1.0		28.0	3.418	3.4180	
	1.5		18.6	2.2705	3.4058	
	2.0		14.0	1.7090	3.418	
	3.0		9.3	1.1353	3.4058	
	4.0		6.8	0.8178	3.2715	

[a]The currents greater than 1 mA were incorrect by some unknown amount. Therefore, the additional data was taken to confirm this supposition and to get reliable data at the higher frequencies, i.e., 0.5 to 0.8 Hz.

$\underline{\eta_T}$

Asymptotic value of $(fA/F_{ss}I)_{\omega>>\omega_n} = 3.609 \pm 0.014$

from plot (Fig. D-3) @ f_n $(fA/F_{ss}I)_{\omega_n} = 2.294 \pm 0.042$

$$\eta_T = \frac{1}{2}\,\frac{(fA/F_{ss}I)_{\omega>>\omega_n}}{(fA/F_{ss}I)_{\omega_n}} = \frac{3.609 \pm 0.014}{2(2.294 \pm 0.042)} = 0.7866 \pm 0.018$$

$\underline{(R_d + R_c)/T_n}$

$$(R_c + R_d)/T_n = \frac{(\eta_T - h)}{(0.707 - h)} \quad \frac{(R_c + R_{dT})}{T_n}$$

$$= \frac{(0.7866 \pm 0.018 - 0.0284)}{(0.707 - 0.0284)} \left(\frac{3884}{42.48}\right) = 102.2 \pm 2.4$$

$\underline{K_d}$

$$K_d^2 = 4\pi M \left(\frac{\ell_o \ell_m}{\ell_s^2}\right) (\eta - h) \quad \frac{(R_c + R_d)}{T_n}$$

from plot (Fig. 5) $K_d = 95.63 \pm 1.12$ N/A

Ratio of Coils

$$\frac{K_s}{K_d} = \frac{35.3}{35.5} = 0.994$$

$$K_s = 95.09 \pm 1.1 \text{ N/A}$$

Calibration Coil Constant

$$G_c = \frac{\ell_m \ell_o}{\ell_c \ell_s} \frac{M}{K_s} \frac{2\pi}{|\bar{T}_{(\omega)}|} \left(\frac{fA}{F_{ss} I}\right)_{\omega >> \omega_n}$$

$$(fA/F_{ss}I)_{\omega >> \omega_n} = 3.609 \pm 0.014$$

$$G_c = \frac{(13.2)}{6.76} \left(\frac{14.07}{14.03}\right) \frac{11.13}{(95.09 \pm 1.1)} 2\pi \quad (3.609 \pm 0.014)$$

$$= 5.20 \pm 0.08 \text{ N/A}$$

Calibration Constant

$$C.C. = \frac{\ell_m}{\ell_o} \frac{M}{G_c}$$

$$= \frac{13.2}{6.76} \left(\frac{11.13}{5.19 \pm 0.08}\right)$$

$$= 4.181 \pm 0.055 \quad \frac{\text{mA} \cdot \text{sec}^2}{\text{mm}}$$

FREQUENCY RESPONSE TEST

STATION Kanab — INSTRUMENT Tangential — SN #4988

Date 9-2-76

System Scale Factor, $F_{ss} = X_1 X_2 X_3 X_4$

$$= (X_1)\left(\frac{100\ \text{Hz}}{5\text{V}}\right)\left(\frac{1.25\ \text{V}}{100\ \text{Hz}}\right)\left(\frac{20\ \text{mm}}{1.25\ \text{V}}\right) = 4\ X_1 \times 10^{-3}\ \frac{\text{mm}}{\text{mV}}$$

$R_{dT} = 3300\ \Omega$ $R_c = 578\ \Omega$ $h = 0.0285$ $T_n = 37.63$ sec

X_1	f(Hz)	I(μA)	A(mm)	$A/F_{ss}I\left(\frac{\mu V}{\mu A}\right)$	$fA/F_{ss}I\left(\frac{Hz\cdot\mu A}{\mu A}\right)$ [a]	$f^2A/F_{ss}I\left(\frac{Hz^2\cdot\mu V}{\mu A}\right)$ [b]
128	0.01	600	16.9	55.013	0.5501	0.00550
	.015		24.5	79.7526	1.1963	.01794
	.02		30.1	97.9818	1.9596	.03919
	.023		31.7	103.19	2.37337	.05459
	.025		32.4	105.469	2.6367	.06592
	.027		32.6	106.12	2.8652	.07736
	.030		32.2	104.818	3.1445	.09434
	.04		28.0	91.1458	3.6458	.1458
	.05		23.3	75.8464	3.7923	.1896
	.06		19.7	64.1276	3.8477	.2309
	.07		17.0	55.339	3.8737	.2712
	.08		15.2	49.479	3.9583	.3167
	.09		13.4	43.6198	3.9258	0.3533
	.10		11.7	38.0859	3.809	
	.13		9.3	30.27	3.936	
	.13	2000	28.6	27.9297	3.6309	
	.16		23.1	22.5586	3.6094	
	.20		18.8	18.3594	3.6719	
	.30		12.5	12.207	3.662	
256	.30		25.0	12.207	3.662	
	.40		18.7	9.131	3.652	
	.50		15.0	7.324	3.662	
512	.50		29.8	7.2754	3.638	
	.60		24.7	6.03	3.618	
	.7		21.4	5.225	3.657	
	0.8		18.7	4.565	3.652	

[a] These data are plotted in Fig. D-4.

[b] These data are plotted in Fig. D-5.

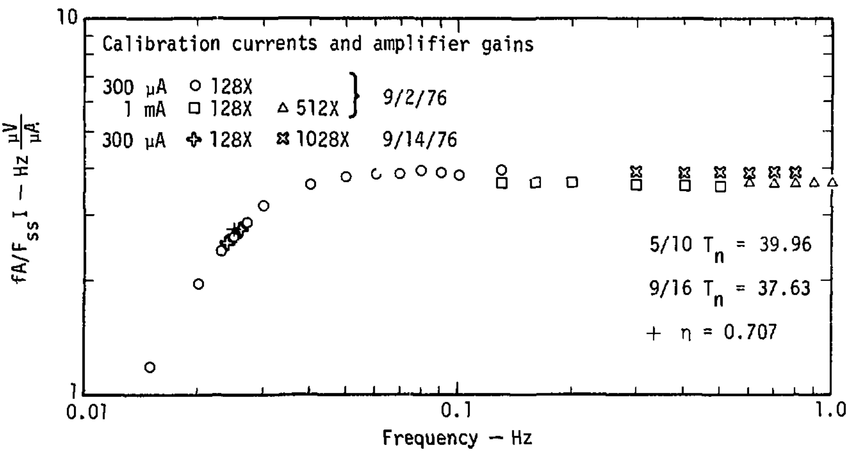


Fig. D-4. Kanab tangential velocity response with trial R_{dT} = 3300 Ω.

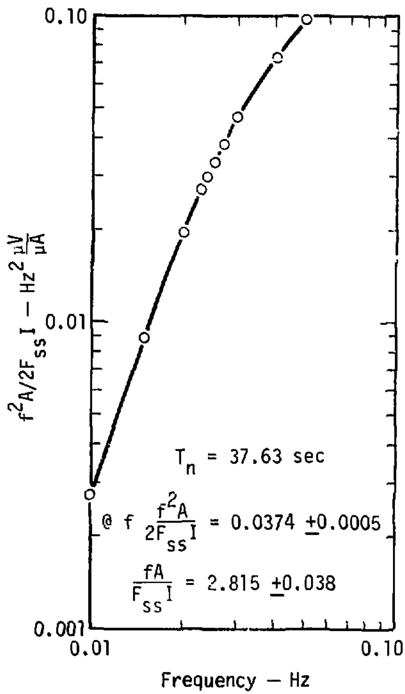


Fig. D-5. Kanab tangential displacement response with trial R_{dT} = 3300 Ω.

X_1[a]	f	F_{ss}	I(μA)	A(mm)	$A/F_{ss}I$	$fA/F_{ss}I$
512	0.900	0.002008	2000	16.6	4.0527	3.6475
↓	1.0	↓	↓	15.0	3.662	3.662
↓	1.3	↓	↓	11.4	2.7832	3.618
↓	1.6	↓	↓	9.2		3.594
↓	2.0	↓	↓	7.3	1.782	3.564
↓	2.0	↓	600	2.4		
↓	2.0	↓	2000	7.8	1.9043	3.808
1024	2.0	0.004096	↓	14.9	1.818	3.638
↓	3	↓	↓	9.8	1.1963	3.589
↓	4	↓	↓	7.1		3.467
↓	5	↓	↓	5.7		3.479

9/14/76

X_1	f	F_{ss}	I(μA)	A(mm)	$A/F_{ss}I$	$fA/F_{ss}I$	$f^2A/F_{ss}I$
128	0.024	0.000512	600	32.1	104.49	2.508	0.06019
↓	.025	↓	↓	32.4	105.47	2.637	.6593
↓	.026	↓	↓	32.5	105.8	2.751	0.07153
1028	.3	0.004096	600	31.7	12.90	3.870	
↓	.4	↓	↓	23.6	9.603	3.841	
↓	.5	↓	↓	19.0	7.731	3.866	RMS =
↓	.6	↓	↓	15.9	6.470	3.882	3.863 ±
↓	.7	↓	↓	13.5	5.493	3.845	0.016
↓	0.8	↓	↓	11.9	4.842	3.874	

[a]The currents greater than 1 mA were incorrect by some unknown amount. Therefore, the additional data was taken to confirm the supposition and to get reliable data at the higher frequencies, i.e., 0.5 to 0.8 Hz.

$\underline{\eta_T}$

Asymptotic value of $(fA/F_{ss}I)_{\omega>>\omega_n} = 3.863 \pm 0.016$

from plot (Fig. D-5) @ f_n $(fA/F_{ss}I)_{\omega_n} = 2.815 \pm 0.038$

$$\eta_T = \frac{1}{2}\,\frac{(fA/F_{ss}I)_{\omega>>\omega_n}}{(fA/F_{ss}I)_{\omega_n}} = \frac{3.863 \pm 0.016}{2(2.815 \pm 0.038)} = 0.6861 \pm 0.012$$

$\underline{(R_d + R_c)/T_n}$

$$(R_c + R_d)/T_n = \frac{(\eta_T - h)}{(0.707 - h)} \quad \frac{(R_c + R_{dT})}{T_n}$$

$$= \frac{(0.6861 \pm 0.012 - 0.0273)}{(0.707 - 0.0273)} \quad \frac{(3878)}{37.63} = 99.89 \pm 1.8\ \Omega/\text{sec}$$

$\underline{K_d}$

$$K_d^2 = 4\ M\left(\frac{\ell_o \ell_m}{\ell_s^2}\right)(\eta - h) \quad \frac{(R_c + R_d)}{T_n}$$

from plot (Fig. 5) $K_d = 94.65 \pm 0.9$ N/A

Ratio of Coils

$$\frac{K_s}{K_d} = \frac{36.2}{36.1} = 1.003$$

$$K_s = 94.91 \pm 0.9 \text{ N/A}$$

Calibration Coil Constant

$$G_c = \frac{\ell_m \ell_o}{\ell_c \ell_s} \frac{M}{K_s} \frac{2\pi}{|\bar{T}_{(\omega)}|} \left(\frac{fA}{F_{ss}I}\right)_{\omega >> \omega_n}$$

$$(fA/F_{ss}I)_{\omega >> \omega_n} = 3.863 \pm 0.016$$

$$G_c = \frac{(13.2)}{6.76} \left(\frac{14.07}{14.03}\right) \frac{11.13}{(94.91 \pm 0.9)} \quad 2\pi \quad (3.863 \pm 0.016)$$

$$= 5.57 \pm 0.07 \text{ N/A}$$

Calibration Constant

$$C.C. = \frac{\ell_m}{\ell_o} \frac{M}{G_c}$$

$$= \frac{13.2}{6.76} \left(\frac{11.13}{5.57 \pm 0.07}\right)$$

$$= 3.899 \pm 0.07 \frac{mA \cdot sec^2}{mm}$$

Appendix E.
Elko Station Calibration

FORCE BALANCE TEST

STATION Elko INSTRUMENT Radial SN #4989

AEC LLL 216803

DATE 4-18-76

Coil Resistances

Lt Coil 580 Ω

Rt Coil 581 Ω

Cal Coil 63.7 Ω

Coil Inductances

Lt Coil 1.3 H

Rt Coil 1.3 H

Cal Coil 23 mH

Natural Damping

$$h_o^2 = 1 \Big/ \left[\left(\frac{\pi(j - i)}{\ell_n A_j/A_i} \right)^2 + 1 \right] \qquad j > 1$$

or for $h_o << 1$

$$h_o \approx - \frac{\ell_n (A_j/A_i)}{(j - i)\pi}$$

from plot (Fig. E-1) $h_o = 0.024$

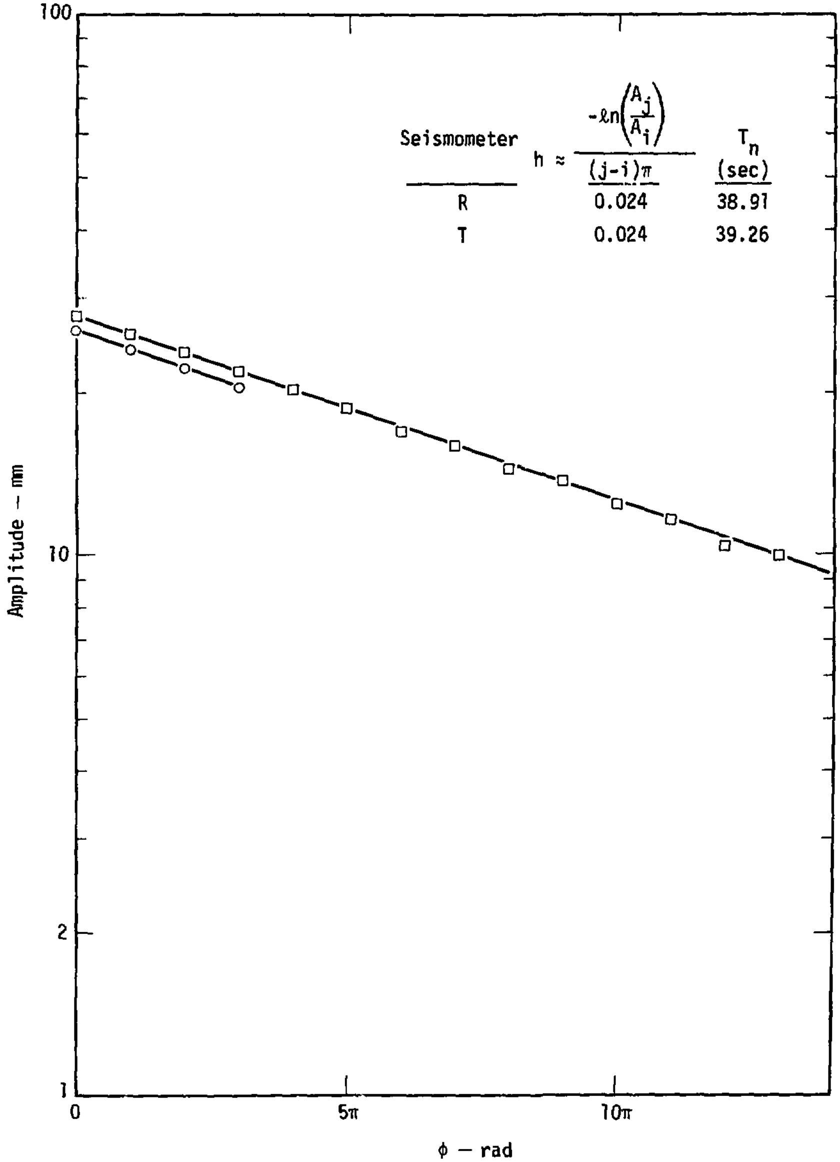


Fig. E-1. Elko station. Natural decay in amplitude while in free period.

Generator Constants

$$K = 9.8 \left(\frac{\ell_w}{\ell_c} \right) \frac{F_{wt}}{I}$$

ℓ_w = 17.0 in.

ℓ_c = 14.03 in. for signal coil
6.76 in. for cal coil

$$F_{wt} = \frac{6}{4.5} \left(\frac{M}{2} \right) = \frac{2}{3} m$$

Coil	m (g)	I (mA)	K(N/A)
Rt	50	4.21	94.02
Lt	50	4.24	93.35
Cal	5	15.8	5.2

Damping Resistance on Left Coil

$$(R_d + R_c)/T_n = \frac{\ell_s^2}{\ell_o \ell_m} \frac{K^2}{4\pi M(0.707 - h_o)}$$

$$= 0.00758 \frac{(93.35)^2}{0.707 - 0.024}$$

$$= 94.94 \ \Omega/\text{sec}$$

FORCE BALANCE TEST

STATION Elko | INSTRUMENT Tangential | SN #4994

AEC LLL 216808

DATE 4-18-76

Coil Resistances

Lt Coil 578 Ω

Rt Coil 572 Ω

Cal Coil 63.5 Ω

Coil Inductances

Lt Coil 1.3 H

Rt Coil 1.3 H

Cal Coil 23 mH

Natural Damping

$$h_o^2 = 1 \Big/ \left[\left(\frac{\pi(j - i)}{\ell_n A_j/A_i} \right)^2 + 1 \right] \qquad j > 1$$

or for $h_o \ll 1$

$$h_o \approx - \frac{\ell_n (A_j/A_i)}{(j - i)\pi}$$

from plot (Fig. E-1) $h_o = 0.024$

Generator Constants

$$K = 9.8\left(\frac{\ell_w}{\ell_c}\right)\frac{F_{wt}}{I}$$

ℓ_w = 17.0 in.

ℓ_c = 14.03 in. for signal coil
6.76 in. for cal coil

$$F_{wt} = \frac{6}{4.5}\left(\frac{M}{2}\right) = \frac{2}{3}\,m$$

Coil	m (g)	I (mA)	K(N/A)
Rt	50	4.15	95.38
Lt	50	4.20	94.24
Cal	5	16.4	5.01

Damping Resistance on Left Coil

$$(R_d + R_c)/T_n = \frac{\ell_s^2}{\ell_o \ell_m}\,\frac{K^2}{4\pi M(0.707 - h_o)}$$

$$= 0.00758\,\frac{(94.24)^2}{0.707 - 0.024}$$

$$= 92.28\ \Omega/\text{sec}$$

FREQUENCY RESPONSE TEST

STATION Elko INSTRUMENT Radial SN #4989

DATE 8-20-76

System Scale Factor, $F_{ss} = X_1 X_2 X_3 X_4$

$$= (256)\left(\frac{100\ \text{Hz}}{5\ \text{V}}\right)\left(\frac{1.25\ \text{V}}{100\ \text{Hz}}\right)\left(\frac{20\ \text{mm}}{1.25\ \text{V}}\right) = 1.024\ \frac{\text{mm}}{\text{mV}}$$

$R_{dT} = 3300\ \Omega$ $R_c = 581\ \Omega$ $h = 0.024$ $T_n = 38.91$ sec

f(Hz)	I(µA)	A(mm)	$A/F_{ss}I\left(\frac{\mu V}{\mu A}\right)$	$fA/F_{ss}I\left(\frac{Hz\cdot\mu A}{\mu A}\right)$ [a]	$f^2A/F_{ss}I\left(\frac{Hz^2\cdot\mu V}{\mu A}\right)$ [b]
0.010	600	33.9	55.18	0.56176	0.0055176
.015	200	16.7	81.543	1.2231	.01835
.02		20.4	99.609	1.9922	.03944
.025		21.7	105.96	2.6489	.066223
.03		21.3	104.00	3.1201	.09360
.04		18.3	89.355	3.5742	.14297
.05		15.2	72.215	3.7109	.18555
.06		12.5	61.035	3.6621	.21973
.06	600	37.5	61.035	3.6621	0.71973
.07		32.3	52.572	3.6800	
.08		28.3	46.061	3.6849	
.09		25	40.690	3.6621	
.10		22.5	36.62	3.662	
.15		15.0	24.414	3.6621	
.20		11.1	18.07	3.6133	
.20	2000	36.9	18.018	3.6035	
.3		24.8	12.109	3.6328	
.4		18.6	9.082	3.6328	
.5		15.1	7.373	3.6865	
.6		12.4	6.0547	3.6325	
.7		10.8	5.2734	3.6914	
.7	6000	32	5.2083	3.6458	
.8		28.2	4.5398	3.6719	
0.9		25	4.0690	3.6621	
1.0		22.3	3.6621	3.6621	

[a] These data are plotted in Fig. E-2.

[b] These data are plotted in Fig. E-3.

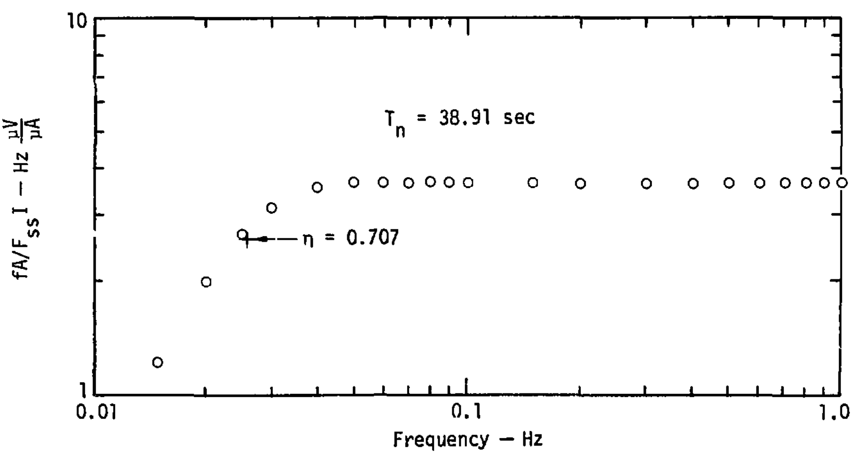


Fig. E-2. Elko radial velocity response with trial R_{dT} = 3300 Ω.

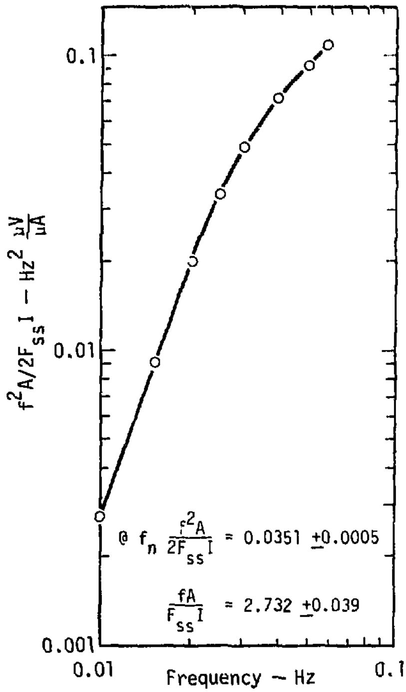


Fig. E-3. Elko radial displacement response with trial R_{dT} = 3300 Ω.

$\underline{\eta_T}$

Asymptotic value of $(fA/F_{ss}I)_{\omega>>\omega_n} = 3.651 \pm 0.027$

from plot (Fig. E-3) @ f_n $(fA/F_{ss}I)_{\omega_n} = 2.732 \pm 0.039$

$$\eta_T = \frac{1}{2}\,\frac{(fA/F_{ss}I)_{\omega>>\omega_n}}{(fA/F_{ss}I)_{\omega_n}} = \frac{3.651 \pm 0.027}{2(2.732 \pm 0.039)} = 0.668 \pm 0.015$$

$\underline{(R_d + R_c)/T_n}$

$$(R_c + R_d)/T_n = \frac{(\eta_T - h)}{(0.707 - h)}\;\frac{(R_c + R_{dT})}{T_n}$$

$$= \frac{(0.668 \pm 0.015 - 0.024)}{(0.707 \pm 0.024)}\;\frac{(3881)}{38.91} = 94.0 \pm 2$$

$\underline{K_d}$

$$K_d^2 = 4\pi M\left(\frac{\ell_o \ell_m}{\ell_s^2}\right)(\eta - h)\quad\frac{(R_c + R_d)}{T_n}$$

from plot (Fig. 5) $K_d = 92.50 \pm 1.0$ N/A

Ratio of Coils

$$\frac{K_s}{K_d} = \frac{30.0}{29.4} = 1.020$$

$$K_s = 93.93 \pm 1.0 \text{ N/A}$$

Calibration Coil Constant

$$G_c = \frac{\ell_m \ell_o}{\ell_c \ell_s} \frac{M}{F_s} \frac{2\pi}{|\tilde{T}_{(\omega)}|} \left(\frac{fA}{F_{ss} I}\right)_{\omega >> \omega_n}$$

$$(fA/F_{ss} I)_{\omega >> \omega_n} = 3.651 \pm 0.027$$

$$G_c = \frac{(13.2)}{6.76} \left(\frac{14.07}{14.03}\right) \frac{11.13}{93.93 \pm 1.0} \; 2\pi(3.651 \pm 0.27)$$

$$= 5.32 \pm 0.1 \text{ N/A}$$

Calibration Constant

$$C.C. = \frac{\ell_m}{\ell_o} \frac{M}{G_c}$$

$$= \frac{13.2}{6.76} \left(\frac{11.13}{5.32 \pm 0.1}\right)$$

$$= 4.09 \pm 0.07 \frac{\text{mA}\cdot\text{sec}^2}{\text{mm}}$$

FREQUENCY RESPONSE TEST

STATION Elko — INSTRUMENT Tangential — SN #4994

DATE 8-20-76

System Scale Factor, $F_{ss} = X_1 \, X_2 \, X_3 \, X_4$

$$= (256) \left(\frac{200 \text{ Hz}}{5 \text{ V}}\right)\left(\frac{1.25 \text{ V}}{100 \text{ Hz}}\right)\left(\frac{20 \text{ mm}}{1.25}\right) = 1.024 \frac{\text{mm}}{\text{mV}}$$

$R_{dT} = 3300 \, \Omega$ $R_c = 578 \, \Omega$ $h = 0.024$ $T_n = 39.76$ °C

f(Hz)	I(μA)	A(mm)	$A/F_{ss}I \left(\frac{\mu V}{\mu A}\right)$	$fA/F_{ss}I \left(\frac{Hz \cdot \mu V}{\mu A}\right)$ [a]	$f^2A/F_{ss}I \left(\frac{Hz^2 \cdot \mu V}{\mu A}\right)$ [b]
0.01	600	32.1	52.246	0.52246	0.005246
.015	200	16.7	81.543	1.2231	.01835
.02		20	97.656	1.9531	.03906
.025		21.2	103.52	2.5879	.064697
.03		20.5	100.10	3.0029	.090088
.04		17.8	86.914	3.476[illegible]	.1391
.05		15.1	73.73	3.6865	.18433
.06		12.9	62.988	3.7793	0.22676
.07		11.2	54.688	3.[illegible]81	
.07	600	30	48.828	[illegible]180	
.08		26.5	43.132	[illegible].4505	
.09		23.5		3.4424	
.10		21		3.4180	
.15		14.5		3.5400	
.20		11		3.5807	
.20	2000	31		3.0273	
.30		21		3.0762	
.4		15.5		3.0273	
.5		12.5		3.0518	
.6		10.5		3.0762	
.7		9		3.0762	
.7	6000	20.5		3.3366	
.8		18.5		3.0111	
0.9		16		2.3438	
1.0		15		1.0667	

[a] These data are plotted in Fig. E-4.

[b] These data are plotted in Fig. E-5.

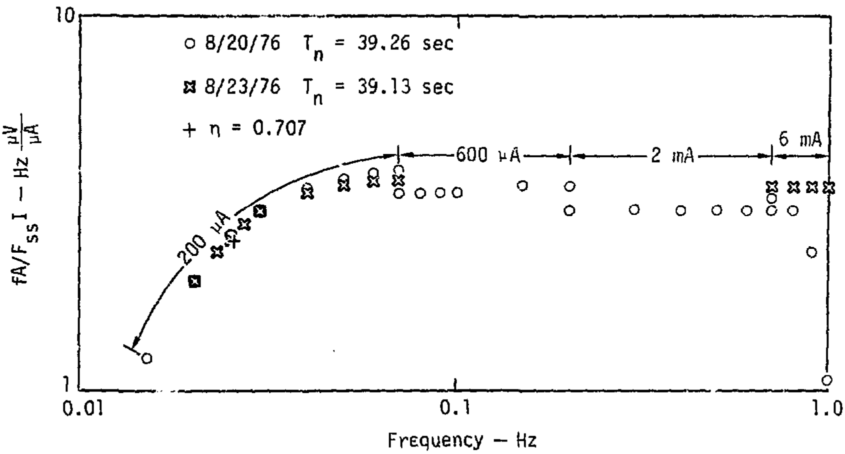


Fig. E-4. Elko tangential velocity response with trial R_{dT} = 3300 Ω.

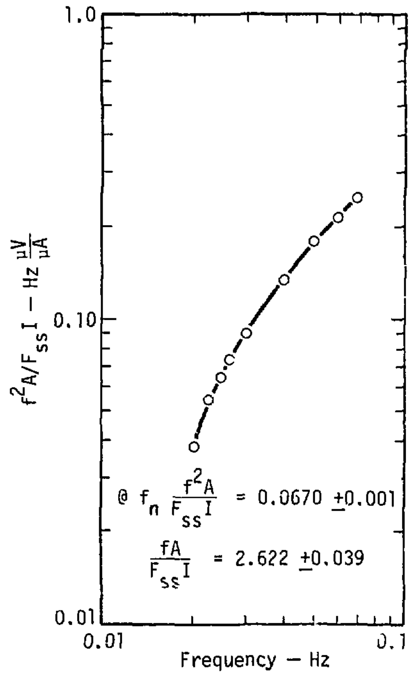


Fig. E-5. Elko tangential displacement *response with trial* R_{3T} *= 3300 Ω.*

f	I	A	$A/F_{ss}I$	$fA/F_{ss}I$	$f^2A/F_{ss}I$
1.5	6000	10		2.4414	
2.0	↓	7		2.2786	
2.0	20,000	14		1.3672	
3.0		9		1.3184[a]	
4.0		7			
5.0	↓	5			
	8/23[b]		T_n = 39.13 sec		
0.020	200	19.9		1.9434	0.03887
.023		21.0		2.3584	.05424
.025		21.0		2.5635	.06409
.027		21		2.7686	.07475
.030		20.6		3.0176	.09053
.040		17.4		3.3984	.1359
.050		14.5		3.5400	.177
.06	↓	12.4		3.6320	.2179
.07	600	31.1		3.5433	0.2480
.70	2000	10.1		3.4521	
.70	6000	30.6		3.4863	
.80		26.8		3.4896	
0.90		23.8		3.4863	
1.0	↓	21.5		3.4993	
			rms = 3.490 ± 0.006		

[a]A wiring error in the calibration chassis was responsible for the erratic behavior on this test.

[b]This test was run after the error was corrected. In the mean time the period had changed due to setting-up of the epoxy used to glue down the glass plates.

$\underline{\eta_T}$

Asymptotic value of $(fA/F_{ss}I)_{\omega>>\omega_n} = 3.490 \pm 0.006$

from plot (Fig. E-5) @ f_n $(fA/F_{ss}I)_{\omega_n} = 2.622 \pm 0.039$

$$\eta_T = \frac{1}{2}\frac{(fA/F_{ss}I)_{\omega>>\omega_n}}{(fA/F_{ss}I)_{\omega_n}} = \frac{3.490 \pm 0.006}{2(2.622 \pm 0.039)} = 0.6655 \pm 0.011$$

$\underline{(R_d + R_c)/T_n}$

$$(R_c + R_d)/T_n = \frac{(\eta_T - h)}{(0.707 - h)} \frac{(R_c + R_{dT})}{T_n}$$

$$= \frac{(0.6655 \pm 0.011 - 0.024)}{(0.707 - 0.024)} \frac{(3878)}{39.13} = 93.1 \pm 1.6$$

$\underline{K_d}$

$$K_d^2 = 4\pi M \left(\frac{\ell_o \ell_m}{\ell_s^2}\right) (\eta - h) \frac{(R_c + R_d)}{T_n}$$

from plot (Fig. 5) $K_d = 91.6 \pm 0.8$ N/A

Ratio of Coils

$$\frac{K_s}{K_d} = \frac{29.1}{28.7} = 1.014$$

$$K_s = 92.9 \pm 0.8 \text{ N/A}$$

Calibration Coil Constant

$$G_c = \frac{\ell_m \ell_o}{\ell_c \ell_s} \frac{M}{K_s} \frac{2\pi}{|\bar{T}_{(\omega)}|} \left(\frac{fA}{F_{ss} I}\right)_{\omega >> \omega_n}$$

$$(fA/F_{ss} I)_{\omega >> \omega_n} = 3.490 \pm 0.006$$

$$G_c = \frac{(13.2)}{6.76} \left(\frac{14.07}{14.03}\right) \frac{11.13}{(92.9)} \quad 2\pi \ (3.490 \pm 0.006)$$

$$= 5.15 \pm 0.05 \text{ N/A}$$

Calibration Constant

$$\text{C.C.} = \frac{\ell_m}{\ell_o} \frac{M}{G_c}$$

$$= \frac{13.2}{6.76} \left(\frac{11.13}{5.15 \pm 0.05}\right)$$

$$= 4.22 \pm 0.07 \ \frac{\text{mA} \cdot \text{sec}^2}{\text{mm}}$$

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