

212  
6/30/77  
Minutes NTIS

UL-60

ERDA/NASA/1004-77-1  
ERDA/NASA/1004-77-1  
NASA TM X-73670

~~Do not stack~~

## VIBRATION CHARACTERISTICS OF A LARGE WIND TURBINE TOWER ON NON-RIGID FOUNDATIONS

Suey T. Yee, Tse-Yung P. Chang, R. J. Scavuzzo,  
David H. Timmerman, and John W. Fenton  
National Aeronautics and Space Administration  
Lewis Research Center  
Cleveland, Ohio 44135

May 1977

**NOTICE**  
This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Prepared for  
ENERGY RESEARCH AND DEVELOPMENT ADMINISTRATION  
Office of the Assistant Administrator for  
Solar, Geothermal, and Advanced Energy Systems  
Division of Solar Energy  
Under Interagency Agreement E(49-26)-1004

MASTER

## **DISCLAIMER**

**This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.**

---

## **DISCLAIMER**

**Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.**

NOTICE

This report was prepared to document work sponsored by the United States Government. Neither the United States nor its agent, the United States Energy Research and Development Administration, nor any Federal employees, nor any of their contractors, subcontractors or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

1. Report No. <b>NASA TM X-73670</b>	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle <b>VIBRATION CHARACTERISTICS OF A LARGE WIND TURBINE TOWER ON NON-RIGID FOUNDATIONS</b>		5. Report Date <b>May 1977</b>	
7. Author(s) Suey T. Yee, Lewis Research Center, Cleveland, Ohio, and Tse-Yung P. Chang, R. J. Scavuzzo, David H. Timmerman, and John W. Fenton, Karl R. Rohrer Associates, Inc., Akron, Ohio		6. Performing Organization Code	
9. Performing Organization Name and Address <b>National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135</b>		8. Performing Organization Report No. <b>E-9194</b>	
12. Sponsoring Agency Name and Address <b>Energy Research and Development Administration Division of Solar Energy Washington, D.C. 20545</b>		10. Work Unit No.	
		11. Contract or Grant No.	
		13. Type of Report and Period Covered <b>Technical Memorandum</b>	
		14. Sponsoring Agency Code Report No. <b>ERDA/NASA 1004/77/1</b>	
15. Supplementary Notes <b>This report was prepared under Interagency Agreement E(49-26)-1004.</b>			
16. Abstract  <b>Vibration characteristics of the Mod-OA wind turbine supported by non-rigid foundations were investigated for a range of soil rigidities. The study shows that the influence of foundation rotation on the fundamental frequency of the wind turbine is quite significant for cohesive soils or loose sand. The reduction in natural frequency can be greater than 20 percent. However, for a foundation resting on well graded, dense granular materials or bedrock, such effect is small and the foundation can be treated as a fixed base.</b>			
<div style="border: 1px solid black; padding: 5px; width: fit-content; margin: auto;"> <b>NOTICE</b>            This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.         </div>			
17. Key Words (Suggested by Author(s))  <b>Wind turbines Natural frequency Non-rigid foundation</b>		18. Distribution Statement  <b>Unclassified - unlimited STAR Category 44 ERDA Category UC-60</b>	
19. Security Classif. (of this report)  <b>Unclassified</b>	20. Security Classif. (of this page)  <b>Unclassified</b>	21. No. of Pages	22. Price*

\* For sale by the National Technical Information Service, Springfield, Virginia 22161

**DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED**

*8/19*

VIBRATION CHARACTERISTICS OF A LARGE WIND TURBINE  
TOWER ON NON-RIGID FOUNDATIONS

by Suey T. Yee, Lewis Research Center, Cleveland, Ohio  
and Tse-Yung P. Chang, R. J. Scavuzzo,  
David H. Timmerman, and John W. Fenton,  
Karl R. Rohrer Associates, Inc., Akron, Ohio

SUMMARY

A study was performed to determine the vibration characteristics of the Mod-OA wind turbine supported by a non-rigid foundation. The foundation considered is a square footing with 34 feet in horizontal dimensions and 4 feet in thickness. Only the rocking action of the foundation was considered in the study.

To establish a reference point, the free vibration of the wind turbine sitting on a fixed base was obtained from the dynamic analysis using the NASTRAN Computer program. Then, a simple model, consisting of three masses and springs representing the tower, machinery and blades, and a torsional spring representing the foundation-soil, was used. The natural frequencies of the wind turbine were obtained for a foundation on soils with various rigidities. From the dynamic analyses, it was found that the influence of foundation rotation on the fundamental frequency is quite significant for soils with elastic moduli less than 5000 psi (e.g., cohesive soil or loose sand) and the reduction in natural frequency can be greater than 20 percent. However, for stiffer soils, such as well graded, dense granular materials or bedrock, the effect of the foundation-soil is not significant and therefore a rigid base can be assumed for dynamic analysis.

In addition, the foundation effect with different tower heights was also studied. The analysis shows that the effect of soil flexibility on the vibration characteristics of the wind turbine is more pronounced if the height of the tower is increased.

## INTRODUCTION

With the declining supplies of oil and natural gas, alternative energy sources and energy conversion systems are being sought and an extensive energy research program is being developed under the direction of Energy Research and Development Administration (ERDA). One of the many energy sources being examined is wind energy which has been utilized by mankind for centuries. The objective of this program is to assess the technology requirements for constructing large wind turbine systems and ultimately to develop a wind energy system which is cost-competitive. In connection with ERDA's effort, NASA Lewis Research Center has designed and constructed a 100-kilowatt wind turbine (called Mod-0) at its Plum Brook Station near Sandusky, Ohio. Larger wind turbines are being designed and will be built in other parts of the country where high wind zones are located.

In designing the Mod-0 Wind Turbine, the tower was assumed to be resting on a rigid foundation; therefore, no interaction effect between the tower and foundation was considered. This assumption was justified by the fact that the Mod-0 tower is supported by fairly rigid concrete caissons sitting on hard shale. For wind turbines located on sites yet to be determined, the assumption of a rigid foundation may not apply. As a result, a study was initiated to determine the vibration characteristics of the wind turbine affected by non-rigid foundations.

The general configuration of the proposed larger wind turbine (called Mod-0A), similar to the one built at Plum Brook, consists of a tower; a nacelle housing the alternator, gear box and other

machinery; and two large rotor blades. The wind turbine is designed to produce 200 kilowatts of electric power in a 22-mph (hub-height) wind at a rotor speed of 40 rpm. In the design of such a large structure, it is necessary to consider the dynamic forces caused by strong wind and forces induced by the rotation of the blades. In addition, the tower must be designed with sufficient rigidity such that its fundamental natural frequency is well above the critical driving frequency of the blades (1.33 Hz for a two blade rotor at 40 rpm). Free vibration analyses of the blades [1] and the wind turbine as a whole [2] have been conducted for the Plum Brook system. A test program [3] was also performed to verify the analyses and it was concluded that the results obtained compare very closely with the test data.

Included in this report is a study on the vibration characteristics of the Mod-0A wind turbine with a square footing on various soil conditions. A simple model consisting of three-lumped masses and springs was used for the study and only the rocking action of the foundation was considered. In order to establish a reference point, the free vibration of the wind turbine on a fixed base was obtained. Then, the description of the simple model and calculation of foundation spring constants were outlined. Finally, the natural frequencies of the wind turbine were obtained for various soil rigidities and the significance of the numerical results was discussed.

### DESCRIPTION OF TOWER

The Mod-OA wind turbine consists of a tower, a nacelle and two rotor blades, shown in Fig. 1, similar to the one built at Plum Brook. The tower is a 93-ft. tall welded truss constructed from steel pipe, angle and tee sections. The sectional view of the tower is a square shape and it varies from 30 ft. wide at the base to 6.7 ft. wide at the top. Elevation of the tower and the sizes of some major members are shown in Fig. 2.

The tower is anchored to a reinforced concrete slab foundation which is 34 feet square in horizontal directions and 4 feet in thickness. The dimensions of the foundation were based solely on the wind and dead loads. A nacelle, which contains the alternator, gear box, and other machinery, is attached to the top of the tower. Connected to the nacelle is a two blade rotor which is 125 feet in diameter. The total weight of the nacelle and blades is approximately 44,400 lbs.

### FREE VIBRATION OF WIND TURBINE ON A FIXED BASE

To establish a reference case for later discussion, free vibration analysis of the wind turbine resting on a fixed base was performed by using the NASTRAN computer program [4]. The computer model consists of all the tower members (represented by beam elements) and the masses due to the nacelle and blades which are evenly distributed at four corner nodes at the top of the tower. A total of 235 elements and 98 nodes were used. From the computer analysis, the first five fundamental frequencies are tabulated in Table 1.

The mode shapes for modes number 1, 2, 4, and 5 are plotted in Fig. 3; mode number 3, representing the vibration of local members, is not shown. The first and second modes represent the cantilever beam action of the tower in two independent x- and y-directions, respectively. The small difference in frequency of these two modes is due to the slight unsymmetric arrangement of the bracing member located at the second platform of the tower. The fourth mode shape represents the torsional action of the tower and the fifth is a higher bending mode.

This analysis shows that the fundamental vibrational mode is the cantilever beam action, particularly due to the heavy mass located at the top of the tower. Analyses were also conducted to investigate the importance of rotary inertia of the blades on the fundamental frequencies of the system. Numerical results indicated that this effect is very small, and therefore it can be ignored in the natural frequency analysis.

### A SIMPLE MODEL

Since the main objective of this study is to determine the effect of a non-rigid foundation on the vibration characteristics of the wind turbine, it is sufficient to use a simplified model consisting of lumped masses and springs for the intended study. Only the rocking action of the foundation is considered. Based on the free vibration analysis of the tower on a fixed base, the fundamental vibrational mode is due to the bending action. Therefore, it was decided that a model, consisting of 3-masses and springs representing the superstructure (the tower, nacelle and blades) and an effective spring representing the foundation-soil (shown in Fig. 4) be used. According to this idealization, the flexibility and mass matrices are derived as follows.

Consider in Fig. 4 that a horizontal force  $F_i$  is applied on the mass  $m_i$  causing a deflection  $u_j$  at  $m_j$ , then

$$u_j = u_j^s + u_j^r, \quad j = 1, 2, \text{ or } 3 \quad (1)$$

where  $u_j^s$  represents the relative deflection at  $j$  before the rigid body motion; and  $u_j^r$ , the rigid body displacement due to the rotation of the tower. From the deflection analysis, one can easily find

$$u_j^s = (f_{ij}^s) F_i \quad (2)$$

and

$$u_j^r = \frac{h_i h_j}{K} F_i \quad (3)$$

where  $f_{ij}^s$  is the flexibility coefficient of the tower structure on a fixed base;  $h_i$ , elevation of the  $i^{\text{th}}$  mass; and  $K$ , the rotational

stiffness of the foundation. From Eq. (1), one can define the flexibility of the system as

$$f_{ij} = \frac{u_j}{F_i} = f_{ij}^s + f_{ij}^r \quad (4)$$

$$f_{ij}^r = \frac{h_i h_j}{K} \quad (5)$$

The above equation consists of two terms: the first term is the fixed-base flexibility and the second term is due to the rotation of the foundation. The terms involving  $f_{ij}^s$  were found from the deflection analyses of the actual tower using NASTRAN by applying unit loads at locations 1, 2, and 3, respectively. For the model considered, the mass points were selected at elevations 93-ft (location 1), 68-ft (location 2), and 38-ft (location 3). In this manner, the terms for  $f_{ij}^s$  and  $f_{ij}^r$  are given by

$$[f_{ij}^s] = \frac{1}{E} \begin{bmatrix} 972 & 456 & 90 \\ & 285 & 49 \\ \text{symmetric} & & 14.5 \end{bmatrix} \frac{\text{in}}{\text{lb}} \quad (6)$$

and

$$[f_{ij}^r] = \frac{1}{K} \begin{bmatrix} 12.45 & 9.10 & 5.10 \\ & 6.65 & 3.72 \\ \text{symmetric} & & 2.08 \end{bmatrix} \frac{\text{in}}{\text{lb}} \quad (7)$$

In the above equation, determination of the foundation stiffness  $K$  will be discussed in the next section.

The mass matrix of the model was determined by distributing the masses of the tower members between two adjacent elevations equally to the respective controlling stations (i.e. 1, 2, or 3).

In addition, the masses of the machinery and blades were lumped at the top, location 1. Therefore, the mass matrix was found to be

$$[M] = \frac{1}{g} \begin{bmatrix} w_1 & 0 & 0 \\ 0 & w_2 & 0 \\ 0 & 0 & w_3 \end{bmatrix} \quad (8)$$

where  $w_1 = 50.10$  kips  
 $w_2 = 10.37$  kips  
 $w_3 = 16.27$  kips  
 $g = \text{gravitational acceleration}$

### EFFECTIVE SPRING CONSTANT FOR FOUNDATION

Consider a circular foundation of radius  $r_o$  resting on an elastic half space as shown in Fig. 5. The foundation is subjected to a constant moment  $T_\psi$ , the corresponding amplitude of dynamic rotation  $A_\psi$  may be evaluated from [5]

$$A_\psi = \psi_s M_\psi \quad (9)$$

where  $\psi_s$  is the static rotation of the foundation and  $M_\psi$  is the dynamic magnification factor. For a circular foundation of radius  $r_o$ , the rotation is related to the moment by

$$\psi_s = \frac{3(1-\nu)}{8} \frac{T_\psi}{Gr_o^3} \quad (10)$$

where  $\nu$  is the Poisson's ratio and  $G$ , the shear modulus of soil. For foundations which are not circular, an effective radius based on equal areas can be found by

$$r_o = \sqrt{\frac{L^2}{\pi}} \quad (11)$$

Thus for a square foundation, Eq. (10) becomes

$$\psi_s = \frac{2.09 (1-\nu)}{G} \frac{T_\psi}{L^3} \quad (12)$$

However, for a square foundation, a more exact expression is given by [5]

$$\psi_s = \frac{1.92 (1-\nu)}{G} \frac{T_\psi}{L^3} \quad (13)$$

Eq. (13) has been used to evaluate the spring constant of the foundation.

According to reference [5] the dynamic magnification factor,  $M_\psi$  in Eq. (9) is a function of the mass ratio  $B_\psi$  given by

$$B_\psi = \frac{3(1-v)}{8} \frac{I_\psi}{\rho r_0^5} \quad (14)$$

and of the dimensionless frequency  $a_0$  defined by the equation

$$a_0 = \frac{\omega r_0}{V_s} \quad (15)$$

Where  $I_\psi$  denotes the mass moment of inertia of the foundation;  $\rho$ , the mass density;  $\omega$ , the forcing frequency; and  $V_s$ , the shear wave velocity of the soil media. Listing the equivalent radius based on Eq. (11), both  $B_\psi$  in Eq. (14) and  $a_0$  in Eq. (15) can be easily determined and the magnification factor  $M_\psi$  calculated. It was found that  $M_\psi$  is near unity \* for the frequency range of interest, i.e.  $f = 2.40$  Hz. Therefore, the dynamic effect can be neglected. From Eq. (13) the effective rotational spring constant  $K$  is

$$K = 0.51 \left( \frac{G}{1-v} \right) L^3 \quad (16)$$

where

$$G = \frac{E}{2(1+v)}, \quad E = \text{Young's modulus}$$

Based on the above equation, the effective spring constant of the foundation can be readily calculated if the Young's modulus  $E$  and Poisson's ratio  $\nu$  of soil is given. In the present investigation, a range of the Young's modulus for typical soils<sup>†</sup> is considered as shown in Table 2. A constant Poisson's ratio of 0.3 is used in all calculations. Typical values of the effective foundation spring-constants vs. various soil moduli and shear wave velocities are tabulated in Table 3.

---

\* Determination of  $M_y$  can be found in Appendix A.

<sup>†</sup> Determination of the foundation-soil parameters is outlined in Appendix B.

### NATURAL FREQUENCY ANALYSIS

Referring to the simple model proposed, the equations of motion for the system can be written as

$$[f] [M] \{u\} + \{u\} = \{0\} \quad (17)$$

where  $u$  denotes the absolute displacement vector, i.e.

$$\{u\} = \{u_1, u_2, u_3\}^T \quad (18)$$

and  $u$  is the corresponding acceleration vector. The matrices  $f$  and  $M$  were defined in Eqs. (4) and (8) respectively.

The natural frequencies can be obtained from the solution to the following linear equation

$$\omega^2 [f] [M] \{\bar{u}\} = \{\bar{u}\} \quad (19)$$

where  $\omega$  is the angular frequency and  $\bar{u}$  is the corresponding vibrational mode shape. The iteration method [6] was employed to calculate the natural frequencies for different shear wave velocities and the results are tabulated in Table 4. As seen in the table, the first fundamental frequency obtained from the simple model on a fixed base is 2.43 Hz which is very close to the value from the NASTRAN dynamic analysis of the tower, i.e. 2.40 Hz.

Define a frequency ratio for the first mode

$$\bar{f}_1 = f_x / f_s \quad (20)$$

where

$f_{1r}$  = First natural frequency of the tower on a  
non-rigid base (spring)

$f_{1s}$  = First natural frequency of the tower on a  
fixed base.

Then, the frequency ratios for various soil rigidities are plotted against the shear wave velocities and elastic moduli in Figs. 6 and 7 respectively. As shown in these two figures, the change in natural frequency of the wind turbine on a non-rigid foundation varies widely with the stiffness of the soil. For dense, well graded granular materials or sound bedrock for which the elastic modulus exceeds 10,000 psi, the effect due to the foundation is very small, and therefore can be ignored in the vibration analysis of the wind turbine. For medium to dense sand ( $E = 7000-12000$  psi), the percentage change in fundamental frequency (as compared with the fixed-base frequency) ranges between 4 and 18%. For cohesive soil or loose sand where the modulus is less than 5000 psi, the foundation effect is quite important and the change in natural frequency can be greater than 20%.

### APPROXIMATE ANALYSIS OF FOUNDATION EFFECTS

The effects of tower flexibility and foundation rotation on the fundamental frequency can be evaluated separately in an approximate manner. By assuming that the tower is fixed at the foundation, the fundamental frequency caused by tower flexibility can be calculated. This condition is obtained when  $v_s = \infty$  and results are listed in Table 4. For the tower investigated  $f_{1s} = 2.40$  Hz.

The effect of tower foundation rotation only on the fundamental frequency of the structure can be evaluated assuming that the tower is rigid. For this case the frequency of the system can be calculated as follows

$$f_{1r} = \frac{1}{2\pi} \sqrt{\frac{K}{I}} \quad (21)$$

where

$K$  = effective foundation rotational stiffness

(Table 3)

$$I = \sum m_i l_i^2$$

For this tower a soil shear velocity of  $v_s = 250$  ft/sec, and frequency as determined from Eq. (19) is  $f_{1r} = 3.13$  Hz.

These two effects, tower flexibility and tower rotation, can be combined using Dunkerly's formula (Ref. 6) to estimate the fundamental frequency of the system.

$$\frac{1}{(f_1)^2} = \frac{1}{(f_{1r})^2} + \frac{1}{(f_{1s})^2} \quad (22)$$

For this case  $f = 1.92$  which is close to the more exact value listed in Table 4.

The advantage of this approximate method is that effects of foundation rotation on the tower fundamental frequency at a particular site can be easily estimated prior to a more complete analysis. These equations show that as the tower becomes taller or stiffer the effect of the soil is more pronounced, if other factors remain constant.

## CONCLUSION

Vibration characteristics of a 200-kilowatt wind turbine system resting on rigid and non-rigid foundations were studied.

Based on the study, the following conclusions can be drawn:

- (1) For a rigid foundation, the fundamental mode of the tower is 2.4 Hz and is caused basically by cantilever bending action of the tower.
- (2) The influence of the foundation rotation on the fundamental frequency is quite significant for soils with elastic moduli less than 5000 psi.
- (3) For stiffer soil, such as dense, well-graded granular materials or sound bedrock, the effect of foundation rotation is not significant and the effect can be ignored.

For other wind turbine tower designs in which the height of the tower is increased while other parameters are unchanged, the approximate analysis in the previous section indicates that the vibration characteristics of the system become more sensitive to changes in soil flexibility. Furthermore, if the tower design is altered to increase its stiffness, the influence of the soil flexibility may dominate the dynamic response of the system.

## APPENDIX A - DETERMINATION OF DYNAMIC MAGNIFICATION FACTOR

The dynamic magnification factor  $M_\psi$  in Eq. (9) is a function of the mass ratio  $B_\psi$  and a dimensionless frequency  $a_o$  defined by

$$B_\psi = \frac{3(1-\nu) I_\psi}{8 \rho r_o^5} \quad (A-1)$$

$$a_o = \frac{\omega r_o}{v_s} \quad (A-2)$$

Since the fundamental frequency of the wind turbine is

$$f_1 = 2.43 \text{ Hz.}$$

and the width and the length of the foundation are

$$L = d = 34'$$

$$\text{Then } \omega = 2\pi f_1 = 15.27 \text{ rad/sec}$$

$$r_o = 230 \text{ in} = 19.17 \text{ ft}$$

The mass ratio,  $B_\psi$ , can be estimated as follows

$$I_\psi \approx \frac{W}{g} \frac{l^2}{12} \quad (A-3)$$

where

$W$  = weight of foundation

$l$  = length

Since the foundation is to be constructed on concrete approximately 4 feet thick and 34 feet square

$$W \approx \gamma \cdot V_s \quad (A-4)$$

$$W = (150) \frac{1b}{ft^3} (34)^2 (4) ft^3$$

$$W = 693,600 \text{ lbs}$$

Thus

$$I_{\psi} = \frac{W}{g} \left( \frac{34^2}{12} \right) = 2.08 \times 10^6 \text{ lb-ft-sec} \quad (A-5)$$

According to Eq. (A-1)

$$B_{\psi} = \frac{(3)(0.7)(2.08)(10^6)}{(8)\left(\frac{100}{32.2}\right)\left(\frac{230}{12}\right)^5} = 0.068 \quad (A-6)$$

Using Figure 7-15 of Reference [1], it can be seen that  $M_{\psi}$  is unity for the frequency range of interest. Therefore, this effect can be neglected.

APPENDIX B - DETERMINATION OF FOUNDATION SOIL PARAMETERS

Typical analysis techniques for determining the response of soil-foundation systems subjected to dynamic loadings consider the soil as some type of equivalent elastic system. Since soil is not a linearly elastic material, an approximation must be made of the elastic modulus,  $E$ , or the shear modulus,  $G$ , and Poisson's ratio,  $\nu$ , which will produce a calculated response within reasonable accuracy.

#### 1. Poisson's Ratio

Poisson's ratio is difficult to determine accurately for soils and can be assumed for most practical calculations based on knowledge of the soil type. Typical values are as follows:

Saturated Clay	$\nu = 0.50$
Unsaturated Clay and Clay with Sand and Silt	$\nu = 0.30$ to $0.40$
Granular Soils	$\nu = 0.30$ to $0.35$

A value of 0.3 has been used in Section 3.0.

#### 2. Elastic Modulus

The elastic modulus and shear modulus vary over a wide range depending on soil type, density and confining pressure. Typical values of the elastic modulus,  $E$ , are listed in Table 2.

### 3. Laboratory Testing and Field Testing

Determination of actual values of E or G can be made from empirical relationships, from laboratory testing or preferably from in situ measurements.

Empirical relationships have been developed by Richart and Hardin [5] for determining the shear modulus, G, for granular soils.

For round-grained sands ( $e < 0.80$ );

$$G = \frac{2630(2.17-e)^2(\bar{\sigma}_o)^{0.5}}{1+e} \quad (B-1)$$

For angular-grained materials:

$$G = \frac{1230(2.97-e)^2(\bar{\sigma}_o)^{0.5}}{1+e} \quad (B-2)$$

where, G = shear modulus - psi

$\bar{\sigma}_o$  = average effective confining pressure - psi

e = void ratio

Laboratory testing is normally performed by the resonant column method. Undisturbed samples of cohesive soils can readily be tested by this method. However, undisturbed samples of granular soils cannot easily be obtained and since G depends on void ratio and confining pressure, determining G from laboratory testing is usually not practical.

Whenever possible, in situ evaluation of the shear modulus should be carried out. This involves determining the average shear wave velocity,  $v_s$ , for the foundation supporting soil. The shear modulus can then be determined from

$$G = \rho v_s^2 \quad (B-3)$$

where  $\rho$  equals the mass density of the soil.

The shear wave velocity,  $v_s$ , can be determined either by direct measurement using seismic survey methods or preferably from steady-state vibration methods. The steady-state vibration method measures Rayleigh wave velocities which for practical engineering purposes can be assumed to equal shear wave velocities. By varying the frequency of the wave generating vibrator, the average shear modulus can be obtained for different depths into the soil for use in analyzing possible soil-foundation systems.

REFERENCES

1. Spera, David A.. Structural Analysis of Wind Turbine Rotors for NSF-NASA Mod-0 Wind Power System. NASA TM X-3198, 1975.
2. Chamis, C. C.; and Sullivan, T. L.: Free Vibrations of the ERDA-NASA 100-KW Wind Turbine. NASA TM X-71879, 1976.
3. Linscott, Bradford S.; Shapton, William R.; and Brown, David: Tower and Rotor Blade Vibration Test Results for a 100-Kilowatt Wind Turbine. NASA TM X-3426, 1976.
4. MacNeal, R. H., ed.: The NASTRAN Theoretical Manual (level 15). NASA SP-221(01), 1972.
5. Richart, F. E., Jr.; et al.: Vibrations of Soils and Foundations. Prentice-Hall Inc., 1970.
6. Thomson, W. T.: Theory of Vibration With Applications. Prentice-Hall Inc., 1972.

TABLE 1  
FUNDAMENTAL FREQUENCIES OF WIND TURBINE ON FIXED BASE

<u>Mode</u>	<u>Description</u>	<u>Value (Hz)</u>
1	Beamwise (x direction)	2.37
2	Beamwise (y direction)	2.40
3	Local Member	6.67
4	Torsional	7.29
5	Beamwise (y direction)	11.07

TABLE 2  
YOUNG'S MODULI OF TYPICAL SOILS

<u>Soil Type</u>	<u>E (psi)</u>
Very soft to soft clay	50 - 500
Medium to hard clay	500 - 2,500
Sandy clay	4,000 - 6,000
Loose sand	1,000 - 4,000
Dense sand	7,000 - 12,000
Dense sand and gravel	14,000 - 28,000

TABLE 3

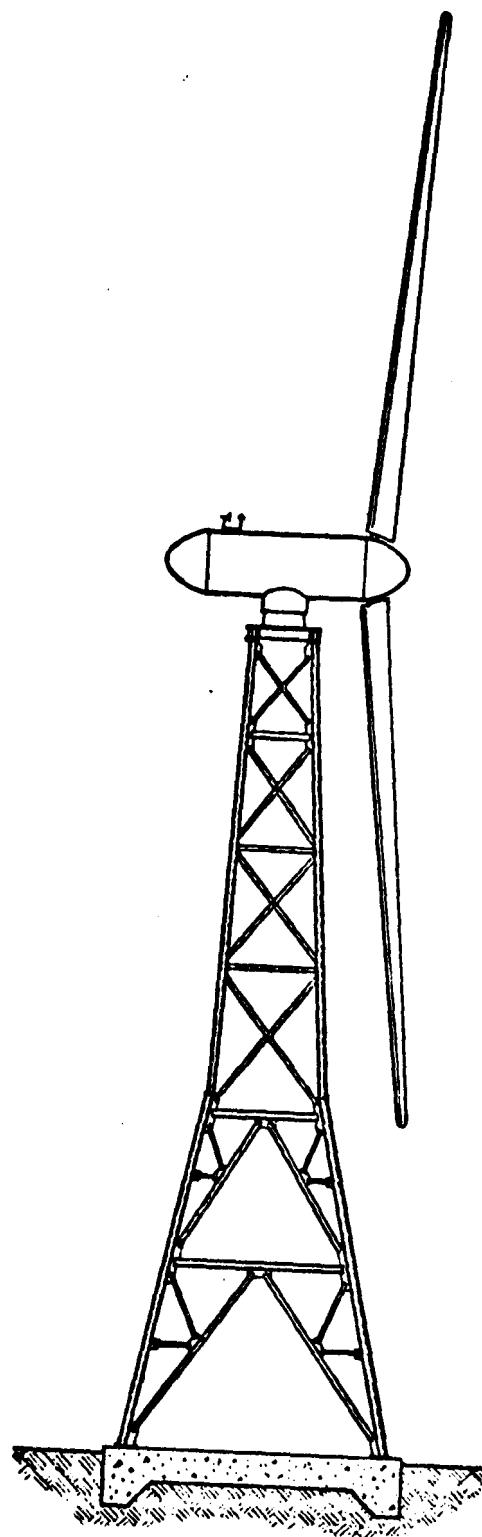
## EFFECTIVE ROTATIONAL SPRING CONSTANTS OF FOUNDATION

<u><math>v_s</math></u> <u>(Ft/Sec)</u>	<u>G</u> <u>(Psi)</u>	<u>E</u> <u>(Psi)</u>	<u>K</u> <u>(Lb-In/Rad)</u>
250	1,350	3,500	$6.81 \times 10^{10}$
500	5,390	14,000	$27.19 \times 10^{10}$
600	7,760	20,200	$39.15 \times 10^{10}$
1000	21,600	56,100	$10.90 \times 10^{11}$
1500	48,500	126,200	$24.47 \times 10^{11}$
2000	86,300	224,300	$43.54 \times 10^{11}$

TABLE 4

## NATURAL FREQUENCIES

<u><math>v_s</math></u> <u>Ft/sec</u>	<u><math>f_1</math></u> <u>(Hz)</u>	<u><math>f_2</math></u> <u>(Hz)</u>	<u><math>f_3</math></u> <u>(Hz)</u>
250	1.89	15.4	29.6
500	2.23	18.5	36.7
1000	2.35	19.8	46.9
1500	2.38	20.1	51.7
2000	2.39	20.1	54.4
$\infty$	2.40	20.3	57.2



**FIG. 1** Mod-OA WIND TURBINE

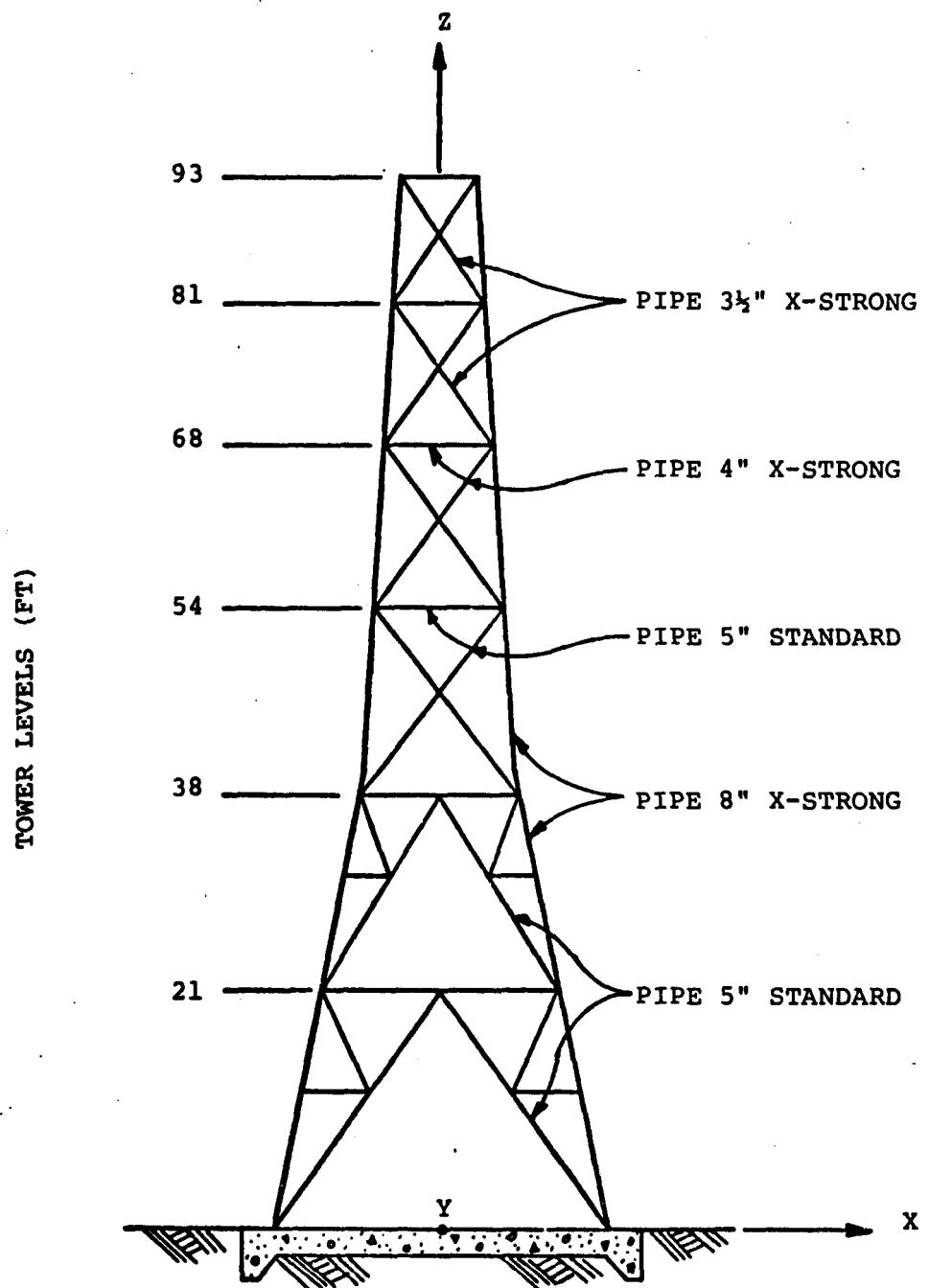


FIG. 2 ELEVATION OF TOWER STRUCTURE

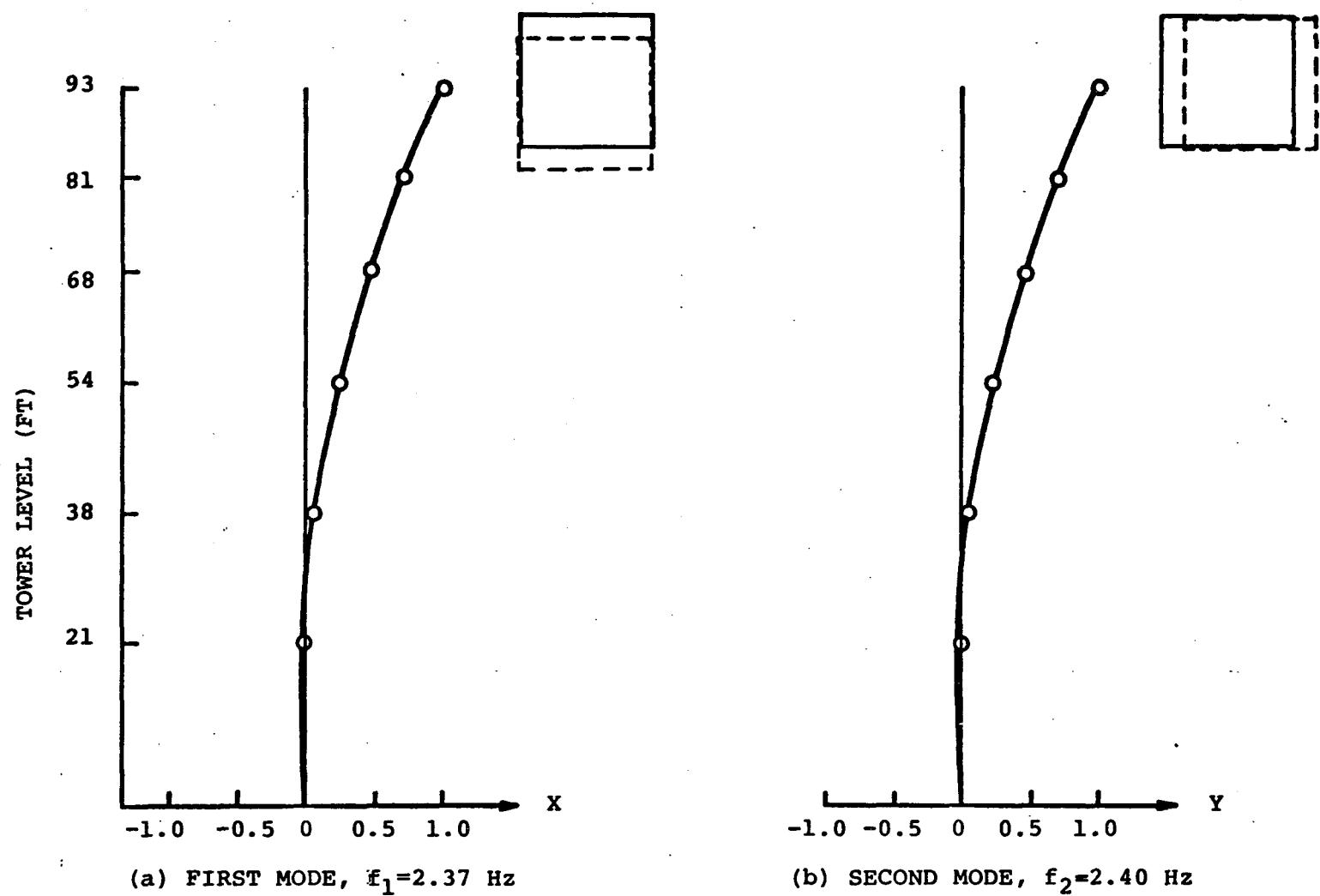
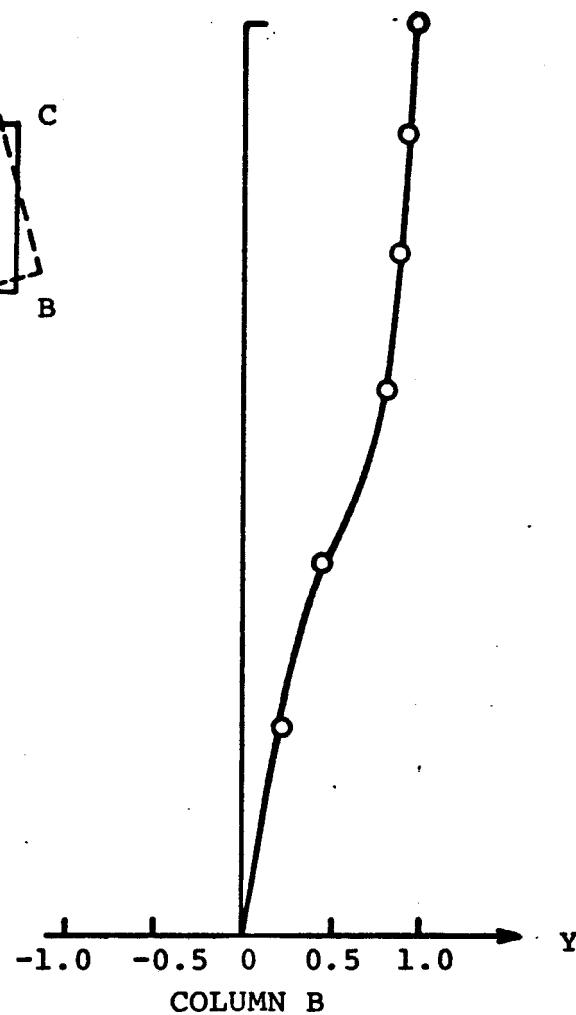
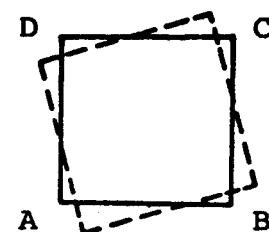
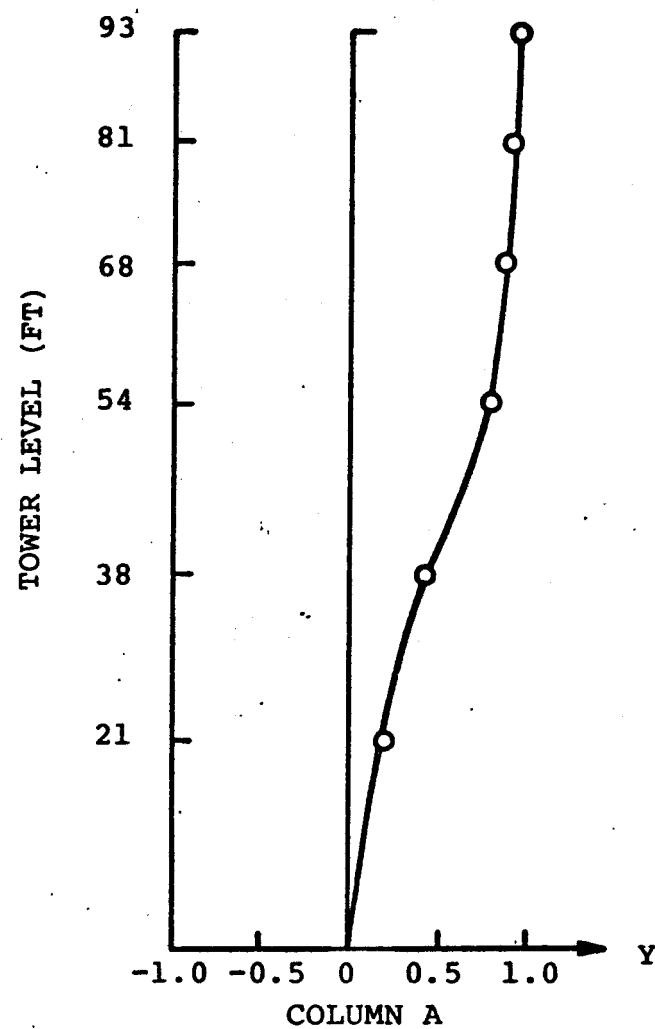
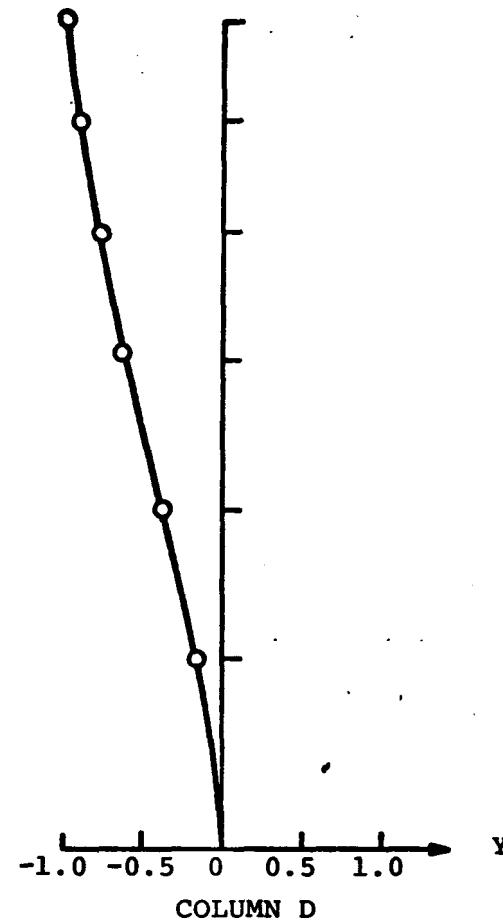
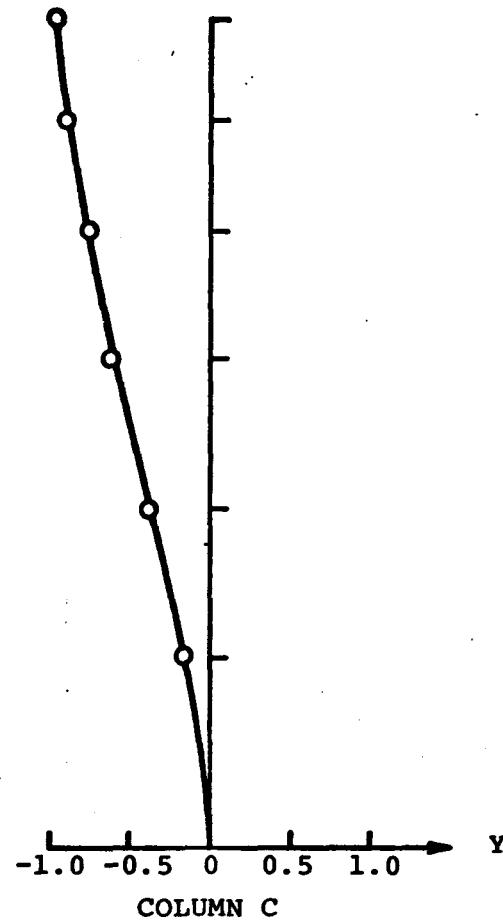


FIG. 3 MODE SHAPES



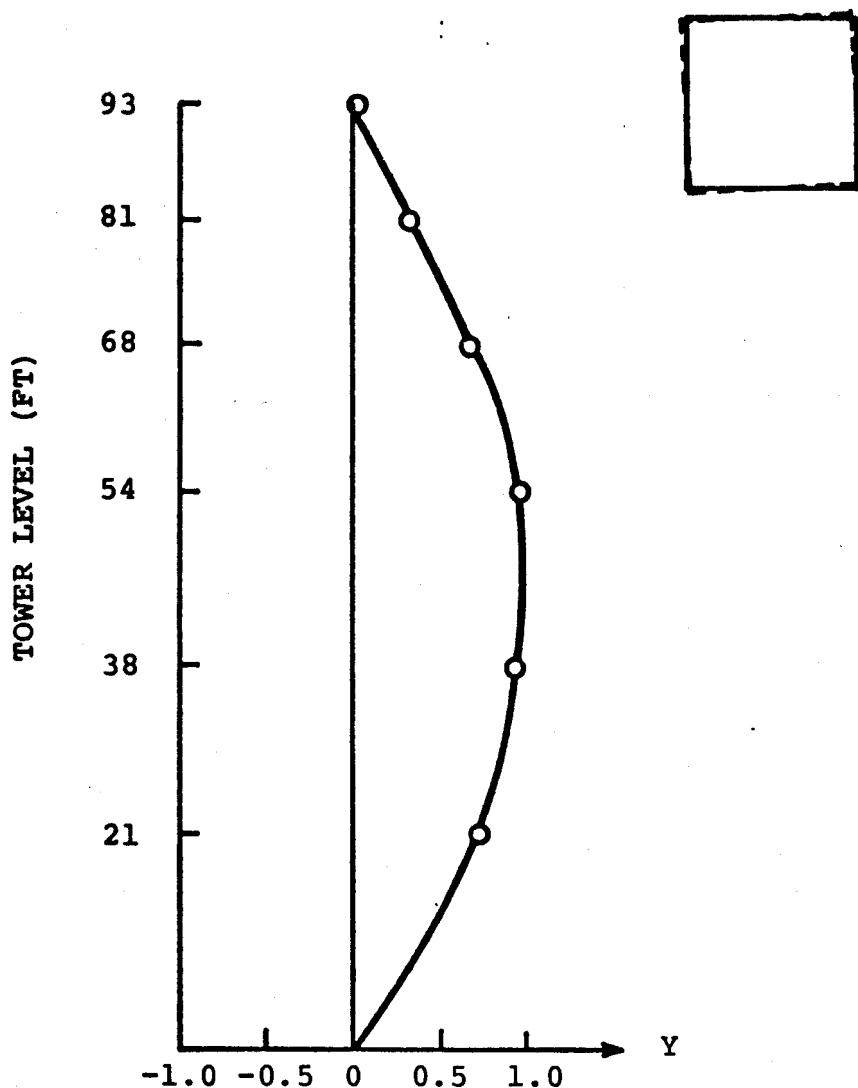
(c) FOURTH MODE (TORSION),  $f_4 = 7.29$  Hz

FIG. 3 MODE SHAPES (CONTINUED)



(c) FOURTH MODE (TORSION)

FIG. 3 MODE SHAPES (CONTINUED)



(d) FIFTH MODE (BEAM-WISE),  $f_5 = 11.07$  Hz

FIG. 3 MODE SHAPES (CONTINUED)

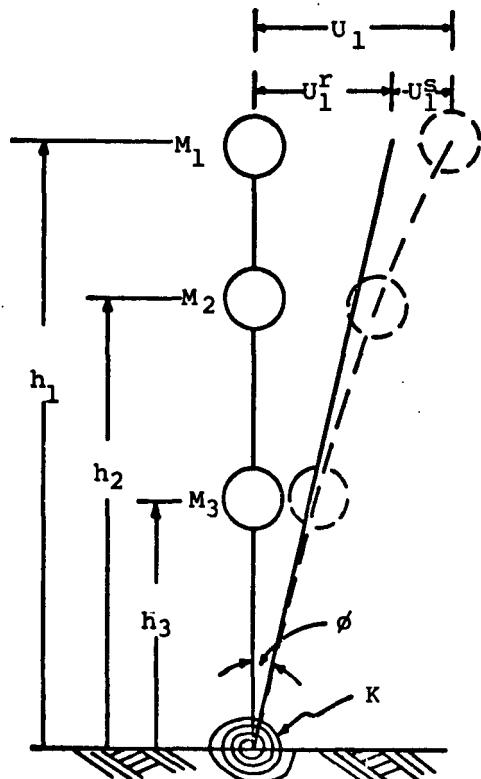


FIG. 4 A SIMPLE MODEL

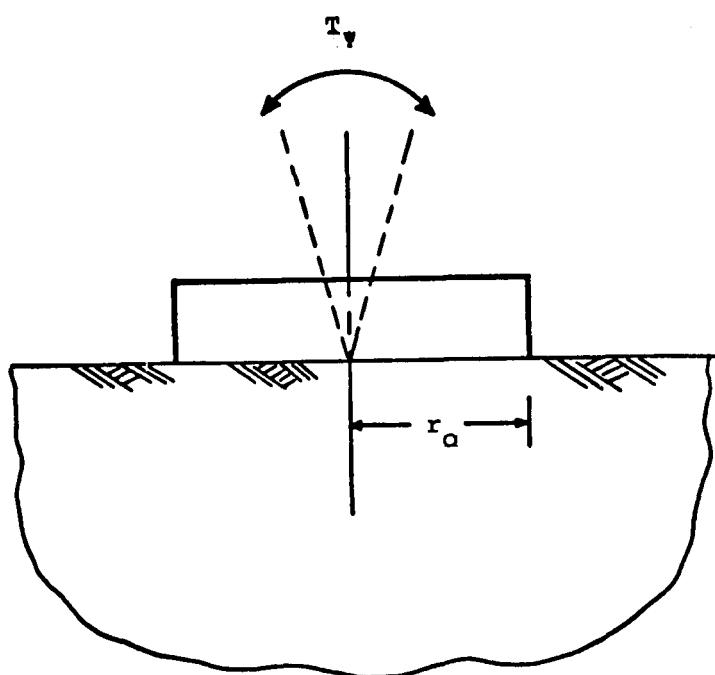


FIG. 5 ROCKING OF RIGID FOOTING  
ON ELASTIC HALF SPACE

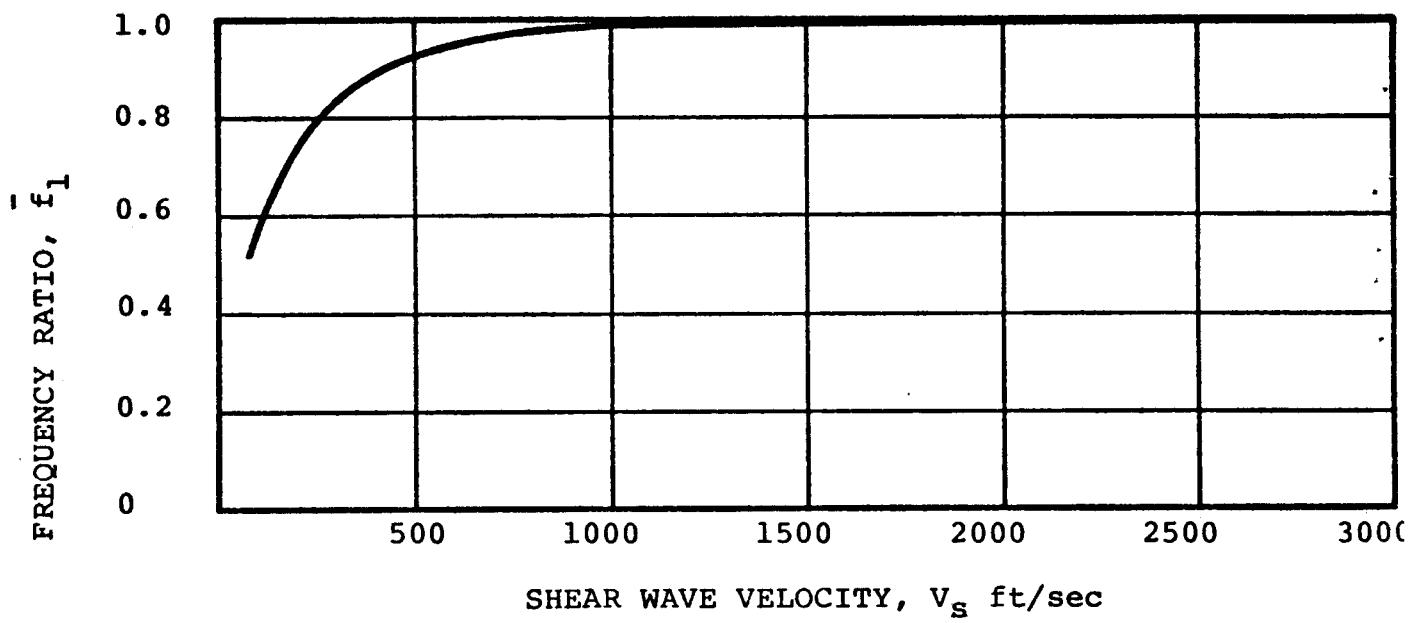


FIG. 6 FREQUENCY RATIO VS. SHEAR WAVE VELOCITY

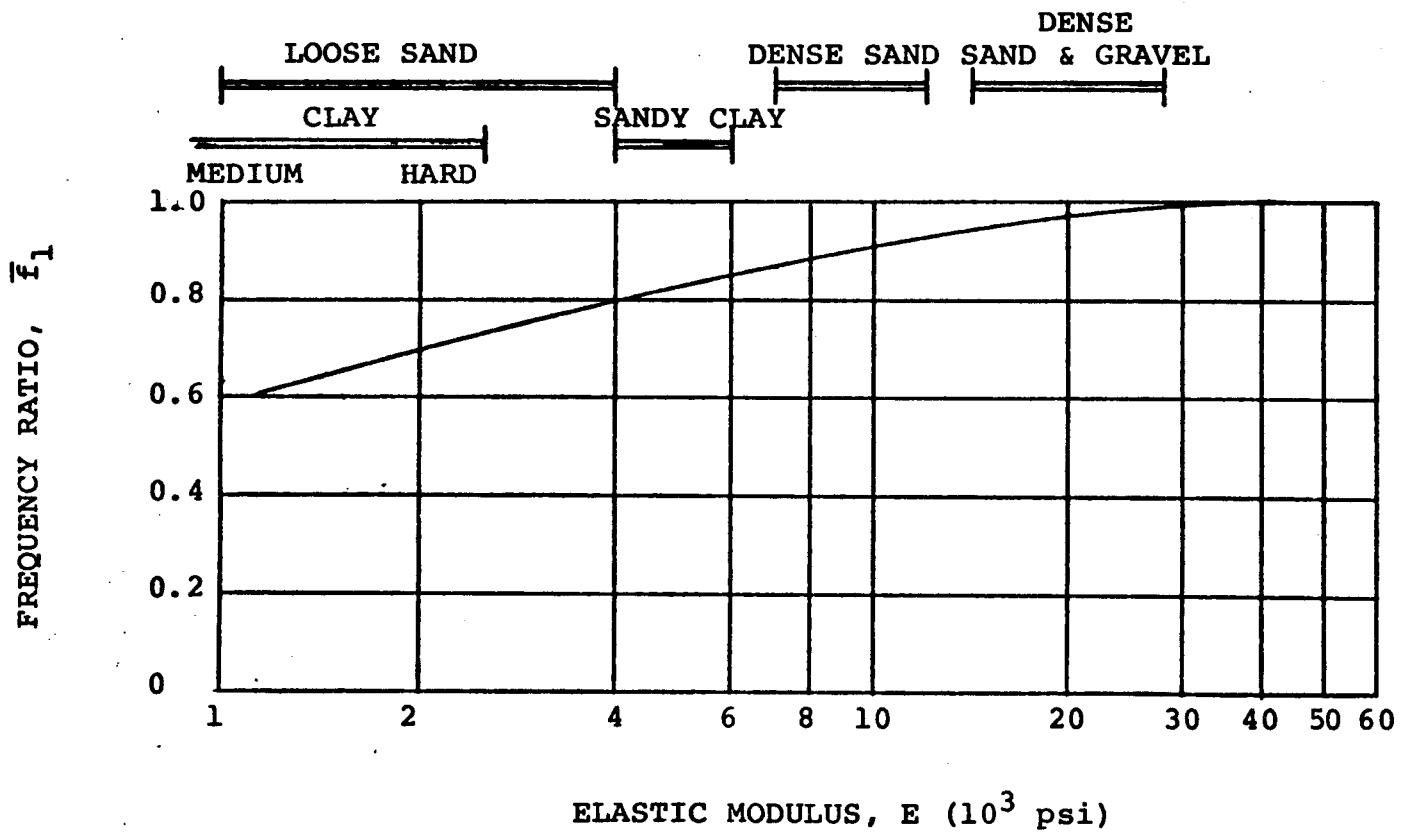


FIG. 7 FREQUENCY RATIO VS. ELASTIC MODULUS