γ

CONF-840530--3

UCRL--90749

DE84 012643

SHELL MODEL STUDY OF $^{90,88}Zr$

J. A. Becker and S. D. Bloom
Lawrence Livermore National Laboratory
University of California
Livermore, CA 94550

E. K. Warburton
Brookhaven National Laboratory
Upton, New York 11973

This paper was prepared for submittal to
InterNational Symposium on In-Beam
Nuclear Spectroscopy
Debrecen, Hungary
May 14-18, 1984

May 1984

SHELL MODEL STUDY OF ^{90,88}Zr

J. A. Becker and S. D. Bloom
Lawrence Livermore National Laboratory
University of California
Livermore, CA 94550

and

E. K. Warburton
Brookhaven National Laboratory
Upton, New York 11973

Conventional spherical shell model calculations have been undertaken to describe ^{90}Zr and ^{88}Zr. Valence orbitals included the $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, and $1g_{9/2}$ for protons, and $1g_{9/2}$ and $2d_{5/2}$ for neutrons. For ^{90}Zr, the number of $1g_{9/2}$ protons was $\leq$ 2. For the high spin even parity states of ^{90}Zr, two calculations were performed, one with $\leq$ 4 (0) $g_{9/2}$ protons (neutron holes) and one with $\leq$ 2 (2) $g_{9/2}$ protons (neutron holes). For ^{88}Zr, the number of particles in the $g_{9/2}$ shell was restricted to $\leq$ 10. For the high spin negative parity states, a calculation was done with up to 11 particles in the $g_{9/2}$ shell. A realistic two-body interaction was employed in this calculation. Predicted excitation energies are compared with experimental results, and for the lower lying positive parity states a comparison of electromagnetic transition rates is also made.

1. INTRODUCTION

The nuclear spectroscopy of ^{88}Zr and ^{90}Zr has been extended to excitation energies $E_x \sim 10$ MeV in a series of experiments done at the Brookhaven National Laboratory Tandem Van de Graaff Facility.[1] In these fusion-evaporation studies, ^{74,76}Ge were bombarded with ^{18}O beams of incident energies between 40 and 80 MeV. Measurements made on the resulting γ-rays included time coincidence, angular distribution, and linear polarization. Nuclear lifetimes were also

deduced using both the Doppler Shift technique and the Recoil-Distance technique. As a result many new nuclear level energies together with their decay modes, transition rates and spin-parity assignments are known to $E_x \sim 12.9$ MeV, $J^\pi = (20)$ in ^{90}Zr, and $E_x \sim 11.2$ MeV, $J^\pi = (21)$ in ^{88}Zr.

2. CALCULATION AND RESULTS

We have undertaken a large-scale shell model calculation of both ^{90}Zr and ^{88}Zr in order to understand the nuclear structure underlying these experimental results. The calculations reported here were done with a local, state independent two-body interaction with a single Yukawa form factor,

$$V_{12} = f(r)[V_0 + V_\sigma \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_\tau \vec{\tau}_1 \cdot \vec{\tau}_2 + V_{\sigma\tau}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\tau}_1 \cdot \vec{\tau}_2)] \quad . \qquad (1)$$

Parameters based on a "realistic" finite range potential[3] are given in Table 1. Single particle energies, adjusted within the model space so that single particle levels in neighboring odd-A nuclei are described, are given in Table 2. Effective charges and magnetic moments were used in the transition rate calculations. E2 rates were done with an additional nucleon charge $\delta = 1$. M1 rates were calculated with the free nucleon spin g factors ($g_s(\pi) = 5.59$, $g_s(\nu) = -3.83$) and an orbital g factor $g_\ell = g_\ell(\text{free}) + \delta g_\ell$, where $\delta g_\ell(\pi) = 0.10$ and $\delta g_\ell(\nu) = -0.05$.

Table 1. Parameters (in MeV) of the two-body potential. Range = 1.0 fm.

V_0	V_σ	V_τ	$V_{\sigma\tau}$
-36.2	6.23	17.8	12.1

Table 2. Single particle energies (MeV) relative to the ^{56}Ni core.

$\epsilon_{f5/2}$	$\epsilon_{p3/2}$	$\epsilon_{p1/2}$	$\epsilon_{g9/2}$	$\epsilon_{d5/2}$
2.30	3.31	5.13	2.98	6.77

^{90}Zr

^{90}Zr has a full $1f_{7/2}$, $1f_{5/2}$, $2p_{3/2}$ and $2p_{1/2}$ proton shell and a full $1g_{9/2}$ neutron shell. Low-lying excitations with positive parity are due to proton excitations from the fp shell to the $g_{9/2}$ orbital. In the model space $\pi[p_{1/2}^{-2} g_{9/2}^{+2}]$ excitations are restricted to $J \leq 8$, and earlier calculations[2] account for energy levels and transition rates within these restrictions. We have chosen to expand the model space in the following ways:

(I) $\pi[(f_{5/2}, p_{3/2}, p_{1/2})^{-2} g_{9/2}^{2}]$ for which $J_{max} = 12$,
(II) $\pi[(f_{5/2}, p_{3/2}, p_{1/2})^{-4} g_{9/2}^{4}]$ + I which has $J_{max} = 18$, and
(III) $\pi[(f_{5/2}, p_{3/2}, p_{1/2})^{-2} g_{9/2}^{+2}]$ $\nu[g_{9/2}^{-2} d_{5/2}^{2}]$ + I,

which was $J_{max} = 24$. Computational restrictions were such that $J_{min} = 8$ and 12 for (II) and (III), respectively. Odd-parity states were described in the model space $\pi[f_5, p_3, p_1)^{-n} g_{9/2}^{+n}]$, n = 1 or 3.

The predicted even- and odd-parity levels are compared with experiment in Figs. 1 and 2, respectively. For even parity levels, only the calculations done with Model I are illustrated in Fig. 1. An adequate description of states with J > 10 requires contributions from Models II and III. For example, the calculated binding energy of the first $J^{\pi} = 12^{+}$ state relative to the ^{56}Ni core is, apart from a constant bias, -222.668, -223.234, and -223.469 MeV for Models I, II, and III respectively. Technically, Model II and III calculations require a shift in main frame computers and we choose to wait for the completion of these calculations rather than present the limited results here. The agreement (within the model space I restriction) between experimental energy levels is reasonable for the even-parity states. The model predicts the level spin sequence correctly. Selected electromagnetic moments and transition rates are presented in Tables 3 and 4, respectively. Agreement is good for the first 8_1^+ state for both quadrupole and magnetic moments. The Model I predictions of B(E2) for the $8_1^+ \rightarrow 6_1^+$, and $2_1^+ \rightarrow 0_1^+$ are good for the $8_1^+ \rightarrow 6_1^+$ transition and off by a factor of 2 for the $2_1^+ \rightarrow 0_1^+$ transition. The electromagnetic transition strengths predicted by Model I are in accord with the γ-ray branching ratios observed for the 10_1^+ and 9_1^+ states; agreement is not good for the decay of the 8_2^+ state. Experimentally the $10_1^+ \rightarrow 8_1^+$ and the $9_1^+ \rightarrow 8_1^+$ γ-ray branches are 100%. The possible $10_1^+ \rightarrow 8_2^+$ and $9_1^+ \rightarrow 8_2^+$ branches are not observed. This is in accord with the transition strengths listed in Table 4. The experimental γ-ray branching of the 8_2^+ state is $[B.R.(8_2^+ \rightarrow 8_1^+)]/[B.R.(8_2^+ \rightarrow 6_1^+)] = 43/67$. The calculation predicts 27/1, not in good agreement.

Table 3. Quadrupole (e^2fm^4) and magnetic moments (μ_0^2) for ^{90}Zr and ^{88}Zr.

$J_\#^\pi$	A	Q(E2) Theory	Q(E2) Exp.	μ_0 Theory	μ_0 Exp.
8_1^+	90	-53.4	$\lvert 51(6) \rvert$[a]	+12.31	+10.85(5)[b]
8_2^+	90	+18.8		+0.31	
8_1^+	88	+25.6	$\lvert 51(6) \rvert$[a]	-3.09	-1.808(4)[c]
8_2^+	88	-72.9		+12.0	

[a] P. Raghavan, private communication.
[b] O. Haüsser, et al., Nucl. Phys. A293, 248 (1977).
[c] T. Faestermann, et al., Hyperfine Interactions 4, 196 (1978).

Table 4. ^{90}Zr electromagnetic transition rates (in Weisskopf units).

$J_i \rightarrow J_f$	Theory B(E2)	Theory B(M1)	Exp. B(E2)	Exp. (BM1)
$10_1^+ \rightarrow 8_1^+$	4.57			
$10_1^+ \rightarrow 8_2^+$	0.42			
$9_1^+ \rightarrow 8_1^+$	3.86	0.10		
$9_1^+ \rightarrow 8_2^+$	0.66	0.03		
$8_2^+ \rightarrow 8_1^+$	0.40	0.29		
$8_2^+ \rightarrow 6_1^+$	2.94			
$8_1^+ \rightarrow 6_1^+$	2.90		3.13(9)[a]	
$6_1^+ \rightarrow 4_1^+$	7.86			
$4_1^+ \rightarrow 2_1^+$	11.40			
$2_1^+ \rightarrow 0_1^+$	2.12		5.38(17)[b]	

[a] O. Haüsser, et al., Nucl. Phys. A293, 248 (1977).
[b] J. Heisenberg, et al., Phys. Rev. C 29, 97 (1984).

The predicted negative parity energy levels are compared with experiment in Fig. 2, normalized to the energy of the 5_1^- state. [The calculated excitation of the 5_1^- state is 1.818 MeV.] There is reasonable agreement with respect to level spacing and spin sequence.

^{88}Zr

Zr^{88} has a full $1f_{7/2}$, $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$ shell for protons and 8 neutrons in the $g_{9/2}$ orbital. We describe the low lying even parity states of ^{88}Zr in the model space $\pi\nu[(f_{5/2}, p_{3/2}, p_{1/2})^{-2} g_{9/2}{}^{10}]$, which has $J_{max} = 20$. Odd-parity states are described for $J \geq 0$ in the model space I $\pi\nu[(f_{5/2}, p_{3/2}, p_{1/2})^{-1} g_{9/2}{}^{9}]$, while for $J \geq 12$ the model space II was $\pi\nu[(f_{5/2}, p_{3/2}, p_{1/2})^{-3} g_{9/2}{}^{11}]$. These models have J_{max} = 15 and 24, respectively. Results for excitation energies are given in Figures 3 and 4, for even- and odd-parity states respectively. The positive parity energy level spectrum is in good agreement with experiment. Note $E_x(9_1^+) > E_x(10_1^+)$ as is observed experimentally. The energy sequence of the negative parity levels has been normalized to the excitation energy of the first 5^- state. [The calculated excitation of the 5^- state is 3.437 MeV.] Model II has been joined onto Model I by normalization to the first 12^- state. [The binding of the 12^- states relative to the ^{56}Ni core is, apart from a constant bias, -203.666 and -204.977 MeV, for Model I and II, respectively.] The spectrum of negative parity states is compressed relative to experiment.

Table 5. ^{88}Zr electromagnetic transition rates (in Weisskopf units).[a]

$J_i \rightarrow J_f$	Theory B(E2)	Theory B(M1)	Exp. B(E2)	Exp. B(M1)
$12_2^+ \rightarrow 12_1^+$	0.51	3.10		0.95(14)
$12_2^+ \rightarrow 11_1^+$	3.79	0.23		
$12_1^+ \rightarrow 11_1^+$	8.09	1.17		
$12_2^+ \rightarrow 10_1^+$	2.84			
$12_1^+ \rightarrow 10_1^+$	27.80		6.45(43)	
$10_1^+ \rightarrow 8_1^+$	0.018		>0.04	
$10_1^+ \rightarrow 8_2^+$	7.56		>15.4	
$9_1^+ \rightarrow 10_1^+$	1.52	6.06		
$9_1^+ \rightarrow 8_1^+$	0.037	5×10^{-4}		
$9_1^+ \rightarrow 8_2^+$	7.44	0.60		
$8_2^+ \rightarrow 8_1^+$	5.4×10^{-4}	6.3×10^{-5}	0.8(6/9)	$8.1(6) \times 10^{-3}$
$8_2^+ \rightarrow 6_1^+$	1.0×10^{-3}			
$8_1^+ \rightarrow 6_1^+$	0.86		5.16(60)[b]	
$6_1^+ \rightarrow 4_1^+$	1.54			
$4_1^+ \rightarrow 2_1^+$	3.16			
$2_1^+ \rightarrow 0_1^+$	2.59			

[a] Experimental values from Ref. 1 unless noted.
[b] M. Ishihama, et al., Phys. Lett. 35B, 398 (1971).

Selected electromagnetic moments and transition rates are presented in Tables 3 and 5, respectively. Agreement for the electric quadrupole moment and magnetic moment for the 8_1^+ state is not very good for ^{88}Zr(8_1^+). There are several electromagnetic transition rates that can be compared: The calculated value of B(E2; $8_1^+ \rightarrow 6_1^+$) is too small by a factor 6. A strong M1 transition $12_2^+ \rightarrow 12_1^+$ is measured; the model predicts B(M1) = 3.10 Wu while the experimental value is 0.95 Wu. The model also predicts B(E2; $10_1^+ \rightarrow 8_2^+$) = 7.56 Wu; the experimental observation is B(E2) $\gtrsim$ 15.4 Wu. The $8_2^+ \rightarrow 8_1^+$ transition has B(M1) = $8.1(6) \times 10^{-3}$; the calculation predicts B(M1) = 6.3×10^{-5}. Some γ-ray branching can also be compared with calculated values. The $10_1^+ \rightarrow 8_1^+$ decay is observed with a 1.8(3)% branch. The model predicts a 2% branch relative to the $10_1^+ \rightarrow 8_2^+$ decay. The 9_1^+ state is observed to decay to the 10_1^+, 8_2^+, and 8_1^+ states with branching ratios 1.8(5), 94.4(9), and 3.8(8)%. The model predicts that the ratio of branching ratios, $[B.R.(9_1^+ \rightarrow 10_1^+)]/[B.R.(9_1^+ \rightarrow 8_2^+)] = 1/23$ and $[B.R.(9_1^+ \rightarrow 8_1^+)]/[B.R.(9_1^+ \rightarrow 8_2^+)] = 1/411$. The 8_2^+ state is observed to decay 100% $8_2^+ \rightarrow 8_1^+$ with no reported $8_2^+ \rightarrow 6_1^+$ branch. The calculation predicts $[B.R.(8_2^+ \rightarrow 8_1^+)]/[B.R.(8_2^+ \rightarrow 6_1^+)] = 134/1$.

3. SUMMARY

A shell model calculation of ^{90}Zr and ^{88}Zr has been done with a "realistic" two-body interaction. A large model space which however does not allow E1 γ-ray transitions was used. Good accounts of the observed electromagnetic moments and transitions of the lower lying positive parity levels are obtained, in particular the decay of the 10_1^+ states for both nuclei. For ^{88}Zr, the strong $12_2^+ \rightarrow 12_1^+$ is identified correctly. Our calculation predicts that the model space required for ^{90}Zr high spin even parity states ($J \geq 11$) requires the breaking of the $g_{9/2}$ neutron shell.

4. ACKNOWLEDGEMENTS

This work was performed in part under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under contract No. W-7405-ENG-48, and in part by the U.S. Department of Energy, Division of Basic Energy Sciences, under contract No. DE-AC02-76CH00016.

5. REFERENCES

[1] E. K. Warburton, J. W. Olness, C. J. Lister, R. W. Zurmühle, and J. A. Becker (to be published).

[2] F. Petrovich, H. McManus, V. A. Madsen, and J. Atkinson, Phys. Rev. Lett. 22, 895 (1969).

[3] R. F. Hausman, Jr., UCRL-52178, (1976).

[4] D. H. Gloeckner and F. J. D. Serduke, Nucl. Phys. A220, 477 (1974).

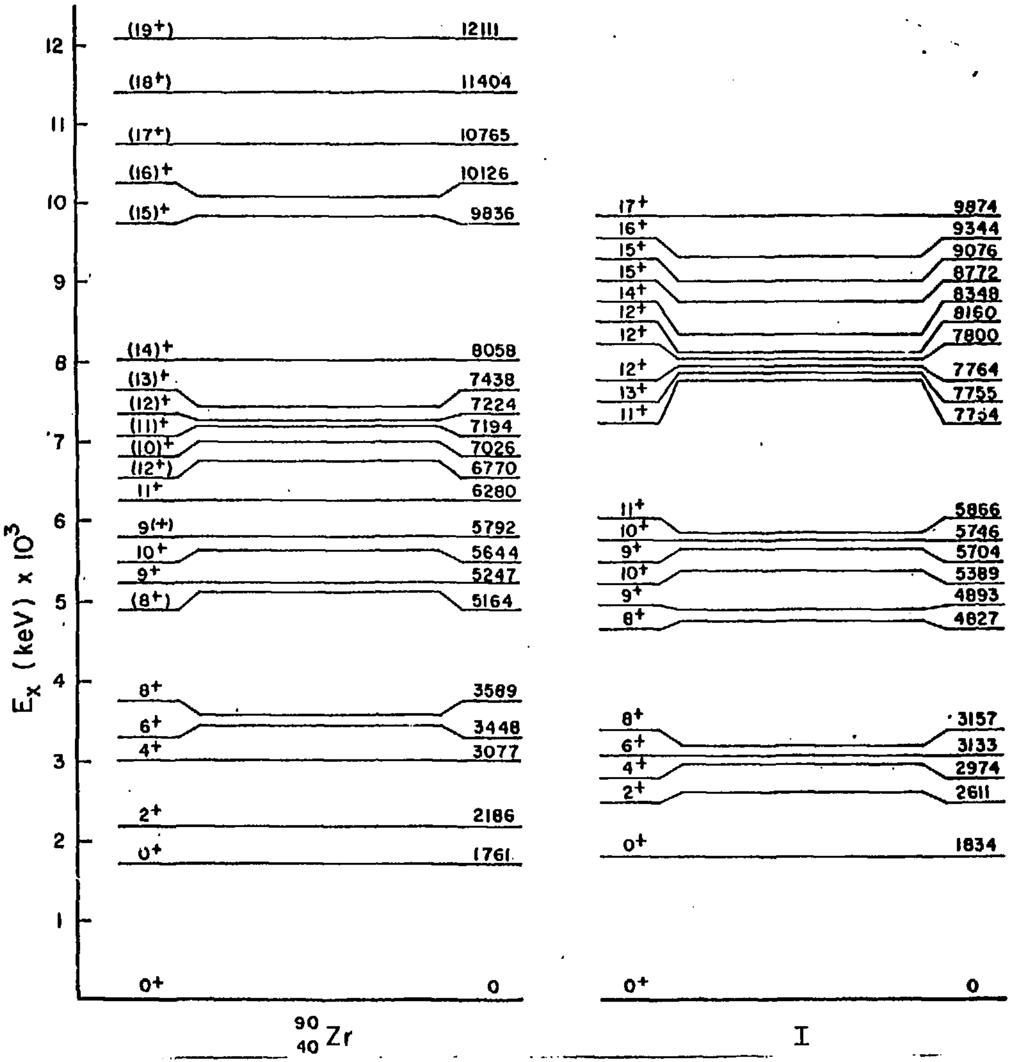

Fig. 1. Experimental and calculated ^{90}Zr partial level schemes for π = +.

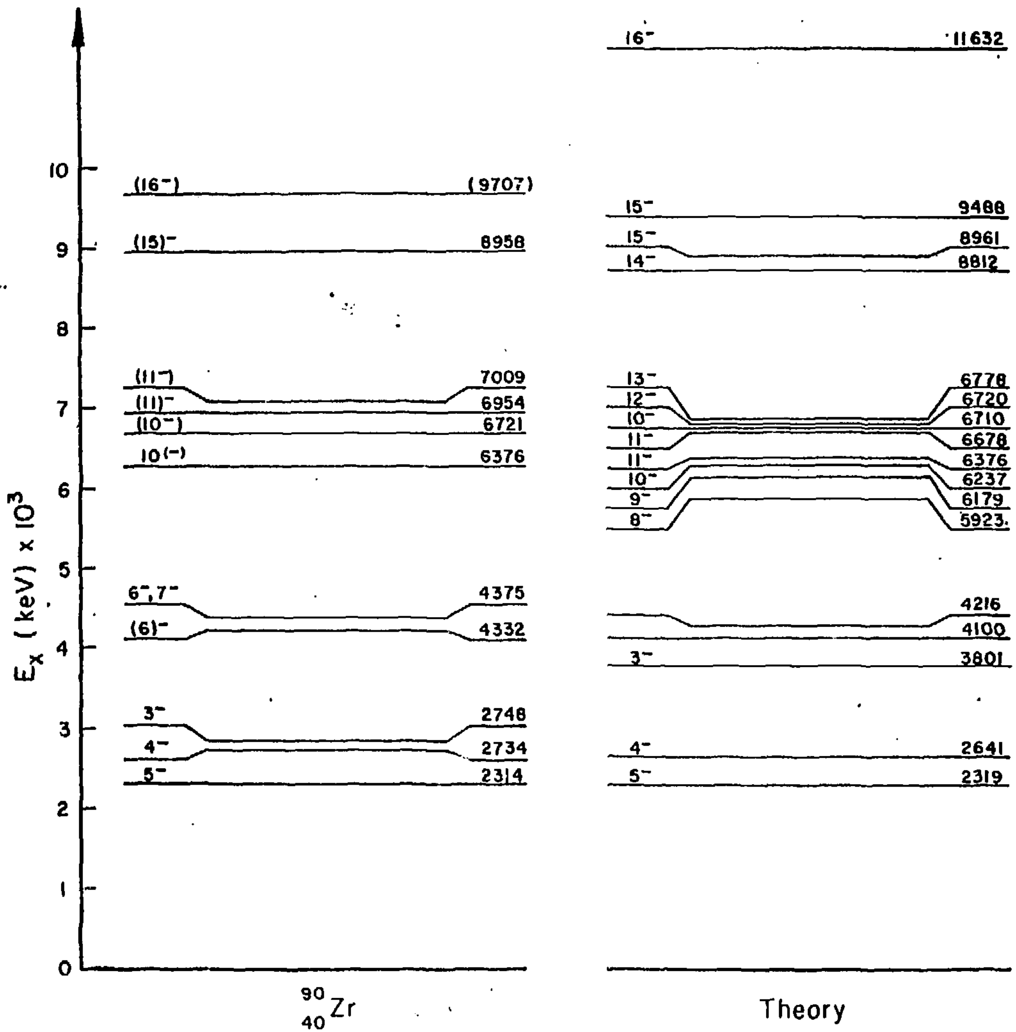

Fig. 2. Experimental and calculated ^{90}Zr partial level schemes for π = –.

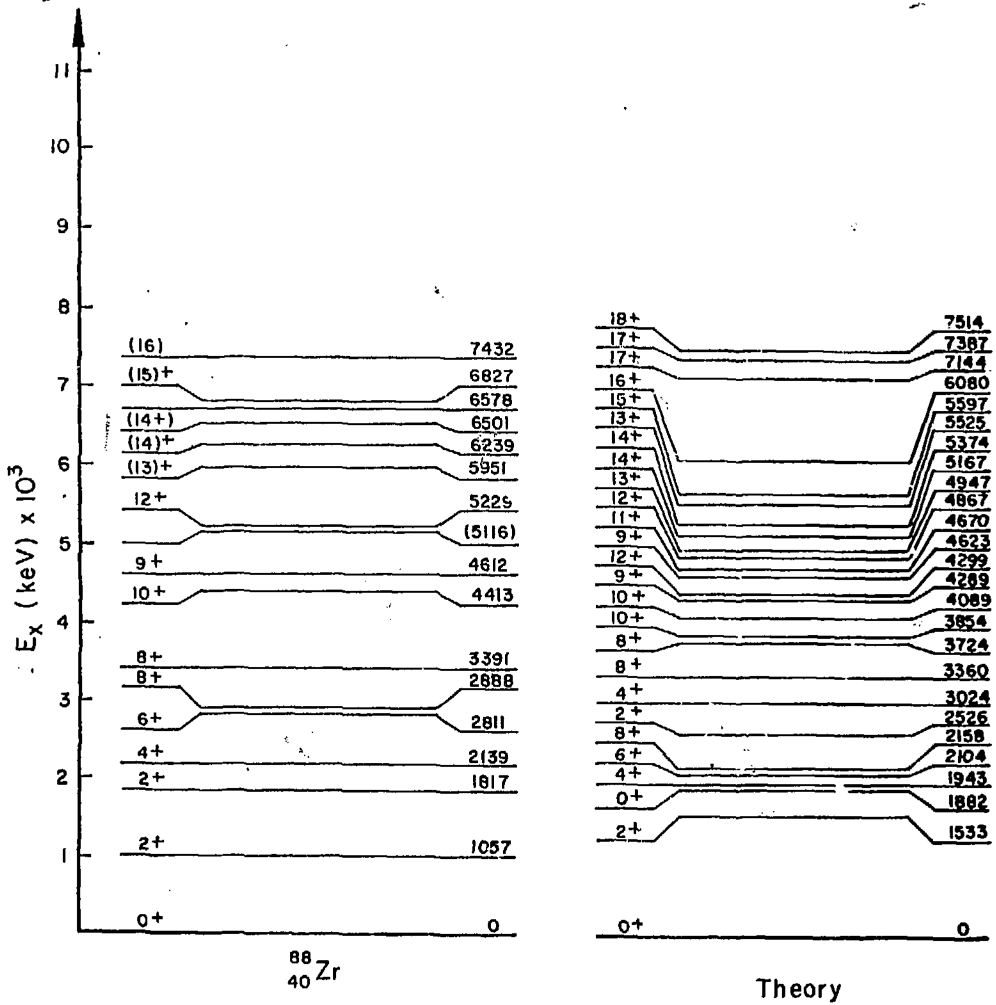

Fig. 3. Experimental and calculated ^{88}Zr partial level schemes for $\pi = +$.

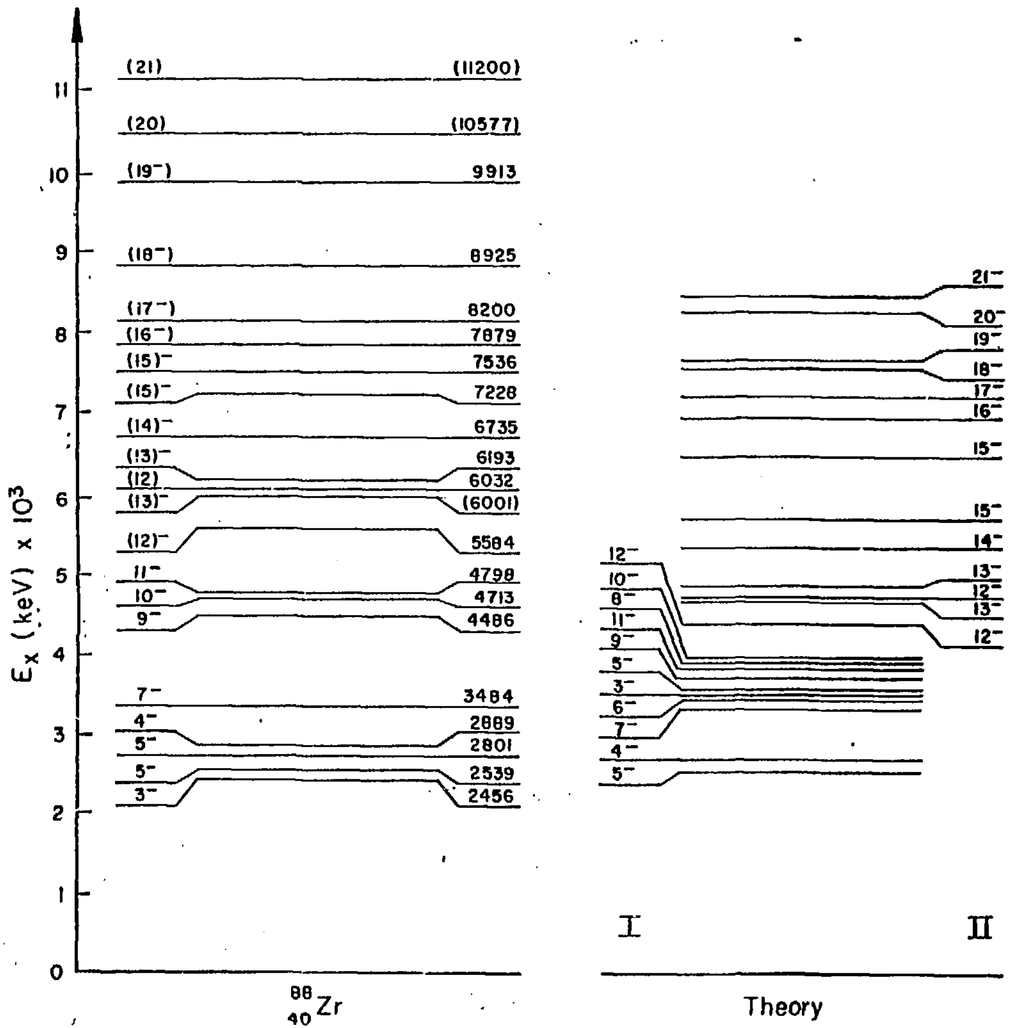

Fig. 4. Experimental and calculated ^{88}Zr partial level schemes for $\pi = -$.