

Progress Report
Sept. 1, 1976 - August 31, 1977

*Study of Structure
of Light Nuclei
with Neutrons*

MASTER

Ohio University

Athens, Ohio

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STUDY OF STRUCTURE OF LIGHT NUCLEI
WITH NEUTRONS

Progress Report

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Ohio University
Athens, Ohio

September 1, 1976 — August 30, 1977

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ABSTRACT

Final differential elastic neutron scattering cross sections have been obtained for a large number of energies and angles for ${}^6\text{Li}$, ${}^7\text{Li}$, ${}^{10}\text{B}$ and ${}^{11}\text{B}$ for energies $4 \text{ MeV} \leq E_n \leq 8 \text{ MeV}$. These results are compared with ENDF/B values and indicate for ${}^7\text{Li}$, at least, a need for revision of that evaluation. The tritium-breeding cross sections for ${}^7\text{Li}(n,n')\alpha$ is deduced from these data on the total cross section for ${}^7\text{Li}$ and the result is compared to other results. The differential neutron inelastic scattering from the 0.478 MeV state in ${}^7\text{Li}$ has been measured for the first time for these energies and was separated from the elastic by an analytical procedure and compared to ENDF/B. The extensive differential cross sections for ${}^{11}\text{B}$ have been analyzed by an improved R-matrix program and new properties of states in the compound nucleus ${}^{12}\text{B}$ were derived. These cross sections are of value to the fusion reactor program in tritium breeding and shielding considerations.

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I. INTRODUCTION

In the course of this third year of the contract a considerable effort was given to the extensive analysis and calculations necessary both to provide final differential cross sections to the nuclear community and data centers, and to obtain maximum analysis and theoretical interpretation value from these data. Recent experiments and those scheduled for the remaining quarter of the contract year have been designed to complete a series of low energy measurements with improved techniques for pulse shape discrimination. This is necessary for maximum effectiveness in the measurement of neutron inelastic and emission spectra down to low energies for the emitted neutrons, and in the remaining low energy elastic scattering measurements.

The final numerical differential elastic scattering cross sections for ^{10}B for $4 \text{ MeV} \leq E_n \leq 8 \text{ MeV}$ were published and widely distributed in the form of an Ohio University Report C00-2490-5,¹⁾ and a condensation of this has been accepted for publication in *Nuclear Science and Engineering*.²⁾

A very comprehensive study of differential elastic scattering cross sections has been made for ^{11}B in the same energy range culminating in a Ph.D. dissertation that developed a new expanded R-matrix program and yielded, from the fitting of data at these energies, several new assignments for the compound nucleus ^{12}B as well as one consistent set of parameters fitting the entire energy range $0.1 \text{ MeV} \leq E_n \leq 8.0 \text{ MeV}$. These experimental and theoretical results were also published in a comprehensive Ohio University Report C00-2490-6³⁾ and circulated to the nuclear community with emphasis on evaluators and groups working on fusion reactor shielding designs, some of which employ boron as a major constituent. Comparison of these integrated elastic cross sections

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for ^{11}B with new high-accuracy total cross sections indicated that the inelastic cross section may, in certain energy intervals, be considerably higher than previously known, which would increase the value of boron ($\approx 80\%$ ^{11}B) as a shielding material. Since hardly any data exist for direct inelastic measurements in this region, these inelastic measurements (along with the ^{10}B) are high on our priorities for the months ahead.

The data on ^7Li were obtained on natural lithium with minor isotope corrections made from simultaneous ^6Li (96%) measurements. While the ^6Li elastic data are free of any further corrections, the first inelastic (0.478) group was not completely separated from the elastic for ^7Li . This was caused not only by the flight-path resolution (which can and will be improved) but to a large measure by the presence of the ^6Li (7.5%) elastic peak which at most energies and angles appears, most unfortunately, in the valley between the elastic and inelastic peaks for ^7Li . After long paper processing and procurement delays at ORNL, highly enriched ^6Li and ^7Li samples have finally been received and resolved differential elastic and inelastic measurements on ^6Li and ^7Li are planned for the months ahead.

The sum of the elastic and first inelastic groups for ^7Li are of good accuracy and are in excellent agreement with the very few other measurements available (which also measure only the sum). Subtraction of our integrated cross sections (for the sum) from known total cross sections can give an alternative check on the important but little-measured $^7\text{Li}(n,n')$ at tritium breeding reaction for fusion reactors. In the course of pursuing this it became apparent that the total cross section for ^7Li in ENDF-B/IV was probably low by as much as 2% between 5 MeV and 7 MeV. We have prevailed upon Jack Harvey at ORNL to run ^7Li at ORELA again for high absolute accuracy and

better energy resolutions for use in a combined consistent analysis of total, elastic and inelastic scattering for ^7Li .

II. DIFFERENTIAL SCATTERING CROSS SECTIONS AND ANALYSES

A. Experimental and Data Reduction Systems

The experimental apparatus and data reduction systems were described thoroughly in our previous Progress Report C00-2490-3 and in our Ohio University Report C00-2490-6³⁾ and we refer the reader to that for a full description. The on-line data reduction program has been improved and we are currently looking into methods to improve our pulse shape discriminator system at low neutron energies to be able to better detect low energy neutrons in the emission spectrum and do some needed low energy elastic scattering, especially on ^{13}C and some other cases where data are sparse or need rechecking.

B. $^6\text{Li}+n$ Differential Cross Sections (H. Knox, R. White)

Final analysis of the $^6\text{Li}+n$ differential elastic cross section data reported earlier⁴⁾ has been completed. A comparison of our integrated elastic cross sections with the ENDF/B-IV evaluated elastic cross section is shown in Figure 1, along with the data of Hopkins et al.⁵⁾ This ENDF evaluation in this region is based on the earlier evaluation of Hopkins et al. and appears to be quite satisfactory up to about 6.9 MeV where the present data lie 6% to 10% below the evaluated curve.

With the elastic measurements completed in this energy region, we plan next to undertake a series of measurements of neutron inelastic (2.18 MeV

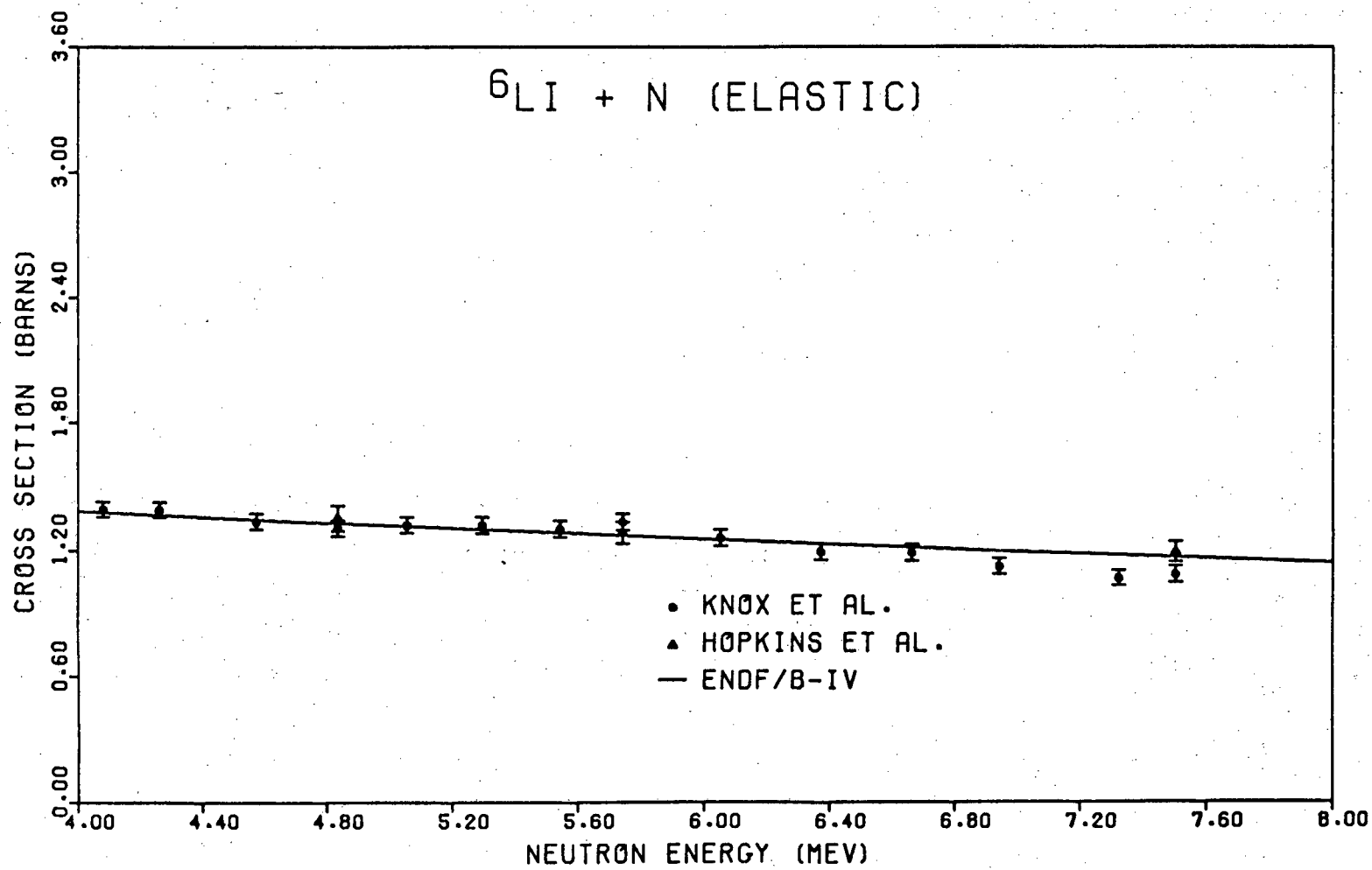


Figure 1.

level) differential cross sections on ${}^6\text{Li}$. In order to obtain the best inelastic data free from ${}^7\text{Li}$ contaminant, we ordered a high enrichment ${}^6\text{Li}$ sample from ORNL. After several months of delays the samples were finally fabricated. We have now received this highly enriched (99.30%) ${}^6\text{Li}$ scattering sample, and will be using it in these upcoming experiments.

C. ${}^7\text{Li}+n$ Differential Cross Sections (H. Knox, R. White)

The final analysis of the ${}^7\text{Li}+n$ differential scattering data reported earlier^{4,6)} has been completed. A plot of the integrated cross sections for elastic plus inelastic (0.478 MeV level) scattering is shown in Figure 2 along with the earlier work of Hopkins et al.⁵⁾ and the current ENDF/B-IV evaluation. The present measurements are in agreement with those of Hopkins et al. and indicate that some revisions in the present evaluation should be made.

In principle, the integrated cross sections shown in Figure 2 can be used to obtain values for the ${}^7\text{Li}(n,n')\alpha t$ reaction or tritium breeding cross section by subtraction from values for the ${}^7\text{Li}$ total neutron cross section. Below about 8 MeV, all neutron reactions on ${}^7\text{Li}$ other than elastic scattering and inelastic scattering to the 0.478 MeV level result in the production of a triton. By subtracting the sum of the integrated elastic and this inelastic cross section from the total cross section, the tritium breeding cross section can be obtained. A problem inherent in this method, however, is that in regions of sharp resonances, the difference cross section could show false structure if the resolutions of the total cross section measurements and the integrated cross section measurements are different. The total cross section measurements on ${}^7\text{Li}$ to date have shown only very

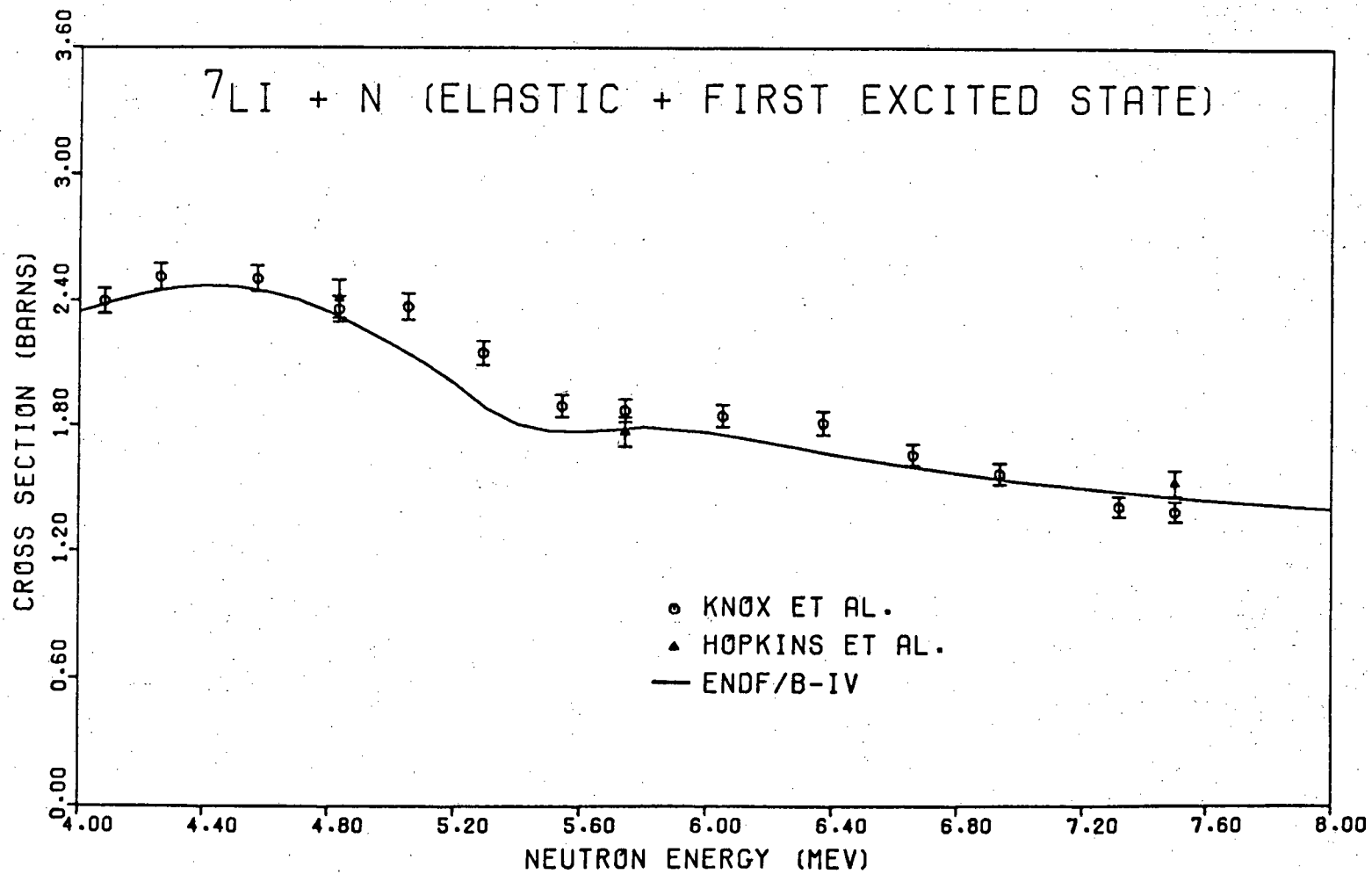


Figure 2.

broad resonance structure in the 4 MeV to 8 MeV energy region. A very narrow state in ^8Li has, however, been observed in charged particle experiments using the $^9\text{Be}(t,\alpha)^8\text{Li}$ reaction.⁷⁾ This state lies at an excitation energy of 6.53 MeV and should be observed at about 5.1 MeV incident energy in neutron experiments. This state has not been observed in neutron total cross section measurements but this is possibly due to insufficient energy resolution.

Figures 3 and 4 show the results of two measurements of the ^7Li total cross section, one by Goulding et al.⁸⁾ at RPI and the other by Jack Harvey⁹⁾ at Oak Ridge. The latter are preliminary and unpublished data. The agreement between these two data sets is remarkably good. The solid curve is the current ENDF evaluation of the total cross section and is 1% - 2% lower than both sets of data at some energies. Figure 5 shows again the data of Goulding et al., but the curve here is a hand drawn curve which fits both sets of data better than the ENDF evaluation.

Using values from this curve for the total cross section and the integrated cross sections from the present measurements, values of the ^7Li tritium breeding cross section have been obtained. These are shown in Figure 6 along with the data of Rosen and Stewart¹⁰⁾ and the ENDF evaluation of this cross section. From 4 MeV to 5 MeV, the present data agree well with the ENDF evaluation. The two points at 5.05 MeV and 5.29 MeV are exceptionally low. These low values are probably caused by the different resolutions of the total cross section measurements and the present differential cross section data over the region of the narrow resonance at 5.1 MeV. In the region from 5.69 MeV to 6.94 MeV, the present results are lower than the ENDF evaluation, but are generally within the errors of the data of Rosen and Stewart. Finally, between 7 MeV and 7.5 MeV, the present data are higher than ENDF,

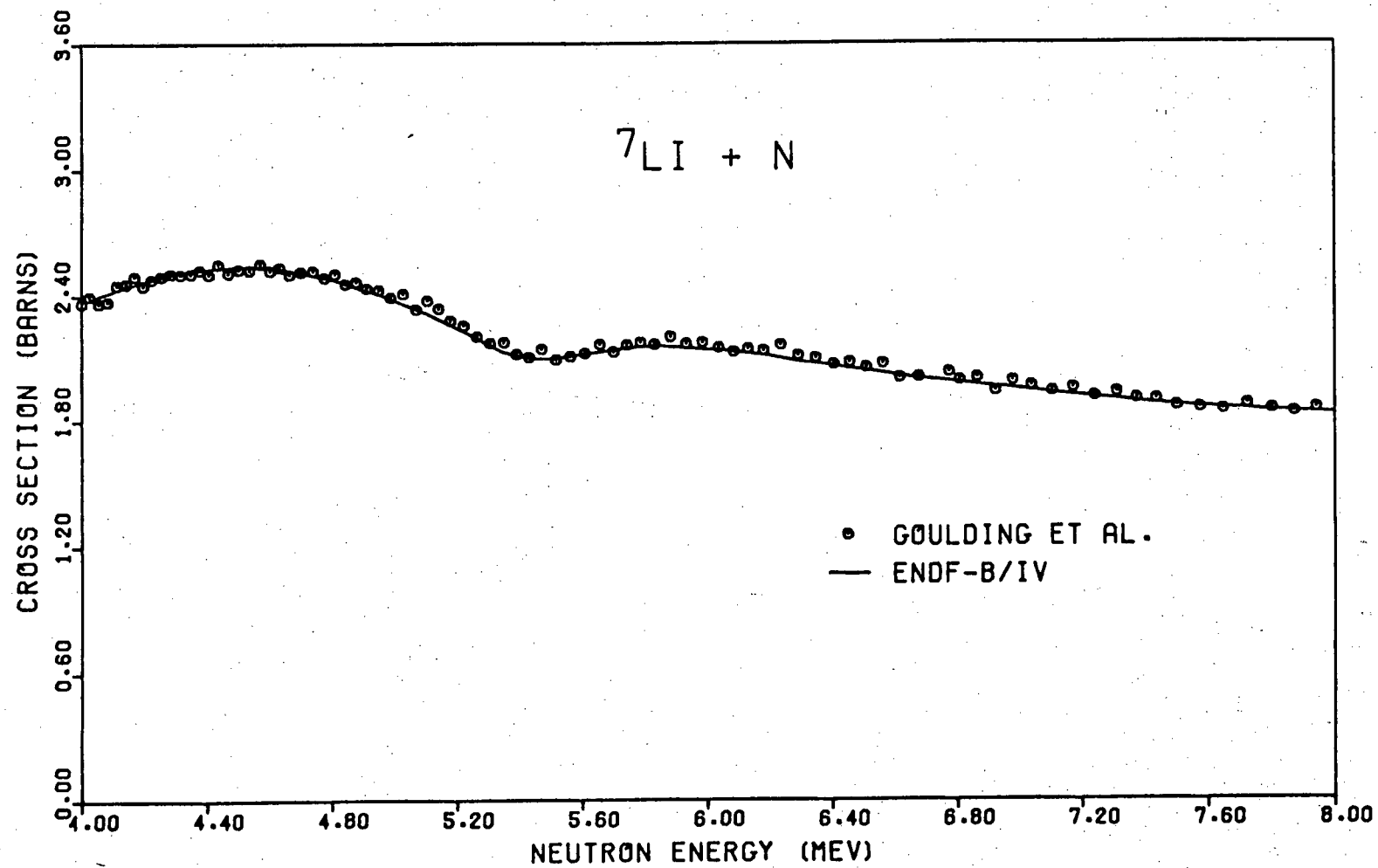


Figure 3.

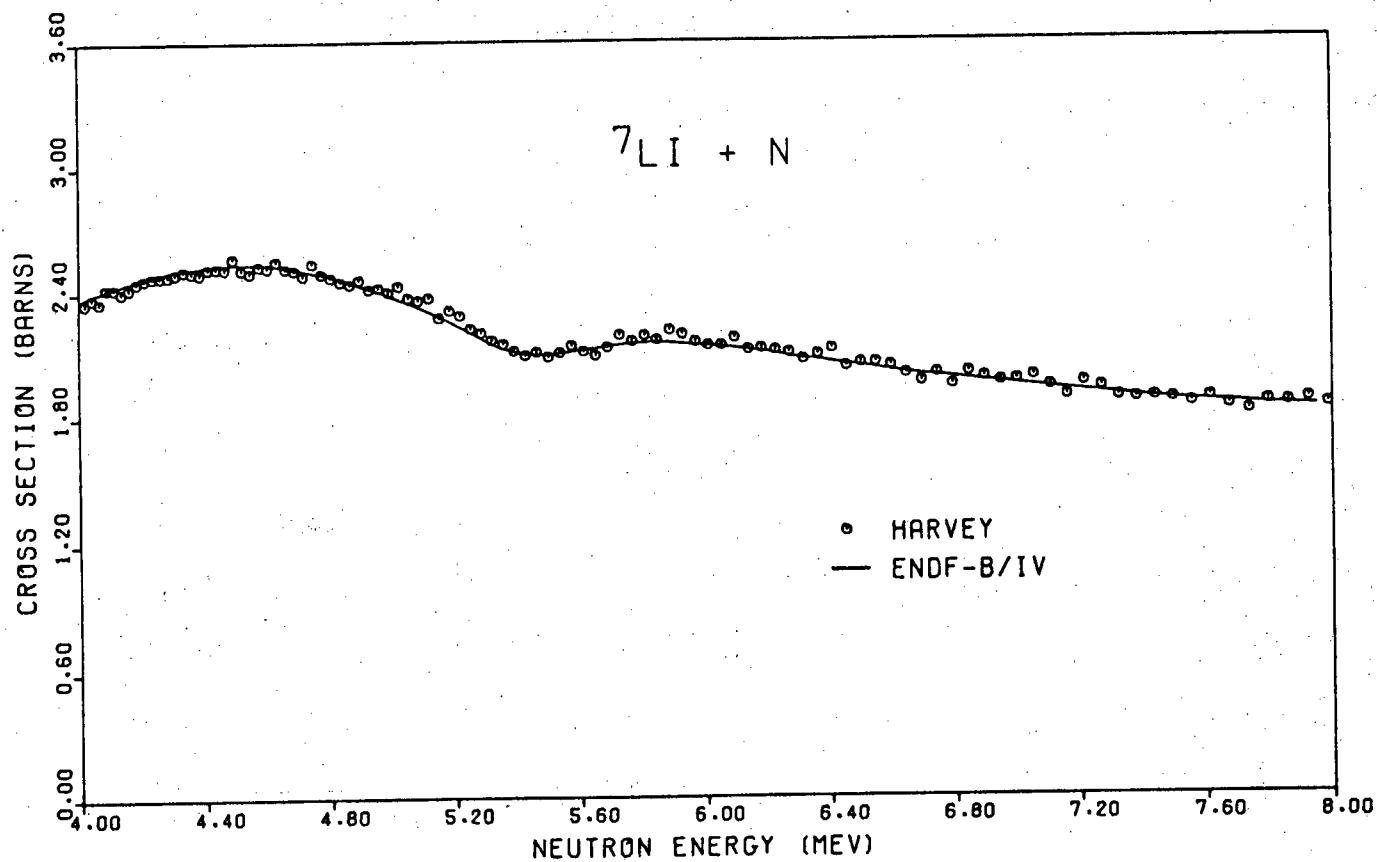


Figure 4.

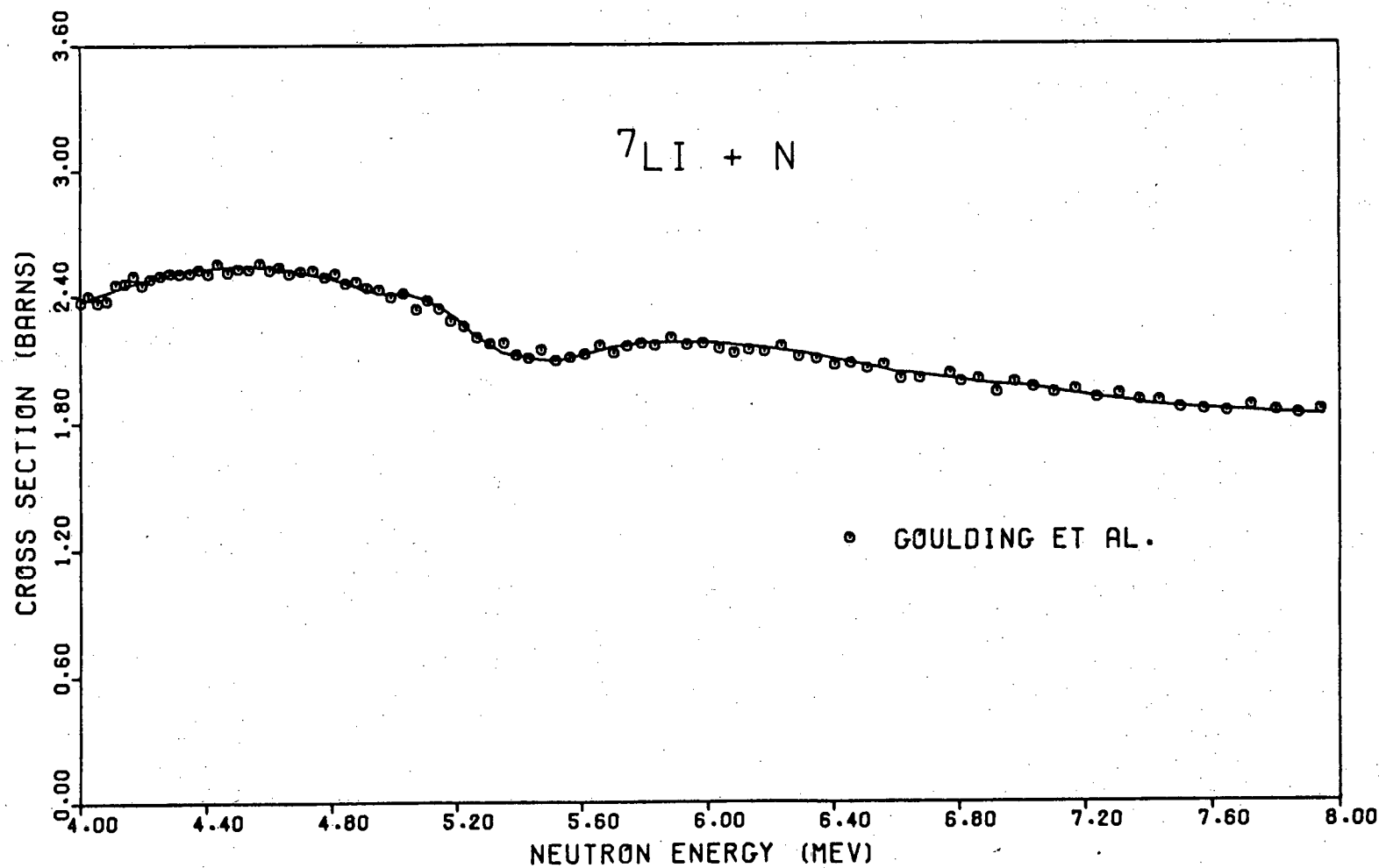


Figure 5.

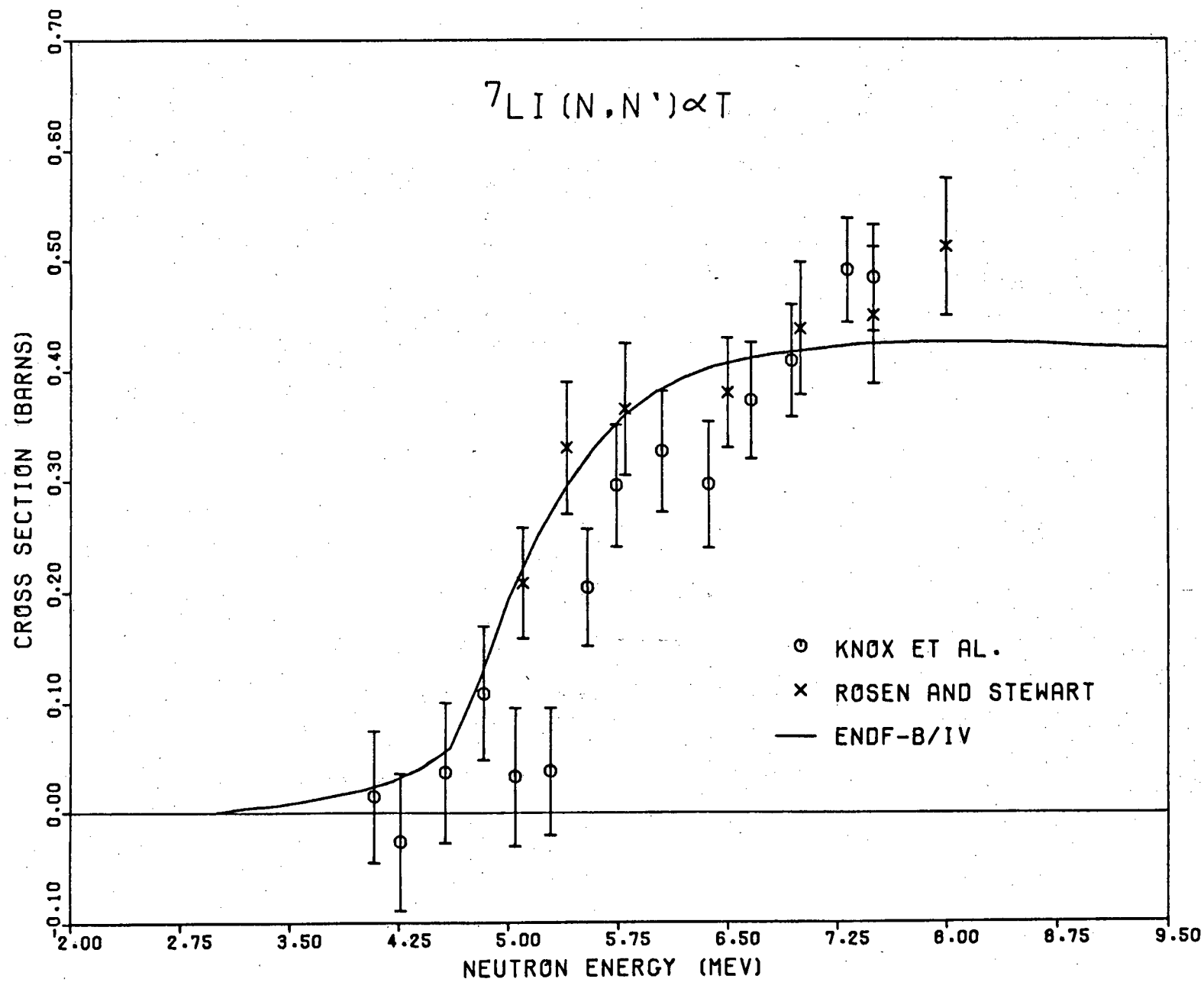


Figure 6.

but this is also the trend of the data of Rosen and Stewart.

These final results are the first values of the ${}^7\text{Li}(n,n')$ at cross section to be obtained using this method. The method requires a high degree of consistency between several different types of data and consequently provides a close check on existing data and evaluations. The present work has shown three results. First, the existing ENDF evaluation of the total neutron cross section is low by as much as 2% in regions between 5 MeV and 7 MeV. Second, there is definite resonance structure in this energy region that can be observed only with measurements having better resolution than those done to date. Toward that end, higher resolution total cross section measurements will be made at Oak Ridge in the next few months by Jack Harvey. Third, in the energy region between 7 MeV and 7.5 MeV, the true cross section for the ${}^7\text{Li}(n,n')$ at reaction is probably higher than the current ENDF evaluation and in agreement with the earlier measurements of Rosen and Stewart.

In our previous progress report, work on obtaining cross sections for neutrons inelastically scattered to the 478 keV level in ${}^7\text{Li}$ was described. Though complete separation of the elastic group and the inelastic group was not possible in these measurements, these cross sections were extracted by curve fitting procedures. The resulting differential cross sections were found to be nearly isotropic at energies near 4 MeV and were characterized by increasingly more negative B_2 Legendre expansion coefficients as the incident neutron energy increases to 7.5 MeV. A comparison of the present integrated inelastic cross section with the current ENDF/B-IV evaluation for this cross section is shown in Figure 7. Counting statistics and experimental uncertainties account for about half of the large error bars (up to 20%) on the present data, while uncertainties in the parameters obtained from the

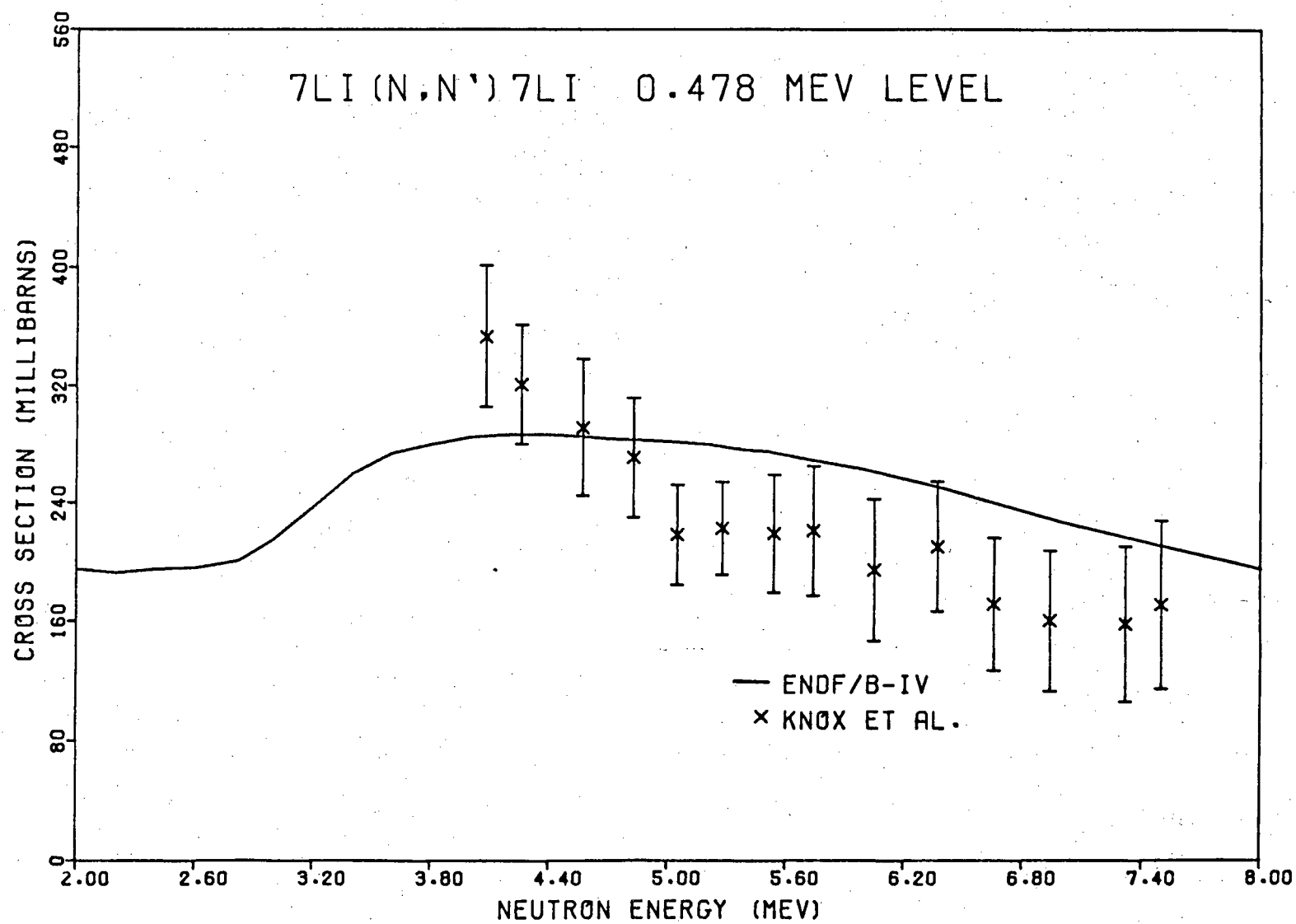


Figure 7.

non-linear fitting procedure account for the remainder. A great deal of the difficulty encountered in the fitting was due to the ^6Li minor isotope contamination in the ^7Li sample, since the ^6Li elastic group falls between the ^7Li elastic and inelastic groups as shown in Figure 8.

In the next few months we plan to begin experiments to re-measure this inelastic cross section. We have obtained a new highly enriched (>99.99%) ^7Li scattering sample from Oak Ridge. This, coupled with longer flight path, better neutron detectors, and better pulse shape discrimination, will greatly aid the improvement of our present measurement of this cross section.

D. $^{10}\text{B}+n$ Differential Cross Sections (H. Knox, R. White)

All the data on differential elastic scattering cross sections for ^{10}B at 30 energies and 9 angles each between 4 MeV and 8 MeV measured at this laboratory have been published in final numerical form as an Ohio University Report C00-2490-5, "Differential Elastic Cross Sections for Neutrons Scattered from ^{10}B for $4 \text{ MeV} \leq E_n \leq 8 \text{ MeV}$," March 1977 (Ref.¹). This report contains a detailed discussion of the corrections, errors, and comparisons of data with those of other groups at the very few points available for comparison. Previous to our work, scattering data in this region of energy have been very sparse. Figures 9 and 10 show the comparison of our results with the only other two reliable sets of data in this energy region.^{11,12)} The agreement is very good. Comparison of the integrated elastic cross section with the ENDF/B-IV evaluation is shown in Figure 11. In the interest of economy, the extensive numerical data and graphs are not reproduced here, and the reader is referred to C00-2490-5 for the full data set. These results have been distributed widely to other workers who may have use for them, e.g.

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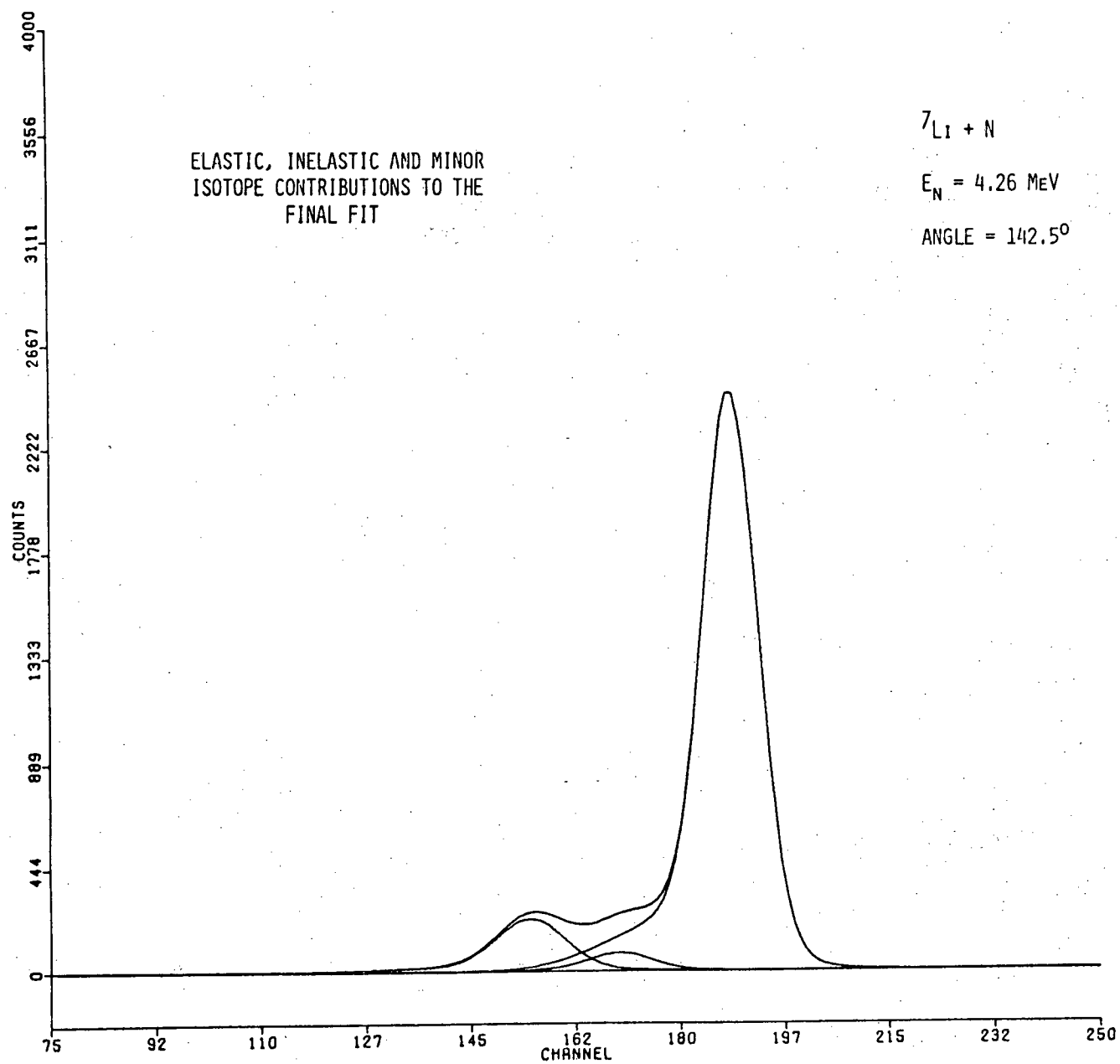


Figure 8.

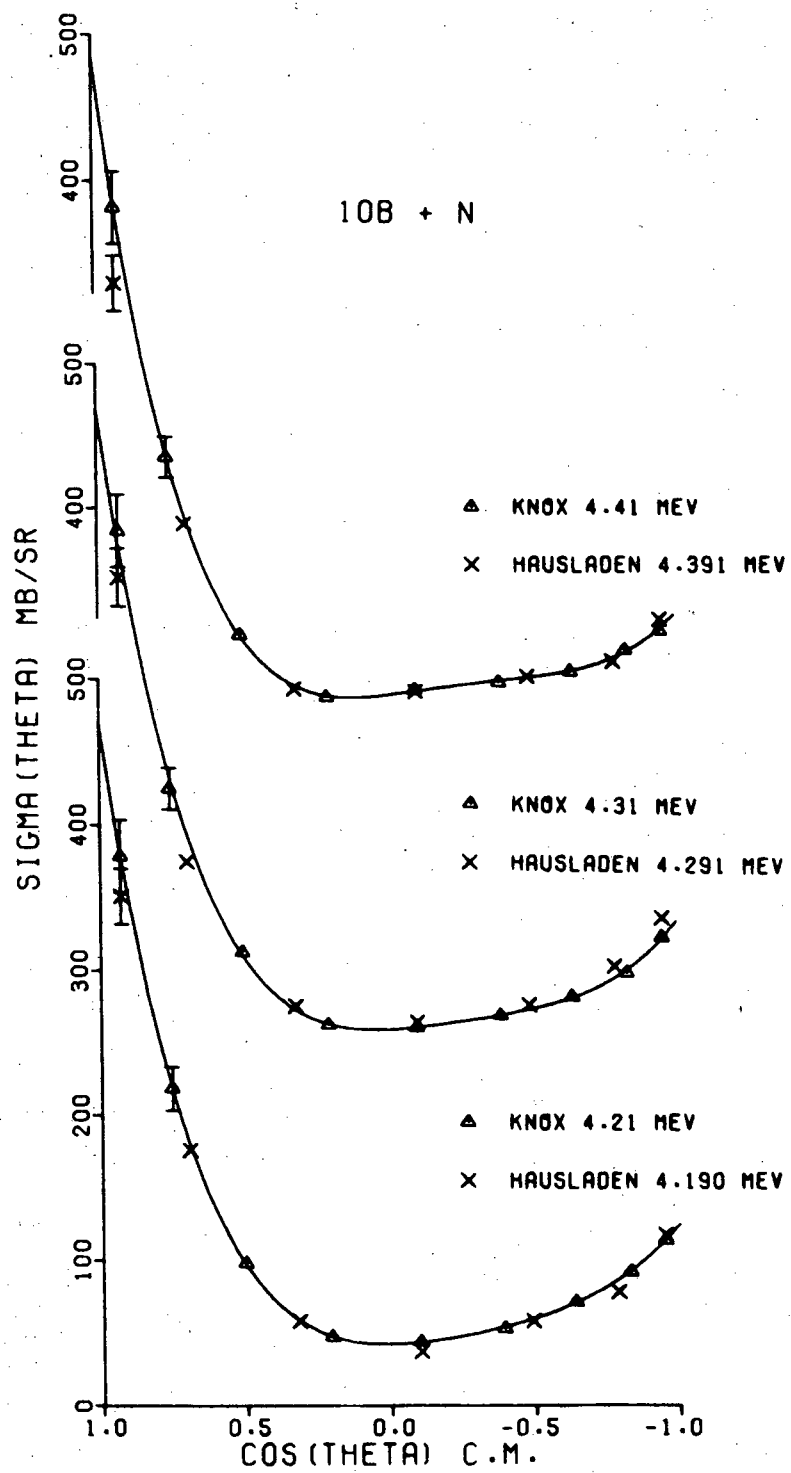


Figure 9.

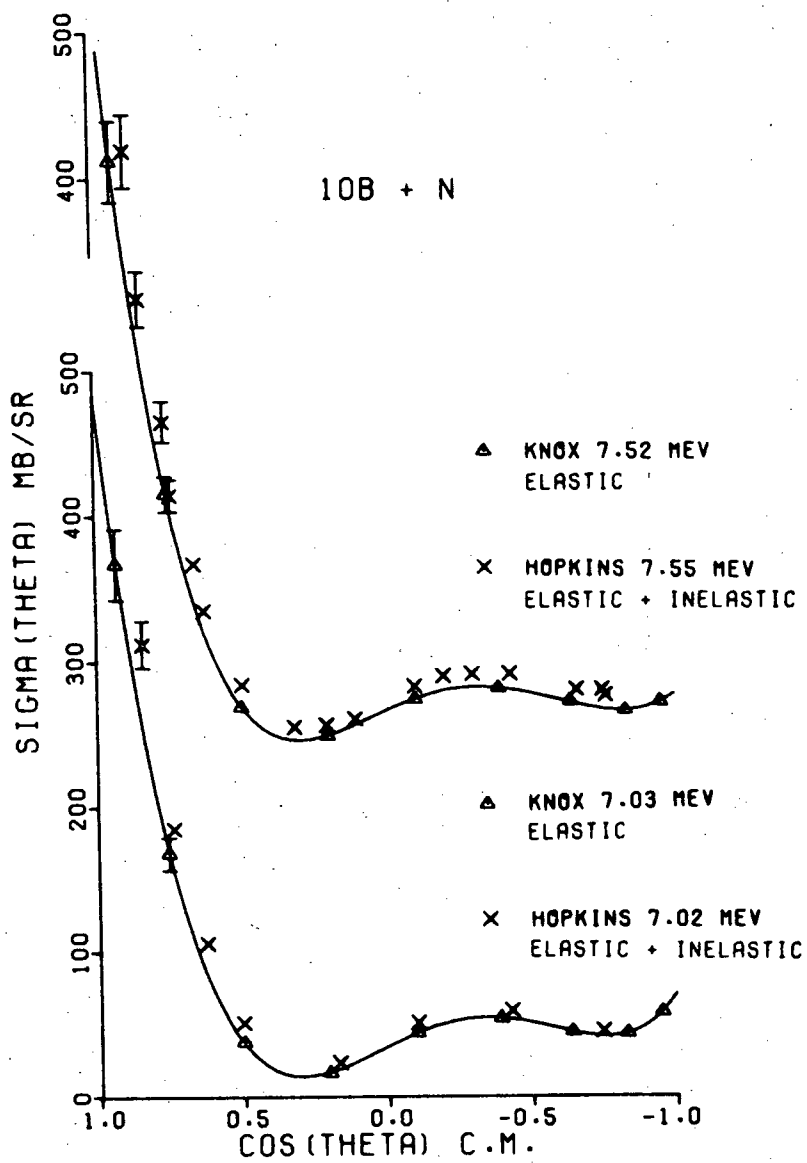


Figure 10.

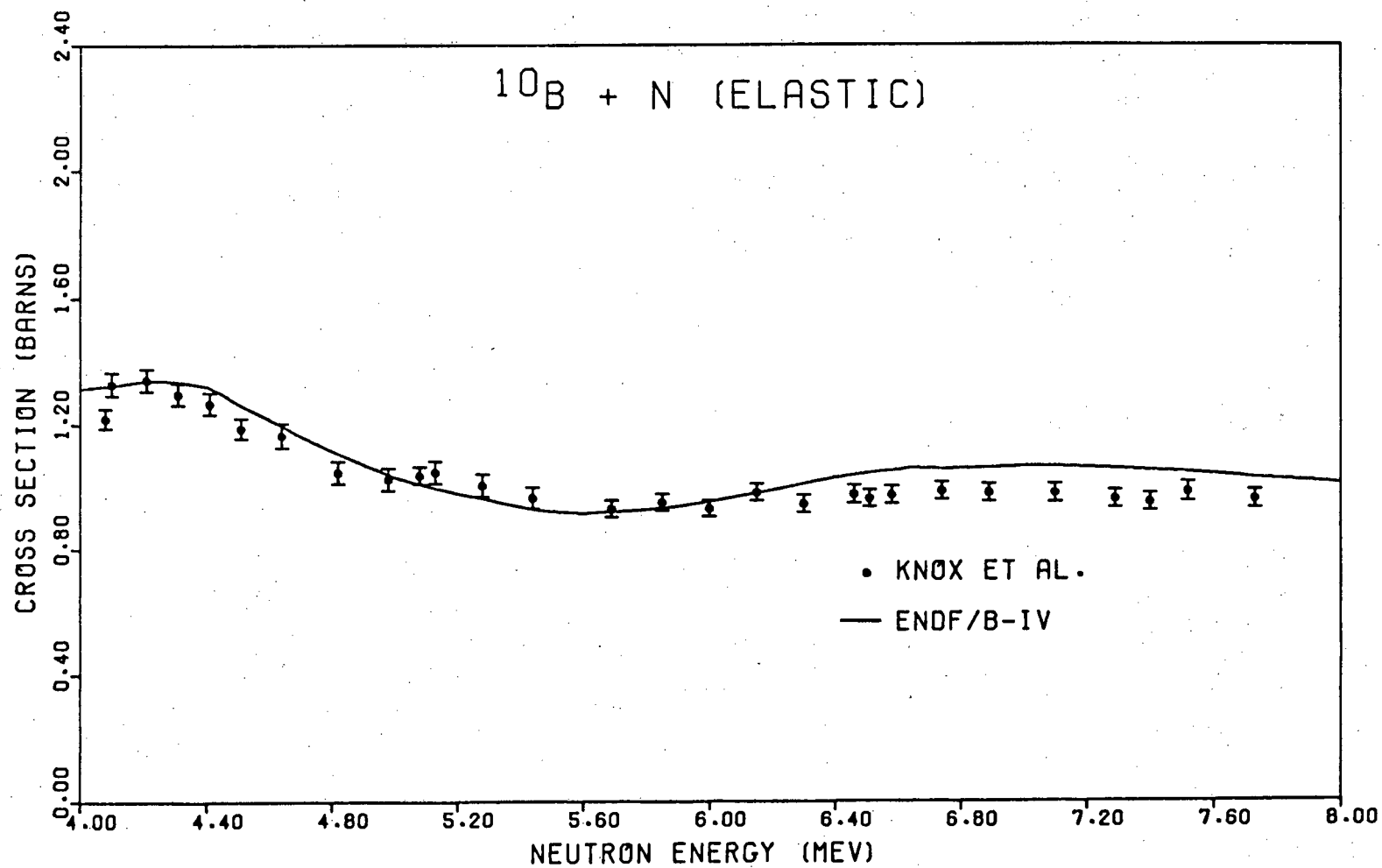


Figure 11.

G. Hale and P. Young of LASL (evaluators for ^{10}B ENDF, R-matrix analysis for standards), F.O. Purser and C.E. Nelson of Duke (doing ^{10}B from 8 MeV up), Bruce Twining of DMFE, M. Abdou of ANL-DMFE and RSIC Shielding Center (for calculations of designs for fusion reactors using ^{10}B for shielding). A condensation of Report COO-2490-5 was accepted for publication by *Nuclear Science and Engineering* (in press).²⁾ It was decided that the best interests of the nuclear community would be served by release of these data at this time for general use, even though an R-matrix analysis of these data has not yet been made by our group. We plan to use our newly purchased high-purity, high-enrichment ^{10}B sample to measure differential cross sections for inelastic groups and include this with any R-matrix analysis, but since this would cause substantial delay we decided upon the more timely release of the data last March in COO-2490-5. G. Hale (LASL) concurred in this decision and invited us to participate in any R-matrix analysis that might be done on our data at LASL if that is undertaken before we complete the inelastic measurements and begin the R-matrix analysis of both elastic and inelastic scattering data.

These results completed from 4 MeV to 8 MeV, together with those previously obtained at this laboratory^{11,13)}, provide a complete data base for elastic scattering from 0.1 MeV to 8.0 MeV for use in evaluated nuclear files, standards applications, and shielding designs for fusion reactors.

E. $^{11}\text{B}+n$ Differential Cross Sections (R. White, H. Knox)

The nucleus ^{12}B is of interest to those who study nuclear structure strictly for the interest in fundamental physics as well as to those who are interested in nuclear measurements for applied purposes. Previous measurements

at this laboratory and elsewhere^{14,15)} of neutron differential cross sections on ^{11}B from 0.1 MeV to 4.4 MeV were completed some time ago. Extensive measurements extending neutron energies from 4.0 MeV to 8.0 MeV were completed on ^{11}B last year and the results were reported in progress report COO-2490-3, covering the period from September 1, 1975 to August 30, 1976.

The nucleus ^{12}B in its various excited states is one of the better of the light nuclei to study from the point of nuclear models when one is using neutrons as probes. States in $^{12}\text{B}^*$ that are not too highly excited can be formed with neutrons of energies in the range of this experiment. Thus, reasonable shell-model calculations can be made which, for various assumptions of interactions between nucleons, can predict neutron cross sections from some reaction formalism such as coupled channels for shell-model states in the continuum. The results of some calculations of this nature are shown below and the comparison to our scattering data are surprisingly encouraging. While the comparisons show only qualitative similarities, this much agreement from relatively few and simple assumptions indicates that a diligent effort to obtain high quality resonance scattering data and a subsequent R-matrix analysis for J^π , γ^2 , E_λ , etc., for states in $^{12}\text{B}^*$ hold good promise of assisting in the development of these important extensions of the shell-model.

In this light, a good portion of this past year has been devoted to developing a practical computer program utilizing the R-matrix theory of nuclear reactions in an attempt to make definitive assignments of angular momentum and parity (as well as width, level energies, etc.) of the various resonances we have observed in the data. In the process, we have used this program to reanalyze the previously measured differential cross sections to verify both the validity of the program and to verify previously assigned

states in the compound nucleus ^{12}B . This effort has resulted in what amounts to an evaluation of all known neutron differential elastic scattering data on ^{11}B , and is reported in considerable detail in our recent laboratory report COO-2490-6, the results of which will only be summarized here. The value of these data for fusion reactor shielding designs has been pointed out in reports by B. Twining¹⁶⁾ and D. Dudziak¹⁷⁾, and these authors recommend that new evaluations for ^{11}B be made. These data and the comprehensive R-matrix analysis essentially provide to the evaluators a ready-made evaluation which incorporates practically all known elastic data to date on ^{11}B . All these results have been sent to the data centers and evaluators P. Young and G. Hale of LASL for use in their new evaluation on ^{11}B scheduled for this fall.

E.(1) R-Matrix Analysis Program

The Ohio University R-matrix analysis program, ORMAP, was written and programmed on the Laboratory's IBM 1800 computer. It is a multichannel, multilevel program which allows the use of either L-S or j-j coupling schemes. The general approach used on the ^{11}B data was to formulate the R-matrix in j-j coupling, form the scattering matrix in j-j coupling from the R-matrix, and then to transform the scattering matrix from j-j to L-S coupling and to use the Blatt and Biedenharm expression¹⁸⁾ to formulate the Legendre expansion coefficients of the differential cross sections. These calculations of Legendre expansion coefficients were compared to the Legendre expansion coefficients of the experimental data and assignments of angular momentum, parity, etc. were made on the basis of these comparisons. This work represents the first time data on $^{11}\text{B}+n$ have been analyzed in j-j coupling and with a change allowed for the ℓ -values between entrance and exit channels.

Considering the ground state spin of ^{11}B and the fact that experimentally only Legendre coefficients through $L=4$ were necessary to fit the data, thus implying that partial waves of $\ell \leq 3$ are negligible in the incident beam, nine possible values of angular momentum and parity can be formed for compound states in ^{12}B from neutron elastic scattering.

E.(2) R-Matrix Results

Analysis of the Region from 0.1 MeV to 2.3 MeV

Figure 12 shows a plot of the R-matrix fit to the data of Lane et al.¹⁴⁾ from 0.1 MeV to 2.3 MeV neutron energy. The 2^+ state at 0.43 MeV neutron energy is well fitted in both shape and magnitude and its previous assignment is confirmed. The broad 2^- state at 1.1 MeV and the 4^- state at 1.26 MeV neutron energy are fitted well in shape and reasonably well in magnitude. The 1^+ state at 1.77 MeV is well fitted in shape and slightly low in magnitude in the B_0 and B_2 terms. However, the fit to the data is certainly sufficient to confirm the assignments of Lane et al. below 2.3 MeV neutron energy.

Analysis of the Region from 2.3 MeV to 4.0 MeV

Figure 13 shows a plot of the R-matrix fit to the data of Nelson et al. from 2.0 MeV to 4.0 MeV neutron energy. The two well-defined resonances at 2.45 MeV and 2.75 MeV neutron energy have been previously assigned 3^+ and 3^- , respectively, and those assignments are confirmed by the present R-matrix fit which is in agreement with the data in both shape and magnitude. However, in the present R-matrix analysis, the broad state reported at 2.65 MeV neutron energy by Nelson et al. could not be fitted assuming a 1^- state as previously

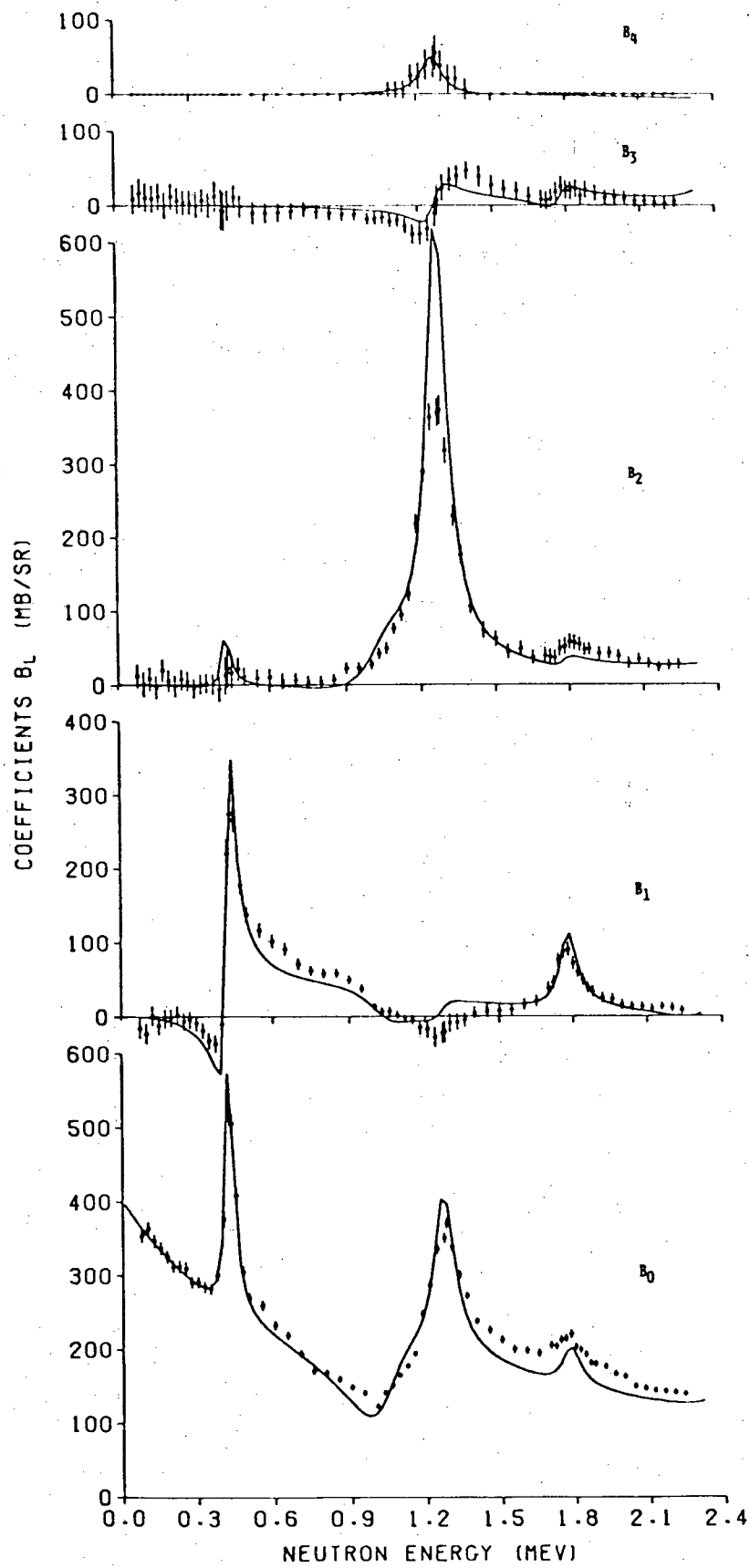


Figure 12.

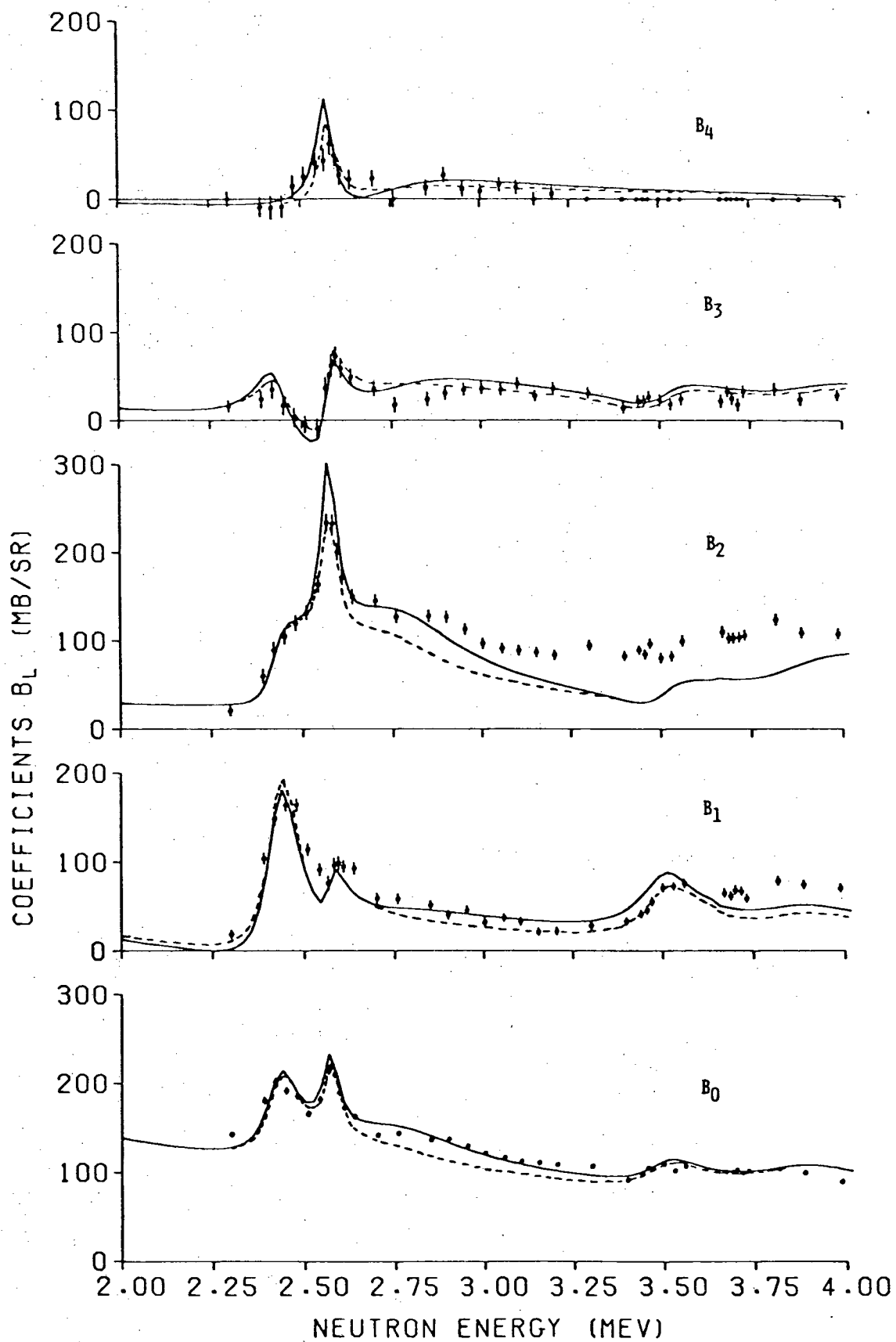


Figure 13.

(tentatively) assigned. A 2^- assignment to this broad resonance produced the solid curve shown in Figure 13 while a 1^- assignment produced the dashed line. It seems clear that the 2^- assignment fits the data better, particularly in view of the fact that the data of Nelson et al. appear to be approximately 10% low in this region. This problem will be discussed later. The present R-matrix fit from 3.0 MeV to 4.0 MeV leaves something to be desired. Hundreds of cases of angular momentum and parity assignments with variations in R-matrix parameters were tried without success in fitting this region. While two 1^+ assignments as previously (tentatively) assigned by Nelson et al. are shown in Figure 13, it is obvious that difficulty exists in fitting the B_1 and B_2 coefficients in this region. Positive parity assignments helped to fit the B_1 terms and negative parity assignments, the B_2 terms, as expected. However, no combination could be found that would yield a respectable fit to this region. It is concluded from hundreds of attempts to fit this region that either some unusually complicated resonance phenomenon is occurring in this region for which the correct R-matrix parameters were not found, or that the data are wrong.

Analysis of the Region from 4.0 MeV to 8.0 MeV

Figure 14 shows a plot of the R-matrix fit to the present data from 4.5 to 7.5 MeV neutron energy. It is apparent from the data and from the fit to the data that this is a much more complicated region with which to deal. First, the region from 4.5 MeV to 5.0 MeV neutron energy is composed of several narrow resonances which appear in the present differential cross section measurements only as a broad composite resonance. The magnitude of the B_2 coefficient in this region is large with the B_3 and B_4 coefficients

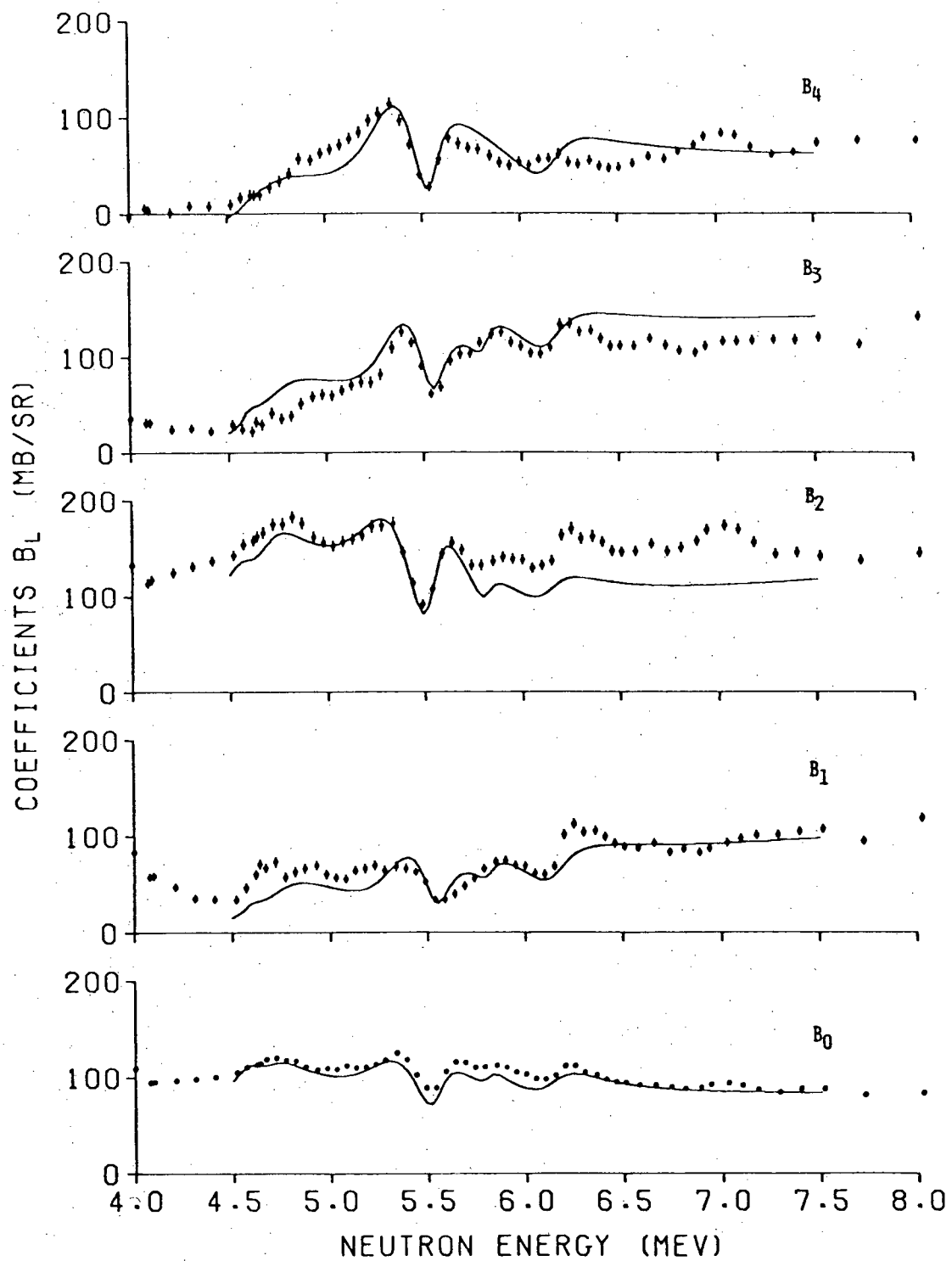


Figure 14.

showing only increasing trends. Numerous combinations of positive and negative parity states were tried, but it was difficult to maintain the shape and magnitude in B_0 and B_1 while increasing the magnitude of the B_2 coefficients, and at the same time limiting the magnitude of the B_3 coefficients. The final fit to this region is only partially successful. Because of the complexity of the problem, it is doubtful that further effort is warranted or would provide any more meaningful results since much of the underlying finer structure here is not resolved in this experiment or even in the higher resolution total cross section measurement of Auchampaugh et al.¹⁹⁾. However, a certain amount of information has been obtained from this effort. First, in order to achieve a general shape fit to this energy region in all coefficients, only states of negative parity gave agreement. Further, no success was achieved in the fitting with negative parity states of angular momentum greater than 2. The fit to the data in Figure 14 in this region consists of 0^- , 1^- , and 2^- states. While these parameters give the best fit to data, they are not to be construed as new assignments. They are undoubtedly not unique, and a different combination of 1^- and 2^- states may give an improved fit to the data. Another difficulty with trying to fit the region from 4.0 MeV to 5.0 MeV is that resonance structure from 3.0 MeV to 4.0 MeV will certainly affect the present fit from 4.0 MeV to 5.0 MeV and the present analysis has been unable to fit the region from 3.0 MeV to 4.0 MeV. In spite of this difficulty, the numerous R-matrix calculations with various combinations of states made in this region suggest that this composite peak is made up largely of 1^- and some 2^- strengths.

One of the main purposes for an R-matrix analysis of the present data was to determine the cause of the large dip in the total cross section at 5.5 MeV neutron energy. This dip has been observed in all total cross

section measurements in this energy region. Such a dip is normally a characteristic effect of the interference between two states of the same angular momentum and parity having roughly the same probability for formation and decay in the same channels. The fact that this dip is seen in all the higher Legendre expansion coefficients is significant because it imposes limits on the angular momentum and parity of the resonance states which cause this effect. Sum rules of the angular momentum coupling coefficients¹⁸⁾ limit the states to 2^- , 3^- , or 4^- assignments. With 2^- assignments at 5.30 MeV and 5.65 MeV neutron energy, the magnitude of the fit was too small to begin to give the significant increase and interference effect seen in the B_2 and B_4 coefficients. With 3^- assignments and equal reduced widths in both $d-3/2$ and $d-5/2$ channels the fit in Figure 14 was obtained. While 4^- assignments gave a better fit to the data than 2^- assignments, they still did not produce as good a fit as the 3^- assignments.

Figure 15 shows the final R-matrix fit to the data of $^{11}\text{B}+n$ for neutron energies from 0.075 MeV to 8.0 MeV. The reader is referred to COO-2490-6 for the final set of R-matrix parameters and discussion of tentative assignments to other states in this region.

E.(3) Comparison with Other $^{11}\text{B}+n$ Measurements

Nelson et al. have measured differential elastic cross sections from 2.4 MeV to 4.4 MeV. A comparison of that measurement at several energies with the present elastic results is shown in Table 1, which is a comparison of the integrated elastic cross sections for various sets of data. While the shape agreement is generally satisfactory, Nelson's measurements at 3.99, 4.09, 4.19, 4.29, and 4.39 MeV average 7% to 15% low in absolute value of the

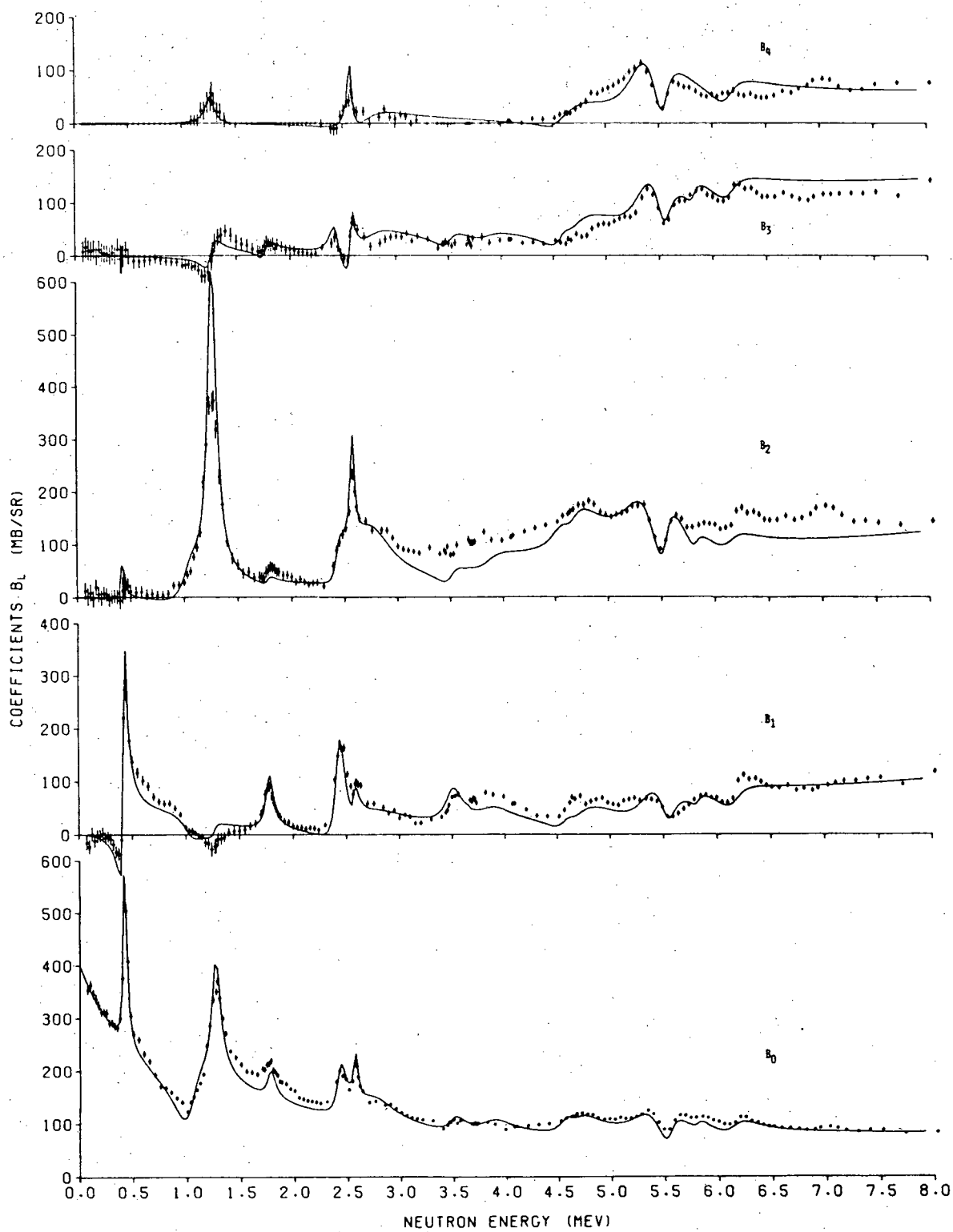


Figure 15.

Table 1

Comparisons of the integrated elastic differential cross sections ($\sigma_{el}=4\pi B_0$) of other measurements with the present data.

Neutron (Lab)	Energy	$\sigma_{el}=4\pi B_0$	Difference
White, <u>et al.</u>	4.00 MeV	1374 mb	
Nelson, <u>et al.</u>	3.99	1131	17.7% low
White, <u>et al.</u>	4.08	1196	
White, <u>et al.</u>	4.10	1202	
Nelson, <u>et al.</u>	4.09	1112	7.3% low
White, <u>et al.</u>	4.21	1221	
Nelson, <u>et al.</u>	4.19	1171	4.1% low
White, <u>et al.</u>	4.31	1235	
Nelson, <u>et al.</u>	4.29	1098	11.1% low
White, <u>et al.</u>	4.41	1260	
Nelson, <u>et al.</u>	4.39	1066	15.4% low
White, <u>et al.</u>	4.31	1235	
Porter, <u>et al.</u>	4.31	1199	
Porter, <u>et al.</u>	4.34	1403	17.0% high
White, <u>et al.</u>	4.82	1475	
Porter, <u>et al.</u>	4.82	1761	19.4% high
White, <u>et al.</u>	7.52	1106	
Hopkins, <u>et al.</u>	7.55	1270	14.8% high

integrated elastic cross section when compared to the present work.

The only other differential scattering cross section measurements in this region are those of Porter et al.²⁰⁾ Table 1 gives the integrated elastic cross sections for these two measurements which are only 30 keV apart in a region where the cross section is known to be slowly varying, yet there exists a 16 percent difference in these two measurements.

The only other measurement in this region is one at 4.82 MeV, also by Porter et al. The shape and absolute value of that measurement at 4.82 MeV also differ in a similar way from the present measurement at 4.31 MeV and suggest Porter's elastic measurements to be approximately 18 percent high in the 4 MeV to 5 MeV region.

There exists one measurement of the neutron differential elastic scattering cross section at 7.55 MeV by Hopkins and Drake.¹²⁾ The comparison with the present data at 7.55 MeV shows general shape agreement but a normalization difference of 15%. Our measurement of $^{12}\text{C}+\text{n}$ at energies in this region plus the consistency of the $^{11}\text{B}+\text{n}$ data lend credence to the measurement at 7.52 reported here.

E.(4) The Integrated Nonelastic Cross Section for $^{11}\text{B}+\text{n}$

Recent high resolution total cross section measurements of $^{11}\text{B}+\text{n}$ by Auchampaugh et al. at Los Alamos Scientific Laboratory are compared in this section with the integrated elastic scattering cross section measurements of Lane et al.¹⁴⁾ from 0.1 to 2.3 MeV neutron energy, Nelson et al. from 2.3 to 4.0 MeV, and the present work from 4.0 to 8.0 MeV. Figure 16 shows the comparison of Lane et al. with Auchampaugh et al. It can be seen that the agreement of the two measurements is quite good as should be expected since the

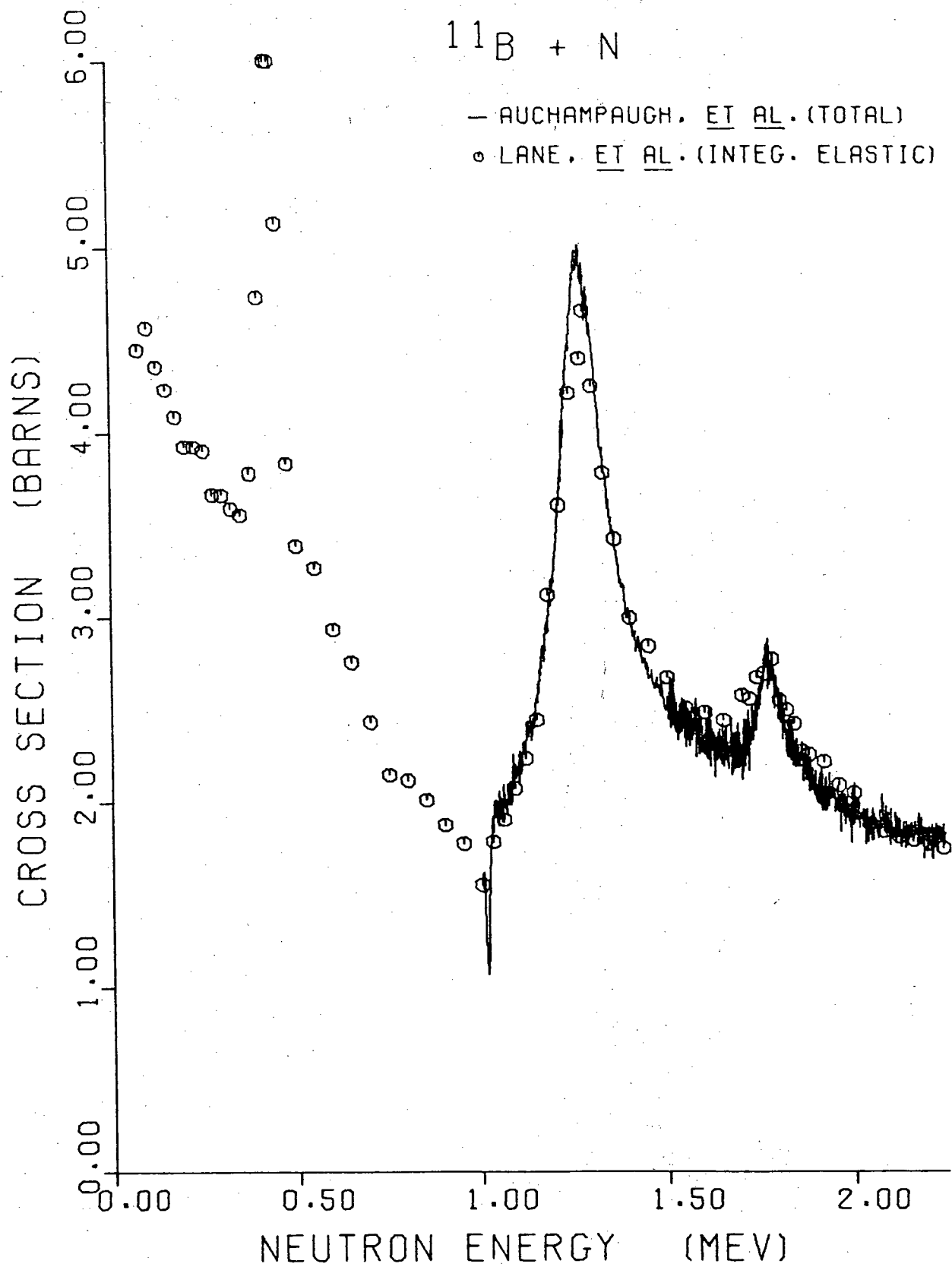


Figure 16.

first inelastic group from the first excited state of ^{11}B is not reached below 2.32 MeV neutron energy.

Figure 17 shows the work of Nelson et al. compared to that of Auchampaugh et al. Even though the first inelastic group is reached at 2.32 MeV, it is unlikely that the magnitude and energy dependence of the inelastic cross section can make up the difference between the total and integrated elastic cross section as shown in Figure 17. In the work of Lind and Day²¹⁾ the excitation function for the 2.14 gamma-ray excited by neutron inelastic scattering in ^{11}B was measured from threshold to approximately 3.2 MeV neutron energy. With the exception of a narrow resonance at 2.6 MeV with a peak height of approximately 40 mb, the cross section up to 3.2 MeV averaged less than 20 mb, an order of magnitude less than that indicated by the difference between the total and integrated elastic scattering cross section in Figure 17. Porter et al. have measured the inelastic differential scattering cross section at 3.18 MeV and 3.74 MeV neutron energy and have given for the integrated inelastic cross sections 11 mb and 45 mb, respectively, while the difference between the total and integrated inelastic cross sections for these same energies from Figure 17 give approximately 200 mb and 250 mb, respectively.

Figure 18 is a comparison of the present work with that of Auchampaugh et al. Several general features are immediately clear. First, the structure agreement of the two sets of data indicates a close agreement in the energy calibration of the two experiments. Second, the change in the inelastic cross section is significant at approximately 4.5 MeV where significant resonance structure is seen in both the total and integrated elastic cross sections. Figure 19 shows a plot of the integrated nonelastic cross section obtained by subtraction of the present data from a computer average of the data

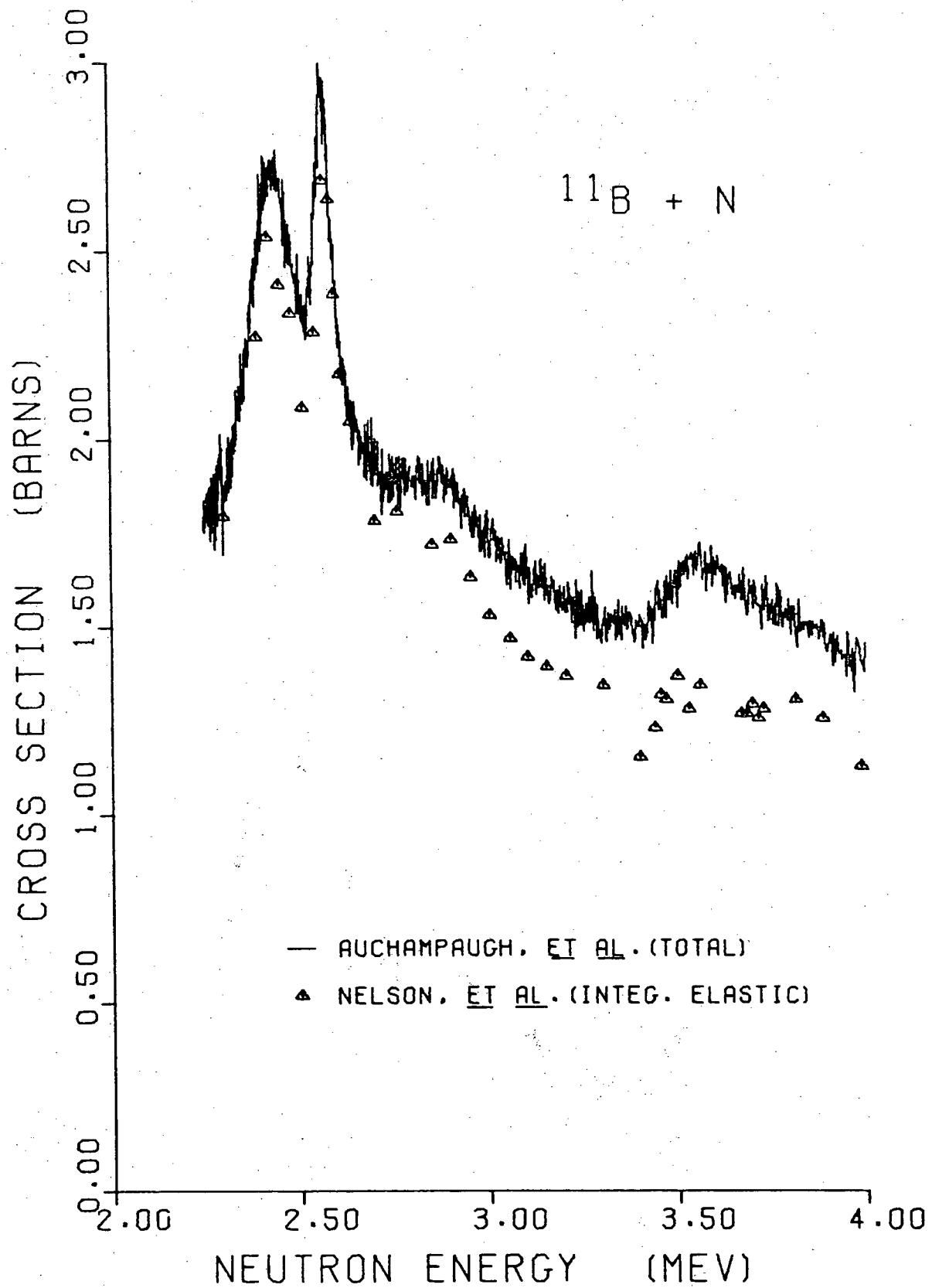


Figure 17.

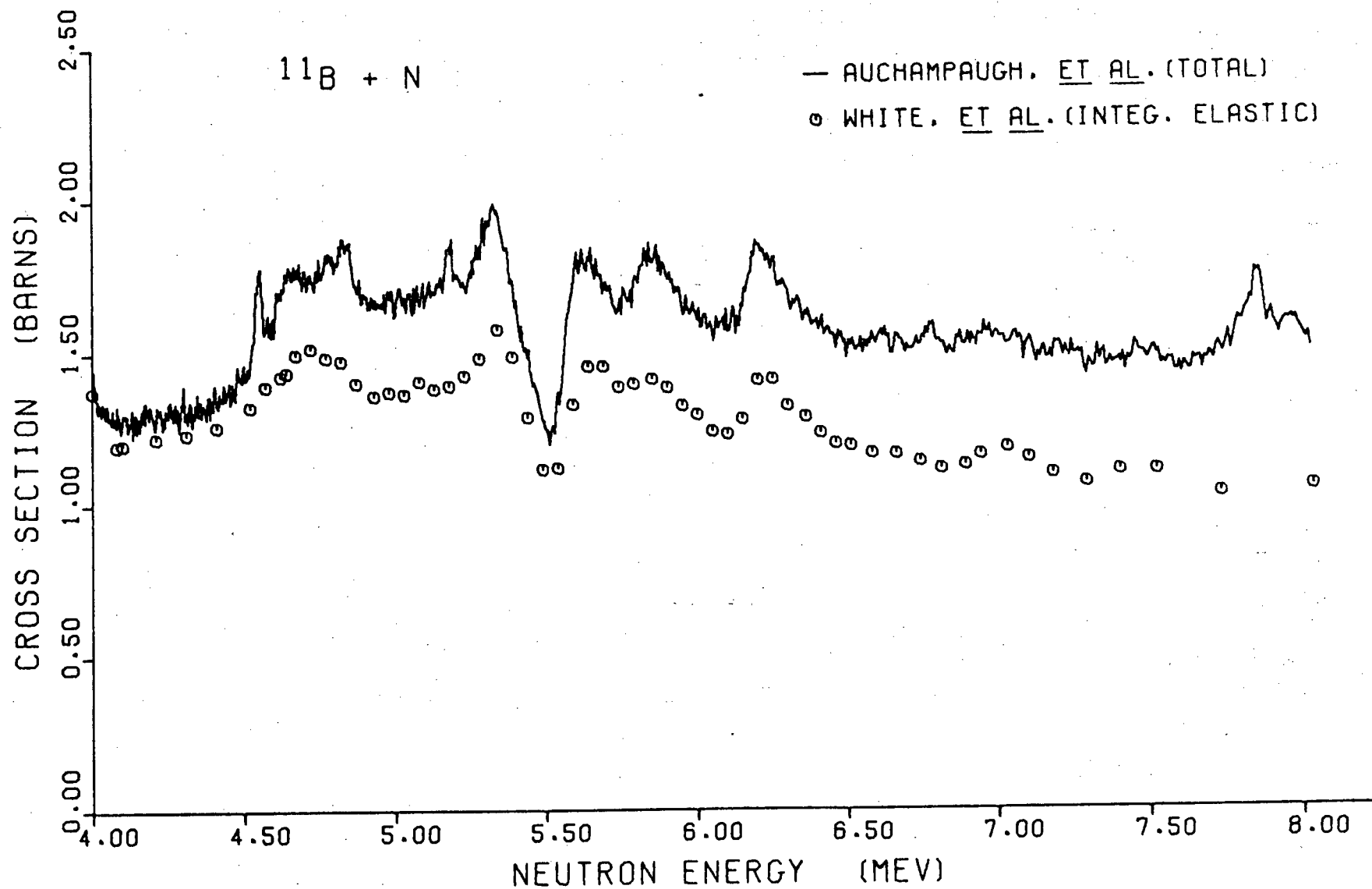


Figure 18.

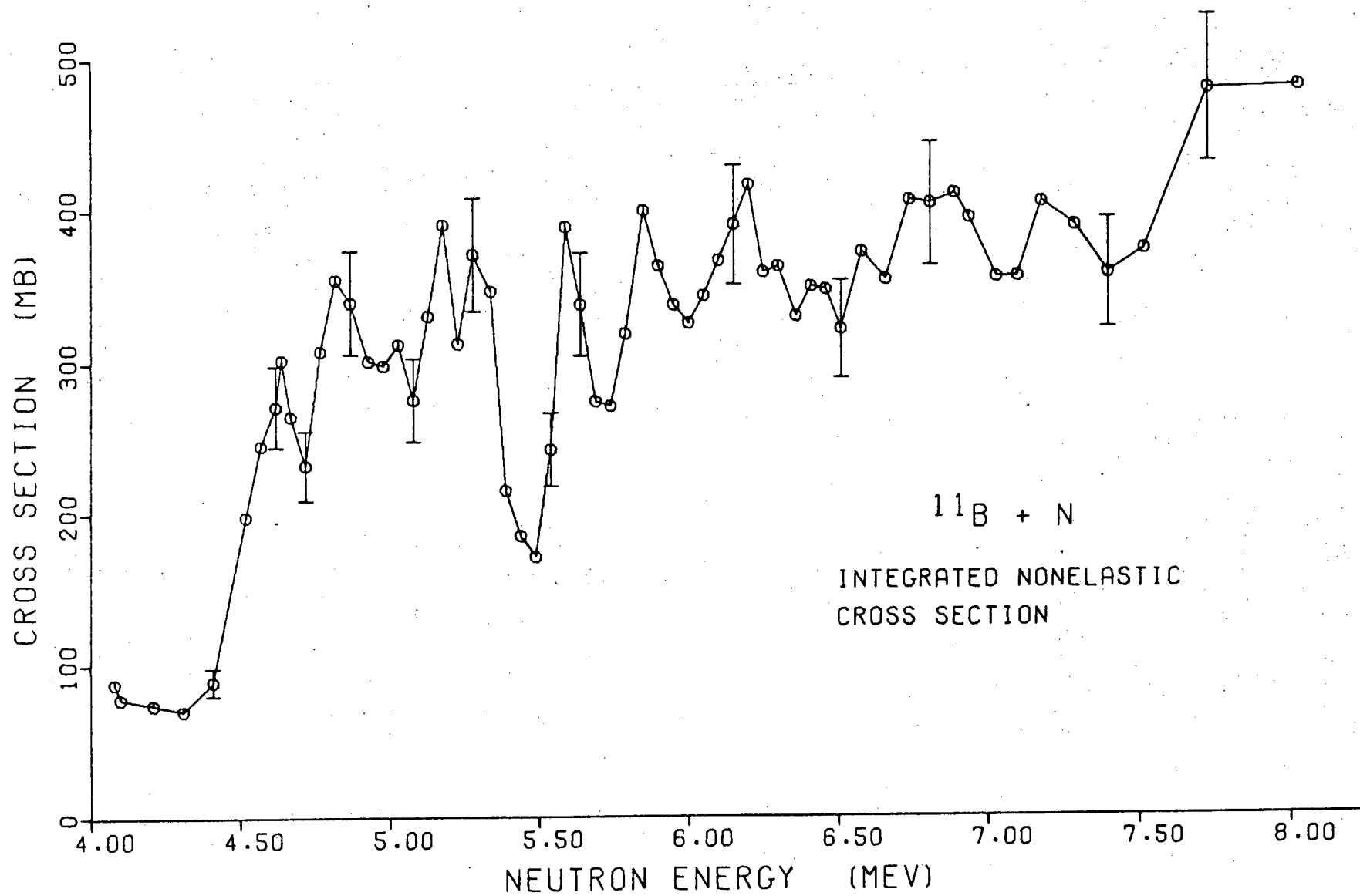


Figure 19.

of Auchampaugh et al. which took into account the difference in resolution of the two experiments. The error bars in Figure 19 were assigned a value of 10% based on the absolute error stated for the total cross section work of Auchampaugh et al. and for the error on the integrated elastic cross sections from the present measurements. It should be noted that while the average error on each data point in the present differential cross section measurements is approximately 4.7%, the average error on the integrated cross section of an angular distribution is only about 1.7%. Since the (n, α) channel is not open below 7.23 MeV, Figure 19 represents the total integrated inelastic cross section up to 7.23 MeV. This plot may be compared to measurements of the inelastic cross section of Porter et al. at 4.31 and 4.82 MeV. Porter et al. obtained 162 ± 10 mb for the integrated inelastic cross section at 4.82 while the present work indicates that quantity to be 350 ± 35 mb. The reason for this factor of two discrepancy between these two sets of data is not clear, and such a large disagreement indicates clearly that careful measurement of the inelastic cross sections should be made directly for 4.0 MeV to 8.0 MeV.

The only other inelastic scattering measurement on ^{11}B has been made by Hopkins et al. at 7.55 MeV. They obtained 52 ± 16 mb and 168 ± 25 mb for the integrated inelastic cross sections from the first and second inelastic groups, respectively. Estimation of contributions from higher levels was 70 ± 40 mb for a total of 290 ± 50 mb for the integrated nonelastic cross section at 7.55 MeV. That value from the present work is 380 ± 40 mb. While the plot on Figure 19 should never be considered to be a substitute for direct measurement of the inelastic cross sections, it does provide valuable information on the average magnitude of the integrated nonelastic cross sections from 4.75 MeV to 7.75 MeV. In addition, it clearly shows the prominent interference

dip at 5.50 MeV. This is significant because the second best R-matrix fit to this interference region in the data was with two 4^- states. However, as can be seen in Figure 20, a 4^- state can only decay by d-waves to the second excited state of ^{11}B ($5/2^-$), and further, cannot decay to the first excited state of ^{11}B ($1/2^-$) for ℓ less than 4.

The outgoing neutrons for the second excited state of ^{11}B (4.45, $5/2^-$) would have only 420 keV of energy and the d-wave penetrability for this energy is very small. Therefore, since the interference from the two resonances at $E_x = 8.24$ MeV and 8.56 MeV is seen in the total integrated inelastic cross section, 4^- assignments can be ruled out. On the other hand, as can be seen in Figure 20, the 3^- assignments allow decay to both excited states, the ℓ -values being consistent with both the penetrabilities and experimental measurement.

Calculations of differential neutron scattering cross sections for ^{11}B have been made by Birkholz and Heil²²⁾ using methods of the shell model in the continuum. Their Legendre coefficients are plotted in Figure 21. They fit many of the low energy negative parity states very well, and show structure for $4 \text{ MeV} \leq E_n \leq 8 \text{ MeV}$ similar to our recent data, especially in the B_4 term where the interference between the two 3^- states is evident.

In summary on the ^{11}B work, a good deal of information useful to evaluators, designers and theorists on ^{12}B has been determined from this work. We have just recently made similar measurements for $3.4 \text{ MeV} \leq E_n \leq 4.08 \text{ MeV}$ to aid in the final R-matrix fitting, and by subtraction from the total cross section, to more accurately indicate the inelastic scattering in this region.

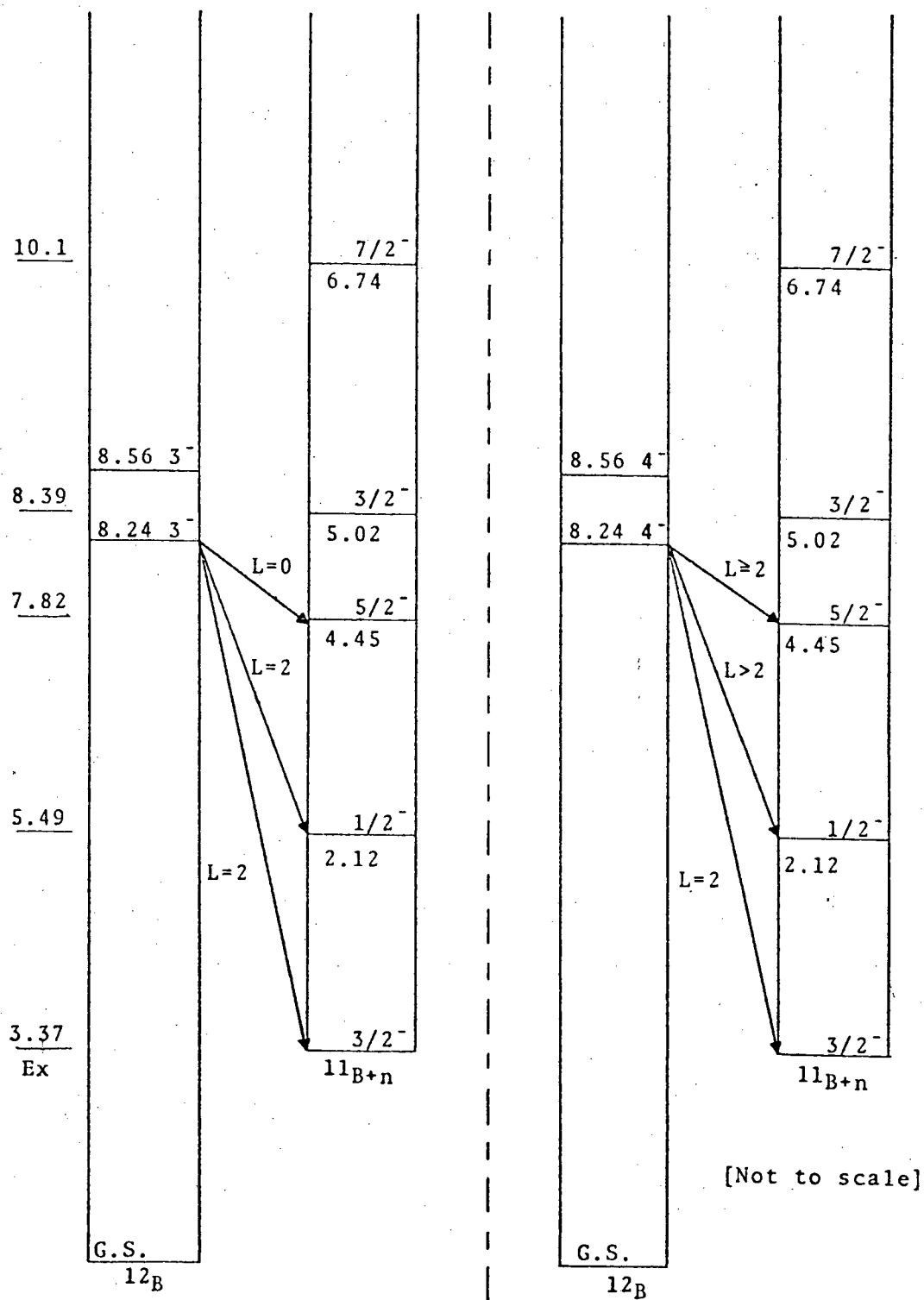


Figure 20.

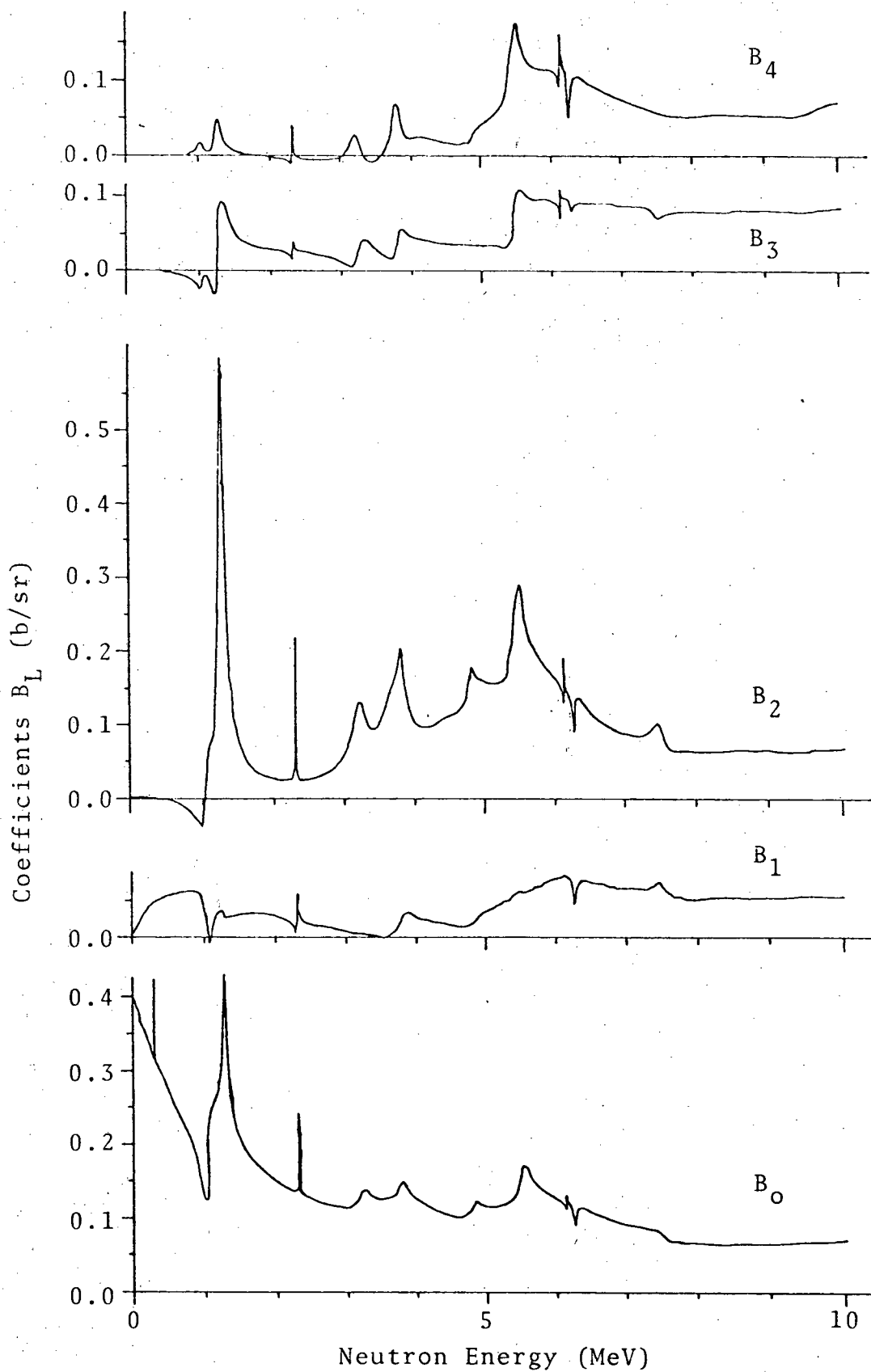


Figure 21.

F. Remaining Quarter of Contract Year

Measurements are scheduled on the Tandem early this fall to finish the low energy ^{11}B data for $2 \text{ MeV} \leq E_n \leq 3.4 \text{ MeV}$ to complete the data for the final R-matrix fit. To do this we are consulting with the LASL group of Auchampaugh and Hill for suggestions to optimize the pulse shape discrimination (PSD) at the low neutron energies. Measurements are also scheduled at Ohio in collaboration with G. Auchampaugh of LASL for ^{13}C elastic scattering, possibly starting at these same energies and progressing upward depending upon the stage of our PSD improvements. LASL is currently working on packing the very fluffy ^{13}C powder to high density in cans. Later in the quarter we plan to begin inelastic scattering measurements on ^{11}B , ^{10}B , ^6Li , and ^7Li for which the PSD improvements at low energies will be very helpful.

III. SCIENTIFIC PUBLICATIONS, Ph.D. DISSERTATIONS AND PROFESSIONAL ACTIVITIES

A. The following is an abstract of an article accepted for publication in *Nuclear Science and Engineering*:

DIFFERENTIAL ELASTIC CROSS SECTIONS OF NEUTRONS SCATTERED FROM ^{10}B FOR $4 \text{ MeV} \leq E_n \leq 8 \text{ MeV}^*$

H.D. Knox, R.M. White, and R.O. Lane

John E. Edwards Accelerator Laboratory
Ohio University, Athens, Ohio 45701

ABSTRACT

Differential cross sections of neutrons elastically scattered from ^{10}B have been measured for approximately thirty incident neutron energies between 4 MeV and 8 MeV. Neutrons inelastically scattered from the 0.717 MeV level were completely resolved from the elastic group except at the most forward angles at energies above 7.5 MeV. Generally, the differential cross sections are slowly varying with energy, indicating little resonance structure in this region.

These data are consistent with earlier lower energy measurements done in this laboratory and provide detailed data in a region where only sparse data were previously available to evaluators and designers in the nuclear energy field.

* Work supported by U.S. Energy Research and Development Administration.

B. The following is an abstract of Ohio University Report COO-2490-5, March, 1977:

DIFFERENTIAL ELASTIC CROSS SECTIONS OF NEUTRONS
SCATTERED FROM ^{10}B FOR $4 \text{ MeV} \leq E_n \leq 8 \text{ MeV}^*$

H.D. Knox, R.M. White, and R.O. Lane

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ABSTRACT

Differential cross sections for neutrons elastically scattered from ^{10}B have been measured for approximately thirty incident neutron energies between 4 MeV and 8 MeV. At each incident energy, measurements were made at nine laboratory angles from 20° to 160° . Comparisons of the present data with previously existing data in this energy range are made.

* Work supported by U.S. Energy Research and Development Administration.

C. The following is an abstract of a Ph.D. dissertation by R.M. White and of an Ohio University Report, COO-2490-6, June 1977:

A STUDY OF THE HIGHER EXCITATION STATES OF ^{12}B
VIA THE $^{11}\text{B}(n,n')^{11}\text{B}$ REACTION

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ABSTRACT

Differential cross sections for neutrons elastically scattered from an isotopically enriched sample of ^{11}B have been measured for laboratory angles from 20° to 160° at 17.5° increments for sixty neutron energies from 4.0 MeV to 8.0 MeV. The data have been corrected

for incident neutron beam attenuation in the sample, air scattering of neutrons, and finite geometry and multiple scattering effects using a Monte Carlo code which included the energy-dependence of differential cross sections of multiple events necessary for light nuclei.

Comparison of the present data is made with data previously measured in this energy region and all known differential neutron data on ^{11}B have been integrated and compared with recent high resolution total cross section measurements to gain information on the neutron inelastic scattering cross section.

All the neutron elastic differential cross section measurements on ^{11}B have been analyzed with a new R-matrix analysis program utilizing j-j coupling with most previous assignments of J^π in the compound nucleus ^{12}B confirmed, and new J^π assignments made in the region of the present measurements. Finally, based on the results of the present measurements and analysis, recommendations for further neutron cross section measurements on ^{11}B are made.

(Prepared for the U.S. Energy Research and Development Administration under Contract EY-76-S-02-2490.A001.)

D. The following two abstracts are for papers presented at the Washington, D.C. meeting of the American Physical Society, April 1977:

THE $^7\text{Li}(n,n')$ at CROSS SECTION FOR $4.0 \leq E_n \leq 7.5 \text{ MeV}^*$

H.D. Knox, R.M. White, and R.O. Lane
Ohio University

Measurements of the differential elastic and inelastic (0.478 MeV level) cross sections for neutrons scattered from ^7Li were reported earlier¹⁾. In the 4 to 7.5 MeV incident neutron energy range studied here, all other reaction channels energetically possible are particle unstable and result in the production of a triton. Using the integrated elastic and inelastic cross sections from the present measurements along with the best available total cross sections, values for the $^7\text{Li}(n,n')$ at reaction have been obtained. Comparisons of these results with previous measurements and comments on existing total cross section measurements will be made.

* Work supported by U.S. E.R.D.A.

1) H.D. Knox, R.M. White, and R.O. Lane, Proc. of the Intl. Conf. on the Interaction of Neutrons with Nuclei, E.R.D.A. CONF-760715-P2, p. 1307.

AN R-MATRIX ANALYSIS OF $^{11}\text{B}(n,n)^{11}\text{B}$ FOR $0.1 \text{ MeV} \leq E_n \leq 8 \text{ MeV}$ *

R.M. White, R.O. Lane, J.M. Cox[†], J.L. Adams and H.D. Knox

Ohio University

Recent measurements¹⁾ of elastically scattered neutrons from ^{11}B at Ohio University have extended neutron differential cross section measurements on that nucleus from 4 to 8 MeV. The entire energy region of 0.1 to 8 MeV has been analyzed for the first time with a multichannel, multilevel R-matrix using j-j coupling. From 0.1 to 2 MeV previous assignments have been verified. From 2 to 4 MeV possible changes in the assignments are being investigated. Difficulties with complex resonances in the 3.3 to 4.1 MeV region will be discussed. The region from 4.5 to 6.5 MeV, containing significant resonance structure, is fitted reasonably well with several new assignments of J^π . The large dip in the total cross section at 5.5 MeV is shown to be an interference effect. Finally, a comparison is made of integrated elastic and total cross section measurements and the total inelastic cross section is inferred.

* Work supported by U.S. E.R.D.A.

[†] Address: Owens-Corning, Granville, Ohio.

1) R.O. Lane, R.M. White, and H.D. Knox, Proc. of the Intl. Conf. on the Interactions of Neutrons with Nuclei, E.R.D.A. CONF-760715-P2, p. 1042.

E. The Principal Investigator has become a member of a group of representatives of laboratories presently funded by the Division of Physical Research of E.R.D.A., who have accepted responsibility for the measurement and analysis of a large scope of nuclear data needs for the DMFE fusion reactor program. The first meeting was November 8-9, 1976 at ORNL under the chairmanship of F.G. Perey. The schedule of measurements made at that meeting extends to 1981 and represents a long-term, on-going commitment of these laboratories to these specific needs of DMFE.

F. The Principal Investigator has been appointed to the Advisory Committee (chaired by R. Ehrlich of G.E.) of the National Nuclear Data Center at BNL, and attended the recent meeting there in May 1977. This committee has

the important responsibility of providing highly expert advice and input to BNL and the Center in its vital function as a national focal point for all nuclear data.

G. Both H. Knox and the Principal Investigator attended the E.R.D.A. Nuclear Data Committee meeting, Gaithersburg (NBS), Md., April 1-2, 1977, chaired by G. Rogosa of E.R.D.A. Ohio University Report COO-2490-5, containing all of our final data on $^{10}\text{B}+n$, and an informal report listing our final tabular data on $^{11}\text{B}+n$ were released and circulated to the Committee at that time.

H. As an invaluable aid to our planning for future measurements and R-matrix calculations to fit data, our group attended the International Symposium on Neutron Standards, Gaithersburg (NBS), Md., March 28-31, 1977, where reviews of the status of standard cross sections and discussions with evaluators (especially the LASL group) were most helpful in our program.

I. The Conference on Neutron Cross Sections at Energies of 10 MeV to 40 MeV, BNL, May 1977, was attended by the Principal Investigator. The needs expressed by speakers and participants in workshops at the conclusion were helpful in indicating new areas of possible investigation for our group.

IV. CONCLUSIONS

In the third year of the contract much of our large volume of elastic scattering data on light nuclei has been published in final form and made available to the nuclear data community and data centers. An improved R-matrix analysis program has been developed and applied to one of the

extensive sets of data to extract theoretical parameters over a larger energy interval than has been done before. Such data and analysis contribute considerably to both the neutron data base for the fusion reactor program and to evaluations being done on these light nuclei for eventual use in both fusion reactor and neutron cross section standards applications. For a small group such as ours, this large amount of high-accuracy data represents a good return to ERDA from funds invested at Ohio University. Because of trends in federal funding of neutron work in recent years, very few young neutron physicists of high quality have been produced lately as evidenced by the extreme difficulty in filling positions that do become available with good candidates. During this contract year a high-quality Ph.D. dissertation was produced and the new young scientist is continuing on as a post doctorate on this contract to apply his expertise to the new phases of inelastic scattering and to ERDA's future needs.

Throughout the first three quarters of the contract year the Principal Investigator devoted 30% of his time to this project and during the fourth (Fall) quarter will devote 100% of his time to this contract. This "interchanges" Fall and Summer (usually 100%) quarters within the same contract year with the same total effort and time spent on the contract, but has certain advantages to ERDA in the staggering of the accelerator schedule for maximum effectiveness.

V. SCIENTIFIC STAFF

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