

Transition Energy Problems in RHIC<sup>1</sup>

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## Abstract:

A tracking program to study the nonlinear effect of the longitudinal motion for particles in the circular accelerator has been written. When the R.F. voltage  $V_0=1.2\text{MV}$  is used, we find that  $\dot{\gamma}_T$  jump,  $\dot{\gamma}_T$  or faster acceleration rate,  $\dot{\gamma}$  is needed in RHIC. The simulation result can be understood analytically via the nonlinear equation of motion. For the  $\dot{\gamma}_T$  jump scheme, there is little bunch shape dilution or deformation in the transition energy crossing. At a bunch area of  $0.3\text{ ev-sec/amu}$ ,  $\Delta\gamma_T=0.6$  in  $30\text{ msec}$  is satisfactory to eliminate the beam loss and preserve the bunch area. By manipulating the momentum aperture,  $\dot{\gamma}$  can be increased substantially at the transition energy region. To eliminate beam loss,  $\pm 0.8\%$  of momentum aperture is needed. On the other hand, when the R.F. voltage is reduced to  $100\text{KV}$ , while maintaining the same acceleration rate, the bunch survival rate across the transition energy is dramatically increased. This is resulted from both smaller momentum spread and longer adiabatic time at the transition energy region due to the smaller voltage.

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## I. Introduction

It is known that the particles in the circular accelerator experience many interesting phenomena, e.g. the microwave instability and nonlinear synchrotron motion at the transition energy region. There are many accelerators that particles can not avoid the transition energy crossing. Therefore the stability of particle motion near the transition energy region is an important problem. For the microwave instability, the problem arises from the vanishing synchrotron frequency, which is needed for the Landau damping. For the nonlinear synchrotron motion, the problem becomes important when the leading linear component vanishes. The nonlinear term can lead to beam loss and bunch dilution and deteriorate the performance of the accelerator<sup>1-5</sup>. It is interesting to find the methods of safe passage through the transition energy. There has been attempt by using the sextupoles distribution to cancel the nonlinear terms in SPS at CERN,<sup>6</sup> and  $\gamma_T$  jump in CPS<sup>7</sup> and AGS.<sup>8</sup>

The Relativistic Heavy Ion Collider(RHIC), proposed at the Brookhaven National Laboratory, has to accelerate the heavy ions through the transition energy ( $\gamma_T \approx 25.4$ ) at a relatively slow acceleration rate( $\dot{\gamma}=1.6/\text{sec}$ ) due to the slow ramping rate of the superconducting magnets. The effect of the transition energy crossing due to the microwave instability has been studied previously<sup>9</sup>. It is important also to note that the kinematical mismatch in the rf acceleration can also be important in the transition energy crossing. Since the lattice design of RHIC can not avoid the transition crossing. It is useful to analyze the effect of the transition energy on the longitudinal beam dynamics.

In this paper, we study the nonlinear longitudinal dynamic in the transition energy region, where the stability of the bunch depends on the proper stable phase relation. Indeed there are interesting analytical studies, where a certain class of nonlinearity can be treated approximately. Since the equation of motion with general nonlinear term can not be solved analytically, we shall study the problem mainly by the particle tracking method to take into account various forms of nonlinear effect. Similar effort has been worked out by MacLachlan, who included space charge effect by using the Fourier analysis for the space

charge induced voltage terms. We shall neglect the space charge effect and microwave instability in order to single out the nonlinear effect. However space charge effect shall be discussed at the end.

The paper is organized in the following: Section 2 reviews the longitudinal dynamics. Section 3 discusses the tracking result with the possible solution of  $\gamma_r$  jump. Section 4 discusses an alternative solution by using the available momentum aperture for the rf programming. Section 5 discusses the effect of the rf parameters in the acceleration process. The conclusion is given in section 6.

## 2. Longitudinal beam dynamics

Acceleration of a bunch of particles in the circular accelerator is accomplished by passing repeatedly through the localized accelerating cavities, where the radio frequency(rf) cavity is synchronize with the circulating particles. The voltage across the accelerating gap in the rf cavity can be expressed as

$$V = V_0 \cdot \sin \phi(t)$$

where  $V_0$  is a slowly varying function of  $t$ , and  $d\phi/dt = \omega$ , the accelerating frequency. The accelerating frequency  $\omega$  is an integral multiple  $h$  of the revolution frequency  $\Omega_0$  of the synchronous particle,

$$\omega = h \cdot \Omega_0 = \frac{h\beta c}{R_0}$$

where  $\beta c$  and  $R_0$  are respectively the velocity and average orbit radius of the synchronous particle, which experiences a constant rf phase  $\phi(t) = \phi_0$ . The energy gain per revolution is  $q_e V_0 \cdot \sin(\phi_0)$ , where  $q_e$  is the charge of the particle. For the non-synchronous particles, the deviations is defined as,

$$\Omega = \Omega_0 + \Delta\Omega; \phi = \phi_0 + \Delta\phi; p = p_0 + \Delta p; E = E_0 + \Delta E; \theta = \theta_0 + \Delta\theta$$

where  $\theta_0$  is azimuthal angle in the machine of the reference angle measuring positive in the direction of motion, or  $\Delta\phi = -h\Delta\theta$ . The equation for the synchrotron motion becomes,

$$\frac{d\phi}{dt} = -h \frac{d\theta}{dt} = -h \Delta\Omega = h\eta\Omega \frac{\Delta p}{p} = \frac{h\eta\Omega}{p_0 R_0} \left( -\frac{\Delta E}{\Omega_0} \right) \quad (1)$$

$$\frac{d}{dt} \left( -\frac{\Delta E}{\Omega_0} \right) = \frac{qeV_0}{2\pi} (\sin\phi - \sin\phi_0) \quad (2)$$

where  $\eta$  is the momentum slip factor,

$$\eta = -\frac{p}{\Omega} \frac{d\Omega}{dp} = 1/\gamma_T^2 - 1/\gamma^2 \quad (3)$$

In the linearized small amplitude synchrotron motion, Eqs. 1 and 2 are equivalent to a simple harmonic motion expressed as following:

$$\Delta\phi = \left[ \frac{\alpha h^2 \eta_0 \Omega_0}{\pi p_0 R_0 \Omega_s} \right]^{1/2} \sin \Omega_s t \quad (4)$$

$$W = \frac{\Delta E}{\Omega_0} = \left[ \frac{\alpha p_0 R_0 \Omega_s}{\pi \eta_0 \Omega_0} \right]^{1/2} \cos \Omega_s t \quad (5)$$

provides that the adiabatic condition is satisfied,

$$|(d\Omega_s/dt)/\Omega_s^2| \ll 1.$$

Here  $\alpha$  is the longitudinal phase space area and  $\Omega_s$  is the the synchrotron oscillation frequency given by

$$\Omega_s = \left[ -\frac{qeV h \eta_0 \Omega_0 \cos\phi_0}{2\pi p_0 R_0} \right]^{1/2} = \left[ -\frac{qeV h \eta_0 \cos\phi_0}{2\pi A \gamma m_0 R_0^2} \right]^{1/2} \quad (6)$$

where  $A$  and  $\gamma$  are respectively the mass number of the ions and the Lorentz energy factor. To obtain a stable synchrotron motion,  $\Omega_s$  in eq.(6) must be a real number. Thus when the energy is below the transition energy, i.e.  $\gamma < \gamma_T$ ,  $\eta < 0$ , the synchronous angle  $\phi_0$  lies between  $[0, \pi/2]$ . At the energy above the transition energy, the stable phase angle should be changed to  $\pi - \phi_0$  to maintain a stable motion.

The amplitude of the synchrotron motion is then given by,

$$\hat{\delta} = \frac{\hat{\Delta p}}{p} = \frac{\hat{W}}{p_0 R_0} = \left[ - \frac{\alpha^2 q e V h \cos \phi_0}{2\pi^3 p_0^3 R_0^3 \Omega_0 \eta} \right]^{1/4} \quad (7)$$

$$\Delta \hat{\sigma} = \left[ - \frac{2 \alpha^2 \Omega_0 \eta}{\pi p_0 R_0 q e V h \cos \phi_0} \right]^{1/4} \quad (8)$$

Note that  $\hat{\delta} \propto \gamma^{-3/4} \cdot \eta^{-1/4}$  and  $\Delta \hat{\sigma} \propto \gamma^{-1/4} \cdot \eta^{1/4}$  are respectively the momentum amplitude and bunch length. When  $\gamma$  approaches  $\gamma_T$ , i.e.  $\eta \approx 0$ , the momentum amplitude of the bunch becomes large and the bunch length becomes small. However at  $\eta \approx 0$ , the transition energy region, the adiabatic condition is violated. The synchrotron motion is not described by the simple harmonic motion mentioned above. When the leading order of  $\eta$  can be expressed as a linear function of time  $t$ , the synchrotron motion of the bunch is governed by an ellipse<sup>3</sup>,

$$A_{\delta\delta} \delta^2 + 2 \cdot A_{\delta\sigma} \delta \Delta \sigma + A_{\sigma\sigma} (\Delta \sigma)^2 = 1 \quad (10)$$

within the non-adiabatic time  $\tau_{NA}$ ,

$$\tau_{NA} = \left[ \frac{m_0 c^2 \gamma_T^4}{2\omega_a^2 h \dot{\gamma} \text{ qeV} |\cos \phi_0|} \frac{2\pi}{\text{qeV} |\cos \phi_0|} \right]^{1/3} \quad (11)$$

Here the coefficients  $A_{\delta\delta}$ ,  $A_{\delta\sigma}$ , and  $A_{\sigma\sigma}$  can be expressed in Bessel and Neumann functions of the order of 2/3. These coefficients depends on rf parameters and  $\dot{\gamma} = d\gamma/dt$  and  $\gamma_T$ . We observe that the ellipse is rotated at the transition energy region. It is however important to point out that Eq.(10) neglects the effect of the nonlinear term in the r.f equation.

There are two nonlinear sources in the rf equation. The first one is the kinematic effect due to the different speed of particles within the bunch. The other is due to the design of the accelerator to be discussed in the following.

For a given circular accelerator, the transition energy is determined by the lattice closed orbit functions,  $X$ . Following Courant and Synder<sup>1</sup>, the equation of motion for the off-momentum particles with momentum  $p+\Delta p$  is given by,

$$\frac{d^2 X}{ds^2} + \frac{1-n}{\rho^2} X = \frac{\Delta p}{\rho p} \quad (12)$$

The difference in the path lengths between this orbit and the reference orbit  $X=0$  is given by,

$$\Delta C = \int_0^C \frac{X}{\rho} ds = \nu \int_0^{2\pi} \frac{\beta X}{\rho} d\psi \quad ; \quad \psi = \int ds/\nu\beta. \quad (13)$$

Using the solution of eq.(12), i.e.,

$$X = \frac{\Delta p}{p} \beta^{1/2} \nu^2 \sum_k \frac{a_k}{\nu^2 - k^2} e^{ik\psi} \quad , \quad (14)$$

with

$$a_k = \frac{1}{2\pi} \int_0^{2\pi} \frac{\beta^{3/2}}{\rho} e^{-ik\psi} d\psi \quad , \quad (15)$$

we obtain

$$\Delta C = 2\pi \nu^3 \frac{\Delta p}{p} \sum_k \frac{|a_k|^2}{\nu^2 - k^2} \quad (16)$$

$$\alpha = \frac{1}{\gamma_T^2} = \frac{\Delta C}{2\pi R \Delta p/p} = \frac{\nu^3}{R} \sum_k \frac{|a_k|^2}{\nu^2 - k^2} \quad (17)$$

Normally, accelerators are designed in such a way that only the  $a_0$  term is important in order to optimize the orbital beam dynamics. Thereby the dispersion function  $X_p = dX/d\delta$ , where  $\delta = \Delta p/p$ , has small variation around the accelerator with  $a_0 \approx (R/\nu)^{1/2}$ , where  $\langle \beta \rangle \approx R/\nu$  is used for the average beta-function. This leads to the result that  $\gamma_T \approx \nu$  for most of the machine. Since the tune of the machine depends on  $\delta$ , according the chromaticity of the lattice, the  $\gamma_T$  depends on the momentum  $\delta$  in a similar fashion. Fig.1 shows the  $\gamma_T$  of RHIC lattice as a function of  $\delta = \Delta p/p$  for  $\beta = 6m$  with chromaticity being adjusted to zero with

sextupoles.

Because of these nonlinearities, eq.(1) should be replaced by

$$\frac{d\phi}{dt} = h\Omega_0 (\eta_0 + \eta_1 \delta + \eta_2 \delta^2 + \eta_3 \delta^3) \delta \quad (18)$$

at the transition energy region. The effect of nonlinear terms are important only in the transition energy region. The coefficients of eq.(18) are given by,

$$\eta_0 = 1/\gamma_T^2 - 1/\gamma^2$$

$$\eta_1 = -\eta_0/\gamma_T^2 + 3\beta^2/2\gamma^2 + \alpha_1/2\gamma_T^2$$

$$\eta_2 = -\beta^2(-.5+2.5\beta^2)/\gamma^2 + \alpha_2/3\gamma_T^2 - (3\beta^2-\alpha_1)/2\gamma^2\gamma_T^2 - \alpha_1/\gamma_T^4 + \eta_0/\gamma_T^4$$

$$\eta_3 = \beta^4(35\beta^2-15)/8\gamma_0^2 + \alpha_3/4\gamma_T^2 + (\beta^2(30\beta^2-6)-9\beta^2\alpha_1+4\alpha_2)/12\gamma^2\gamma_T^2 - (8\alpha_2+3\alpha_1^2)/12\gamma_T^4 + (3\beta^2-2\alpha_1)/2\gamma^2\gamma_T^4 + 3\alpha_1/\gamma_T^6 - \eta_0/\gamma_T^6$$

and where the coefficients  $\alpha_1, \alpha_2, \alpha_3$  are obtained from the expansion of the momentum compaction factor  $\alpha$ ,

$$\alpha = \frac{d(R/R_0)}{d\delta} = \alpha_0 (1 + \alpha_1 \delta + \alpha_2 \delta^2 + \alpha_3 \delta^3 + \dots) \quad (19)$$

with  $\alpha_0 = 1/\gamma_T^2$ . Only two terms,  $\eta_0$  and  $\eta_1 \delta$ , in eq.(18) are important to the particle motion, unless extremely large bunch area and/or small  $\gamma_T$  are encountered. As an example, we consider the RHIC lattice for  $\beta = 6m$  and zero chromaticity,  $\eta_1 \approx 1.8/\gamma_T^2$  at the transition energy region. Assuming that the rf voltage  $V_0 = 1.2MV$  and  $\phi_0 = 0.04$ , the particle distribution would fall within the momentum window of  $-.005 < \delta < +.005$  for a bunch area of 0.3 evsec/amu of Au beam. Because of the nonlinear term in eq. (18),  $\eta_1 \delta$ , those particles at momentum  $\delta = \hat{\delta} = 0.005$  change sign while the synchronous particle at energy  $\gamma$  below the transition energy  $\gamma_T$  by  $\gamma - \gamma_T \approx -0.9\gamma_T \hat{\delta} \approx -0.11$ . When particles are accelerated from below toward the transition energy, large momentum particles experienced the effect of the transition energy crossing while the synchronous particles still  $-0.11$  away from the transition energy. The premature sign change

In the rf equation (18) gives rise to unstable motion.

Thus the difficulty involved in the transition energy crossing lies in the fact that part of the bunched beam crossed the transition energy while the stable phase angle  $\phi_0$  has not been jumped. This lead to the unstable motion for those particles within the time lapse proportional to the momentum spread of the bunch. When the acceleration rate is large, e.g.  $\dot{\gamma}=74/\text{sec}$  in AGS, particles with unstable phase angle can be recaptured into the stable phase bucket with certain dilution of the bunch area. When the acceleration rate is small, these unstable particles are drifted away from the retainable bucket area. To eliminate the leading nonlinear term, i.e.  $\eta_1=0$ ,  $\alpha_1$  should be set to  $-1.5$ , which corresponds to the chromaticity of 1.5 times the tune of the machine. Such an accelerator would be difficult to operate. Particle motion under the influence of the nonlinear equation in the transition energy region shall be studied in next section with numerical simulation.

### 3. Tracking results of the longitudinal phase space.

Although analytic estimate can be made to predict the behavior of particle motion for eqs.(18) and (2), analytical solution are not known at present when  $\eta$  depends on  $\delta$  in the complicated way. We choose to simulate the bunch with the Monte-Carlo numerical calculation. There exists a longitudinal phase space tracking code, developed by MacLauchlin<sup>10</sup> in FNAL. Since the space charge is an not important issue in RHIC energy and the collective instability has been analyzed earlier<sup>9</sup>, we will neglect these effect in the tracking study until section 6. Our tracking code is to solve Eqs (18) and (2) in the difference equation with the constraint of symplectic condition. The rf parameter is taken to be  $V_0=1.2\text{MV}$  and  $\phi_0=0.04$  rad., which corresponds to  $\dot{\gamma}=1.6/\text{sec}$  for Au ion or the magnetic field ramping rate of  $\dot{B}=0.05$  Tesla/sec.<sup>11</sup> An initial Gaussian distribution for the longitudinal bunch. Each bunch is populated by 1000 or more particles. Fig.2 shows an example of tracking 1000 particles through the transition energy  $\gamma_T=25.4$ . We found that large percentage of beam is lost due to small acceleration rate( $\dot{\gamma}=1.6/\text{sec}$ ). Fig. 3 shows the percentage loss for the Au ion at the acceleration rate( $\dot{\gamma}=1.6/\text{sec}$ ) as a function of the phase

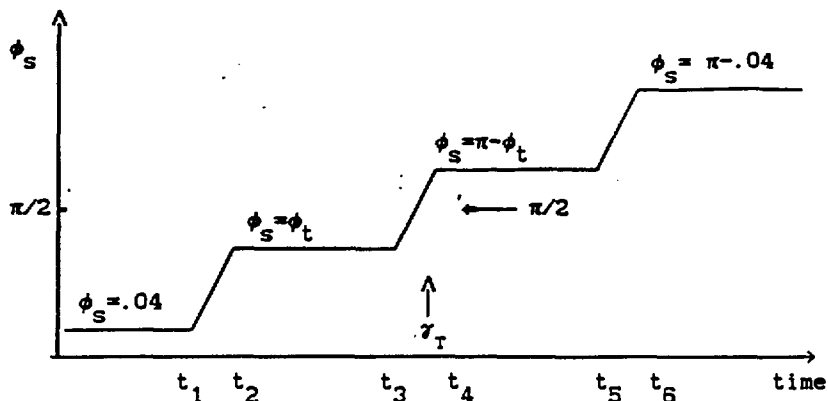
space area. It is clear that higher acceleration rate across the transition energy is needed in order to maintain the beam quality in the longitudinal phase space. This can be achieved by the transition energy jump, which is a routine operation in CPS and AGS in the future operation mode.

Fig. 4 shows the tracking result in RHIC with  $\gamma_T$  jump. We have found that  $\Delta\gamma_T$  of 0.6 and  $\Delta t=30$  msec or equivalently  $\dot{\gamma} = 20/\text{sec}$  can maintain minimum phase space area blow-up and no loss in the transition crossing. Once the  $\gamma_T$  jump is employed, the bunch shape remains undiluted across the transition energy. Figs.5 and 6 shows the projection onto the longitudinal and momentum coordinates during the transition energy region.

It is noted that the  $\gamma_T$  jump scheme requires the lattice function of the accelerator being changed rapidly by a special set of tuning quadrupoles. To accomplish this goal, the dispersion function is changed substantially<sup>12</sup>. Thus the orbital beam dynamic becomes an important issue.

#### 4. Fast transition crossing by increasing $\dot{\gamma}$ with rf program.

A higher acceleration rate through the transition energy region can be reached by using the momentum aperture. Since the dipole magnets are ramped at a constant rate of .05 Tesla/sec, the beam particles under higher acceleration rate will move out into the momentum aperture as  $X_p \cdot \Delta p/p$ . Fortunately the beam size is small during the acceleration period, the available aperture may be used to manipulate the acceleration rate of the bunch.



Note here that during the time  $t_2$  to  $t_5$  in the above rf program, particles in the accelerator have been accelerated much faster than the magnet ramping rate. The bunch will move outward onto a closed orbit with larger radius. The amount of radius excursion depends on the time required in the fast acceleration period,  $\Delta t = t_5 - t_2$ . On the other hand, when  $\Delta t$  is increased, the effective of the nonlinear term in the rf equation becomes less important. Based on the estimate of section 2, we find that the nonlinear term becomes dominate at  $|\gamma - \gamma_T| \leq 0.1$  for phase space area of 0.3 ev-sec/amu. Thus a minimum momentum aperture requirement would be  $\Delta p/p \approx \pm 0.004$ . One should however also take into account the increase of  $\Delta p/p$  amplitude when  $\eta$  becomes very small (see eq.7). The aperture requirement would be increased. To find out the aperture requirement, Numerical simulation for the longitudinal motion according the rf program mentioned in the above diagram. Fig. 10 shows the particle survival rate vs the time for the rf stable phase equal to  $\phi_t$ , where  $\phi_t$  is chosen to be  $45^\circ$ ,  $60^\circ$  and  $90^\circ$  respectively and the time  $t$  is equal to  $t_3 - t_2$ . On the top of Fig.10, we also show the aperture requirement in  $\Delta p/p$ . To reach 98% survival through transition crossing, the beam would make an excursion from  $-0.8\%$  to  $+0.8\%$  during the fast acceleration period of time. Since the bunch has  $\pm 0.5\%$  of momentum spread, the required aperture would be  $\pm 1.3\%$  during the transition energy crossing. It also interesting to note that the aperture

requirement of Fig. 17 is independent of  $\phi_t$  so long as  $\phi_t$  larger than  $45^\circ$ . This indicate that the remaining loss of 2% may result from the increment of momentum window when  $\gamma$  is approaching  $\gamma_T$ .

The aperture requirement for the edge of the beam is therefore  $X_p \cdot \Delta p/p \approx 20$  mm, where  $X_p = 1.5$ m and  $\Delta p/p = 1.3\%$ . Since the emittance of the beam is relatively small, it appears that this is also a viable solution. The possible problems is that the beam stays off centered in the arc region for considerable amount of time ( $\approx 11$  msec) in the transition energy region. The impedance of the chamber that the particle see may vary as a function of time. Careful evaluation is needed. It is interesting to note that the beam does not experience much radial excursion in the insertion region because of small dispersion function in this region

#### 5. Aperture requirements for $\dot{\gamma}_T$ and $\dot{\gamma}$ schemes

We have studied the longitudinal motion at the transition energy region in RHIC. It is clear that the slow acceleration rate of the superconducting magnet is the main reason of the beam loss in the transition energy region. To minimize or eliminate the beam loss in the transition energy region, a faster rate of transition crossing is needed. This can be achieved by either  $\gamma_T$  jump in the lattice or a faster acceleration rate by manipulating the momentum aperture. We found that  $\Delta\gamma_T = 0.6$  in 30 msec or  $\dot{\gamma} = 20/\text{sec}$  is needed to eliminate the beam loss for a phase space area of 0.3ev-sec/amu. In the  $\gamma_T$  jump scheme, the large beam size requirement is due to the large dispersion function. On the other hand, the momentum aperture requirement is  $\pm 0.8\%$  in order to obtain the fast acceleration across the transition energy region. Table 1 summarizes the various cures for the nonlinear effect in the longitudinal motion at the transition energy region.

Table 1 Comparison of transition crossing schemes

	$\dot{\gamma}_T$ ( $\Delta\gamma_T=0.6$ )		$\dot{\gamma}$
	unmatched	matched	
momentum aperture (mm)	19.5	7.5	19.5
betatron aperture (mm)	7	7	7
Advantage	2 power supplies		rf program
		good beam dynamics	good beam dynamics
Disadvantage	large $X_p$ in insertion region 1/2 integer stopband	complicated power supply system	limited available momentum aperture

6. Effect of the rf parameters on the transition energy crossing.

As we have discussed in section 2 eq.(18), the effect of the nonlinear term becomes more important with increasing the momentum width,  $\hat{\delta}$ . Eq.(7) indicates that  $\hat{\delta} \propto (\alpha^2 V_0 h \cos \phi_0)^{1/4}$ . One method to minimize the effect is to minimize the phase space area of the bunch. Fig. 2 showed the loss rate decrease very fast with decreasing phase space area. An alternative method of getting smaller  $\hat{\delta}$  is to decrease  $V_0$  while keeping  $V_0 \sin \phi_0$  constant. For example, using  $V_0=100KV$  and  $\sin \phi_0=0.48$ , i.e. same acceleration rate, would decrease  $\hat{\delta}$  by a factor of 2 in comparison with the scenario of  $V_0=1.2MV$  and  $\sin \phi_0=0.04$ . At the same time, the synchrotron frequency is also decreased. The nonlinear effect become less important. Fig.8 shows the example of the phase space evolution in the 100KV rf scenario. Similarly, the loss rate is greatly reduced. Dashed curve of Fig.2 shows the loss rate.

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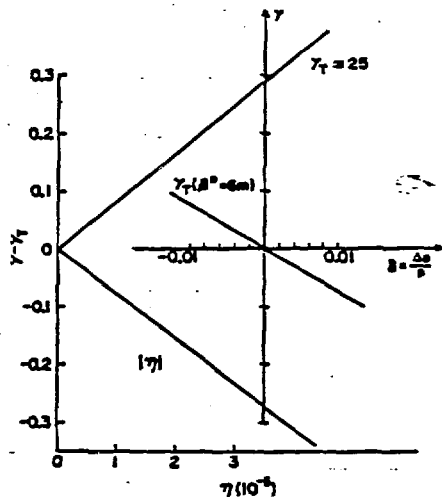


Fig 1.  $\gamma_T$  as a function of  $\delta = \Delta p/p$  for RHIC lattice with  $\beta^2=0.6$  and chromaticity 0.

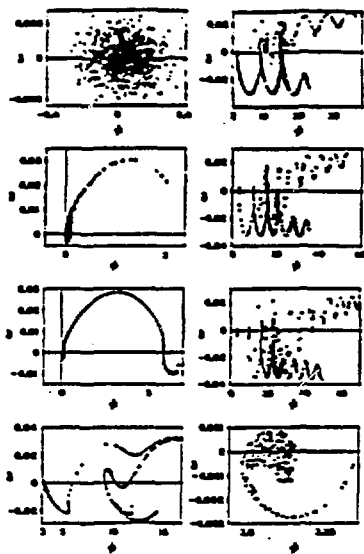


Fig 2. Particle tracking of longitudinal motion for  $V_0=1.2\text{MV}$   $\phi_0=0.04$  across the transition energy region. The time sequence is first left column from up to down, and then right column up to down. Each frame is separated by 51 msec.

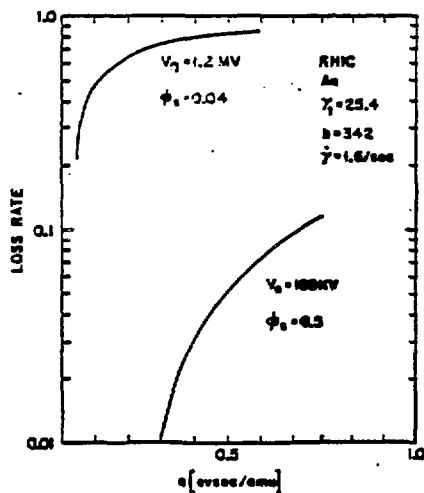


Fig 3. Loss vs phase space area is shown for two r.f. cases

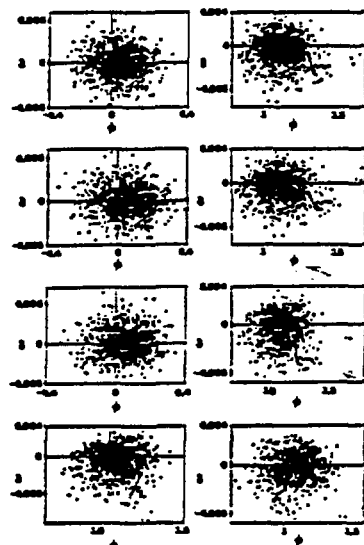


Fig 4. See the caption of Fig. 2 but with  $\gamma_T$  jump for  $\Delta\gamma_T=0.6$  in 30 msec.

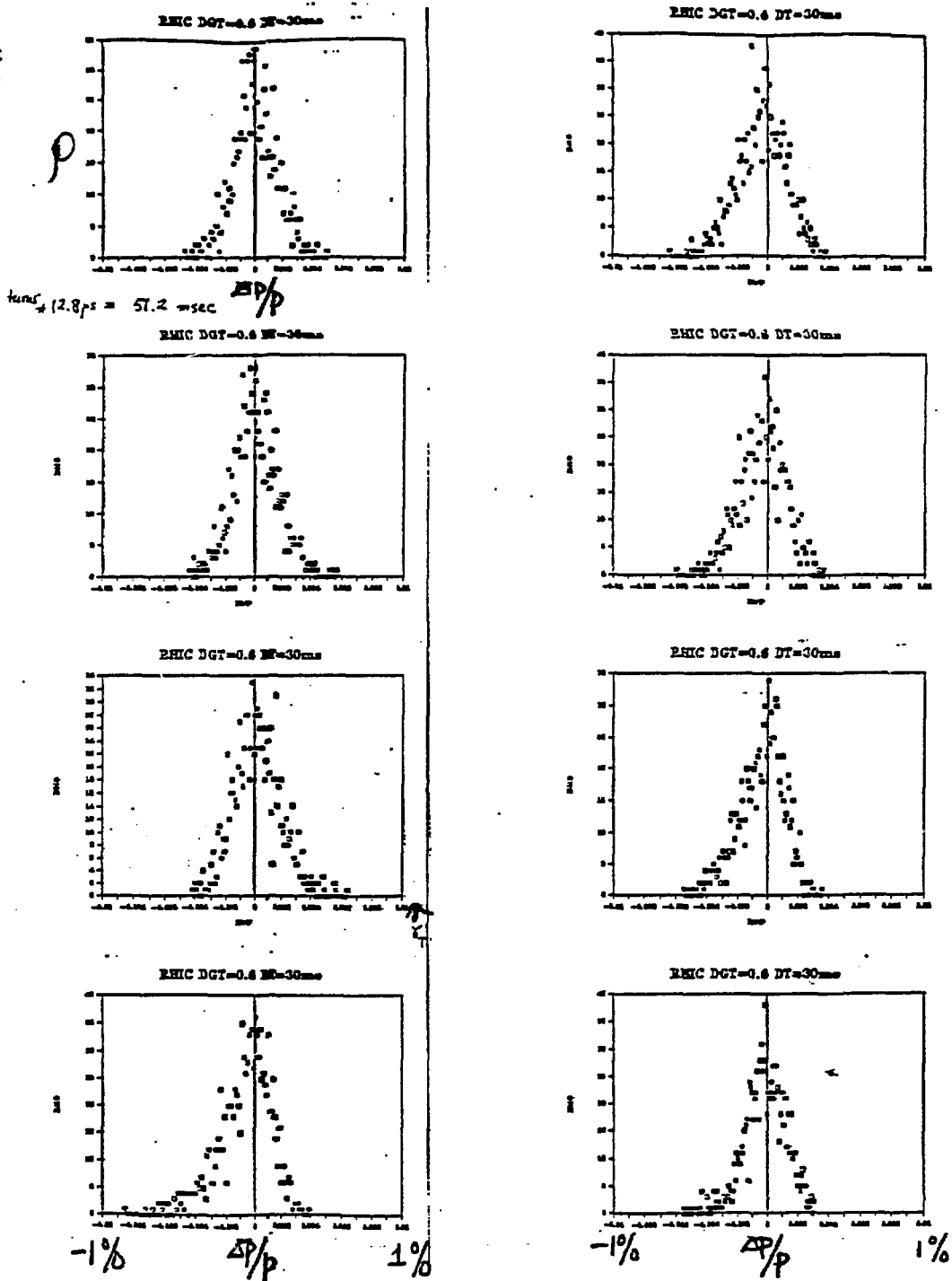


Fig. 5 Projection of the bunch distribution of Fig. 4 onto the momentum space,  $\Delta P/P$

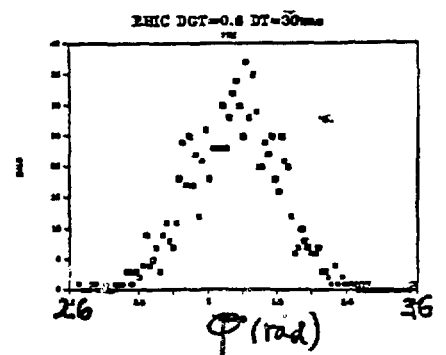
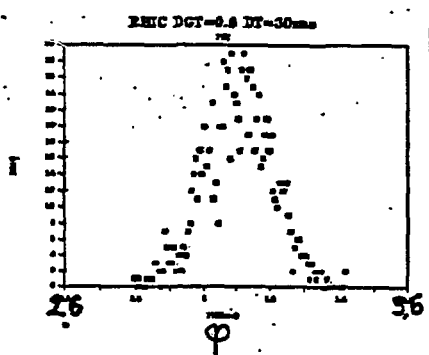
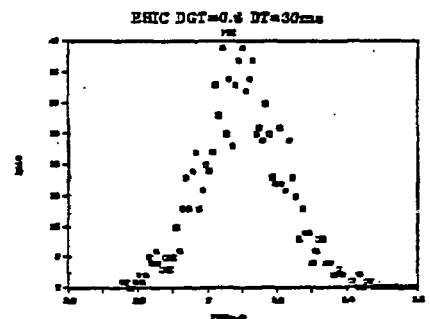
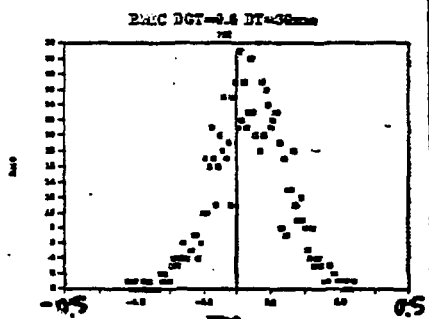
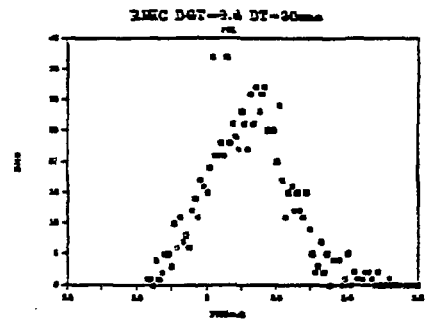
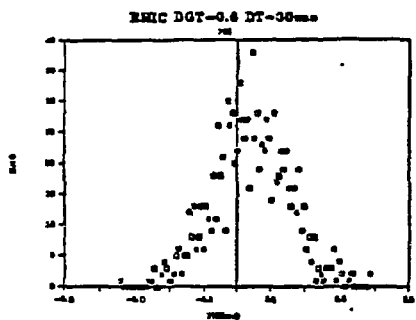
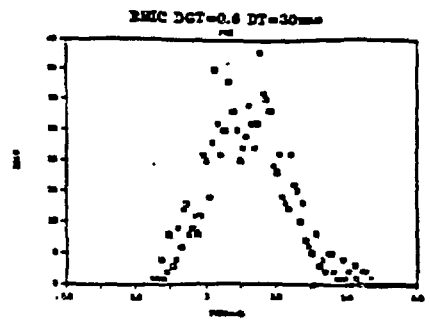
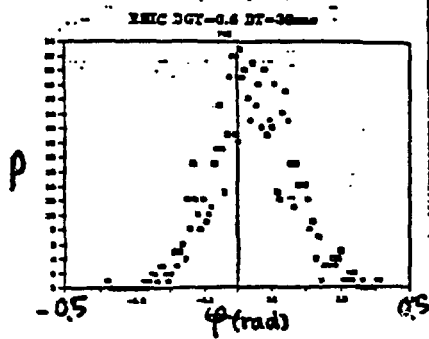


Fig.6 PROJECTION ONTO THE LONGITUDINAL COORDINATE  
From Fig.4 with a  $\chi_T$  jump.

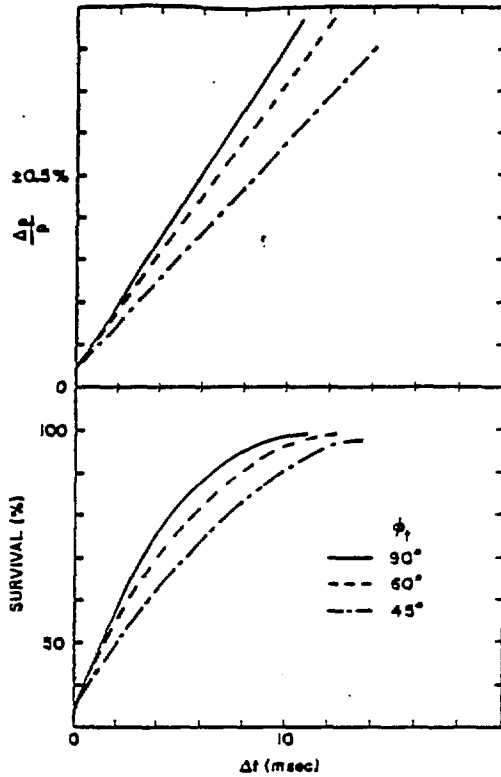


Fig 7. Survival through transition crossing is plotted as a function of fast acceleration time  $\Delta t$ .

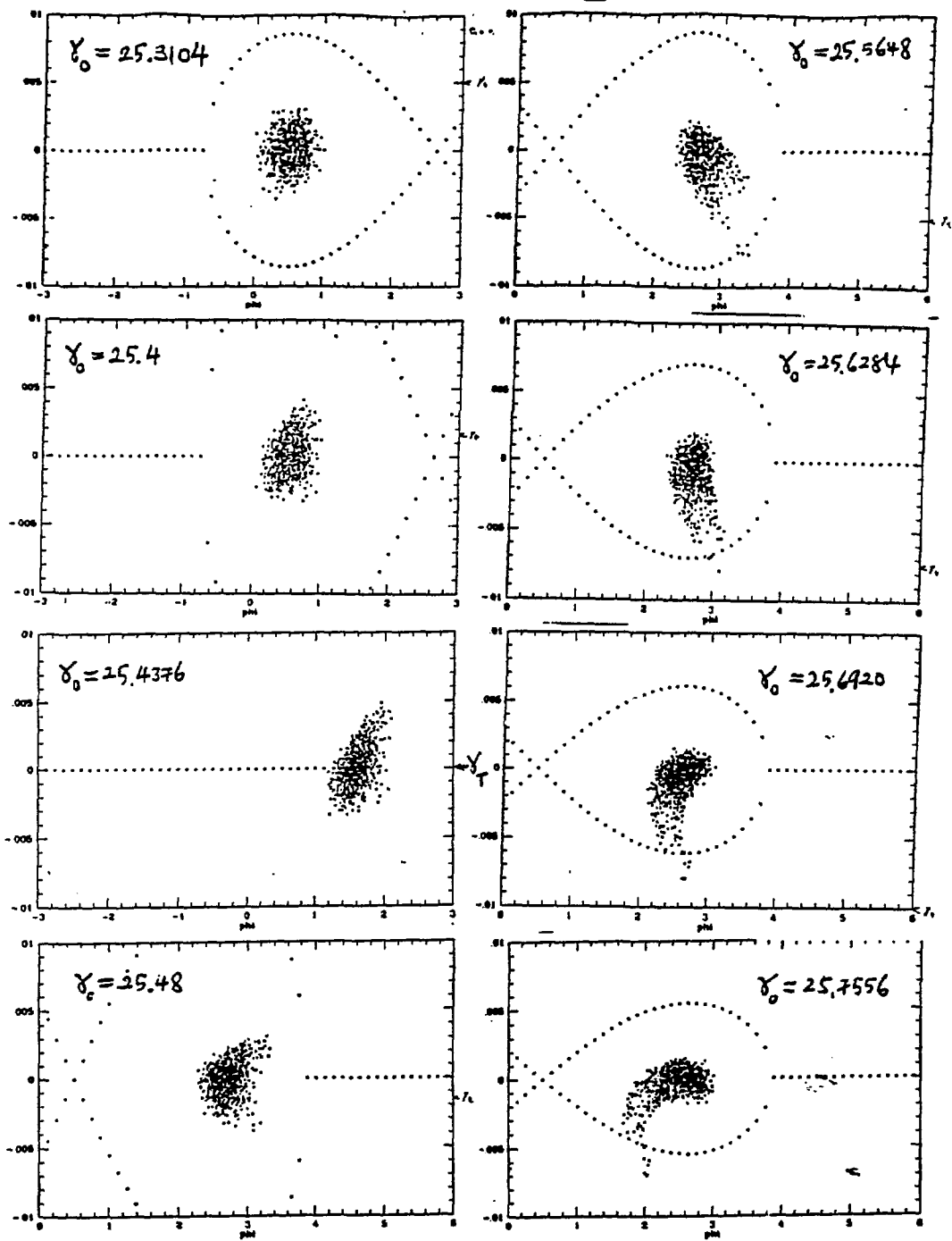


Fig. 8 Similar tracking result of that of Fig. 1 with  $V=100$  KV and  $\sin \theta = 0.48$ . THE loss rate becomes 1%.