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Real and Imaginary Parts of Hadronic Amplitudes, Diffractive Contribution, and the Chew-Rosenzweig Pomeron*

by

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Abstract:

We show that when the f' and the diffractive elastic (and quasi-elastic) contributions are included within the Chew-Rosenzweig type solution for the Pomeron, one can explain reasonably well the π and $(2K-\pi)$ total cross sections as well as the ratios of the real over imaginary parts of the forward π and K amplitudes.

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A promising approach to hadronic physics is the method of "dual unitarization and topological expansion."[1-4] Within this approach, Chew and Rosenzweig (C-R)[3] have made an interesting proposal that the Pomeron P is just the f-trajectory, renormalized and mixed with the planar f'-trajectory via the cylinder correction to the planar approximation. Veneziano[5] has pointed out that the "strict" Harari-Freund Pomeron[6] is incompatible with the topological expansion (TE)[1] and that the C-R Pomeron is the simplest realization of the TE. Some phenomenological analyses[7,8] have already shown that the C-R Pomeron with α_P slightly less than one can pass certain experimental checks at moderate energies (10 GeV/c $\leq p_{Lab} \leq$ 50 GeV/c). However, it has been stated[8,9] that the difference of total cross sections, (2K-π), where

$$K \equiv \frac{1}{4}(\sigma_{K^+p} + \sigma_{K^-p} + \sigma_{K^+n} + \sigma_{K^-n}) \quad \text{and} \quad \pi \equiv \frac{1}{2}(\sigma_{\pi^+p} + \sigma_{\pi^-p})$$

should fall with energy in the C-R scheme with $\alpha_P < 1$, whereas the data show a rise, although Chew-Rosenzweig-Stevens[8] pointed out that a proper consideration of the strange threshold may account for this rise. Furthermore, Freund and Romão[10] have recently claimed that it is impossible for the C-R scheme to simultaneously explain the experimental data on both (2K-π) and the ratio ρ_π of the real over imaginary part of the forward π amplitude. In particular, they argued that in the C-R scheme if (2K-π) rises (falls) with energy, then $\rho_\pi > 0$ (<0), whereas the data show (2K-π) rises and $\rho_\pi < 0$. Since the C-R solution is the simplest realization of the TE, Freund and Romão questioned the whole TE approach.

In this paper we answer the criticism of Freund and Romão. We want to point out two things. The first is that the renormalized f'-trajectory giv s an unambiguous negative contribution to (2K-π) and will be shown to cause (2K-π) to rise slightly with energy at moderate energies even if $\alpha_p \leq 1$. The second is that the TE up to the cylinder level as presently formulated has not included the whole elastic and quasi-elastic (called elastic for short) contributions; the part of the elastic contribution which has been included corresponds to a Regge exchange, which of course is only a small part of the whole elastic contribution in the energy range of interest. The missing piece, the diffractive contribution, will be shown to give a substantial negative contribution to ρ_π. In this paper, we take into account both the f' and diffractive contributions and show that the C-R Pomeron is consistent with the π, (2K-π), and ρ_π data; we also make a prediction for ρ_K, the ratio of real over imaginary part of the forward K amplitude.

Our starting point is the C-R matrix A up to the cylinder level for the even charge conjugation isoscalar trajectories

$$A = \begin{pmatrix} a & b \\ b & c \end{pmatrix}^{-1} = \begin{pmatrix} J-\alpha_0-2k & -\sqrt{2}\,k \\ -\sqrt{2}\,k & J-\alpha_3-k \end{pmatrix}^{-1} , \qquad (1)$$

where α_0 and α_3 are the planar $(f\text{-}\omega\text{-}\rho\text{-}A_2)$ and $(f'\text{-}\phi)$ intercepts and k is the cylinder coupling.[3] The output pole intercepts, given by the zeros of the determinant D of A, are

$$\alpha_{P,f'} = \tfrac{1}{2}[\alpha_0 + \alpha_3 + 3k \pm \sqrt{(\alpha_0-\alpha_3+k)^2 + 8k^2}] \quad . \qquad (2)$$

The SU(3) mixing angles, given by[3,7] $\tan\theta_i = -\frac{b(J=\alpha_i)}{c(J=\alpha_i)}$, are

$$\tan\theta_P = -\mathrm{ctn}\,\theta_{f'} = \frac{2\sqrt{2}\,k}{\alpha_0 - \alpha_3 + k + \sqrt{(\alpha_0-\alpha_3+k)^2 + 8k^2}} \ . \qquad (3)$$

The pole residues, given by[7] $\gamma_i = \left.\frac{(J-\alpha_i)[a(J) + c(J)]}{D(J)}\right|_{J=\alpha_i}$, gives

$$\gamma_P = \gamma_{f'} \equiv \gamma \quad . \qquad (4)$$

Using the additive quark model for coupling the quark states $q\bar{q}$ to the external meson and nucleon, we can then write[7]

$$\pi_M = \beta[(\frac{s}{s_0})^{\alpha_P-1} + (\frac{s}{s_0})^{\alpha_{f'}-1} \tan^2\theta_P] , \quad (5a)$$

$$(2K-\pi)_M = \sqrt{2}\,\beta \tan\theta_P[(\frac{s}{s_0})^{\alpha_P-1} - (\frac{s}{s_0})^{\alpha_{f'}-1}] , \quad (5b)$$

where $\beta \equiv \frac{3\gamma G}{2(1+\tan^2\theta_P)}$ with G being the product of the Reggeon-$q\bar{q}$ couplings, and the subscript M reminds us that what we have calculated is the multiperipheral (or nondiffractive) contribution, to which we must add the diffractive elastic and quasi-elastic contributions. Note that in Eq. (5b), the f' gives a negative contribution.[F1] The diffractive elastic contributions π_D and K_D are given by

$$\pi_D = \frac{C_\pi s^{2\alpha_P-2}}{B_\pi} , \quad \text{with } B_\pi = 2b_\pi + 2\alpha_P' \ln s , \quad (6a)$$

$$K_D = \frac{C_K s^{2\alpha_P-2}}{B_K} , \quad \text{with } B_K = 2b_K + 2\alpha_P' \ln s , \quad (6b)$$

where in our calculation we use a nominal value of 0.5 for α_P'. At the highest energy of the present model (50 GeV/c), we make use of the Pomeron dominance approximation for the elastic cross section, and taking the extrapolated experimental values[F2] at this energy of $\pi_D \approx 3.1$ mb, $K_D \approx 2.3$ mb,[11] $B_\pi \approx 8.3\ (\text{GeV/c})^{-2}$, and $B_K \approx 7.8\ (\text{GeV/c})^{-2}$,[12] we can determine from Eq. (6) the quantities b_π, b_K, C_π, and C_K. Equation (6) then gives the energy dependence of π_D and K_D. The total cross sections are then given by

$$\pi = \pi_M + \lambda\pi_D , \quad (7a)$$

$$(2K-\pi) = (2K-\pi)_M + \lambda(2K-\pi)_D , \quad (7b)$$

where the factor λ takes into account the diffractive quasi-elastic contribution. In our calculation we set it at the reasonable value of $\lambda = 1.5$.[F3] Before presenting our quantitative results, we first discuss ρ_π and ρ_K.

We preent the derivation for ρ_π. The full amplitude has two contributions corresponding to the multiperipheral and diffractive components. The forward elastic amplitude A_π is related to π by the Optical Theorem, $\text{Im } A_\pi = s\pi$, with $A_\pi \equiv A_\pi^M + A_\pi^D$. Defining $\rho_\pi^M \equiv \frac{\text{Re } A_\pi^M}{\text{Im } A_\pi^M}$ and $\rho_\pi^D \equiv \frac{\text{Re } A_\pi^D}{\text{Im } A_\pi^D}$, one can easily show that

$$\rho_\pi = (1-\eta_\pi)\rho_\pi^M + \eta_\pi\rho_\pi^D , \qquad (8a)$$

where $\eta_\pi \equiv \lambda(\frac{\pi_D}{\pi})$, i.e., the ratio of the diffractive cross section over the total cross section. In our model, ρ_π^M is given by

$$\rho_\pi^M = - \left[\frac{\text{ctn}(\frac{\pi\alpha_P}{2})\cos^2\theta_P(\frac{s}{s_0})^{\alpha_P} + \text{ctn}(\frac{\pi\alpha_{f'}}{2})\sin^2\theta_P(\frac{s}{s_0})^{\alpha_{f'}}}{\cos^2\theta_P(\frac{s}{s_0})^{\alpha_P} + \sin^2\theta_P(\frac{s}{s_0})^{\alpha_{f'}}} \right] , \qquad (9a)$$

which of course reduces to the usual $-\text{ctn}(\frac{\pi\alpha_P}{2})$ when f' is neglected. We can also write

$$\text{Im } A_\pi^D = s(\lambda\pi_D) \approx \frac{\lambda}{16\pi}\int_{-\infty}^{0} dt \left(\frac{\beta(t)}{\sin\frac{\pi\alpha_P(t)}{2}}\right)^2 s^{\varepsilon(t)} ,$$

where $\varepsilon(t) \equiv 2\alpha_P(t) - 1$.[F4] Now since A_π is, by construction, crossing even, and A_π^M is also crossing even as only vacuum trajectories contribute, this leads to a crossing even A_π^D. From a dispersion relation calculation similar to the one used in obtaining the usual even signature factor, the full diffractive amplitude is then given by

$$A_\pi^D = - \frac{\lambda}{16\pi}\int_{-\infty}^{0} dt \left(\frac{\beta(t)}{\sin\frac{\pi\alpha_P(t)}{2}}\right)^2 \frac{s^{\varepsilon(t)}\, e^{-\frac{i\pi\varepsilon(t)}{2}}}{\sin\frac{\pi\varepsilon(t)}{2}} .$$

Due to "non-sense zeros" in $\beta(t)$, the quantity in parenthesis does not have poles. Since the integrand is expected to peak exponentially at $t = 0$, and since $\alpha_P(0) \approx 1$, we may ignore the slowly varying t-dependence in $\sin\frac{\pi\varepsilon(t)}{2}$ near $t = 0$ and take it outside the integral.[F5] As in deriving (6), the quantity in parenthesis has the form $e^{b_\pi t}$. This gives

$$A_\pi^D \approx - \frac{\lambda\beta_0^2}{16\pi \sin\frac{\pi\varepsilon(0)}{2}} \int_{-\infty}^{0} dt\; e^{2b_\pi t}\, s^{\varepsilon(t)} \left[\cos\frac{\pi\varepsilon(t)}{2} - i \sin\frac{\pi\varepsilon(t)}{2}\right] .$$

Upon doing the integrals,[13] we get

$$\rho_\pi^D = - \left[\frac{(2b_\pi + 2\alpha_P' \ln s)\sin\pi\delta + \pi\alpha_P'\cos\pi\delta}{(2b_\pi + 2\alpha_P' \ln s)\cos\pi\delta - \pi\alpha_P'\sin\pi\delta}\right] , \qquad (10a)$$

where $\delta \equiv 1 - \alpha_P(0)$. Similarly, for the kaon case, if $\eta_K \equiv \lambda(\frac{K_D}{K})$, we get

$$\rho_K = (1-\eta_K)\rho_K^M + \eta_K\rho_K^D \quad , \qquad (8b)$$

$$\rho_K^M = - \left[\frac{\operatorname{ctn}\frac{\pi\alpha_P}{2}\cos^2\theta_P(1+\sqrt{2}\tan\theta_P)(\frac{s}{s_0})^{\alpha_P} + \operatorname{ctn}\frac{\pi\alpha_{f'}}{2}\sin^2\theta_P(1-\sqrt{2}\operatorname{ctn}\theta_P)(\frac{s}{s_0})^{\alpha_{f'}}}{\cos^2\theta_P(1 + \sqrt{2}\tan\theta_P)(\frac{s}{s_0})^{\alpha_P} + \sin^2\theta_P(1 - \sqrt{2}\operatorname{ctn}\theta_P)(\frac{s}{s_0})^{\alpha_{f'}}}\right], \qquad (9b)$$

$$\rho_K^D = - \left[\frac{(2b_K + 2\alpha_P' \ln s)\sin\pi\delta + \pi\alpha_P'\cos\pi\delta}{(2b_K + 2\alpha_P' \ln s)\cos\pi\delta - \pi\alpha_P'\sin\pi\delta}\right] . \qquad (10b)$$

Equations (2)-(10) determine π, $(2K-\pi)$, ρ_π, and ρ_K in terms of α_0, α_3, β, k, and s_0. Since α_0 is the planar (f-ω-ρ-A_2) intercept and the cylinder does not renormalize the ρ-A_2 trajectories, we expect $\alpha_0 \approx 0.5$. As discussed in Ref. 7, planar bootstrap leads to an equal spacing rule and implies $(\alpha_0-\alpha_3)$ is twice the separation of the ρ and K* intercepts, which we take as 0.2,[14] i.e., $\alpha_3 \approx 0.3$. The quantity β is a normalization constant which we determine by using the experimental π cross section at 30 GeV/c (mid-point of energy range of interest). Therefore, our model has essentially two free parameters, k and s_0.

The results of our calculation for $\alpha_0 = 0.5$ and $\alpha_3 = 0.3$ are shown in Figs. 1 and 2 with $k = 0.18$ and $s_0 = 1.5$ GeV^2. These correspond to $\alpha_P = 0.97$, $\alpha_{f'} = 0.35$ and $\text{ctn}\theta_P = 2.0$. We see the calculated ρ_π is large and negative and in excellent agreement with the data. Notice the (2K-π) curve does not fall but actually rises slightly over this energy region. Both the π and (2K-π) curves are in approximate agreement with the data. We want to emphasize the point raised in Ref. 8, i.e., the energy dependence of the strange threshold has not been taken into account in the model. This threshold effect has the consequence of causing the π cross section to fall faster (slower) and the (2K-π) cross section to be smaller (larger) at the lower (higher) end of our energy range and therefore should further improve the agreement with the data. So we conclude that our model is consistent with the data. In Fig. 2a we also plotted separately the multiperipheral contribution $(1-\eta_\pi)\rho_\pi^M$ and the diffractive contribution $\eta_\pi\rho_\pi^D$. Note that the diffractive component gives a sizeable negative contribution. In the K case, the negative ρ_K value comes essentially from the diffractive contribution. The theoretical curve is expected to be between ρ_{K^+p} and ρ_{K^-p}. Although there is not sufficient data, our result is consistent with the existing $\rho_{K^\pm p}$ data.[F6]

We also found a similar quality of fits for $\alpha_0 = 0.55$, $\alpha_3 = 0.35$ with $k = 0.16$ and $s_0 = 1.0$ GeV^2. These correspond to $\alpha_P = 0.98$, $\alpha_{f'} = 0.40$, $\text{ctn}\theta_P = 2.1$. Note that these and the previously mentioned values are quite close to those of solution 2 in Ref. 7.

Our results tell us that the inclusion of the f'-contribution and the diffractive elastic (and quasi-elastic) contributions are of crucial importance in explaining the (2K-π) and ρ_π data. Their inclusion allows us to conclude that the C-R Pomeron is a viable scheme.

Footnotes

F1. This negative sign is not due to a particular sign convention in the couplings or due to the convention in defining the mixing angle $\theta_{f'}$. Both Eqs. (5a) and (5b) follow directly from the additive quark model expressions given in Ref. 7,

$$\pi_M = G[\tfrac{3}{2}\gamma_P s_P \cos^2\theta_P + \tfrac{3}{2}\gamma_{f'} s_{f'} \cos^2\theta_{f'}]$$
$$K_M = G[\tfrac{3}{4}\gamma_P s_P \cos^2\theta_P (1 + \sqrt{2}\tan\theta_P) + \tfrac{3}{4}\gamma_{f'} s_{f'} \cos^2\theta_{f'} (1 + \sqrt{2}\tan\theta_{f'})] \quad ,$$

together with Eqs. (3) and (4).

F2. Our results are insensitive to small changes in those values given. Also there is no elastic cross section data for $K^{\pm}n$. We assume the sum of $K^{\pm}n$ elastic cross sections to be the same as that of $K^{\pm}p$.

F3. Of course λ for π may be slightly different from λ for K. To make the discussion simpler and to reduce the number of parameters, we set $\lambda_\pi = \lambda_K \equiv \lambda$. Our conclusions do not depend sensitively on this choice or on the specific λ value chosen.

F4. Technically speaking, the quantity $\lambda^{1/2}\left(\frac{\beta(t)}{\sin\frac{\pi\alpha_P(t)}{2}}\right)$ should be replaced by some pertinent triple Regge vertex $g(0,t,t)$.

F5. The factor $\sin\frac{\pi\varepsilon(t)}{2}$ has zeros at $\varepsilon(t) = 0, -2, \ldots$ or at $t \approx -1, -3, \ldots$ $(\mathrm{GeV}/c)^2$. In general these zeros might be cancelled by corresponding zeros in $g(0,t,t)$ of Footnote 4 although such cancellation does not occur in the usual dual resonance model. Even for no cancellation, these zeros only give rise to some finite contribution of the order of s^0, s^{-2}, ... (up to lns factors).

F6. To make more quantitative check of our prediction for ρ_K, one needs $K^{\pm}n$ data.

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Figure Captions

Fig. 1. Theoretical curves compared with: (a) the π data, and (b) the $(2K-\pi)$ data. The data are from Refs. 11 and 15. Note the different scales for (a) and (b).

Fig. 2. (a) Theoretical curve (solid) compared with the ρ_π data of Ref. 16. Also shown are the curves of diffractive and multiperipheral contributions. (b) Theoretical ρ_K curve, and the ρ_{K^+p} data (x) and ρ_{K^-p} data (•) of Ref. 17. Note that (b) uses the right vertical scale.

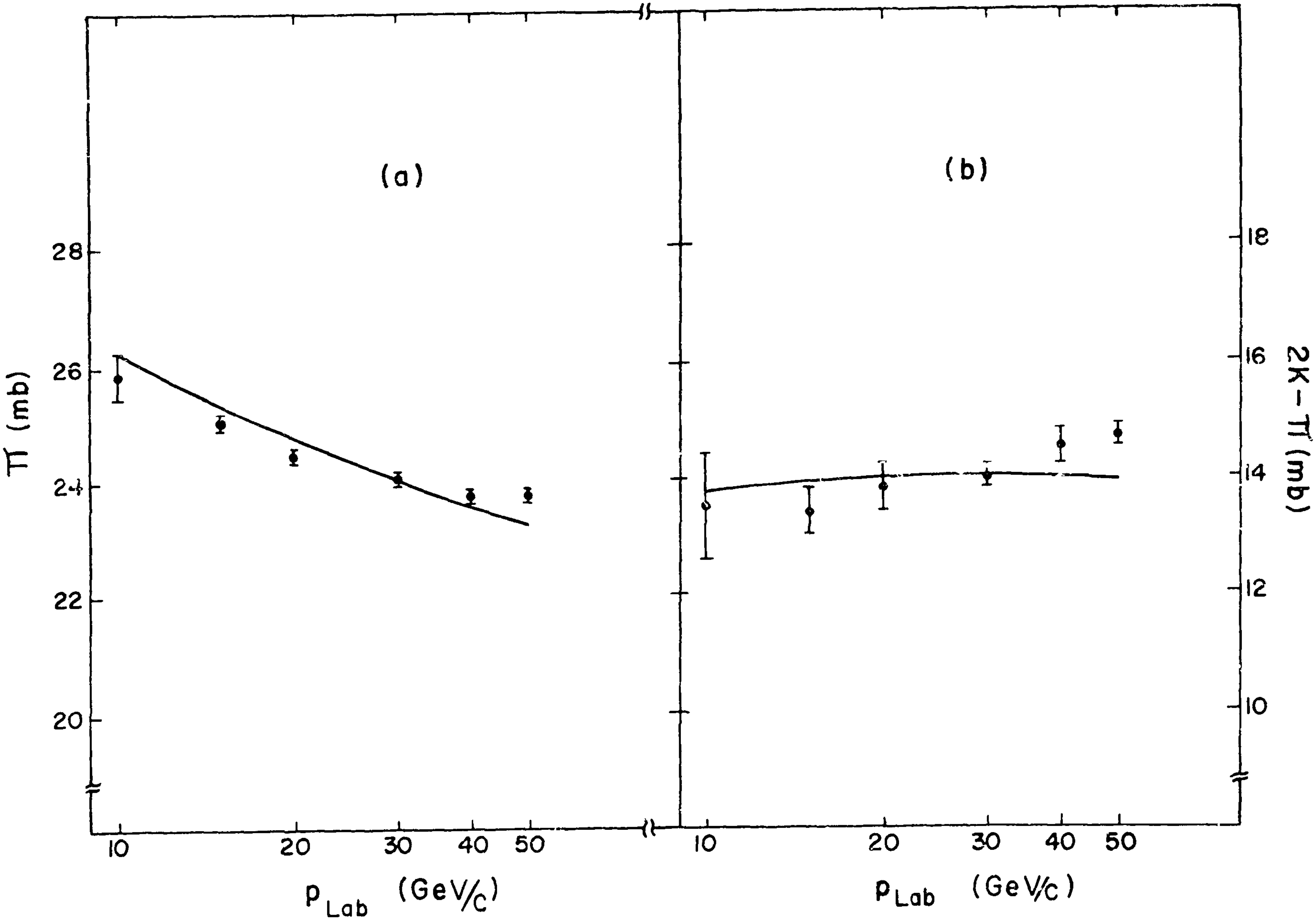
(a)
(b)
28
26
24
22
20
σ (mb)
18
16
14
12
10
2K − σ (mb)
10
20
30
40
50
p Lab (GeV/c)
10
20
30
40
50
p Lab (GeV/c)

Figure 1

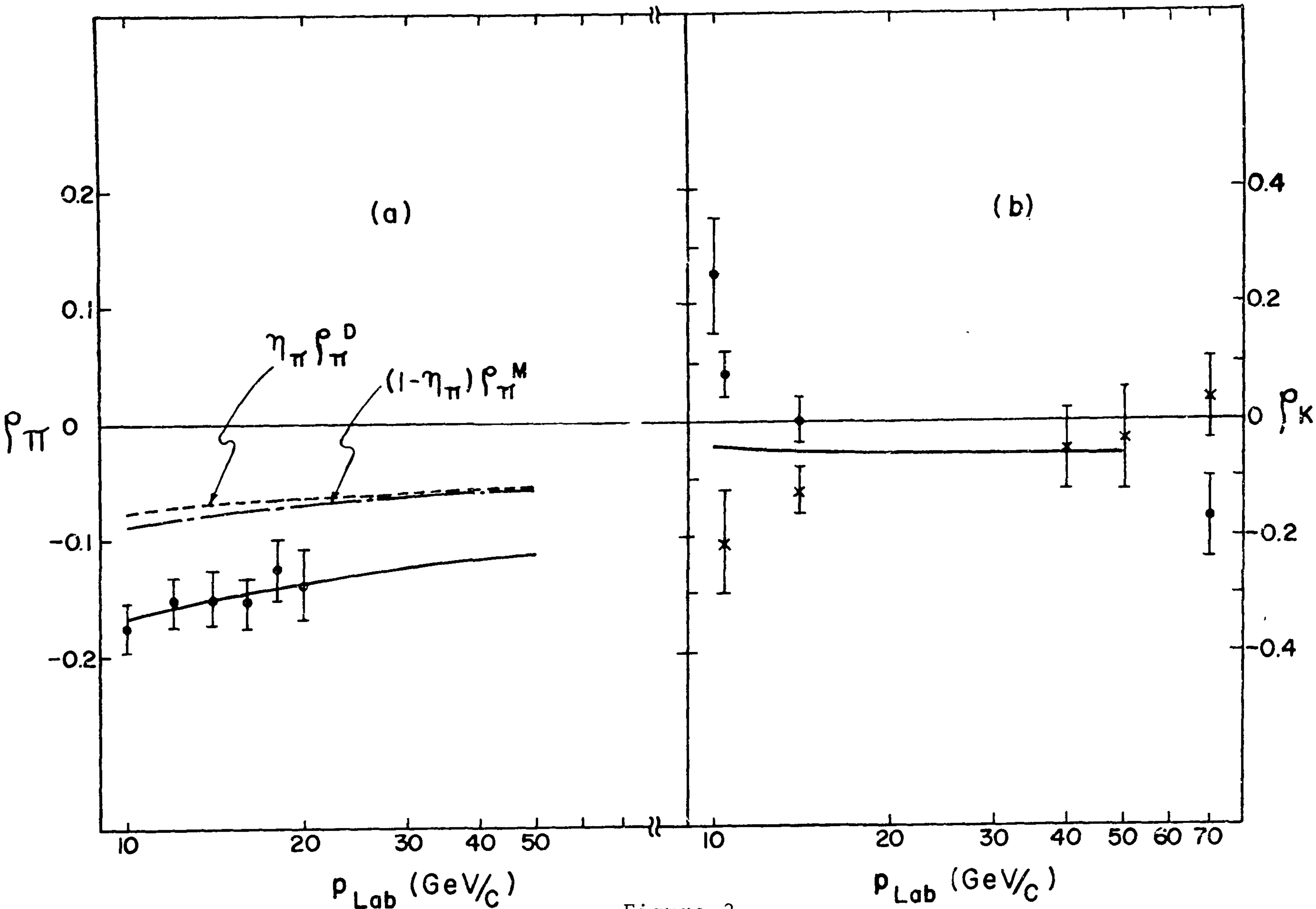
(a)
(b)
0.2
0.1
0
-0.1
-0.2
0.4
0.2
-0.2
-0.4
ρ_π
ρ_K
$\eta_\pi \rho_\pi^D$
$(1-\eta_\pi)\rho_\pi^M$
10
20
30
40
50
60
70
p_{Lab} (GeV/c)

Figure 2