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TITLE SOLAR OSCILLATION FREQUENCY AND SOLAR NEUTRINO PREDICTIONS

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SOLAR OSCILLATION FREQUENCY AND SOLAR NEUTRINO PREDICTIONS

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ABSTRACT The light and velocity variations of the Sun and solar-like stars are unique among intrinsic variable stars. Unlike all other standard classes, such as Cepheids, B stars, and white dwarfs, the pulsation driving is caused by coupling with the acoustic noise in the upper convection zone. Each global pulsation mode is just another degree of freedom for the turbulent convection, and energy is shared equally between these g^--modes and the solar oscillation modes. This driving and damping, together with the normal stellar pulsation mechanisms produce extremely low amplitude solar oscillations. Actually, the surface layer radiative damping is strong, and the varying oscillation mode amplitudes manifest the stochastic convection driving and the steady damping. Thus stability calculations for solar-like pulsations are difficult and mostly inconclusive (Cox, Chitre, Frandsen, and Kumar, 1990), but calculations of pulsation periods are as straightforward as for all the other classes of intrinsic variable stars. The issue that is important for the Sun is its internal structure, because the mass, radius, and luminosity are extremely well known. Conventionally, we need the pulsation constants for each of millions of modes. Unknown parameters for constructing solar models are the composition and its material pressure, energy, and opacity, as well as the convection mixing length. We treat the nuclear energy and neutrino production formulas as sufficiently well known. The presence of weakly interacting massive particles (WIMPs) (Faulkner and Gilliland, 1985, Spergel and Press, 1985, and Gilliland Faulkner, Press, and Spergel, 1986) orbiting the solar center affects the predicted oscillation frequencies so that they do not agree with observations as well as those for models without WIMPs (Cox, Guzik, and Raby, 1990). Results will be presented for neutrino outputs and oscillation frequencies for variations of the convection zone and complete ionization equations of state, and for variations of the opacity at the surface, at the bottom of the convection zone, and at the solar center. The conclusions are that solar oscillation frequencies are predicted to be very close to those observed, but solar neutrino flux predictions continue to be much larger than observed.

INTRODUCTION AND BACKGROUND

Solar evolution and oscillation studies for the past 10 years reveal that there is no important confrontation between stellar pulsation and stellar evolution theories in the case of main sequence stars near the solar mass. However, there still remain the more ordinary conflicts between some predictions and observations. From the point of view of the solar interior, the only large problem is the continuing prediction that the solar interior should generate about four times as many neutrinos as actually observed by the chlorine detector. To further compound the problem, there are only rough guesses about how the neutrino output might vary with the sunspot cycle and even with the time of year. Since this is a review about stars with variable light output, and not about varying neutrino fluxes, I leave the neutrino problems for others. I show that solar oscillations refine our knowledge of the solar structure and evolution.

TABLE I Some Recent Standard Solar Models

Authors	Journal	Y	SNU
Christensen-Dalsgaard	MNRAS, 1982	0.247	5.4
Ulrich	Ap. J., 1982	.264	9.0
Shibahashi, Noels, Gabriel	A. A., 1983	.251	-
Noels, Scuflaire, Gabriel	A. A., 1984	.274	-
Faulkner, Gilliland	Ap. J., 1985	.243	9.2
Lebreton, Maeder	A. A., 1986	.282	13.3
Gilliland, Faulkner, Press, Spergel	Ap. J., 1986	.252	5.7
Bahcall, Ulrich	RMP, 1988	.271	7.9
Lebreton, Dappen	Tenerife, 1988	.278	7.6
Guenther, Sarajedini	Ap. J., 1988	.24	-
Lebreton, Berthomieu, Provost, Schatzman	A. A., 1988	.287	8.0
Lebreton, Berthomieu, Provost Schatzman	A. A., 1988	.291	8.4
Turck-Chieze, Cahen, Casse, Doom	Ap. J., 1988	.276	5.8
Christensen-Dalsgaard, Dappen, Lebreton	Nature, 1988	.237	-
Korzennik, Ulrich	Ap. J., 1989	.271	8.2
Guenther	Ap. J., 1989	.282	-
Guenther, Jaffe, Demarque	Ap. J., 1989	.28	-
Cox, Guzik, Kidman	Ap. J., 1989	.291	11.4
Cox, Guzik, Raby	Ap. J., 1990	.28	9.0
Sackmann, Boothroyd, Fowler	Ap. J., 1990	0.278	7.7

Table I gives some of the recent papers that have been published about standard solar evolution. This means mostly that there is no allowed internal mixing of the elements produced by nuclear reactions. The only mixing is in the deep surface convection zone and slightly deeper by overshooting. Nonradial oscillation frequencies are calculated in less than half of these studies, probably because the procedures to obtain data for the mode frequencies and linear theory growth rates are rather intricate and have been developed by only a few research teams. In this review I discuss only the recent p-mode results by Christensen-Dalsgaard, Dappen, and Lebreton and those calculated by me and Joyce Guzik together with Russell Kidman and Stuart Raby at Los Alamos. Earlier pulsation studies are already out of date.

The table gives the helium content required in each of the 20 models to obtain the current solar age, luminosity, and radius for the well known solar mass. All these investigations assume an original homogeneous composition throughout. The highest helium mass fraction, Y=0.291, is obtained by Lebreton et al. and by our Los Alamos team. The lowest is Y=0.237 from Christensen-Dalsgaard, Dappen and Lebreton. I do not readily understand this low value, but I do know that our high Y value is due to an equation of state error that made our pressure almost one percent too large. When the error was corrected and the extra helium was no longer needed to make up mass without producing as much pressure as hydrogen, our required Y went down to 0.273. The pressure equation of state is extremely important in precise solar modeling.

Also given in the table is the neutrino output presented in the papers. The high values usually go with the high Y, because these models have larger central temperatures to produce the required luminosity with less central hydrogen. The correction of our equation of state pressure error reduced the solar neutrino output from 11.4 to 8.7 SNU. One can see from the SNU values that the generally agreed predictions seem to be very close to 8 SNU, almost 4 times the observed average over the last 20 years with the Homestake mine chlorine detector.

The procedure to calculate these current solar models involves selecting the Y value and a mixing length for standard convection theory (Cox and Giuli, 1968) so that after approximately 4.5×10^9 years, the model has the well known solar radius and luminosity. During this evolution, about half of the central hydrogen is transmuted to helium, but the helium fusion decreases to none at all about eighty percent of the mass from the center. Typically from tens to hundreds of discrete time steps are taken to calculate the hydrogen fusion and the slow shrinking of the matter because the helium produced does not have as much pressure per gram as hydrogen.

It is extremely important that the matter pressure and energy and their derivatives with respect to temperature and density be well known for the varying composition throughout the model. This varying composition mostly comes from the hydrogen fusion, but recent work by Cox, Guzik, and Kidman (1989) has considered the small effects of diffusive settling of helium and the heavier elements and the floating of hydrogen. These effects are small, and affect only the convection zone composition, because the time scales for the deeper regions are much longer than the solar age. It seems that the diffusion coefficients we used in that work were a bit too large, as suggested by some experts, and that seems reasonable since the oscillation frequencies for the no-diffusion model agree better with observations than for the model with the diffusion effects. The problems of element separation remain of interest for many stellar evolution problems, but at least for the Sun, pulsation frequency and mode stability predictions need only to consider standard solar models. We most recently have considered only models without any helium or heavy element settling.

SOLAR MODELS

Our Los Alamos solar modeling has been done mostly using the Iben (1963, 1965, 1975) equation of state and opacity procedures. We have found that calibrating these with actual Los Alamos Astrophysical Opacity Library tables has resulted in models that produce p-mode oscillation frequencies very close to those observed. However, use of the new MHD equation of state (Däppen *et al.*, 1988) is certainly better than the simple Saha approximation in the Iben procedure, and we have found that our predicted frequencies for MHD solar models are as good or better.

The earlier work by Cox, Guzik, and Kidman (1989) used the Iben equation of state directly, but it was noticed that there was about a one percent pressure error. This small error gave more intrinsic pressure for matter than it really should have, and to compensate for the problem, the lower pressure per unit mass helium was needed in the models that matched the solar mass radius, and luminosity at the current solar age. Then the solar center was too hot, because a higher temperature was needed to produce the correct luminosity with less hydrogen fuel. It was finally discovered that the original Iben molecular weights for hydrogen and helium were set to exactly 1 and 4. When the almost one percent higher, correct molecular weights were used in the evolution and pulsation model programs, the corrected Iben models produced oscillation frequencies much closer to those observed.

TABLE II Convection Zone Iben and MHD Equation of State

$T(10^6 K)$	$\rho(g/cm^3)$	$\Gamma_3 - 1$ Iben	$\Gamma_3 - 1$ MHD	Γ_1 Iben	Γ_1 MHD	$b'(10^8 erg/g/K)$ Iben	$b'(10^8 erg/g/K)$ MHD	$c_v(10^8 erg/g/K)$ Iben	$c_v(10^8 erg/g/K)$ MHD
1.4818	9.539E-2	0.662	0.657	1.667	1.662	1.321	1.327	2.04	2.05
1.0178	5.420E-2	0.661	0.659	1.668	1.665	1.316	1.323	2.04	2.04
0.4974	1.843E-2	0.657	0.658	1.668	1.670	1.304	1.312	2.07	2.05
0.3026	8.668E-3	0.647	0.649	1.664	1.667	1.289	1.298	2.12	2.09
0.2024	4.618E-3	0.617	0.626	1.644	1.651	1.270	1.281	2.26	2.18
0.1012	1.327E-3	0.529	0.518	1.565	1.554	1.213	1.232	2.66	2.63
0.0898	1.056E-3	0.540	0.525	1.577	1.560	1.202	1.222	2.58	2.61
0.0794	8.429E-4	0.551	0.542	1.590	1.578	1.191	1.212	2.52	2.50
0.0702	6.738E-4	0.547	0.551	1.591	1.593	1.179	1.201	2.55	2.45
0.0576	4.627E-4	0.498	0.521	1.550	1.576	1.155	1.181	2.89	2.65
0.0499	3.396E-4	0.440	0.467	1.495	1.532	1.134	1.161	3.39	3.05
0.04009	1.932E-4	0.351	0.370	1.406	1.439	1.094	1.123	4.48	4.10
0.03011	7.612E-5	0.272	0.281	1.326	1.356	1.030	1.056	6.12	5.78
0.02007	1.282E-5	0.182	0.182	1.239	1.251	0.910	0.924	9.75	9.83
0.01002	3.591E-7	0.180	0.178	1 222	1.221	0.667	0.664	4.97	5.02
0.00598	2.591E-7	0.633	0.633	1.636	1.636	0.640	0.636	1.02	1.01

Table II gives a comparison between the equation of state quantities needed for our evolution and pulsation studies. Four thermodynamic quantities from the corrected Iben and fine MHD tables are given for temperature, density pairs in the convection zone. The V is PV/T. The differences are typical of two different equation of state calculations, and they both result in the very good agreement between theoretical and observational oscillation frequencies for low degree modes that are not highly concentrated in the upper convection zone. For high degree modes differences are greater. This is not surprising since at 50,000K the Γ_1 values differ by 0.037, and at 40,000K the pressure given by the two equations of state differ by about 3 %.

One can further note that the older elaborate EXOP equation of state values from Los Alamos (Cox, 1965) seem to be closer to the Iben simple Saha type of equation of state. Thus the amount of new physics for the MHD data must be considerable.

Opacities are also needed for the models discussed in this paper. At Los Alamos, we have decided that an easy way to allow for the composition varying in space and time is to use the Iben opacity procedures that we can calibrate to conform to the latest data. Since the Iben fit was made to the original Cox-Stewart (1965) opacities, and in the late 1960's the solar iron abundance was increased by a factor of 10, these opacities are low. The only important region where this composition change is important is at the solar center where a single iron line and a single iron absorption edge comprises 1/3 of the total opacity. The Iben procedure has a term (κ_e)that we multiply by 1.5 for our two models to allow for the increased iron.

As discussed by Cox, Guzik, and Kidman (1989), solar oscillation frequencies match observations if the Los Alamos Astrophysical Opacity Library values are increased by about 15-20 percent in the region around 2 to 7 million kelvin, just below the surface convection zone. Just one year ago we heard that this increase can come naturally from previously neglected same-shell transitions in the M shell of highly ionized iron. Iglesias and Rogers (1990) get an 18 percent opacity increase over the Library, and that is exactly what we need. To mimic this effect, we multiply an Iben term (A_t) by 1.3 for our corrected Iben equation of state model in this paper, and we multiply the term by 1.27 (an improved estimate) for the MHD model.

Finally, we need to allow for the number of mostly neutral iron lines at the solar surface that should have been much larger in earlier Cox-Tabor opacities. More comprehensive calculations for the Opacity Library, which are now possible since single elements are calculated separately, have a much better representation of the true solar surface opacities. We allow for this large effect by multiplying the Stellingwerf (1975ab) opacity fit to the Cox-Tabor opacities everywhere by a factor of 3 for the corrected Iben equation of state model and a factor of 2 for the MHD model.

The global quantities of mass, radius, surface effective temperature, luminosity, and the internal composition structure are used to calculate a special 1700 mass zone model that is used for our pulsation analyses. In this model the mass shells are very thin both at the surface and at the center so that we can obtain good spatial resolution for both high radial order p- and g-modes. The construction of this model is done by integrating the equations of pressure balance (momentum equation) and luminosity conservation (energy equation) from the surface to the center in only one pass. Just slight adjustments in the Y values (typically much less than 0.001) and in the convection mixing length are needed to assure that the evolution model is well tracked and that the center is reached with all the space and mass accounted for. The integration from the surface works well in this case only because we use exactly the same physics in both the evolution and the pulsation model building codes. More details can be found in the Cox, Guzik, and Kidman (1989) and Cox, Guzik, and Raby (1990) papers.

Our new models with the corrected Iben equation of state and with the MHD equation of state produce a neutrino output for the chorine detector of 8.7 and 8.4 SNU, respectively, very similar to the average for all the recent models of Table I.

PULSATION MECHANISMS

In the Sun and all stars the properties of hydrogen and helium give a rapid increase of opacity with temperature and a strong decrease in the Γ_1 and $\Gamma_3 - 1$ in the ionization regions. Thus if the mass level in the stars for these κ and γ effects involves enough mass, but is not so deep that the energy flow time scale is too long compared to the period of the eigenmode, pulsational destabilization can occur. This mass level for the Sun is between 10^{-9} and 10^{-10} of the mass from the surface at a temperature of about 9000K.

We describe here six mechanisms that are known to operate in the solar envelope, just as they operate in classical variable stars. The most important feature of pulsation driving is whether it can overwhelm the radiative and turbulent damping that exist in stars and cause the stars to have growing amplitudes with time. We assume, as seems to be correct for the many classes of variable stars, that to exhibit light and velocity variations, the stars must be pulsationally driven in the very small amplitude (linear) regime. More details of stellar models and pulsation theory can be found from Cox and Giuli (1968), Cox (1980) and Christensen-Dalsgaard (1986).

The first and most important mechanism is the κ effect, caused by the stellar opacity increasing when the star experiences a local compressive perturbation. Almost always, the opacity of the stellar material increases with density, as electrons are often forced back onto atoms to make them more absorptive or as free-free absorption increases with density. However, an increase in temperature usually decreases the abundance of absorbing material like partially ionized atoms, and the opacity usually decreases. However, in the hydrogen and helium ionization zones there is a temperature range where an increase in temperature produces an opacity increase. This is mostly caused by the energy of the photons in the field increasing and moving into the photon energy range that hydrogen and helium can absorb effectively. Anyway, an increase of opacity with compression and the natural temperature increase that goes with the compression, makes the outward flow of luminosity in a star decrease. A cyclical perturbation in density then dams-up the luminosity at maximum compression, only to have the luminosity subsequently increase during the reexpansion part of the cycle. This lagged flow of luminosity produces a lagged pressure history and the conversion of radiative luminosity to mechanical motions.

A parallel mechanism that also operates in the hydrogen and helium ionization zones is the γ effect. With energy going into ionization instead of into the kinetic energy of the particles, compression does not increase the matter temperature as much as when there is no ionization sink. Another way of saying the same thing is that with the low γ in an ionization zone, the temperature excursions are less than with a γ of completely ionized matter as 5/3. Similarly as above, a cyclical perturbation will produce cyclical luminosity variations, but with the luminosity lagged a bit because it is partially hidden during the compression stages.

A third, simple geometric pulsation mechanism was discussed by Baker (1966). This radius effect locally causes driving, because at maximum compression, a mass shell frequently has a smaller radiating area than its mean value. Thus luminosity is impeded to be lagged to a later expansion phase where the resulting pressure increment can produce mechanical motions.

In the equation for the radiative luminosity another important factor is the temperature gradient. The amplitude of the cyclical variation of the temperature varies from level to level in the solar envelope. The solution of the pulsation equations, including nonadiabatic effects, results in the relative amplitude of the sinusoidal variations being smaller at the surface than deeper at the top of the convection zone. Thus at this important level where enough of the solar mass is involved, the temperature variations are considerably larger than at the surface. This gives an increase in the temperature gradient at compression, promoting radiation leaking and pulsation damping.

In addition there are the Cowling mechanisms. The original Cowling (1957) mechanism, for nonradial motions only, involved a strong magnetic field in the presence of a superadiabatic gradient. A rapid adiabatic displacement, say, upward in a convection zone would cool the displaced material, but it still would be hotter than its surroundings. The strong magnetic field would resist the displacement and force the material back to its original position. But, when the material returns to its original position before the small perturbation, it will be cooler than its original surroundings because of the heat it lost on the upward excursion. With rapid pressure equilibration, cooler element would be heavier and continue to sink. Then, on its downward excursion, it would be cooler than its surroundings there, and gain heat. The magnetic field again forces the element back to its original position, but now it is too hot for its original surroundings. Analysis shows that looping in the P-V diagram would be clockwise, just as it is for the κ and γ effects, and the energy in the material would be converted to motions.

Even without the magnetic field, there are Cowling mechanisms. The Kato (1966) mechanism in semiconvection zones in evolved stars has the composition gradient as the restoring force. But the radiative and convective Cowling or δ (diffusion) mechanisms (Moore and Spiegel, 1966, and see Unno et al., 1989, for more details) that occur in the Sun need no restoring force. A cyclical displacement in a convection zone would have this sidewise heat flow either by radiation or even by convection. Since often numerical calculations do not allow any change in the convection flux with time, the convective Cowling mechanism is not allowed to occur in them. The radiative Cowling mechanism does operate in current calculations, but usually the flux is so small, for moderate l values, as shown by Ando and Osaki, that no effect can be recognized.

To be complete, there are other envelope pulsation driving mechanisms and several that occur in the deep interior. Very strong magnetic fields as seen in the A_p star pulsators can act as a restoring force to select high order, low degree nonradial modes. In the case of very strong convection with the bottom of the convection zone at the proper mass depth, the blocking of radiative luminosity can cause pulsations as observed in the white dwarf variables. Another relevant deep mechanism for the Sun, with no convective core or semiconvective zone, is the Eddington ϵ mechanism involving the nuclear burning of H and especially ^3He. Cox, Guzik, and Kidman (1989) find that these last two mechanism do not have enough driving for any solar mode, even the g modes.

One possible g-mode excitation mechanism has been named convection blocking by Cox, Starrfield, Kidman and Pesnell (1987) and Pesnell (1987) in their discussions of white dwarf star pulsations. If the time scale for the convection is long compared to the pulsation period, convection will continue to carry its mean outward luminosity without regard to the input luminosity and configuration variations. At a time during the pulsation cycle when the luminosity entering the bottom of the convection zone is higher than its mean over the cycle, the convection zone will not respond to this increased input. Thus luminosity will be blocked. If this is a time of compression of the material (as it usually is), the convection blocking will cause driving of any small perturbation. This is because as the luminosity into the bottom of the slowly adapting convection zone is dammed-up, it increases the pressure during expansion over that which would obtain for a purely adiabatic excursion. When the luminosity is less than its mean, the convection zone will still carry its mean luminosity, drawing radiation out from the radiative region below. For these expanded material phases then, the pressure will be lower than for adiabatic motion, and there will be less resistance to a collapse. Convection blocking then takes energy out of the environment and puts it into pulsation motions.

It is also possible that there could be a convective Cowling mechanism operating inside but near the bottom of the convection zone. Here the sidewise flow of convective luminosity might be destabilizing as the g-mode motions decay in the evanescent region of the convection zone. It seems that this effect is very small in unpublished calculations by Cox.

There are other aspects of the nonadiabatic effects in solar oscillations and they have been discussed in detail by Cox, Chitre, Frandsen, and Kumar (1991).

Even though these pulsation mechanisms are strong in the Sun, they are overwhelmed by radiative damping processes that operate less deep in the Sun at lower temperatures and densities. Thus, the Sun is cooler than the normal red edge of the hydrogen ionization (Cepheid) instability strip and not an intrinsic variable in the usual meaning of the word. Yet observers can detect maybe 10 million radial and nonradial modes in the period range from about 2 to 15 minutes. These oscillation modes are mostly excited, not by the normal mechanisms, but by simply coupling to the convective eddies that also have approximately the same spatial and temporal structures. While other stars like Procyon, α Centauri A, ϵ Eridani, or β Hydrae may eventually exhibit to observers these solar-like oscillations, at present the Sun is a unique variable star.

COMPARISONS OF FREQUENCIES WITH OBSERVATIONS

The 1700 zone model is analyzed using a nonradial mode instability program developed by Pesnell (1990). This nonradial program follows the concepts of matrix solutions for the linear problem developed earlier by Castor (1971). Both adiabatic and nonadiabatic results from previous models have been discussed in considerable detail by Cox, Guzik, and Kidman (1989). The models are constructed from evolution runs that match the observed solar data close enough to expect oscillation frequencies with theoretical uncertainties of less than one microhertz.

One result of interest is that the small flow of energy cyclicly in and out of the Lagrangian mass shells of the pulsation model reduce the mode frequencies typically a few microhertz. Thus to compare with observations that have accuracies to typically 0.1 microhertz, these nonadiabatic effects must be included.

Comparisons of oscillation frequencies with observations using the most modern material property data were first given by Christensen-Dalsgaard, Dappen and Lebreton (1988). They nicely show that use of the older Cox-Tabor (1976) opacities and the older Eggleton, Faulkner, and Flannery (1973) equation of state give much worse agreement than when using the Los Alamos Astrophysical Opacity Library and the new MHD equation of state. Cox, Guzik, and Kidman (1989) have shown some of the same effects, and this report gives the most up-to-date status.

Since the Sun is close enough to actually image the surface, it is possible to see directly the spherical harmonic patterns for all l values up the about 1000, where the node line spacing gets close to the granulation scale. For most stars, the light variations from the dark and bright areas cancel rather effectively when l get larger than 2 or 3. Note that for PG 1159-035, even with 125 observed modes, none of them have been identified with $l=3$.

An extremely interesting comparison between theory and observations involves the frequencies of the solar g-modes. These modes that derive their restoring force by buoyancy rather than pressure, have amplitudes large in the stellar interior and small at the surface. Thus their frequencies sample the deep structure, and even the very stellar center. These modes cannot exist in convection zones, and therefore, for any observation of them we must rely on the small tunneling that occurs through the solar surface convection zone. Predictions are that these modes should be very small in amplitude, but the Arizona and Stanford (and possibly also the Birmingham) teams report seeing these modes. There are numerous controversies about these observations, but the frequencies that are reported by the first two teams agree well with those expected from standard solar models that I have discussed in this review.

WIMP FREQUENCIES

An interesting result of the Cox, Guzik, and Raby (1990) solar oscillation studies is that standard solar evolution theory gives models, which produce pulsation frequencies that easily agree with the observed frequencies. The long-standing puzzle that solar neutrino output predictions are much higher than those observed with the Homestake mine chlorine detector apparently cannot be solved by having the solar center cooled by an unconventional method.

A popular method for cooling the solar center has been to have weakly interacting massive particles (WIMPs) orbit the solar center out to about ten percent of the mass and ten percent of the solar radius. These particles of about 5 proton masses would occasionally interact with the solar material at the very center, and occasionally interact with matter ten percent further out. The very weak interaction then promotes efficient conduction that normal matter does not have, since it must rely on the slow diffusion of photons.

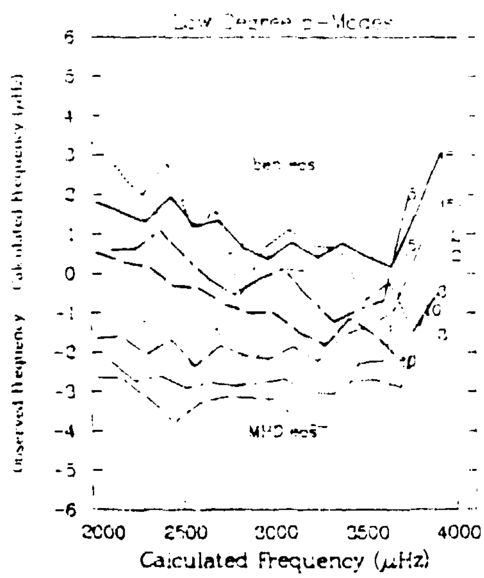
The Cox, Guzik, and Raby (CGR) paper presents two Cosmion models with these WIMPs doing the conduction, but both give frequencies that do not accord as well with observations as those from conventional models. Actually, asymptotic theory by Tassoul (1980) and others predicts that the difference between radial and quadrupole mode frequencies that are for modes separated by one radial order should be almost zero. The small higher order difference then should then decrease with mode order in a way determined almost entirely by the central solar structure. Models that have cool solar centers and have low neutrino outputs produce this frequency difference that is considerably smaller than observed for modes of radial order 11 to 33.

The CGR paper also discusses other models that cool the solar center and reduce the neutrino output. An important result, which is not generally appreciated, is that factors on the opacity like 1000 are needed for a significant temperature reduction at the solar center. Thus the CGR model that considers that the iron is condensed-out to produce a lower opacity solar material does not really give much temperature and neutrino reduction.

CORRECTED IBEN AND MHD EQUATION OF STATE RESULTS

Figure 1 is a plot of the observed minus calculated oscillation frequencies for low degree modes in our two models that have motions throughout the entire Sun. The observed frequencies are given by Duvall et al. (1988) and Libbrecht and Kaufman (1988). Only the nonadiabatic frequencies have been used here for both the model using the corrected Iben equation of state and the one using the most recent MHD elaborate equation of state. For the highest frequencies there is a positive trend for these differences that must indicate some small error in the models at the top of the convection zone. Since both the simple Saha (Iben) and elaborate MHD equations of state are valid for well ionized materials, the fact that both sets of frequencies agree well with the observations is reasonable.

Figure 2, however, shows the observed minus calculated frequencies for modes of degree up to 200. At this degree, the peak of the weighting for the period determination in the linear nonadiabatic theory is at that mass level where the temperature is 40,000K. Now one can see that the simple Saha ionization of the Iben procedure is not adequate to produce a model that yields the observed oscillation frequencies. The MHD equation of state here gives better agreement with observations, but surprisingly, only those modes in the region from $l=200$ to the granulation scale of $l=1000$ really need this more accurate material data.



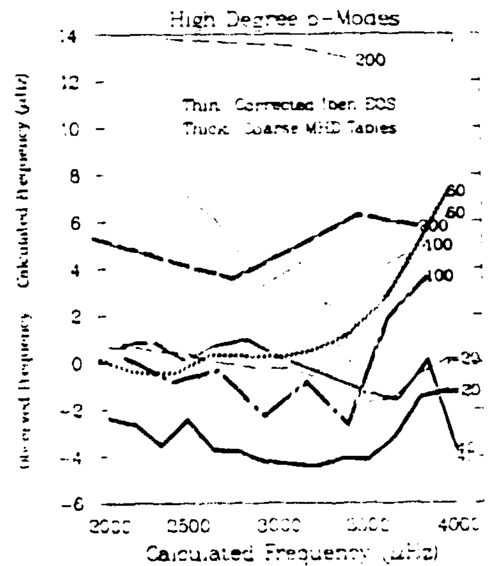


Fig. 1. Observed minus calculated low degree p-modes using the corrected Iben and the MHD equations of state.

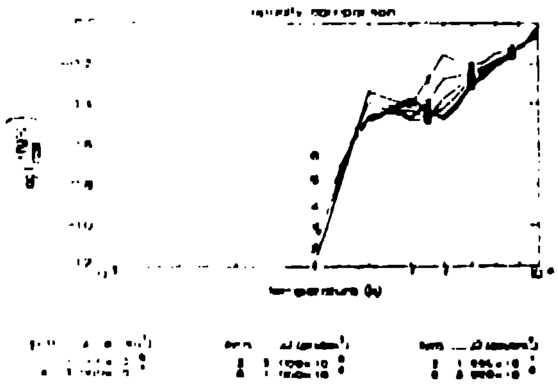
Fig. 2. Observed minus calculated high degree p-modes using the corrected Iben and the MHD equations of state.

Figure 3 shows the logarithm of the ratio of the Cox-Tabor mixture King IVa table opacity to the Opacity Library mixture Ross-Aller 1 table opacity versus temperature in the region of the solar model in or above the convection zone. Factors of 2 or 3 on the Stellingwerf (1975ab) fit that matches reasonably well the King IVa table are reasonable. The observed minus calculated oscillation frequencies using the corrected Iben equation of state and for Stellingwerf factors of 1 and 3 are given in Figure 4. It appears that with everything else held fixed, solar oscillations can constrain solar surface opacities.

There is a possibly important problem for the solar oscillations that probe the very upper layers of the convection zone and the photosphere. To obtain accurate eigenfrequencies, it is necessary to include layers out to an optical depth as thin as 10^{-4}, because the sound speed is small there for the p-mode waves. However, for this important part of the solar structure, we always use a temperature gradient set by the diffusion of photons rather than the real transport that occurs. At least the general structure is correct, but our approximations may give wrong results, especially for those very high frequency modes that are actually seen beyond the acoustic cutoff.

CONCLUSIONS

Our recent studies of solar evolution and pulsation models have led us to the conclusion that all recent solar evolution calculations are reasonably consistent with one another, and that all predict 6-10 SNUs for the chlorine detector neutrinos. The allowance for element diffusion during the solar evolution is a small effect for oscillations. The results of Cox, Guzik, and Kidman (1989) probably overestimate the effects. Problems with equations of state and opacities have mostly been solved with the elaborate MHD data, and small calibration corrections to the Los Alamos Astrophysical Opacity Library data. However, for the very high observed frequencies, more accuracy in both the equation of state and in the opacities and even monochromatic absorption coefficients may be required. No unconventional mixing of the material in the Sun has occurred in its evolution as verified by the good agreement of standard model predictions with observed oscillation frequencies. WIMPs do not seem necessary to supply a source of conduction at the solar center. This leaves the solar neutrino problem exactly where it has been for over 20 years, but I expect that the small number of observed neutrinos points to the MSW effect that will reduce both the high energy chlorine neutrinos, and also those emitted during the basic hydrogen burning. Oscillations of the Sun have taught us a great deal about our nearest variable star.



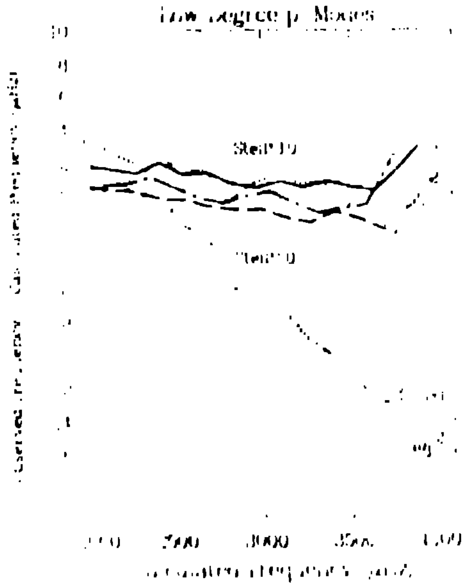


Fig. 3. The logarithm of the ratio King IVa/Ross Aller 1 table values.
Fig. 4. Observed minus calculated low degree p-modes using the Stellingwerf fit times one and times three.

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DISCUSSION

WALLERSTEIN: Regarding diffusion, what about radiative diffusion that might lift elements that contribute to the continuous opacity?
COX: We found that time scales for element separation are long, especially at the solar center. Levitation by absorbing photon momentum can move elements outward when they are in a position to absorb the photon and keep the momentum in the atom. Actually we have done a model that resembles the case where the central iron is blown outward, but in our case our concept was that the iron was absent from the gas because it was condensed-out as a liquid. Then with the smaller gas opacity, the central temperature was reduced, but not anywhere near enough to reduce significantly the neutrino output. As Cox, Guzik, and Raby (1990) discuss, oscillation frequencies are only slightly affected.
MATTHEWS: The (O-C) results from your models show a systematic difference from zero, although they have little or no slope in frequency. (This is similar to results from some European groups.) A difference of 5 μHz is large compared to observational precision. Are there ideas of how to resolve this?
COX: Both of the corrected Iben and MHD differences can be brought exactly to zero by adjusting the opacity below the solar convection zone just a few percent, but we have not felt such a calculation was worth the time. Our opacities below the convection zone were originally adjusted to improve agreement with oscillation frequencies, but more recently, we have adjusted the Los Alamos Astrophysical Opacity Library opacities to allow for the now known multitude of iron lines that our Livermore colleagues have actually calculated for us.
MATTHEWS: Is the enhanced opacity effect on eigenfrequencies important only in the outer layers, since low-degree p-modes are sensitive to the mid-interior?
COX: Actually the solar oscillation frequencies depend on the opacities very near the surface, as discussed in this paper, below the convection zone, as I just mentioned, and at the solar center. The solar center case is related to the Wallerstein question. If a single strong iron line at 15 million kelvin is deleted from the opacity by the iron being absent from the solar mixture, then the overall opacity is lower, and lowest degree and order oscillation frequencies are changed a little.
TEAYS: There have been some searches for neutrino oscillations, and as far as I know, they have not been detected. Do you disregard these results when you conclude that oscillations are the most likely answer to the neutrino problem.
COX: The searches that you mention are all for very large Δm^2 between the flavors of the neutrinos. Searches in the part of parameter space for the neutrino matter oscillations that are of interest for the solar MSW effect have not been made. An interesting verification of the MSW effect can come soon when the lower energy gallium neutrinos are detected. The MSW effect would greatly reduce their number also.
WELCH: I'd just like to point out that the Canadian Government completed funding of the Sudbury Neutrino Observatory earlier this year. It will detect about 30 neutrinos per day when operating. This should provide vastly better time resolution and sensitivity than current experiments.