Light-Front Dynamics of Elastic Electron-Deuteron Scattering*

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ABSTRACT

Measurements of the deuteron form factors over a wide range of momentum transfer can provide important clues to the role of subnucleon degrees of freedom in nuclear dynamics. For a meaningful calculation of the form factors it is essential that the current density operators and the deuteron wave function transform under Lorentz transformations in a mutually consistent manner. Standard nucleon-nucleon interactions can be used to construct unitary representations of the Poincaré group on the two-nucleon Hilbert space. Deuteron wave functions represent eigenstates of the the four-momentum operator. Existing parametrizations of measured single-nucleon form factors are used to construct a conserved covariant electromagnetic current operator. The light-front symmetry of the representation allows a clean separation of the effects of one- and two-body currents for arbitrary momentum transfers. Comparison with data indicates that for $Q^2 < \mathrm{GeV}^2$ the elastic cross sections are not dominated by by two-body currents.

1. INTRODUCTION

High energy electron scattering by hadronic targets is perhaps the most useful probe of the target structure. The electron-hadron interaction is sufficiently weak to be treated perturbatively, and its form is invariant under Lorentz transformations and space-time translations (Poincaré invariance). Cross sections are determined by matrix elements of the four vector current density $I^\nu(x)$, which are definite functions of Lorentz invariant form factors. To relate the current operator to the structure of the target involves the strong interactions that bind the target, and requires the construction of nonperturbative models. Nonrelativistic models are inherently unsatisfactory and inconclusive even when they agree with data. A model of the deuteron structure should respect the the principles of quantum theory and Poincaré invariance, that is, the representations of the current density and the deuteron states must both transform consistently under the same unitary representation of the Poincaré group. It should also be sufficiently well defined that failure to agree with data provides significant information.

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DISCLAIMER

For the models described here[1] the nuclear dynamics is obtained by explicitly constructing a unitary representation of thePoincaré group on the tensor product of two single-nucleon Hilbert spaces. This representation must be in agreement with the deuteron binding energy and the nucleon-nucleon scattering data. The intent is to achieve a Lorentz invariant separation of one-body, and two- body current effects in the deuteron form factors.

One-body operators are operators with domain and range in an single-nucleon Hilbert space, and on the two-nucleon Hilbert space they are defined by taking the tensor product with the identity. Current-density operators are sums of one-body operators that are completely specified by the form factors of the nucleons and two- body operators that vanish on states of widely separated nucleons. The separation of the current into one- and two-body parts is not invariant under general unitary transformations and he one-body operators alone cannot be covariant under all Poincaré transformations.

It is possible, however to construct current operators for which all two-body matrix elements are generated from one-body currents by dynamic Lorentz transformations and an explicit knowledge of these two-body currents is not need for the calculation of deuteron form factors, which are then unambiguously determined by the nucleon form factors and the deuteron wave function. The implicit effects of subnucleon degrees of freedom, mesons and/or quarks, must show up in additional two-body currents, which make separately Lorentz invariant contributions to the deuteron form factors.

2. Form Factors and Covariance of Current Operators

For any particle of spin j and mass m the states $|p,\mu\rangle$, $(-j \le \mu \le j$, $p^2 = -m^2)$ transform under Lorentz transformations according to

$$U(\Lambda)\,|p,\mu\rangle = \sum_{\mu'} |\Lambda p,\mu'\rangle\,\langle\mu'|\,\mathcal{R}_w(\Lambda,p)\,|\mu\rangle\,, \tag{1}$$

where the matrix $\mathcal{R}_w$ represents a rotation (Wigner rotation) that depends on the momentum p, and on the choice of a Lorentz transformation $L(p)$ that reduces the vector p to $\{m,0,0,0\}$. It will be convenient to chose $L(p)$ such that it leaves the light front $x^+ \equiv x^0 + \vec{n}\cdot\vec{x} = 0$ invariant. The Wigner rotation $\mathcal{R}_w(\Lambda,p)$ is the identity for all Lorentz transformations Λ that leave the light front invariant.

The transformation properties of the matrix elements $\langle\mu',p'|\,I^\nu(0)\,|p,\mu\rangle$ of the current density follow from the covariance of the current,

$$U^\dagger(\Lambda)I^\rho(x)U(\Lambda) = \Lambda^\rho{}_\sigma I^\sigma(\Lambda^{-1}x)\ , \tag{2}$$

and the transformation properties (1) of the state vectors $|p,\mu\rangle$. Current conservation requires that

$$\langle \mu', p' | \, Q \cdot I(0) \, | p, \mu \rangle \ , \tag{3}$$

where $Q \equiv p' - p$. For any value of Q^2 all matrix elements of the current operator are related by Lorentz transformations to $2j+1$ form factors which are conveniently chosen standard matrix elements. For spacelike Q it is always possible to chose the direction of the vector $\vec{n}$ such that $Q^+ = 0$, and therefore $\vec{Q}_\perp^2 = Q^2$. For $Q^+ = 0$ the matrix elements $\langle \mu', p' | \, I^+(0) \, | p, \mu \rangle / p^+$ are invariant under Lorentz transformations that leave the light front $x^+ = 0$ invariant, except for rotations about the longitudinal axis. They are therefore independent of p^+ and the sum $\vec{p}_\perp + \vec{p}_\perp{}'$ and I will use the abbreviated notation

$$\langle \mu' | \, I^+(0) \, | \mu \rangle \equiv \langle \mu', p' | \, I^+(0) \, | p, \mu \rangle / p^+ \ , \tag{4}$$

with the understanding that these matrix elements are functions of $\vec{Q}_\perp$. The coordinate axis are chosen such that $\vec{n} = \{0, 0, 1\}$ and $\vec{Q}_\perp = \{\sqrt{Q^2}, 0, 0\}$. Invariance of I^+ under rotations about the longitudinal axis and under time reversal plus reflection on the plane perpendicular to $\vec{n}$ implies the relations

$$\langle \mu' | \, I^+(0) \, | \mu \rangle = (-1)^{(\mu' - \mu)} \, \langle \mu | \, I^+(0) \, | \mu' \rangle \ , \tag{5}$$

and

$$\langle \mu' | \, I^+(0) \, | \mu \rangle = (-1)^{(\mu' - \mu)} \, \langle -\mu' | \, I^+(0) \, | -\mu \rangle \ . \tag{6}$$

The properties just listed make it convenient to use the matrix elements $\langle \mu' | \, I^+(0) \, | \mu \rangle$ to define the form factors. For spin $\frac{1}{2}$ the form factors F_1 and F_2 defined by

$$F_1(Q^2) \equiv \langle +\tfrac{1}{2} | \, I^+(0) \, | +\tfrac{1}{2} \rangle \ , \qquad F_2(Q^2) \equiv \frac{1}{\sqrt{\tau}} \langle -\tfrac{1}{2} | \, I^+(0) \, | +\tfrac{1}{2} \rangle \ , \tag{7}$$

with $\tau \equiv Q^2/4m^2$, are identical to the usual Dirac and Pauli form factors. For spin 1 conventional form factors $G_0(Q^2)$, $G_1(Q^2)$ and $G_2(Q^2)$, are related to the matrix elements $\frac{1}{2}\left(\langle +1|\, I^+(0)\, |+1\rangle + \langle 0|\, I^+(0)\, |0\rangle\right)$, $\langle +1|\, I^+(0)\, |0\rangle$ and $\langle +1|\, I^+(0)\, |-1\rangle$ by

$$
\begin{aligned}
G_0(Q^2) &= \frac{1}{1+\eta}\Big[\tfrac{1}{2}(1-\tfrac{2}{3}\eta)\big(\langle +1|\, I^+(0)\, |+1\rangle + \langle 0|\, I^+(0)\, |0\rangle\big) \\
&\quad - \tfrac{1}{6}(1-4\eta)\,\langle +1|\, I^+(0)\, |-1\rangle + \tfrac{5}{3}\sqrt{2\eta}\,\langle +1|\, I^+(0)\, |0\rangle\Big] \\
G_1(Q^2) &= \frac{1}{1+\eta}\Big[\langle +1|\, I^+(0)\, |+1\rangle + \langle 0|\, I^+(0)\, |0\rangle - \langle +1|\, I^+(0)\, |-1\rangle \\
&\quad - (1-\eta)\sqrt{\frac{2}{\eta}}\,\langle +1|\, I^+(0)\, |0\rangle\Big] \\
G_2(Q^2) &= \frac{\sqrt{8}}{3(1+\eta)}\Big[\sqrt{2\eta}\,\langle +1|\, I^+(0)\, |0\rangle \\
&\quad - \tfrac{1}{2}\eta\big(\langle +1|\, I^+(0)\, |+1\rangle + \langle 0|\, I^+(0)\, |0\rangle\big) - \langle +1|\, I^+(0)\, |-1\rangle\Big] \;,
\end{aligned}
\tag{8}
$$

where $\eta \equiv Q^2/4M_d^2$.

The difference, $\langle +1|\, I^+(0)\, |+1\rangle - \langle 0|\, I^+(0)\, |0\rangle$ of the diagonal matrix elements is determined by the requirements of rotational invariance of the charge density.

$$
\begin{aligned}
\langle 0|\, I^+(0)\, |0\rangle - \langle +1|\, I^+(0)\, |+1\rangle = \frac{1}{1+\eta}\big[&\eta\big(\langle +1|\, I^+(0)\, |+1\rangle + \langle 0|\, I^+(0)\, |0\rangle\big) \\
&+ \langle +1|\, I^+(0)\, |-1\rangle - \sqrt{8\eta}\,\langle +1|\, I^+(0)\, |0\rangle\big] \;.
\end{aligned}
\tag{9}
$$

The observable electric and magnetic structure functions $A(Q^2)$, $B(Q^2)$ and the tensor polarization T_{20} are related to the form factors G_0, G_1, G_2 by

$$
\begin{aligned}
A(Q^2) &= G_0^2(Q^2) + G_2^2(Q^2) + \tfrac{2}{3}\eta G_1^2(Q^2) \;, \\
B(Q^2) &= \tfrac{4}{3}\eta(1+\eta)G_1^2(Q^2) \;, \\
T_{20}(Q^2,\theta) &= -\frac{G_2^2 + \sqrt{8}G_0G_2 + \tfrac{2}{3}\eta G_1^2[\tfrac{1}{2} + (1+\eta)\,\tan^2(\theta/2)]}{\sqrt{2}\,[A + B\,\tan^2(\theta/2)]} \;.
\end{aligned}
\tag{10}
$$

For sufficiently large Q^2, when perturbative QCD is applicable, the matrix element

$\langle 0|\, I^+(0)\, |0\rangle$ gives the leading contribution.[2] In that approximation

$$A \approx \frac{1 + \frac{4}{3}\eta(1+\eta)}{4(1+\eta)^2} \langle 0|\, I^+(0)\, |0\rangle^2$$

$$\frac{B}{A} \approx \frac{\frac{16}{3}\eta(1+\eta)}{1 + \frac{4}{3}\eta(1+\eta)} , \tag{11}$$

and

$$T_{20}(Q^2, \theta) \approx \frac{\frac{4}{3}\eta(1-2\eta) - \frac{8}{3}\eta(1+\eta)\tan^2(\theta/2)}{\sqrt{2}\left[1 + \frac{4}{3}\eta(1+\eta) + \frac{16}{3}\eta(1+\eta)\tan^2(\theta/2)\right]} . \tag{12}$$

In the region of interest, $Q^2 < 10\text{GeV}^2$, it would be inappropriate to expand these expressions in powers of η. However, the η dependent factor of the structure function A is approximately constant since it has a broad minimum at 7GeV^2.

3. Deuteron Wave Functions

Eigenfunctions of the four momentum operator that transform unitarily under Poincaré transformations can be constructed using conventional deuteron wave functions. Conventional bound-state wave functions of a nucleus are eigenfunctions of the energy and spin operators for the nucleus *at rest*, that means they can be interpreted as eigenfunctions of mass and spin operators. Eigenfunctions of the four-momentum can then be generated by choosing three components of the four- momentum to be conserved kinematically, while the remaining component is determined by the mass and the three kinematic components. In the unitary representation of the Poincaré group, generated in this manner, at most the representation of a subgroup (the kinematic subgroup) may be independent of the interactions.If I choose the light-front momentum $\mathbf{P} \equiv \{P^+, \vec{P}_\perp\}$ to be kinematically conserved then the kinematic subgroup leaves the light front $x^+ = 0$ invariant, and the deuteron wave function $\Psi_{P_d,\mu_d}(\mathbf{p}_1, \mathbf{p}_2, \mu_1, \mu_2)$ has the form

$$\Psi_{P_d,\mu_d}(\mathbf{p}_1, \mathbf{p}_2, \mu_1, \mu_2) = \sqrt{\frac{\partial(\xi, \vec{k}_T, \mathbf{P})}{\partial(\mathbf{p}_1, \mathbf{p}_2)}}\, \chi_{\mu_d}(\xi, \vec{k}_T, \mu_1, \mu_2)\delta^3(\mathbf{P} - \mathbf{P}_d)\sqrt{P_d^+}, \tag{13}$$

where

$$\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2 , \qquad \xi = \frac{p_1^+}{P^+} , \qquad \vec{k}_\perp = \vec{p}_{1\perp} - \xi\vec{P}_\perp , \tag{14}$$

and

$$\frac{\partial(\xi,\vec{k}_T,\mathbf{P})}{\partial(\mathbf{p}_1,\mathbf{p}_2)}=\frac{1}{P^+} \tag{15}$$

is the Jacobian of the variable transformation $(\mathbf{p}_1,\mathbf{p}_2)\to(\xi,\vec{k}_T,\mathbf{P})$.

The matrix elements $\langle\mu_d'|I^+(0)|\mu_d\rangle$ that determine the deuteron form factors, can be calculated if the deuteron wave function $\Psi_{P_d,\mu_d}(\mathbf{p}_1,\mathbf{p}_2,\mu_1,\mu_2)$ and the associated representation $\langle\mu_2',\mu_1',\mathbf{p}_2',\mathbf{p}_1'|I^+(0)|\mathbf{p}_1,\mathbf{p}_2,\mu_1,\mu_2\rangle$ of the current operator are known.

$$\begin{aligned}\langle\mu_d'|I^+(0)|\mu_d\rangle=&\sum_{\mu_1',\mu_1}\sum_{\mu_2',\mu_2}\int d^3\mathbf{p}_1'\int d^3\mathbf{p}_1\int d^3\mathbf{p}_2'\int d^3\mathbf{p}_2\\&\times\Psi^*_{P_d',\mu_d'}(\mathbf{p}_1',\mathbf{p}_2',\mu_1',\mu_2')\Psi_{P_d,\mu_d}(\mathbf{p}_1,\mathbf{p}_2,\mu_1,\mu_2)/P_d^+\\&\times\langle\mu_2',\mu_1',\mathbf{p}_2',\mathbf{p}_1'|I^+(0)|\mathbf{p}_1,\mathbf{p}_2,\mu_1,\mu_2\rangle\ .\end{aligned} \tag{16}$$

To assume that only one-body currents,

$$\begin{aligned}\langle\mu_2',\mu_1',\mathbf{p}_2',\mathbf{p}_1'|I^+(0)|\mathbf{p}_1,\mathbf{p}_2,\mu_1,\mu_2\rangle=&\langle\mu_1',\mathbf{p}_1'|I_1^+(0)|\mathbf{p}_1,\mu_1\rangle\ \delta_{\mu_2',\mu_2}\delta(\mathbf{p}_2'-\mathbf{p}_2)\\&+\langle\mu_2',\mathbf{p}_2'|I_2^+(0)|\mathbf{p}_2,\mu_2\rangle\ \delta_{\mu_1',\mu_1}\,\delta(\mathbf{p}_1'-\mathbf{p}_1)\ ,\end{aligned} \tag{17}$$

contribute explicitly to to the deuteron form factors may neglect important physical effects but it does not involve any formal inconsistency because all the matrix elements (17) that occur in (16) are related to each other and to the nucleon form factors by kinematic Lorentz transformations. The deuteron form factors can then be calculated by evaluating the integrals

$$\begin{aligned}\langle\mu_d'|I^+(0)|\mu_d\rangle=&\sum_{\mu_1,\mu_2}\int d^2k_T'\int d^2k_T\int d\xi\ \delta\left[\vec{k}_T'-\vec{k}_T-(1-\xi)\vec{Q}\right]\\&\times\left[F_{1N}(Q^2)\,\chi^*_{\mu_d'}(\xi,\vec{k}_T',\mu_1,\mu_2)\,\chi_{\mu_d}(\xi,\vec{k}_T,\mu_1,\mu_2)\right.\\&\left.-\sqrt{\tau}F_{2N}(Q^2)\sum_{\mu_1'}\chi^*_{\mu_d'}(\xi,\vec{k}_T',\mu_1',\mu_2)\,\langle\mu_1'|\,i\sigma_2\,|\mu_1\rangle\ \chi_{\mu_d}(\xi,\vec{k}_T,\mu_1,\mu_2)\right].\end{aligned} \tag{18}$$

Results obtained by calculating form factors using Eq. (18) [1,3] are presented in the next Section.

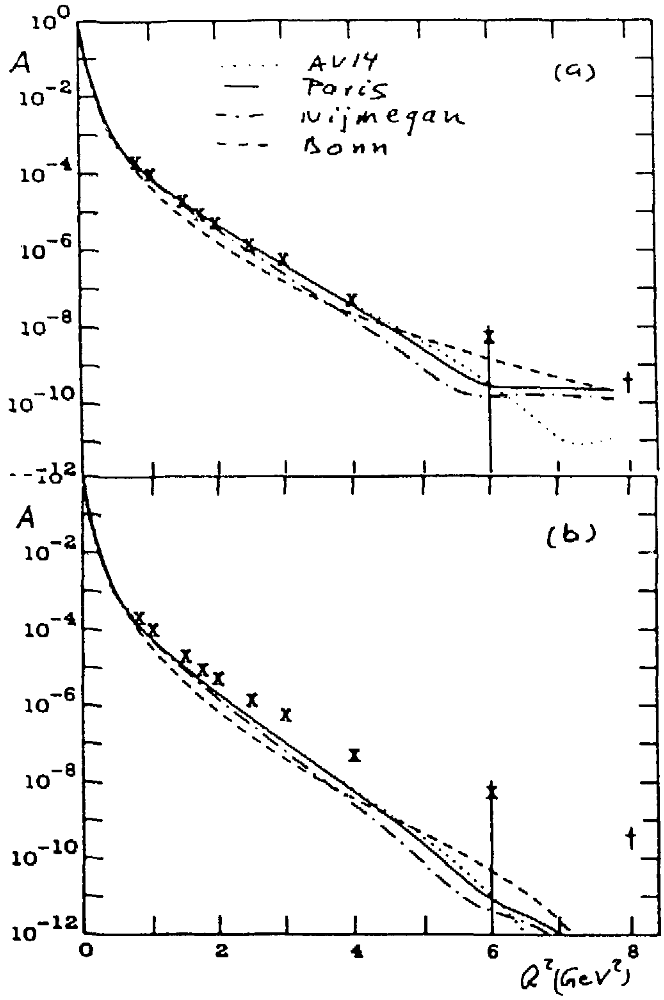

Fig. 1. The deuteron structure function $A(Q^2)$ for different deuteron wave functions compared to data[7]. Precise low energy data[6] are indistinguishable from the curve on the scale of this figure. (a) Gari-Krümpelmann nucleon form factors. (b) Höhler nucleon form factors.

4. Numerical Results and Conclusions

The form factors obtained from Eq. (18) can be readily compared to the the nonrelativistic form factors calculated with the same deuteron wave functions and the same nucleon form factors. The difference are generally quite small for $Q^2 < 4\text{GeV}^2$, but this does not mean that reliable relativistic results can be obtained by adding $1/m^2$ corrections to a nonrelativistic calculation. In such correction terms, obtained by expansion, the high-momentum tail of the wave function tends to generate erroneous contributions. For instance, the relativistic correction to the quadrupole moment obtained by expansion to order $1/m^2$ has the wrong sign as

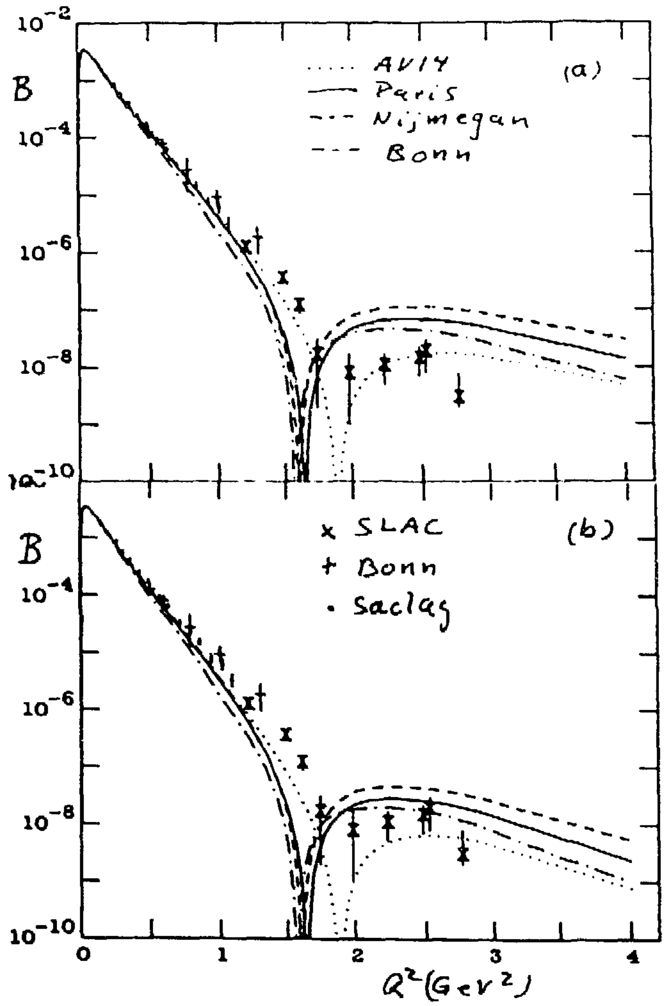

Fig. 2. The deuteron structure function $B(Q^2)$ for different deuteron wave functions compared to data.[8-10] (a) Gari-Krümpelmann nucleon form factors. (b) Höhler nucleon form factors.

well as the wrong magnitude.[4] For $Q^2 < 2$ the relativistic effect in the structure function A is a decrease by less than 2%.[1] This small effect is nevertheless significant when when measurements of A are used to determine the electric form factor of the neutron.[5,6] The relativistic effect in tha calculation of A increases the "measured " value of the neutron form factor.

The dependence of the elastic structure functions A and B on the deuteron wave function and the nucleon form factors are illustrated in Figs. 1. and 2. The wave functions used are, in order of decreasing D-state probability, Argonne v_{14} (AV14), Paris, Nijmegan, and the "energy independent relativistic" Bonn . Dif-

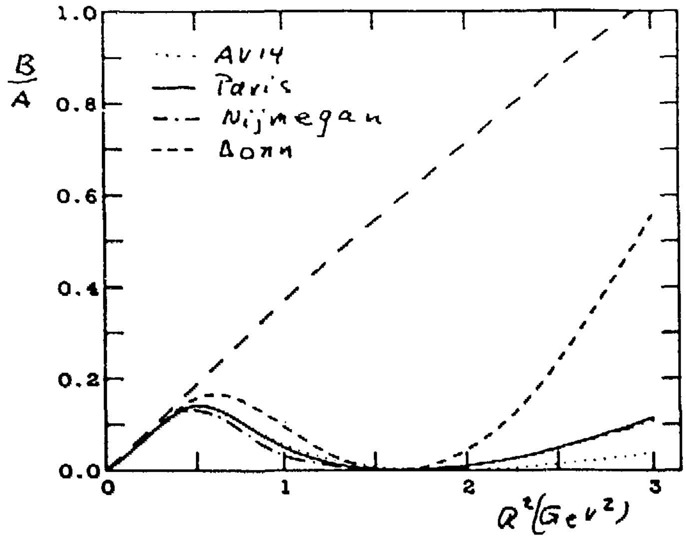

Fig. 3. The ratios B/A for different deuteron wave functions and Gari-Krümpelmann form factors. The long dashes represent the right hand side of Eq. (11) .

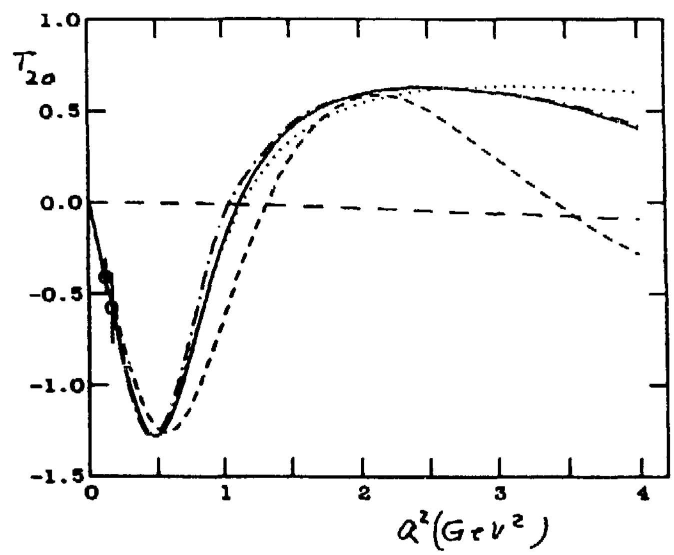

Fig. 4. The tensor polarization T_{20} for different deuteron wave functions and Gari-Krümpelmann form factors.The long dashes represent the right hand side of Eq. (12) .

ferent parameterizations of the nucleon form factors by Höhler[11) and by Gari and Krümpelmann (GK)[12) are used to illustrate the uncertainty in the empirical nucleon form factors. The AV14 potential with GK nucleon form factors gives satisfactory agreement with the data for both A and B. Disagreement with data indicates the need for two-body currents which were not included in in these calculations. The need for two-body currents depends most strongly on the nucleon form factors and to a lesser degree on the choice of the deuteron model.

Calculated ratios B/A are shown in Fig. 3 and compared to the expression (11) which obtains under the assumption that a single current matrix element, $\langle 0|I^+(0)|0\rangle$, give the dominant contribution. This assumption is not in agreement with the data for B.

The tensor polarization T_{20} for $\theta = 70°$ is shown in Fig. 4. It is not very sensitive to changes in the deuteron wave function, but the asymptotic expression (12) shows a very different behavior. Existing data[13] do not discriminate between models. New data at higher Q^2 might well exhibit substantial two-body current effects.

The principal result is that the present data do not require that subnucleon degrees of freedom produce dominant two-body current effects in the elastic structure functions.

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