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A REVIEW OF MATTER OSCILLATIONS AND SOLAR NEUTRINOS

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Abstract

We review the theory of the Mikheyev-Smirnov-Wolfenstein effect, and examine its consequences for the solar neutrino problem. Using a two-flavor model, we discuss the solutions in the Δm^2-$\sin^2 2\theta$ parameter space for the $^{37}C\ell$ experiment, and describe their predictions for the ^{71}Ga experiment and for the spectrum of electron-neutrinos arriving at earth.

1. Introduction

In this talk we shall briefly review the basic physics of the MSW (Mikheyev, Smirnov, Wolfenstein) effect[1,2] and the resulting enhancement of oscillations for neutrinos travelling through a medium of constant density. We then discuss the case of a medium with varying density, such as the sun, and outline the conditions for the validity of the principal approximations which have been used in theoretical analyses.

To apply the MSW effect to the solar neutrino problem,[3] we determine those parameters which give the requisite reduction of the $^{37}C\ell$ signal, especially in the small mixing angle regime. We then examine the implications such parameters will have for the ^{71}Ga experiment, and we emphasize the need for new experiments which will measure the energy spectrum of electron neutrinos arriving at earth.

2. The Physics of MSW

The two essential ingredients of the MSW effect are[2]: (1) the prior existence of neutrino mixing; and (2) the charged-current scattering of electron neutrinos by electrons. Neutrino mixing means that the flavor eigenstates associated with weak interactions are linear superpositions of the eigenstates of the mass matrix, and so "in vacuo" oscillations can take place. In the standard electroweak model, all neutrinos can scatter from electrons (and also from quarks) by means of the neutral current (Z° exchange) interaction, but only electron-type neutrinos can scatter from electrons by means of charged-current interactions (W^--exchange); this means that the coherent, forward scattering amplitude for electron-neutrinos differs from those for muon- and tau-neutrinos, and hence it gives rise to a different index of refraction, or effective mass as the electron neutrino propagates through matter.

We express the flavor eigenstates in terms of mass eigenstates through

$$[\nu]_{flavor} = W[\nu]_{mass}$$

$$W^+W = WW^+ = I \quad . \tag{2.1}$$

In the mass eigenstate basis, each neutrino has a given momentum p, and its energy is

$$E_i \cong p + \frac{m_i^2}{2p} \quad , \quad (p >> m) \quad . \tag{2.2}$$

The differential equation governing the time development of phase differences between the mass eigenstates is

$$i\frac{d}{dt}[a_\nu]_{mass} = H_{diag}[a_\nu]_{mass} \quad , \tag{2.3}$$

where $[a_\nu]_{mass}$ represents the probability amplitudes for all eigenstates in the mass basis and

$$H_{diag} \equiv \begin{bmatrix} m_1^2/2p & 0 \;\ldots\ldots \\ 0 & m_2^2/2p \\ \cdot & \\ \cdot & \\ \cdot & \end{bmatrix} \quad m_1 < m_2 < \ldots . \tag{2.4}$$

Transforming to the flavor basis, we have

$$i\frac{d}{dt}[a_\nu]_{flavor} = WHW^+[a_\nu]_{flavor} \quad . \tag{2.5}$$

The charged-current diagram generates a difference in the effective mass of electron neutrinos as compared with other flavors[4]

$$(\delta m)_{\nu e} = \sqrt{2}\, G_F N_e \quad , \tag{2.6}$$

where G_F is the Fermi constant for β-decay and N_e is the density of electrons. Including this effect, we find that Eq. (2.5) is replaced by

$$i\frac{d}{dt}[a_\nu]_{flavor} = \{WHW^+ + \sqrt{2}\, G_F N_e \hat{J}\}[a_\nu]_{flavor} \tag{2.7}$$

where $\hat{J}$ is a matrix with 1 in the (e,e) position and zeros everywhere else.

To illustrate this formalism, let us consider the two-flavor case with ν_e and ν_x, where x represents another family (muon, tau, or a fourth generation) but does not correspond to a sterile neutrino. The mixing matrix is

$$\begin{bmatrix} \nu_e \\ \nu_x \end{bmatrix} = \begin{bmatrix} c, s \\ -s, c \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} \tag{2.8}$$

where $c \equiv \cos\theta$ and $s \equiv \sin\theta$ and the time development equation is

$$i\frac{d}{dt}\begin{bmatrix} a_e(t) \\ a_x(t) \end{bmatrix} = \begin{bmatrix} A, B \\ B, D \end{bmatrix} \begin{bmatrix} a_e(t) \\ a_x(t) \end{bmatrix} \tag{2.9}$$

where

$$A = \frac{1}{2p}(m_1^2 c^2 + m_2^2 s^2) + \sqrt{2} G_F N_e \quad ,$$

$$D = \frac{1}{2p}(m_1^2 s^2 + m_2^2 c^2) \quad , \tag{2.10}$$

$$B = \frac{1}{2p}(\Delta m^2) cs \quad , \quad \Delta m^2 = m_2^2 - m_1^2 > 0 \quad .$$

with the appropriate choice of electron density, we can "tune" the effective mass matrix so that

$$A = D \quad . \tag{2.11}$$

The eigenstates of the matrix will then be equal admixtures of ν_e and ν_x, and we shall have maximal mixing between the flavor eigenstates. The condition for this can be written as

$$\sqrt{2}\, G_F N_e = \frac{\Delta m^2}{2p} \cos 2\theta \quad , \tag{2.12}$$

and since, in the standard electro-weak model, G_F is positive, Eq. (2.12) requires that the electron neutrino be dominantly composed of the lighter of the two mass eigenstates, namely m_1.[4]

Let us now suppose that the neutrino is travelling through a medium of constant density. We define a "matter oscillation length" L_o as[2]

$$L_o = \frac{2\pi}{\sqrt{2}\, G_F N_e} = \frac{(1.77) \times 10^7}{\rho_e} \text{ meters} \quad , \tag{2.13}$$

where ρ_e is the density of electrons in units of Avogadro's Number:

$$N_e = 6 \times 10^{23} \rho_e \quad . \tag{2.14}$$

Typical values of ρ_e on earth are in the range of 2-4, although it can reach as high as 13 at the center of the earth.[5] In the solar core ρ_e is of the order of 100.

In vacuo, the neutrino has a mixing angle θ (Eq. 2.8) and an oscillation length L_v:

$$L_v = \frac{4\pi p}{\Delta m^2} = 2.5 \left[\frac{(p/\text{MeV})}{(\Delta m^2/\text{eV}^2)}\right] \text{ meters} \quad . \tag{2.15}$$

but in the medium it oscillates with modified parameters θ_m and L_m where[2]

$$\sin^2 2\theta_m = \sin^2 2\theta / \{\sin^2 2\theta + (L_v/L_o - \cos 2\theta)^2\} \tag{2.16}$$

$$L_m = L_v / \sqrt{\{\sin^2 2\theta + (L_v/L_o - \cos 2\theta)^2\}} \quad .$$

Two properties are important in the formula for the modified mixing angle: first that, no matter how small the "in vacuo" angle θ may be, the "in medio" angle θ_m will have its maximum value ($\sin^2 2\theta_m = 1$) when the ratio of oscillation lengths happens to satisfy a relation

$$(L_v/L_o) = \cos 2\theta \quad , \tag{2.17}$$

which is just the A=D condition (Eq. 2.11, 12) in another form. In other words, as long as θ is not zero, there is always a density for which the neutrino will oscillate with maximal mixing. However, the oscillation length becomes much longer than before, namely $(L_v/\sin 2\theta)$.

The second property is that the width of the $\sin^2 2\theta_m$ curve as a function of (L_v/L_o) is proportional to $\sin 2\theta$: in fact the full width at half maximum is given by[1]

$$2\Delta(L_v/L_o) = 2|L_v/L_o - \cos 2\theta| = 2\sin 2\theta \quad . \tag{2.18}$$

Thus the smaller the angle θ, the narrower the peak; and so for very small angles, the peak becomes a sharp spike. Outside the peak, θ_m tends to zero for high densities ($L_v/L_o \to \infty$) and to its in vacuo value θ for low densities ($L_v/L_o \to 0$).

3. Varying Density: The Sun

In the sun, the density of electrons decreases steadily from a value of $\rho_e \cong 115$ at the core to $\rho_e \cong 0$ at the edge.[3,6] Consequently, for every $p/\Delta m^2$ within a wide range, there exists a density somewhere inside the sun for which enhancement condition (Eqs. 2.11,12,17) is satisfied. In the vicinity of this density, we expect large oscillation effects to occur.

For the purposes of this discussion, we use an exponentially falling solar density

$$\rho_e(x) = \rho_{core}\, e^{-x/R_c} \tag{3.1}$$

with

$$\rho_{core} \cong 115 \quad , \quad R_c \cong \frac{1}{10} R_{sun} \cong 7 \times 10^7 \text{m} \quad . \tag{3.2}$$

Outside the core region (the first 5% of the solar radius), this provides a good approximation to the density profile of the sun. The enhancement condition is satisfied when

$$(p/\Delta m^2) = \frac{0.7 \times 10^7}{\rho_e} \cos 2\theta \quad , \tag{3.3}$$

and so the range of applicability for neutrino parametars is approximately

$$10^4 \le (p/\Delta m^2) \le 10^9 \quad , \tag{3.4}$$

where we measure p in MeV and Δm^2 in $(\text{eV})^2$.

The travel history for a typical neutrino born in the core can be divided into three parts. Initially, the neutrino finds itself in a region of high density for which $L_v \gg L_o$: the effective mixing angle is much smaller than the in vacuo angle (Eq. 2.16) and so oscillations are suppressed. The neutrino then moves into a region of intermediate density for which $L_v \cong L_o$ and, since $\sin^2 2\theta_m \cong 1$, oscillations are enhanced. Finally it passes into a region of low density where $L_v \ll L_o$ and "in vacuo" oscillations set in.

In the adiabatic approximation,[7] the eigenvectors of the equations of motion change very slowly during the passage through the sun, and in the slab,[8] or sudden approximation changes take place in an extremely small region. The criterion distinguishing between these cases comes from a comparison between the physical size $2\Delta x$ of the region in which enhanced oscillations can take place and the effective oscillation length L_m at the actual point of enhancement. When $2\Delta x$ is much greater than L_m, the adiabatic approximation is valid, and when it is much smaller than L_m, the slab (sudden) approximation comes into play.

The size of the enhancement region within the sun itself is

$$2\Delta x = 2(\tan 2\theta/h_o) \quad ; \quad h_o = \left| \frac{1}{\rho}\frac{d\rho}{dx} \right|_{\text{enhancement}} \qquad (3.5)$$

For the exponentially falling density distribution of Eqs. (3.1 and 2), the scale height h_o is a constant,

$$h_o = 1/R_c \cong 1/7 \times 10^7 m \quad , \qquad (3.6)$$

and thus for small mixing angles, the enhancement region is a small fraction of a solar radius:

$$2\Delta x \cong (0.2)(2\theta)\ R_{sun} \cong 2(2\theta) \times 7 \times 10^7 m \quad . \qquad (3.7)$$

For the adiabatic approximation to be valid, the enhancement region must be larger than the effective oscillation length L_m at the point of enhancement. This condition translates into a bound on $p/\Delta m^2$:

$$(p/\Delta m^2) << \frac{\sin 2\theta\ \tan 2\theta}{2\pi\ h_o} \quad . \qquad (3.8)$$

The essential feature of the adiabatic approximation is that the eigenvectors of the "Hamiltonian" matrix of Eqs. (2.9 and 10) change so slowly that, for all practical purposes, the neutrino remains in the same eigenstate as it crosses the enhancement region; however, the meaning of the eigenstate in terms of neutrino flavor changes. An electron neutrino born in the core of the sun is dominantly in the "heavier" of the two eigenstates, but when the neutrino emerges from the sun, the heavier neutrino is the muon one! Thus, by remaining in the same eigenstate, the neutrino has changed flavor from electron-type to muon-type.

Several authors[7] have calculated the probability for ν_e to remain ν_e at earth in the adiabatic approximation:

$$P^{ad}(\nu_e \to \nu_e \text{at Earth}) = \cos^2\phi_o\ \sin^2\theta + \sin^2\phi_o\ \cos^2\theta, \qquad (3.9)$$

where $(\cos\phi_o, -\sin\phi_o)$ is the "heavier" eigenvector of the Hamiltonian (Eqs. 2.9, 10) at the point of birth of the neutrino. For high density, or for large $(p/\Delta m^2)$, ϕ_o approaches zero, and for low density ϕ_o becomes $(\pi/2+\theta)$ where θ is the in vacuo mixing angle. The typical behavior of the probability $P^{ad}(\nu_e \to \nu_e$ at Earth) for small angles as a function of $(p/\Delta m^2)$ is that it remains close to unity in the region of 10^4-10^5 and then falls rapidly to its asymptotic value of $\sin^2\theta$ as $p/\Delta m^2$ increases[7]; at the value of $p/\Delta m^2$ corresponding to the point of enhancement it is always equal to 1/2. The actual probability for ν_e to remain ν_e cannot remain at $\sin^2\theta$ indefinitely because, at some value of $p/\Delta m^2$ (see Eq. 3.7), the adiabatic approximation begins to break down; however, the larger the angle θ, the longer it is before the breakdown occurs.

When the adiabatic approximation does break down we move into the regime of the sudden, or slab approximation,[8] the criterion for which is exactly the reverse of Eq. (3.7) namely

$$(p/\Delta m^2) >> \frac{\sin 2\theta\ \tan 2\theta}{2\pi\ h_o} \quad . \qquad (3.10)$$

In this case the probability that the neutrino will make a sudden transition from one eigenstate to the other (and thus preserve its flavor) grows. A naive model for this behavior, especially in the case of small mixing angles[8], is to assume that in the high and low density regions of the sun, for which L_v/L_o is either much greater than, or much less than unity, the neutrino does not oscillate. Its only oscillations take place in the enhancement region, which, given Eq. (3.9), is much smaller than the oscillation length at enhancement, L_m. Thus one catches only a fraction of the wave and predicts that

$$P^{slab}(\nu_e \to \nu_e \text{ at Earth}) = \cos^2\left(\frac{\Delta m^2}{2p}\ \frac{\sin 2\theta\ \tan 2\theta}{h_o}\right) \quad . \qquad (3.11)$$

This formula gives the correct qualitative behavior of the direct computations I shall describe below, but it does not work well in a quantitative sense. A much better expression, in fact one whose agreement with the computations is remarkable, has been obtained by Haxton[9] and by Parke[10] using the Landau-Zener formula:

$$P^{slab}(\nu_e \to \nu_e \text{ at Earth}) = \exp\left(-\frac{\pi}{2}\cdot\frac{\Delta m^2}{2p}\ \frac{\sin 2\theta\ \tan 2\theta}{h_o}\right) \quad . \qquad (3.12)$$

Both expressions in Eqs. (3.10 and 11) have the property that as $(P/\Delta m^2)$ increases, the probability for ν_e to remain ν_e steadily increases from $\sin^2\theta$ (the adiabatic limit) back to one.

4. Calculations for the ^{37}Cl and ^{71}Ga Experiments

We now apply these ideas to the experiment of Davis and coworkers[11] in which they attempt to observe the energetic components (principally from ^8B and ^7Be) of the solar neutrino spectrum through the reaction

$$\nu + {}^{37}Cl \to {}^{37}Ar + e^- \quad . \qquad (4.1)$$

Our general approach is to assume that the diminution of the observed signal (2.1±0.3 SNU) by a factor between 2 and 4 as compared with the signal (5.9±2.2 SNU) predicted on the basis of the standard solar model[12] is due to the MSW effect. We then compute those values of $\sin^2 2\theta$ and Δm^2 in a two-flavor model that yield the desired reduction, and for each such set of parameters we predict the rate that should be observed in the gallium solar neutrino experiment,

$$\nu + {}^{71}Ga + e^- \to {}^{71}Ge \quad , \qquad (4.2)$$

which is sensitive principally to the low energy, but much more abundant, pp neutrinos. In addition, we calculate the probability spectrum for ν_e to remain ν_e at Earth as a function of energy, and we argue that this spectrum will be an important tool for distinguishing between different explanations of the solar neutrino problem.[8] Throughout this discussion our emphasis will be on small mixing angles,

$$10^{-4} \le \sin^2 2\theta \le 10^{-1} \quad , \qquad (4.3)$$

although we shall comment on the large-angle case.

There are two classes of solution for the ^{37}Cl experiment: one in which Δm^2 remains in the neighborhood of 10^{-4} (eV)^2 for small mixing angles; and the other for which the product $(\Delta m^2) \times (\sin^2 2\theta)$ is approximately equal to $10^{-7.5}$ (eV)^2. Both solutions are implicit in the original work of Mikheyev and Smirnov[2]; Bethe[13] has elaborated upon the first one, and Rosen and Gelb[8] upon the second.

The predictions for the ^{71}Ga experiment are shown in Table 1, where the circled values correspond to oscillation paramaters, which reduce the ^{37}Cℓ signal by a factor 3. The upper row of circled values corresponds to the first class of solutions, and the numbers represent the percentage of the standard solar model signal that is expected to be seen in the gallium experiment. Likewise the lower row of circled figures in Table 1 corresponds to the second class. From the table we see that the first solution for ^{37}Cℓ leads to the prediction that we should see 100% of the standard model signal in gallium, whereas the second solution tends to predict a reduced signal for gallium, the reduction being as much as a factor of 10 in some cases.

To understand the differences between the two ^{37}Cℓ solutions, we have computed the probability for ν_e to remain ν_e at Earth, as a function of neutrino energy, $P(\nu_e \to \nu_e; E)$. For a given (small) value of $\sin^2 2\Theta$, there are two possible values of Δm^2, which yield a reduction of 1/3 in the ^{37}Cℓ signal; one corresponds to the first solution and the other to the second one. As emphasized by Bethe,[13] the first solution has the property that low energy neutrinos remain as electron neutrinos while high energy ones are almost totally converted to brand X. The division between "low" and "high" energy lies somewhere in the vicinity of 5 to 7 MeV depending on the value of $\sin^2 2\Theta$. Since the pp neutrinos responsible for most of the ^{71}Ga signal are "low" energy, they will always, in the upper solution, yield 100% of the standard solar model signal.

By contrast, the second solution has the property that neutrinos of all energies are converted to brand X, but the conversion is much stronger for low energies than for high ones. In this case the pp neutrinos can suffer a strong conversion to muon- or tau-neutrinos, and the gallium signal will correspondingly be reduced, as shown in Table 1.

An important implication of this analysis is the need to measure the spectrum of electron neutrinos arriving at earth, especially those from ^{8}B decay in the sun. This measurement can be used to confirm the MSW effect and also to resolve ambiguities of interpretation that might arise once the gallium experiment has been carried out. By way of confirming the MSW effect, we note that changes in the standard solar model, which serve to lower the temperature of the core, will reduce the overall normalization of ^{8}B neutrinos, but will not change their spectral shape. Likewise non-MSW oscillation solutions with large $\sin^2 2\Theta$ and small Δm^2 (either too small for MSW or of the wrong sign) tend not to affect the shape of the spectrum, except possibly at the high energy and where $P(\nu_e \to \nu_e; E)$ could come close to one. MSW, as we have just shown, does change the spectrum in one of two characteristic ways. Hence, a measurement of the spectrum would enable us to confirm, or to reject MSW as an explanation of the ^{37}Cℓ experiment.

Depending upon the outcome of the ^{71}Ga experiment, there might be serious ambiguities in its interpretation. If, for example, the gallium signal turns out to be close to that predicted by the standard solar model, we will have to choose between the upper MSW solution and some modification of the solar core temperature[14] as the explanation of the Davis experiment. Significant changes in the energy spectrum of electron neutrinos will support the former possibility, while no significant change will support the latter.

Another conceivable outcome might be that the gallium signal is found to be about 1/3 of the standard model prediction. In this case we can definitely conclude that neutrino oscillations are taking place, but without a spectral measurement, we cannot choose between oscillations of the MSW variety with a small mixing angle, and non-MSW oscillations with a large mixing angle as the correct explanation. A modified spectrum will point to MSW with small mixing angles, and an unmodified one will indicate the non-MSW alternative. But even in the latter case there is a residual ambiguity which may be hard to remove.

Parke[10,15] has recently emphasized that, in addition to the two small angle solutions of the Davis experiment mentioned above, there is a third, large angle MSW solution. It arises when the "suppression gap" for $P(\nu_e \to \nu_e)$ is large enough to include essentially all of the solar neutrino spectrum and when the asymptotic value of the adiabatic solution, $\sin^2\Theta$ (see discussion below Eq. (3.8)), is roughly 1/3 (i.e. $\sin^2 2\Theta \cong 0.9$). In this case, we again obtain an essentially unmodified spectral shape for ^{8}B neutrinos. Now the large angle MSW solution tends to have a larger $\Delta m^2 (10^{-7} - 10^{-5} (eV)^2)$, than a non-MSW solution, which either has the wrong sign for Δm^2, or a value of $10^{-8} (eV)^2$ or smaller. This puts the $(p/\Delta m^2)$ value for the large-angle MSW solution in a range such that day-night and winter-summer asymmetries[16-18] may show up in the gallium, and other proposed neutrino experiments. These asymmetries, estimated to be of order 15%,[18] will resolve, at least in principle, the ambiguity between large angle MSW and non-MSW solutions.

To draw this part of the discussion to a close, we note that should there be found in the gallium experiment a definite suppression of the signal as compared with the standard model prediction, and should this suppression be much greater than, or much less than the suppression in the ^{37}Cℓ experiment, then we can definitely conclude that MSW oscillations are taking place. This would be a result of enormous significance for neutrino physics in particular, and for particle physics in general.

TABLE 1

Predictions for the ^{71}Ga experiment for parameters (circles) which yield a 1/3 reduction in the ^{37}Cℓ experiment.

	$\sin^2\theta$: $10^{-3.0}$	$10^{-2.5}$	$10^{-2.0}$	$10^{-1.5}$	$10^{-1.0}$	$10^{-0.5}$
1.1E-4	100	100	100	100	100	100
1.0E-4	100	100	100	100	100	100
9.5E-5	100	100	100	100	100	100
5.8E-5	100	100	100	100	100	100
5.0E-5	100	100	100	100	100	100
1.7E-5	100	100	100	100	100	95
3.6E-6	65	60	60	60	50	45
1.1E-6	45	15	10	10	20	25
3.5E-7	70	40	10	5	5	20
1.0E-7	85	70	40	10	5	20

5. Final Comments

Several groups[16-18] have observed that when $p/\Delta m^2$ is in the range 10^6-10^7, there can be significant enhancement effects for neutrinos passing through the earth, which has a density of order $\rho \cong 13$ at its core, and an average of order 2-4. In particular, solar neutrinos which have been converted to muon- or tau-neutrinos could be reconverted to electron-type when they pass through the earth. Thus, one anticipates significant differences between the day and night signals, and also between winter (longer nights) and summer (shorter nights) signals.

It is quite possible that such asymmetries could be observed either before the gallium experiment is completed, or at least before the ν_e-spectral measurements are made. Such observations would provide strong evidence for the MSW effect. There is, however, one possible snag, namely that values of $p/\Delta m^2$ in the range 10^6 to 10^7 correspond to oscillation lengths of order of the diameter of the earth. This means that large mixing angle, non-MSW oscillations with the appropriate Δm^2 could also give significant day-night effects. Again one might need a spectral measurement to settle the issue.

In conclusion, we just declare our own particular prejudice that the MSW effect is so elegant that it ought to be true. Should it indeed prove to be the correct explanation of the "solar neutrino problem," then solar neutrinos will be the only practical source from which we can learn about neutrino masses and mixings.

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