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## Transition from Resistive- $G$ to Eta- $i$ Driven Turbulence in Stellarator Systems

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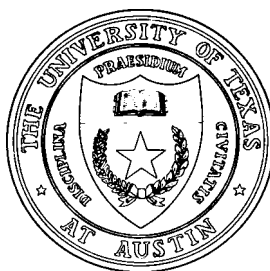
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# Transition from Resistive- $G$ to Eta- $i$ Driven Turbulence in Stellarator Systems

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## Abstract

By an electromagnetic incompressible two fluid model describing both ion temperature gradient drift modes ( $\eta_i$  modes) and resistive interchange modes ( $g$  modes), a new type of  $\eta_i$  mode is studied in cylindrical geometry including magnetic shear and an averaged curvature of Heliotron/Torsatron. This  $\eta_i$  mode is destabilized by the coupling to the unstable  $g$  mode. Finite plasma pressure beta increases the growth rate of this mode and the radial mode width also increases with plasma pressure beta indicating large anomalous transport in the Heliotron/Torsatron configuration. The transport from  $\eta_i$  mode exceeds that from resistive  $g$  when the mean-free-path exceeds the machine circumference. For plasma beta above two to three times the Suydam limit the  $m = 1/n = 1$  growth rate increases from the  $\eta_i$  mode value to the MHD value.

# I. Introduction

Recently ECRH heating experiments<sup>1</sup> in the Heliotron E showed that the ion temperature did not increase when the electron density increased at constant heating power, although the power input to the ions from the electrons was strongly enhanced. Since the electron density profile is fairly flat in the outside region and the  $\eta_i$  parameter is probably larger than one, the anomalous ion thermal transport driven by the  $\eta_i$  modes is a good candidate to explain this result. In this work we demonstrate that the  $\eta_i$  modes are also destabilized by the bad average curvature, and that they couple to the  $g$  modes in the Heliotron/Torsatron. This means that the ion heat transport becomes anomalous due to the  $\eta_i$  mode turbulence, when  $\eta_i$  becomes large. The difference between these two modes comes from the electron dynamics. For the  $\eta_i$  modes the electrons satisfy the adiabatic (Boltzmann) relation,  $\tilde{n}/n_0 \simeq e\tilde{\varphi}/T_e$  in the regime of  $k_{\parallel}^2 v_{Te}^2 > \omega \nu_{ei}$ , and for the  $g$  modes the electrons behave isothermally,  $\tilde{n}/n_0 = (\omega_{*}/\omega)(e\tilde{\varphi}/T_e)$  in the regime of  $k_{\parallel}^2 v_{Te}^2 < \omega \nu_{ei}$ . Here  $\tilde{n}$  and  $\tilde{\varphi}$  are the density and electric potential perturbations, respectively, and  $k_{\parallel}$  is a typical parallel wavenumber,  $\omega$  a characteristic frequency and  $\nu_{ei}$  the electron-ion collision frequency. Hence, the mode structure in a collisional plasma with both magnetic shear and bad average magnetic curvature will be strongly affected by the force driving the  $g$  mode in the inner region (sufficiently close to the rational surface  $k_{\parallel} = 0$ ) and by the force driving the  $\eta_i$  mode in the outer region in the case of  $\eta_i \geq 1$ . We find that the  $\eta_i$  mode is further destabilized by the coupling to the resistive  $g$  mode in the sheared slab model.<sup>2</sup> Cordey *et al.*<sup>3</sup> studied a similar situation in the levitron configuration and showed that the coupling between the  $\eta_i$  mode and the  $g$  mode produces a single, strongly destabilized mode in the electrostatic limit.

Here we study the coupling of the  $\eta_i$  mode to the resistive  $g$  mode in the cylindrical plasma with both magnetic shear and curvature of the Heliotron/Torsatron magnetic field, which is an extension of the slab model analysis showing a new type of the  $\eta_i$  mode.<sup>2</sup> Two

types of eigenmodes occur in the system for the same parameters and mode numbers: one mode is radially localized and the other mode is radially global. Both the mode localized near the mode rational surface and the global mode (extending over the radius of the plasma) are studied. The global mode was not found in the slab model<sup>2</sup>; however, it seems similar to the nonlocal resistive drift waves obtained in the cylindrical plasma model of Heliotron E.<sup>4</sup> The mode with the larger growth rate is found to be the localized mode, but the global mode tends to have the larger  $\gamma(\Delta r)^2$  which measures the transport. Also, we investigate the effect of finite plasma pressure beta on this new  $\eta_i$  mode in Heliotron/Torsatron by solving the two coupled second order differential eigenvalue equations for the electrostatic and parallel vector potentials.

Section II introduces the two component dissipative hydrodynamic equations we use to derive the two coupled second order differential equations for the electromagnetic stability problem. Numerical solutions are given in Sec. III for a cylindrical plasma of the Heliotron-E. Conclusions are given in Sec. IV.

## II. Model Equations

We use incompressible two fluid equations to describe the coupling of the  $\eta_i$  mode and the resistive  $g$  mode in the Heliotron/Torsatron. Using the normalizations  $e\Phi/T_e \equiv \varphi$ ,  $n_i/n_0 \equiv n$ ,  $\Omega_{ci}(\rho_s/a)^2 t \equiv t$ ,  $r/a \equiv r$  and  $z/R \equiv z$ , we obtain the fluid model equations

$$\begin{aligned} \frac{\partial}{\partial t} \nabla_{\perp}^2 \varphi + [\varphi, \nabla_{\perp}^2 \varphi] + \nabla_{\perp} \cdot [p_i, \nabla_{\perp} \varphi] \\ = -\frac{2\epsilon}{\beta_e \rho^2} \nabla_{\parallel} \nabla_{\perp}^2 A + \frac{1}{\rho^2} [p_i + n, \Omega] \end{aligned} \quad (1)$$

$$\frac{\partial}{\partial t} v_{\parallel i} + [\varphi, v_{\parallel i}] = -\frac{\epsilon}{\rho^2} \nabla_{\parallel} p_i - \frac{1}{\rho^2} \frac{\partial A}{\partial t} - \frac{\epsilon}{\rho^2} \nabla_{\parallel} \phi + \frac{2\nu_e}{\rho^2 \beta_e} \nabla_{\perp}^2 A \quad (2)$$

$$\frac{\partial n}{\partial t} + [\varphi, n] = -[\varphi - n, \Omega] - \epsilon \nabla_{\parallel} v_{\parallel i} - \frac{2\epsilon}{\beta_e} \nabla_{\parallel} \nabla_{\perp}^2 A \quad (3)$$

$$\frac{\partial p_i}{\partial t} + [\varphi, p_i] = 0 \quad (4)$$

$$\frac{\partial A}{\partial t} = -\epsilon(\nabla_{\parallel}\varphi - \nabla_{\parallel}n) + \frac{2\nu_e}{\beta_e} \nabla_{\perp}^2 A, \quad (5)$$

where  $\epsilon = a/R$ ,  $\rho = \rho_s/a$ ,  $\beta_e = 8\pi n_0 T_e/B_0^2$  and  $\nu_e = \nu_{ei}/\Omega_{ce}$ . In Eqs. (1)–(5), the convective nonlinearities are written using the Poisson bracket

$$[f, g] \equiv \nabla f \times \nabla g \cdot \hat{\mathbf{z}} \quad \text{and} \quad \nabla_{\parallel} f \equiv \frac{\partial}{\partial z} + \frac{1}{B_0} [\Psi_0 - A, f]. \quad (6)$$

The function  $\Psi_0(r)$  is the equilibrium magnetic flux function obtained from the helical magnetic fields which produce a rotational transform with the relation to a poloidal magnetic field  $\mathbf{B}_p = \hat{\mathbf{z}} \times \nabla \Psi_0$ . The four-field equations with different normalization in the electrostatic limit are given in Ref. 2.

## A. Equilibrium

The force  $\nabla \Omega(r)$  represents the averaged curvature of the magnetic field line due to the stellarator field. Since we have the average curvature without toroidicity we employ a cylindrical geometry where all equilibrium quantities  $n_0$ ,  $\Psi_0$  and  $\Omega$  depend only on  $r$ . Then the magnetic flux function  $\Psi_0$  and the average curvature term  $\Omega$  are related by

$$\Psi_0(r) = \int_0^r r' \iota(r') dr' \quad (7)$$

and

$$\Omega(r) = \frac{N\epsilon^2}{\ell} \left( r^2 \iota(r) + 2 \int_0^r r' \iota(r') dr' \right), \quad (8)$$

where  $\iota(r)$  is a rotational transform,  $\ell$  is the poloidal number, and  $N$  is the pitch number in Stellarator/Heliotron devices.

## B. Fluctuation Energy Density and Transport

Equations (1)–(5) have an energy conservation relation in the case of  $\nu_e = 0$  given by

$$\frac{\partial}{\partial t} \int dv \left[ \frac{1}{2} (\nabla_{\perp} \phi + \nabla_{\perp} p_i)^2 + \frac{1}{2} v_{\parallel i}^2 + \frac{1}{2} n^2 + \frac{1}{2\beta_e} (\nabla_{\perp} A)^2 + n p_i \right] = 0. \quad (9)$$

The nonlinear evolution of the instabilities with the invariant of Eq. (9) is for a future work. Here we give the background evolution or balance equations obtained from Eqs. (1)–(5) after averaging over the  $\theta - z$  dependence of the fluctuations. Using the notation  $\bar{f} = \langle f(r, \theta, z) \rangle_{\theta, z} = f_{0,0}(r)$  and adding the background sources and sinks, the ambient or mean field equations become

$$\begin{aligned} \frac{\partial}{\partial r} (r v_{\theta}) + \frac{1}{r} \frac{\partial}{\partial r} (\Pi_{\theta}(r, t)) &= -r j_r^{na} B / \rho + r F_{\theta}^{\text{ext}} / \rho \\ \frac{\partial v_{\parallel}}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \Pi_{\parallel}) &= F_{\parallel}^{\text{ext}} / \rho + \mu_{\parallel} \nabla^2 v_{\parallel} \\ \frac{\partial N}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \Gamma) &= S_n \\ \frac{3}{2} \frac{\partial}{\partial t} P_i + \frac{1}{r} \frac{\partial}{\partial r} (r Q_i) &= P_i^{\text{ext}} \\ \frac{\partial A}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r F_A) &= \frac{c^2 \eta}{4\pi} \nabla^2 A \end{aligned} \quad (10)$$

The anomalous fluxes in Eqs. (10) are given by

$$\begin{aligned} \Pi_{\theta}(r, t) &= r \overline{(v_{\theta} \delta v_{ri})} = - \overline{\frac{\partial \varphi}{\partial r} \frac{\partial}{\partial \theta} (\varphi + \tilde{p}_i)} \\ \Pi_{\parallel}(r, t) &= \overline{v_{\parallel} \delta v_{rE}} = - \overline{\frac{v_{\parallel}}{r} \frac{\partial \varphi}{\partial \theta}} \\ \Gamma(r, t) &= \overline{n \delta v_{rE}} = - \overline{\frac{n}{r} \frac{\partial \varphi}{\partial \theta}} \\ Q_i(r, t) &= \frac{3}{2} \overline{p_i \delta v_{rE}} = - \frac{3}{2} \overline{\frac{p_i}{r} \frac{\partial \varphi}{\partial \theta}} \\ F_A(r, t) &= \overline{A \delta v_{re}} = - \overline{\frac{A}{r} \frac{\partial}{\partial \theta} (\varphi - \tilde{n})} \end{aligned} \quad (11)$$

where the radial transport velocities are the ion fluid velocity  $v_{ri}$ , the electron fluid velocity  $v_{re}$  or the  $\mathbf{E} \times \mathbf{B}$  velocity depending on the quantity being transported.

### III. Linear Eigenmodes and their Stability

To examine the linear electromagnetic stability problem, we linearize Eqs. (1)–(5) assuming the perturbed quantities have the form of  $\exp(-i\omega t + im\theta - inz)$ . Following earlier work<sup>5</sup> we derive the fluctuating parallel vector potential  $A_{\parallel}$  from a new potential  $\psi$  by writing  $\left(\frac{i\omega}{c}\right) A_{\parallel} = \mathbf{b}_0 \cdot \nabla \psi$  which makes

$$E_{\parallel} = -i k_{\parallel}(\phi - \psi) .$$

Analysis in terms of  $\phi$  and  $\psi$  simplifies the equations and makes the MHD polarization  $\psi_{m,n}(r) \cong \phi_{m,n}(r)$  easy to recognize.

We obtain the following two coupled second order differential equations,

$$\begin{aligned} & \left(1 - \frac{\omega_{*pi}}{\omega}\right) \rho^2 \nabla_r^2 \phi - \left(1 - \frac{\omega_{*pi}}{\omega}\right) \frac{m^2 \rho^2}{r^2} \phi \\ &= \frac{\omega_{De} - i\nu_{\parallel}}{\omega - \omega_{De} + i\nu_{\parallel}} \left[ \frac{k_{\parallel}^2 \epsilon^2}{\omega^2 \rho^2} \left(1 - \frac{\omega_{*pi}}{\omega}\right) (\phi - \psi) - \left(1 - \frac{\omega_{*e}}{\omega}\right) \left\{ \phi - \left(1 - \frac{\omega_{De}}{\omega}\right) \psi \right\} \right] \\ &+ \frac{\omega_{De}}{\omega} \left[ \left(1 - \frac{\omega_{*pi}}{\omega}\right) \phi - \left(1 - \frac{\omega_{*e}}{\omega}\right) \psi \right] \end{aligned} \quad (12)$$

$$\begin{aligned} & -\frac{\nabla_r^2 k_{\parallel} \psi}{k_{\parallel}} + \frac{m^2}{r^2} \psi = \frac{\beta_e}{2} \frac{\omega^2}{k_{\parallel}^2 \epsilon^2} \frac{i\nu_{\parallel}}{\omega - \omega_{De} + i\nu_{\parallel}} \left[ \frac{k_{\parallel}^2 \epsilon^2}{\omega^2 \rho^2} \left(1 - \frac{\omega_{*pi}}{\omega}\right) (\phi - \psi) \right. \\ & \left. - \left(1 - \frac{\omega_{*e}}{\omega}\right) \left\{ \phi - \left(1 - \frac{\omega_{De}}{\omega}\right) \psi \right\} \right] \end{aligned} \quad (13)$$

where

$$\begin{aligned} \nabla_r^2 &= \frac{1}{r} \frac{d}{dr} r \frac{d}{dr} \quad , \quad k_{\parallel}(r) = m \iota(r) - n \quad , \quad \nu_{\parallel} = \frac{k_{\parallel}^2 \epsilon^2}{\nu_e} \\ \omega_{De} &= \frac{m}{r} \frac{d\Omega}{dr} = mg \quad , \quad \omega_{*e} = \frac{m}{r} \frac{1}{r_n} \quad , \quad \omega_{*pi} = -\frac{T_i}{T_e} (1 + \eta_i) \omega_{*e} \end{aligned}$$



with  $\nu_e = \nu_{ei}/\omega_{ce}$ . In the limit of the electrostatic approximation, Eqs. (12) and (13) reduce to

$$\begin{aligned} (\omega - \omega_{*pi})\rho^2 \nabla_r^2 \phi - \left\{ (\omega - \omega_{*pi}) \frac{m^2 \rho^2}{r^2} - \frac{\omega_{*pi} \omega_{De}}{\omega} + i\nu_{||} \right. \\ \left. + (\omega_{De} - i\nu_{||}) \left[ \frac{\omega_{*e} - \omega_{De} + i\nu_{||} + \frac{k_{||}^2 \epsilon^2}{\omega \rho^2} \left( 1 - \frac{\omega_{*pi}}{\omega} \right) \right] \right\} \phi = 0 \end{aligned} \quad (14)$$

In the slab geometry Eq. (14) was solved in Ref. 2 and shown that the  $\eta_i$  mode is further destabilized by the coupling to the resistive  $g$  mode. Equation (14) in the cold ion limit was studied in the cylindrical plasma by Sugama *et al.*<sup>4</sup> and it was found that there are two branches of the unstable modes: one localized to the mode rational surface (identified there as the resistive interchange mode) and the other is a global eigenmode, not localized to a mode rational surface (the resistive drift wave).

In the collisionless limit, with the local approximation, Eqs. (12)–(13) are analyzed for the tokamak case by Horton *et al.*<sup>5</sup> and it was shown that as the plasma pressure ( $\beta$ ) increases the electrostatic  $\eta_i$  mode is strongly coupled with the FLR-MHD mode. In Sec. III, numerical solution of Eq. (14) is shown in Figs. 1–4 and the electromagnetic effect is shown in Fig. 5 by solving Eqs. (12) and (13).

## IV. Numerical Results

Equations (12) and (13) are solved numerically for their eigenvalues and eigenfunctions using the shooting method. The parameters for the calculation are  $\epsilon = a/R = 0.1$ ,  $\rho = \rho_s/a = 0.02$ , the background density and pressure profile  $n_0 \propto p_0 \propto \exp(-2r^2)$  such that  $\eta_i = 1$ , and the rotational transform profile of  $\iota(r) = 0.51 + 1.69 r^{2.5}$ , which is similar to that of

Heliotron E.<sup>6</sup> For these parameters, the resonant surface of the  $m = 1/n = 1$  mode which seems to be most dangerous mode in the Heliotron E is at  $r_0 = 0.61$ .

We use the numerical shooting method to find the eigenmodes due to the cylindrical geometry and the two characteristic scale regions around the mode rational surface. The inner most layer is defined by from the resistive diffusion of the electrons by  $x_r^2 = \nu_{ei}(\omega - \omega_{De})/k_{||}^2 v_e^2$  and the outer layer by the coupling to the ion acoustic waves with  $x_i^2 = \omega(\omega - \omega_{*e})/k_{||}^2 c_s^2$ . Some analytic results from asymptotic matching are given in Ref. 2.

## A. Electrostatic Limit

Figure 1 shows the growth rate of the mode with a radial node number of  $\ell = 0, 1, 2$  and the global mode as a function of collision frequency  $\nu_e (= \nu_{ei}/\Omega_{ce})$ . Here  $g = 0$  corresponds to the slab  $\eta_i$  mode, and  $g \neq 0$  corresponds to the toroidal  $\eta_i$  modes within the cylindrical model. Both the global and the slab  $\eta_i$  mode show a weak destabilizing dependence on the collision frequency. Increasing the collision frequency enhances the growth rate of the localized mode driven by a bad magnetic curvature. When the collision frequency is small the mode with a higher radial node number ( $\ell = 2$ ) has a larger growth rate, but as  $\nu_e$  increases the  $\ell = 0$  mode has the largest growth rate. Also, the radial mode width of the localized  $\ell = 0$  mode increases with collision frequency due to the coupling to the resistive  $g$  mode indicating a stronger anomalous convective transport across the 1/1 rational surface. This tendency for the radial mode width to increase with collisionality was already found in the slab model.<sup>2</sup>

Figure 2 shows the dependence of the growth rate on the average curvature of the Heliotron/Torsatron. Parameters are the same as in Fig. 1 except now  $\nu_e = \nu_{ei}/\omega_{ce} = 5 \times 10^{-4}$ . The growth rate of the global mode has a weak dependence on average curvature parameter  $g$ . For the  $\ell = 0$ , localized mode, the growth rate  $\gamma$  increases when  $g > 0$  and the effect of the negative curvature is weak.

In Fig. 3 we change the ion temperature gradient  $\eta_i$  parameter. In the collisionless limit

(Fig. 3(a)), the threshold value is given by  $\eta_c \simeq -1$  which is the usual prediction of simple, incompressible, fluid theory.<sup>7</sup> It was shown that improved fluid theory gives  $\eta_c \simeq 2/3$ , which is comparable with the kinetic theory.<sup>8</sup> When  $\nu_e = 5 \times 10^{-4}$ , (Fig. 3(b)), the growth rate is enhanced by a factor of two-three, and the mode remains unstable at  $\eta_i = \eta_c = -1$  due to the coupling to the resistive  $g$  mode. Also, the  $\ell = 0$  mode growth rate is dominant in the resistive regime.

The effect of shear on the growth rate is shown in Fig. 4. There is a stabilizing effect for the  $\ell = 0$  mode in the collisionless and collisional case, but the effect is weak. The radial mode width depends strongly on shear with the mode width decreasing as the shear increases, indicating a strong dependence on  $s$  of the anomalous transport. Turbulent transport theory for the resistive  $g$  mode<sup>9,10</sup> and the  $\eta_i$  mode<sup>5,11</sup> give the anomalous transport rates for  $Q_i = \frac{3}{2} \overline{p_i \delta v_{rE}} \simeq -n_i \chi_i dp/dr$  in Eq. (11) for resistive  $g$  as

$$\chi_i^{r-g} = \frac{c^2 \eta}{4\pi} \frac{\beta_p}{s^2} \left( \frac{r^2}{L_p R_c} \right) \simeq \nu_{ei} \rho_e^2 \left( \frac{L_s^2}{L_p R_c} \right) \quad (15)$$

where  $\beta_p$  is the poloidal  $\beta$ ,  $s = r q' / q = q R / L_s$  is the shear parameter,  $L_s$  is the scale length of shear,  $R$  is the major radius,  $q = 2\pi / \iota$  is the safety factor and  $R_c$  is the radius of curvature and for the  $\eta_i$  mode in the weak shear-strong toroidicity branch<sup>5</sup>

$$\chi_i^{\eta_i} \cong \left( \frac{\rho_s L_s}{R_c^{1/2} L_{Ti}^{3/2}} \right) \left( \frac{c T_e}{e B_p} \right) \quad (16)$$

and for the strong shear-weak toroidicity branch<sup>11</sup> as

$$\chi_i^{\eta_i} \cong \left( \frac{\rho_s}{L_{Ti}} \right) \left( \frac{c T_i}{e B} \right) \exp \left( \frac{-4 L_{Ti}}{L_s} \right) . \quad (17)$$

For all modes formulas (15)–(17) indicate that strong magnetic shear is the most effective mechanism for controlling the transport in both the collisional and collisionless regimes. The scaling from Eq. (15) is  $\chi^{r-g} \sim n T_e^{-1/2} B_t^{-2} L_s^2 L_p^{-1} R_c^{-1}$  compared with the collisionless scaling  $\chi^{\eta_i} \sim T_e^{3/2} B_p^{-1} B_T^{-1} L_s R_c^{-1/2} L_p^{-3/2}$  for (16). With increasing  $T_e^2 / n_e \sim \lambda_{mfp} / R$  the

collisionless transport from Eqs. (16) and (17) quickly exceeds the collisional transport from Eq. (15).

## B. High Beta Electromagnetic Stability

Electromagnetic effect from finite plasma pressure beta on the growth rate is shown in Fig. 5 for both the collisionless and collisional case. As  $\beta$  increases, both the growth rate increases and the radial mode width increases, implying large  $\gamma(\Delta r)^2$  which measures the anomalous transport. This destabilizing effect of the finite pressure is different from the tokamak case where the finite beta effect is related to ballooning modes; here, for the Heliotron/Torsatron, the pressure limit appears to be related to the Suydam criterion for the interchange mode in cylindrical plasma. For the same model configuration as used in Fig. 5, Sugama and Wakatani<sup>12</sup> find that the Suydam instability appears for  $\beta(0) \geq \beta_{\text{crit}} = 0.016$  and the growth rate becomes substantial for  $\beta(0) \gtrsim 2\beta_{\text{crit}} = 0.03$ . It is noted that the Suydam criterion also indicates the existence of low- $m$  mode interchange instabilities with the 1/1 mode growth rate at about  $0.6\gamma$  (Suydam) when  $\beta = 0.03$  according to Ref. 12.

Dominguez *et al.*<sup>13</sup> also show in toroidal stability theory that the Mercier criterion gives  $\beta = 0.016$  for the resonant surface of  $m = 1/n = 1$  using the numerical MHD toroidal equilibrium. Thus our cylindrical approximation appears good for the electromagnetic  $\eta_i$  mode analysis.

We find that the eigenfunction  $\psi$  increases from zero at  $\beta_e = 0$  to  $\psi_{1,1} \sim \frac{1}{2} \phi_{1,1}$  at  $\beta_e = 0.01$  to the MHD polarization with  $\psi_{1,1}(r) \simeq \phi_{1,1}(r)$  at  $\beta_e = 0.02$ .

Finally we find that the compressibility effect on the new  $\eta_i$  modes is not important, except near marginal stability for finite  $g$  case as demonstrated in Ref. 2. For other low- $m$   $\eta_i$  mode cases such as  $m = 3/n = 2$  and  $m = 5/n = 3$  no essential difference appears from the present  $m = 1/n = 1$  study.

## V. Conclusions

The stability analysis of the radial eigenmode equations presented here shows that there is a new type of  $\eta_i$  mode which is further destabilized by the coupling to the resistive  $g$  modes in the Heliotron/Torsatron system. Both the localized and the global modes are found to be unstable in the cylindrical plasma. Including the electromagnetic component of the electromagnetic fields further enhances the growth rate of this new  $\eta_i$  mode implying that the instability becomes stronger with auxiliary heating in the high density plasma. We conclude that the  $\eta_i$  mode is a good candidate to explain the anomalous ion thermal transport in the Heliotron E experiments. In view of these stability results we are proceeding to the nonlinear studies of the turbulence governed by Eqs. (1)–(5).

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## Figure Captions

1. Growth rate  $\gamma$  in units of  $\Omega_{ci} \frac{\rho_s^2}{a^2}$  versus collisionality  $\nu_{ei}/\Omega_{ce}$ .
2. Growth rate  $\gamma$  in units of  $\Omega_{ci} \frac{\rho_s^2}{a^2}$  versus average curvature  $g$  (arbitrary unit) for  $\nu_{ei}/\Omega_{ce} = 5 \times 10^{-4}$ .
3. Growth rate  $\gamma$  in units of  $\Omega_{ci} \frac{\rho_s^2}{a^2}$  versus ion temperature gradient  $\eta_i$  for collisionless limit (a) and  $\nu_{ei}/\Omega_{ce} = 5 \times 10^{-4}$  (b).
4. Growth rate  $\gamma$  in units of  $\Omega_{ci} \frac{\rho_s^2}{a^2}$  versus shear parameter  $s$  (arbitrary unit).
5. Growth rate  $\gamma$  in units of  $\Omega_{ci} \frac{\rho_s^2}{a^2}$  versus plasma pressure  $\beta$  for  $\ell = 0$  mode.

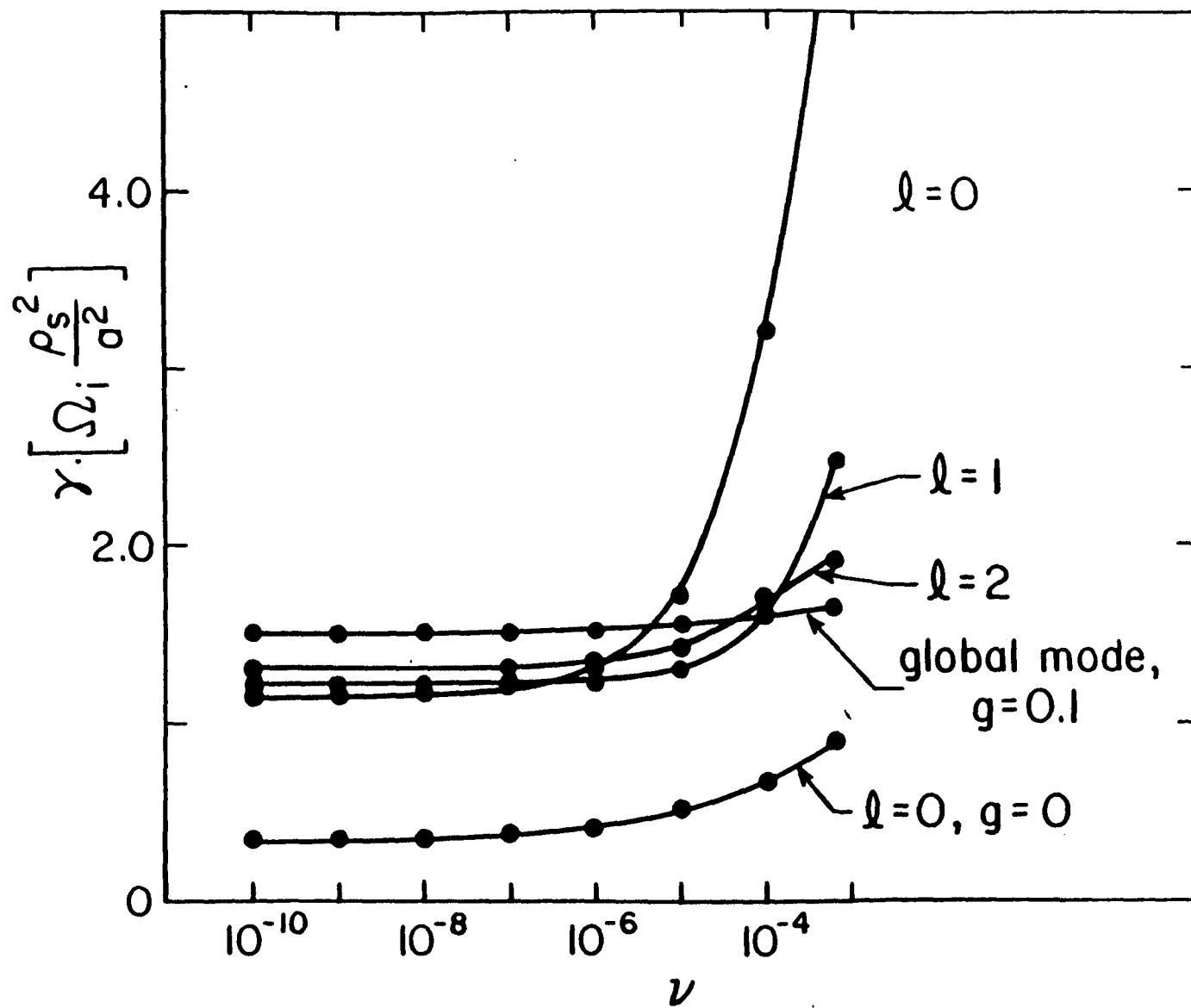


Fig. 1



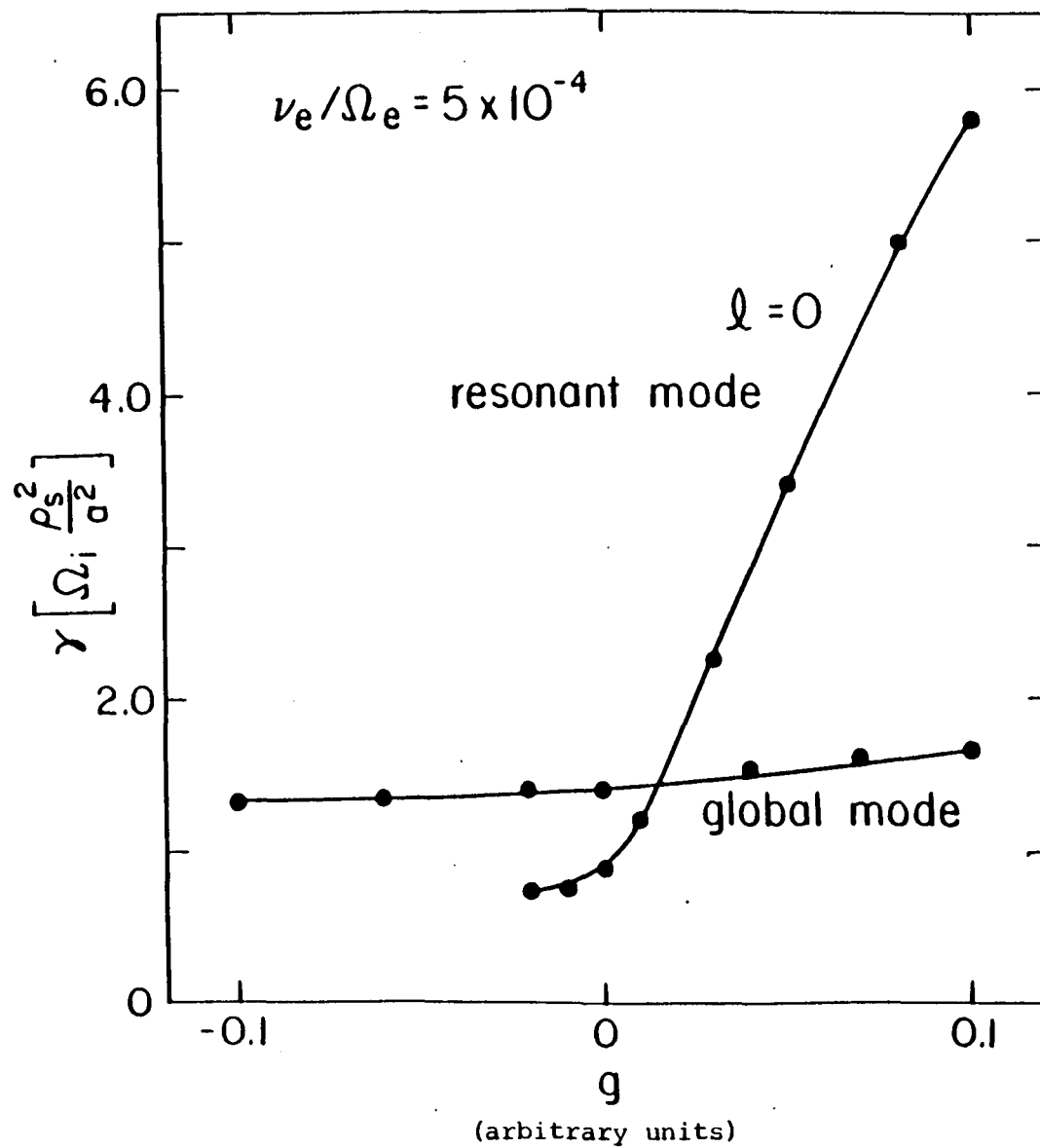


Fig. 2

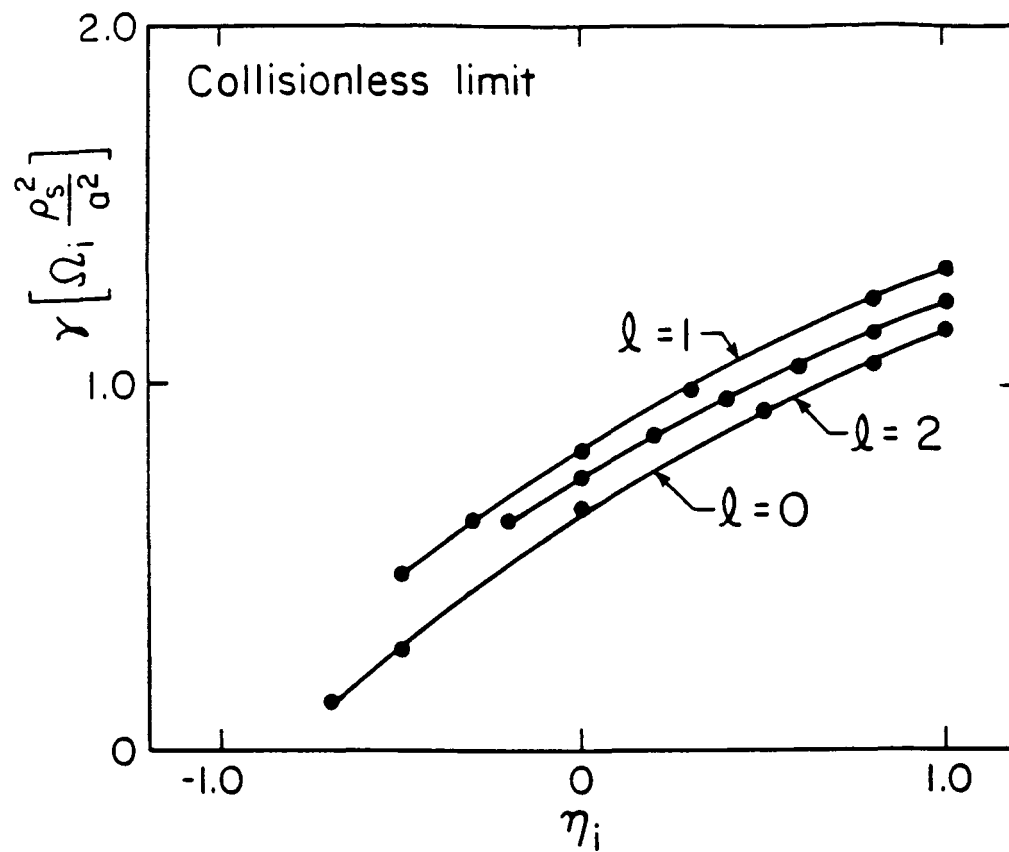


Fig. 3(a)

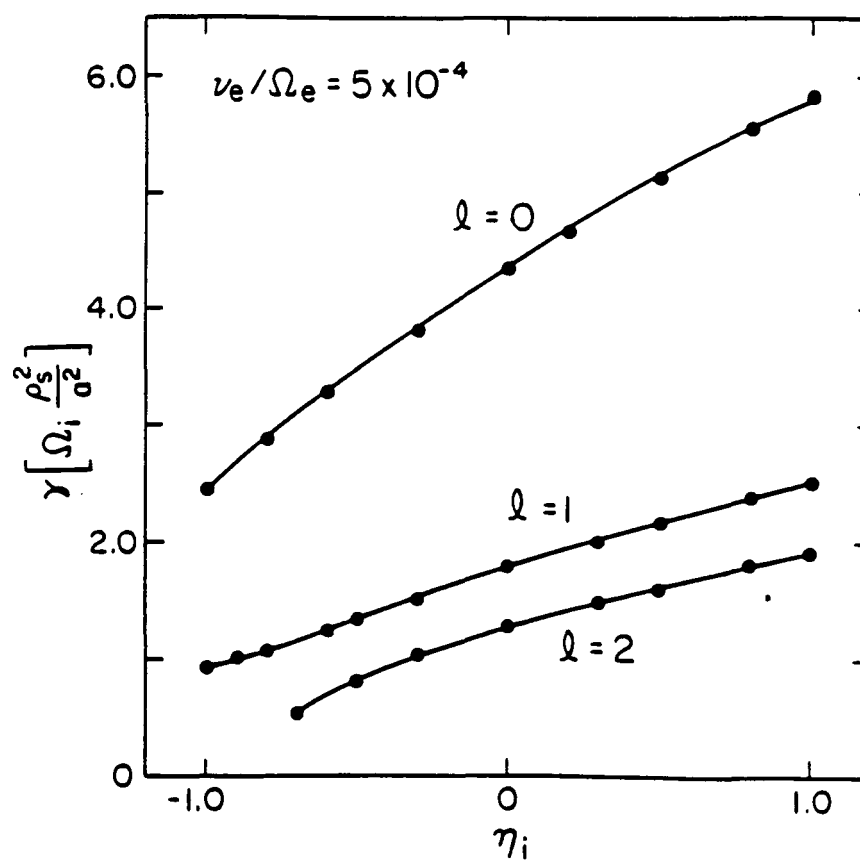


Fig. 3(b)

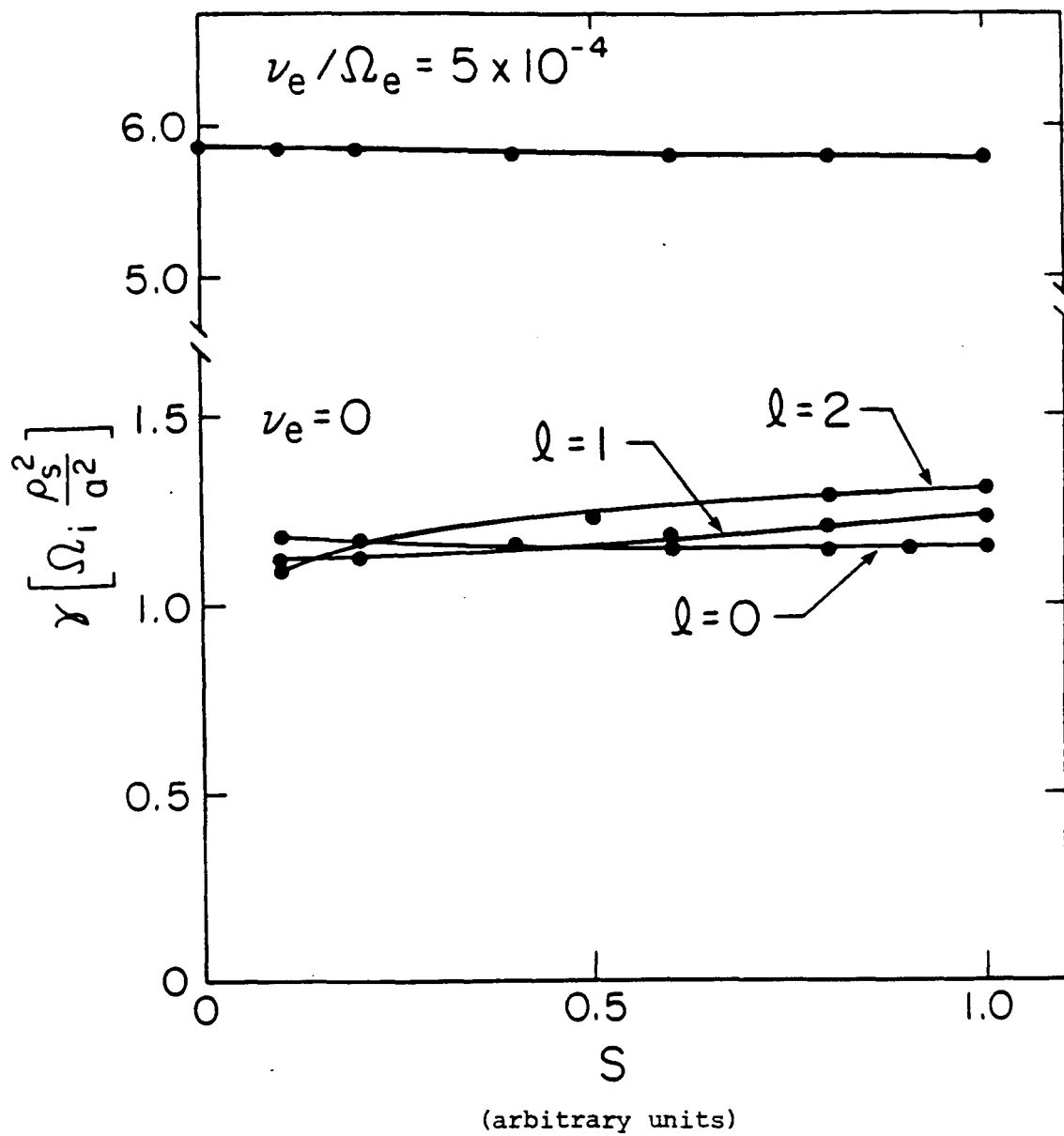


Fig. 4

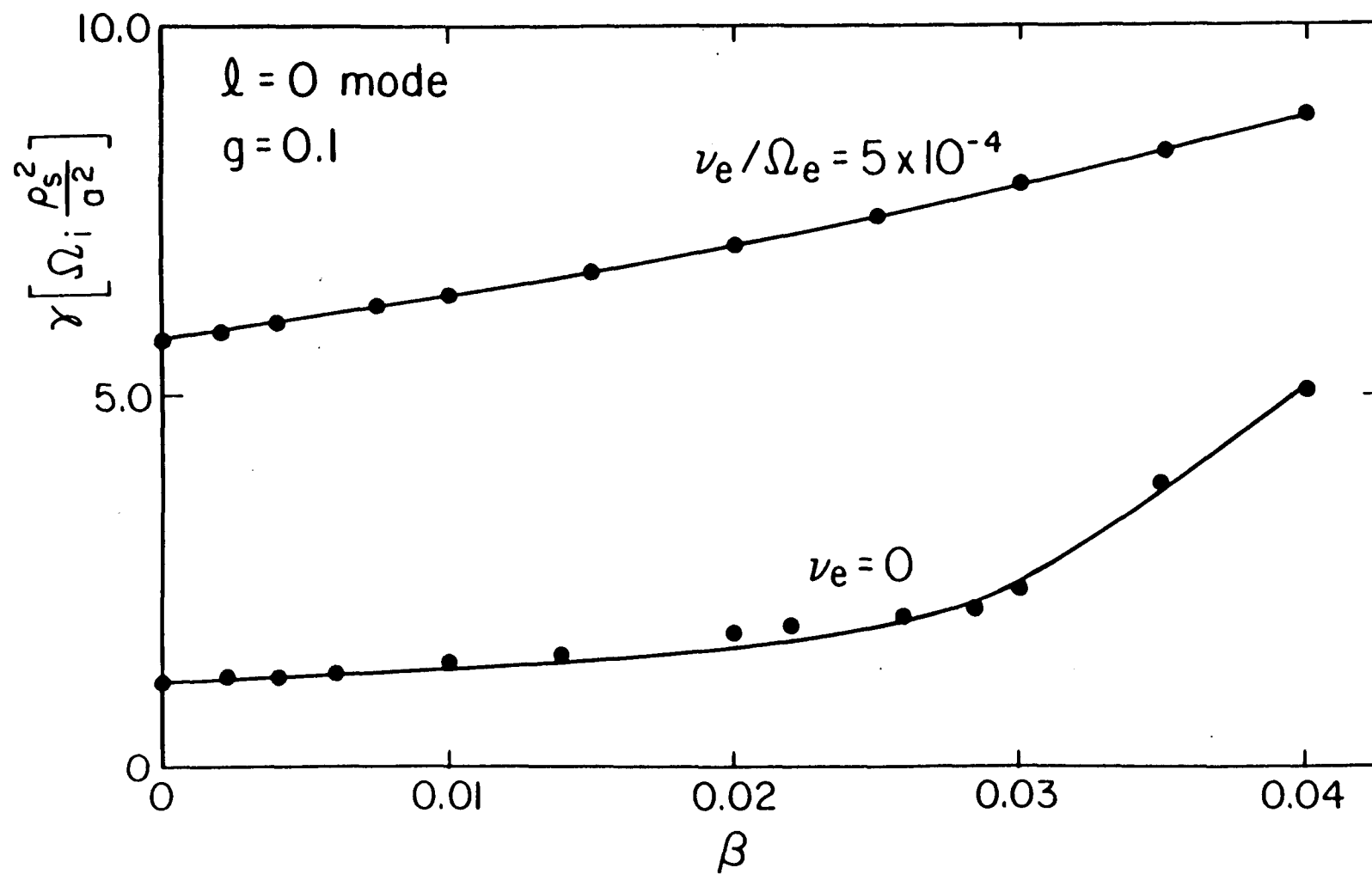


Fig. 5