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Effects of single fractures on seismic wave propagation

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ABSTRACT: Detection and characterization of fractures, joints and faults remains an important problem in mining and geotechnical engineering, as well as in petroleum reservoir engineering. A theoretical model has been developed which, from a single set of assumptions, predicts the amplitude and group time delay of the transmitted, reflected, and converted waves resulting from a plane wave incident upon a single fracture. The theory represents a fracture as a zero-thickness, non-welded interface of infinite areal extent between two elastic half-spaces. The elastic properties of the half-spaces may be the same or different, and it is assumed that seismic stresses are continuous across the interface. Seismic particle displacements, however, are assumed to be discontinuous. For completely dry conditions, the magnitude of the displacement discontinuity is given by the ratio of the seismic stress to the stiffness, κ, of the fracture. If a fluid is present in the fracture, we postulate that, in addition to the displacement discontinuity, a velocity discontinuity exists which is equal to the ratio of the seismic stress to specific viscosity, η. The wave equation has been solved for two sets of boundary conditions, with each set incorporating both specific stiffness and specific viscosity. Because of their similarity to well known rheological models, these boundary conditions have been designated as Kelvin and Maxwell models. Theoretical results based on the models show that fractures cause both a frequency dependent loss of amplitude and a time delay in the transmitted wave even for the completely elastic system in which only a displacement discontinuity is present. Results of field measurements show qualitatively the same effects and results from laboratory seismic measurements on a single fracture under dry conditions show that the effects of a fracture on seismic wave propagation can be quantitatively related to its specific stiffness. The effect of fluid viscosity is to add a dissipative factor, and to alter, depending on the assumed model, the frequency dependent characteristics of the theory. Results indicate that this new approach to modelling of fractures will lead to significant improvements in the use of seismic data for determination of the geotechnical properties of fractures and joints.

INTRODUCTION

Seismic waves propagated through a fractured rock mass are both slowed and attenuated. It is commonly accepted that seismic velocities in fractured rock will be lower than in unfractured rock because the fractures reduce the effective elastic modulus of the rock. Proposed mechanisms to explain observed attenuation include frictional sliding (Walsh, 1966), fluid flow or "pumping" (in partially or fully saturated media) (Biot, 1956; Mavko and Nur, 1979) and scattering (Hudson, 1981). An assumption in most models is that the fractures are small in areal extent relative to the seismic wavelength, resulting in effective media properties.

The problem of detection and characterization of fractures, joints and faults in geotechnical and petroleum reservoir engineering has become of great importance in recent years. Increasing emphasis is being placed on the use of higher frequency seismic energy to locate and characterize specific features at smaller scales than those traditionally dealt with in seismology. In many cases it is difficult, if not inappropriate, to use models based on effective media properties because the features of interest are large in areal extent, thin and sparsely spaced relative to the seismic wavelength.

The purpose of this paper is to discuss, as an alternative to effective media models, the theory for seismic wave propagation across a displacement discontinuity. While we believe the theory to be of general application to seismic wave propagation in fractured rock we will limit discussion to the effects of a single fracture, assuming it is of large areal extent relative to wavelength and infinitesimal thickness. The basic premise of the model is that seismic particle displacements are discontinuous across a fracture while the average seismic stresses remain continuous. From a single set of assumptions, the theory predicts changes in group time delay, and reflection and transmission coefficients of seismic waves.

WAVE PROPAGATION ACROSS DRY FRACTURES

To analytically predict the seismic responses of a single fracture under dry conditions, the fracture is modelled as a displacement discontinuity at the boundary between two half-spaces. The ratio between the seismic stress and the magnitude of the displacement discontinuity produced across the boundary is defined as the specific stiffness of the fracture. The specific stiffness of a fracture is a mechanical property which is determined, under pseudo-static conditions, from measurement of fracture displacement as a function of load, as described by Goodman 1976, Bandis et al., 1983 and others.

If the boundary between two elastic half-spaces lies in the x-y plane, the boundary conditions for an incident compressional (P-) wave or shear wave polarized in the x-z plane (S_v– wave) are:

$$u_{zI} - u_{zII} = \tau_{zz}/\kappa_z \;,\; \tau_{zzI} = \tau_{zzII} \;, \qquad (1)$$

$$u_{xI} - u_{xII} = \tau_{zx}/\kappa_x \;,\; \tau_{zxI} = \tau_{zxII}$$

where

$$\tau_{zz} = \lambda \left[\frac{\partial u_x}{\partial x}\right] + \left[\lambda + 2\mu\right] \left[\frac{\partial u_z}{\partial z}\right] \;.$$

$$\tau_{zx} = \mu \left[\left[\frac{\partial u_z}{\partial x}\right] + \left[\frac{\partial u_x}{\partial z}\right]\right] \;.$$

and

u = particle displacement

τ = stress

κ = specific stiffness of the displacement discontinuity in the x or z directions

λ, μ = Lame's constants for media on either side of displacement discontinuity

I,II = subscripts referring to media above and below displacement discontinuity

For an incident shear wave with polarization in the x-y plane (S_h-wave) the boundary conditions are:

$$u_{yI} - u_{yII} = \tau_{zy}/\kappa_y \;,\; \tau_{zyI} = \tau_{zyII} \;, \qquad (2)$$

where

$$\tau_{zy} = \mu \left[\frac{\partial u_y}{\partial z}\right].$$

The general solution of the wave equation for arbitrary angles of incidence and materials of different seismic impedance above and below the discontinuity has been given by Schoenberg (1980), Pyrak, 1988 and others. For seismic waves normal to the displacement discontinuity with the same material properties in both half-spaces, the reflection ($R(\omega)$) and transmission ($T(\omega)$) coefficients for P, S_v and S_h waves are:

$$R_p(\omega) = \frac{i\omega}{-i\omega + 2(\kappa_z/z_p)} \;,\; T_p(\omega) = \frac{2(\kappa_z/z_p)}{-i\omega + 2(\kappa_z/z_p)}$$

$$R_{sv}(\omega) = \frac{-i\omega}{-i\omega + 2(\kappa_x/z_s)} \;,\; T_{sv}(\omega) = \frac{2(\kappa_x/z_s)}{-i\omega + 2(\kappa_x/z_s)} \qquad (3)$$

$$R_{sh}(\omega) = \frac{-i\omega}{-i\omega + 2(\kappa_y/z_s)} \;,\; T_{sh}(\omega) = \frac{2(\kappa_y/z_s)}{-i\omega + 2(\kappa_y z_s)}$$

where

ω = angular frequency

z = $\rho\alpha$ for P-waves, or $\rho\beta$ for S-waves

α = $\sqrt{\lambda + 2\mu/\rho}$

β = $\sqrt{\mu/\rho}$

ρ = density

From the phase of $T(\omega)$ a group time delay, t_g, can be found. For normally incident waves and the same materials in both half-spaces, the group time delay for the transmitted wave, t_{gT}, is given by:

$$t_{gT} = \frac{2(\kappa/z)}{4(\kappa/z)^2 + \omega^2} \qquad (4)$$

These equations show that both the reflection and transmission coefficients, as well as the group time delay are dependent upon the specific stiffness of the displacement discontinuity and the frequency of the propagating wave. In addition, for all angles of incidence

$$|R(\omega)|^2 + |T(\omega)|^2 = 1 \qquad (5)$$

Thus energy is conserved at a displacement discontinuity.

Figure 1 presents a numerical example to illustrate the effect of a single fracture in a homogeneous medium on a normally incident wave. For a particular incident pulse, (shown at extreme left) transmitted pulses were calculated for a range of specific stiffness values. Shown on the right in the figure are the amplitude spectra of the transmitted pulses. For a high specific stiffness, (κ/z = 500 in this case) the transmitted pulse is essentially identical to the incident pulse, i.e., $|T| = 1$. Since energy is conserved from Eq. (5), $|R| = 0$. Thus, the case of infinite specific

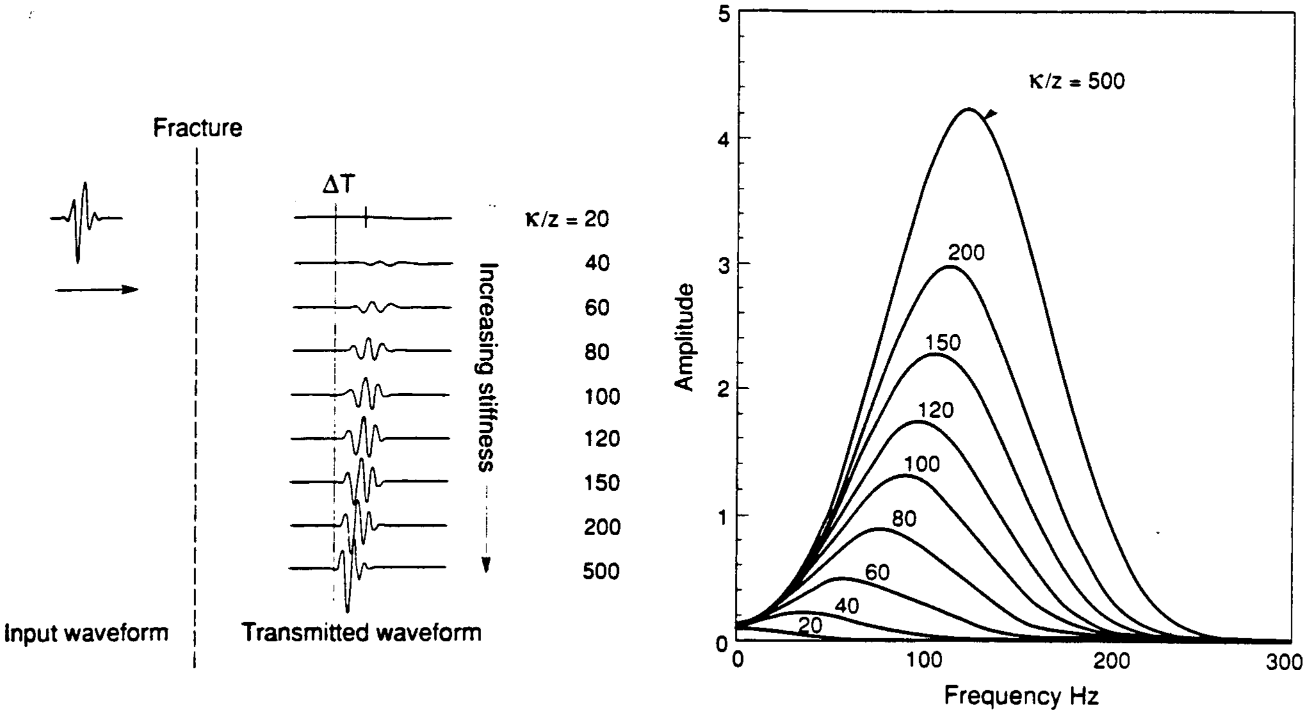

Fig. 1. Numerical example illustrating the effect of a single displacement discontinuity on the transmitted waveform and corresponding amplitude spectra (after Majer et al., 1990).

stiffness corresponds to the "welded" boundary conditions normally assumed in seismologic analyses of multilayer systems. As the specific stiffness decreases, the transmitted wave is both slowed and attenuated. The attenuation is characterized by both decreasing amplitude and filtering of the high frequency components of the pulse. This filtering is evidenced in the amplitude spectra by a shift in the frequency of the peak spectral amplitude. For zero specific stiffness no energy is transmitted; it is all reflected. Thus, in this limit the fracture acts as a free surface.

The theoretically predicted changes in transmitted pulses in response to a reduction in specific stiffness are qualitatively similar to those observed in field data such as those shown in Figure 2. The field data are from a crosshole experiment conducted by King et al. (1986) in which seismic waves were propagated between four horizontal drillholes in the wall of a drift in a basalt rock mass (see insert, Fig. 2) at a depth of about 46 m in a test site above the water table. The primary joint set, nearly vertical, was formed by the basalt columns which ranged in thickness from about 0.2 m to 0.4 m. The propagation path of the top trace in the fracture was vertical and parallel to the predominate jointing while the path of the bottom trace was horizontal, across the predominant jointing. The vertical scale on the lower trace has been amplified by a factor of 100, so the actual amplitudes of the waves propagating across the joints (between boreholes A3-A4) were much smaller than those propagating along the intact columns (between A1-A2). The apparent effect of the predominantly vertical fracturing was to filter out the high frequencies, slow, and reduce the amplitude of the waves propagating across the joints. The difference between the two waveforms is qualitatively very similar to the differences in Figure 1 between a wave propagated across a stiff fracture and

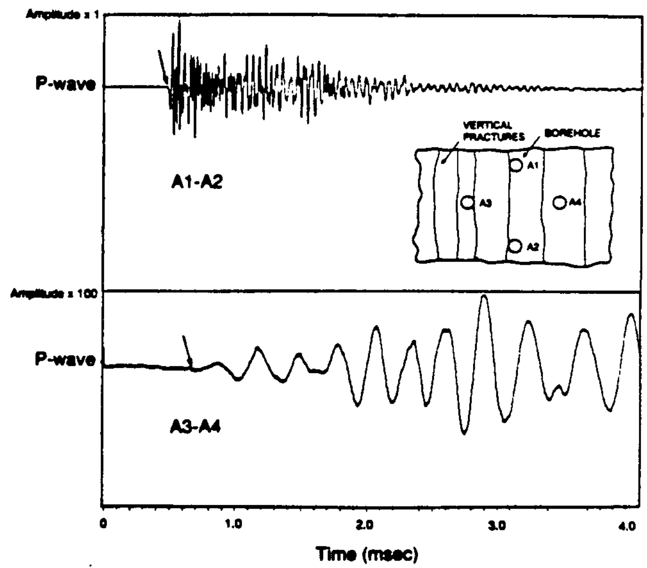

Fig. 2. Example of field data comparing P-waves propagated parallel to (A1-A2) and across (A3-A4) predominate jointing. Arrows indicate first arrival; no difference in amplitude scales.

one which has propagated across a compliant fracture. Because the wavelength of the P-waves was about 0.1 m, it is also apparent that the observed effects were occurring at the fractures, rather than as a result of an average or effective property of the medium.

Results of laboratory tests on specimens containing a single natural fracture provide a more quantitative comparison between theory and observed data (Pyrak-Nolte et al., 1990). In these tests the specimens were uniaxially compressed between seismic transducers such that a normal stress existed across the fracture. P- and S-wave pulses were collected as a function of stress under dry and saturated conditions. Reference signals were also obtained under the same conditions on specimens of intact rock cut from the same core in close proximity to the fractured specimens. Frequency spectra were obtained by performing a Fast Fourier Transform on signals which had been tapered to isolate the initial pulse.

Typical results shown in Figure 3 for two stress levels under dry conditions show lower spectral amplitudes for the fractured specimen and also a slight shift in the frequency of the peak spectral amplitude. The behavior of the fractured specimen is interpreted to be a consequence of the effect of the fracture on the seismic displacements. These displacements, which are a function of the specific stiffness of the fracture, are additional to those which would occur if the rock were intact and are localized in the very near vicinity of the fracture plane. Consequently they represent a discontinuity in the seismic displacement field.

To obtain the predicted spectra shown in the figure, the observed spectra for the companion intact rock specimen were multiplied by $|T(\omega)|$ obtained from Eq. (3). Values of specific stiffness were determined by trial and error to give the "best fit." Good agreement between observed and predicted results is achieved using values of specific stiffness of the same order of magnitude, though somewhat higher than those measured on the same specimens under static loading conditions (Pyrak-Nolte et al., 1987).

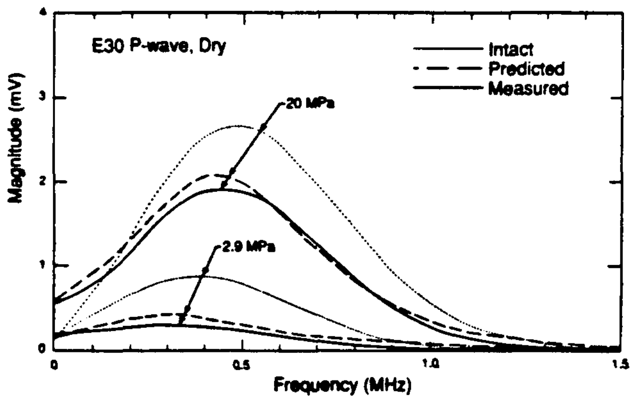

Fig. 3. Example results of laboratory tests showing amplitude spectra for fractured and intact specimens at the same stress level, and predicted curves.

Results of seismic measurements on the same specimens under saturated conditions suggested that fluid in a fracture may have two effects on seismic wave propagation (Pyrak-Nolte, 1989). First, the relative (to air) incompressibility of a fluid in the fracture void space will increase the specific stiffness of the fracture, particularly for P-waves. Secondly, the fluid may introduce viscous coupling between the surfaces of the fracture. A viscous layer will cause dissipation of energy in the propagating wave but, if thin, it can also lead to enhanced transmission relative to dry conditions. In order to study the effects of combining specific stiffness with viscosity of a thin liquid layer, a generalized theory, summarized below, has been developed.

GENERALIZED SEISMIC THEORY

The analytic model of a single fracture filled with fluid assumes a velocity discontinuity in addition to a displacement discontinuity at the boundary between two elastic half-spaces. The velocity discontinuity, which accounts for viscous coupling between the fracture surfaces, is equal to the ratio of the seismic stress to specific viscosity, η, of the fracture. The units of η are viscosity per length.

The boundary conditions for the wave equation are formulated in two ways, which, because of their similarity to well known rheological models are referred to as the Kelvin and Maxwell models. By analogy, the boundary conditions in Eqs. (1) represent the elastic model.

Assuming the Kelvin model, for an incident P- and S_v-wave, the boundary conditions are:

$$\kappa_z (u_{zI} - u_{zII}) + \eta_z (\dot{u}_{zI} - \dot{u}_{zII}) = \tau_{zz} , \qquad (6)$$

$$\kappa_x (u_{xI} - u_{xII}) + \eta_x (\dot{u}_{xI} - \dot{u}_{xII}) = \tau_{zx} ,$$

$$\tau_{zzI} = \tau_{zzII} \qquad (7)$$

$$\tau_{zxI} = \tau_{zxII}$$

where

$$\tau_{zz} = \lambda \left[\frac{\partial u_x}{\partial x} \right] + (\lambda + 2\mu) \left[\frac{\partial u_z}{\partial z} \right] ,$$

$$\tau_{zx} = \mu \left[\left[\frac{\partial u_z}{\partial x} \right] + \left[\frac{\partial u_x}{\partial z} \right] \right] ,$$

and the dot refers to time derivative. For an incident S_h-wave the boundary conditions are:

$$\kappa_y (u_{yI} - u_{yII}) + \eta_y (\dot{u}_{yI} - \dot{u}_{yII}) = \tau_{zy} , \qquad (8)$$

$$\tau_{zyI} = \tau_{zyII} \qquad (9)$$

where

$$\tau_{zy} = \mu \frac{\partial u_y}{\partial z} \, .$$

Assuming the Maxwell model, Eqs. (6) for an incident P- and S_v-wave become:

$$(\dot{u}_{zI} - \dot{u}_{zII}) = \dot{\tau}_{zz}/\kappa_z + \tau_{zz}/\eta_z \qquad (10)$$

$$(\dot{u}_{xI} - \dot{u}_{xII}) = \dot{\tau}_{zx}/\kappa_x + \tau_{zx}/\eta_x$$

and for an incident S_h-wave, Eq. (8) becomes:

$$(\dot{u}_{yI} - \dot{u}_{yII}) = \dot{\tau}_{zy}/\kappa_y + \tau_{zy}/\eta_y \qquad (11)$$

For waves normally incident upon the fracture where the two half-spaces have the same elastic properties, the reflection and transmission coefficients for P-, S_v-, and S_h-waves are, for the Kelvin model:

$$R_p(\omega) = \frac{i\omega Z_p}{2\kappa_z - i\omega\,(2\eta_z + Z_p)} \, , \; T_p(\omega) = \frac{2(\kappa_z - i\omega\eta_z)}{2\kappa_z - i\omega\,(2\eta_z + Z_s)} \qquad (12)$$

$$R_{sv}(\omega) = \frac{-i\omega Z_s}{2\kappa_x - i\omega(2\eta_x + Z_s)} \, , \; T_{sv}(\omega) = \frac{2(\kappa_x - i\omega\eta_x)}{2\kappa_x - i\omega(2\eta_x + Z_s)}$$

$$R_{sh}(\omega) = \frac{-i\omega Z_s}{2\kappa_y - i\omega(2\eta_y + Z_s)} \, , \; T_{sh}(\omega) = \frac{2(\kappa_y - i\omega\eta_x)}{2\kappa_y - i\omega\,(2\eta_y + Z_s)} \, .$$

For the Maxwell model the reflection and transmission coefficients are:

$$R_p(\omega) = \frac{-\dfrac{Z_p}{2\eta_z}\left[1 + \dfrac{Z_p}{2\eta_z}\right] + \dfrac{\omega Z_p}{2\kappa_z}\left[i - \dfrac{\omega Z_p}{2\kappa_z}\right]}{\left[1 + \dfrac{Z_p}{2\eta_z}\right]^2 + \left[\dfrac{\omega Z_p}{2\kappa_z}\right]^2} \, , \qquad (13)$$

$$T_p(\omega) = \frac{\left[1 + \dfrac{Z_p}{2\eta_z}\right] + \dfrac{i\omega Z_p}{2\kappa_z}}{\left[1 + \dfrac{Z_p}{2\eta_z}\right]^2 + \left[\dfrac{\omega Z_p}{2\kappa_Z}\right]^2} \, ,$$

$$R_{sv}(\omega) = \frac{\dfrac{Z_s}{2\eta}\left[1 + \dfrac{Z_s}{2\eta_x}\right] + \dfrac{\omega Z_s}{2\kappa_x}\left[\dfrac{\omega Z_s}{2\kappa_x} - i\right]}{\left[1 + \dfrac{Z_s}{2\eta_x}\right]^2 + \left[\dfrac{\omega Z_s}{2\kappa_x}\right]^2} \, ,$$

$$T_{sv}(\omega) = \frac{\left[1 + \dfrac{Z_s}{2\eta_x}\right] + \dfrac{i\omega Z_s}{2\kappa_x}}{\left[1 + \dfrac{Z_s}{2\eta_x}\right]^2 + \left[\dfrac{\omega Z_s}{2\kappa_x}\right]^2} \, ,$$

$$R_{sh}(\omega) = \frac{\dfrac{Z_s}{2\eta_y}\left[1 + \dfrac{Z_s}{2\eta_y}\right] + \dfrac{\omega Z_s}{2\kappa_y}\left[\dfrac{\omega Z_s}{2\eta_y} - i\right]}{\left[1 + \dfrac{Z_s}{2\eta_y}\right]^2 + \left[\dfrac{\omega Z_s}{2\kappa_y}\right]^2} \, ,$$

$$T_{sh}(\omega) = \frac{\left[1 + \dfrac{Z_s}{2\eta_y}\right] + \dfrac{i\omega Z_s}{2\kappa_y}}{\left[1 + \dfrac{Z_s}{2\eta_y}\right]^2 + \left[\dfrac{\omega Z_s}{2\kappa_x}\right]^2} \, .$$

It should be noted that for both the Maxwell and Kelvin models, $|R(\omega)|^2 + |T(\omega)|^2 \neq 1$ because of viscous losses in the fluid layer.

The magnitude of the transmission coefficient (of similar form for P-, S_v- and S_h- waves) for the Kelvin model is plotted in Figure 4. In the limit when either κ or η become very large, all energy is transmitted, ($|T| \rightarrow 1$), while if both approach zero, none is transmitted ($|T| \rightarrow 0$). If $\kappa = 0$ but η remains non-zero, $|T|$ is independent of frequency. Thus, as seen in the figure, curves for non-zero values of η/z approach a horizontal slope when the solution is dominated by terms involving η. The curve for $\eta/z = 0$ (specific viscosity of zero) is equivalent to that which would be obtained using Eqs. (3) for the elastic model. Compared to the solution for the elastic model, the effect of finite values of specific viscosity is to reduce energy transmitted at low frequencies and increase the energy transmitted at high frequencies. For a given value of specific stiffness and at a particular frequency, increasing the specific viscosity results in larger values of $|T|$.

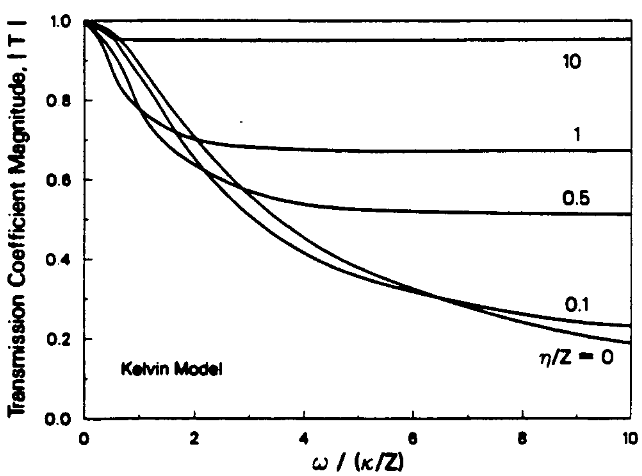

Fig. 4. Magnitude of the transmission coefficient as predicted by the Kelvin model as a function of normalized frequency for a range of normalized specific viscosities.

For comparison with the Kelvin model the magnitude of the transmission coefficient as predicted by the Maxwell model is plotted in Figure 5. In this case it is seen that very high ($\eta/Z \rightarrow \infty$) specific viscosity values in combination with finite specific stiffness values yield a curve similar to that for a the elastic model. Reducing the value of specific viscosity (while holding specific stiffness constant) has the effect of reducing energy transmitted at low frequencies relative to that transmitted at high frequencies. When terms involving η dominate the solution $|T|$ becomes frequency independent.

It is clear that these two models represent in a sense, bounds on the effect of fluid filled fractures on seismic wave propagation; in the Kelvin model the effects of η dominate when it is large and in the the Maxwell model when it is small. Laboratory work is currently underway to evaluate and compare the models with observations of the seismic behavior of interfaces filled with fluids with different rheologies. From a practical standpoint the significance of these models is that they include, explicitly both the mechanical and fluid properties of a fracture. Hence, there is the potential not only to use seismic methods to locate joints and fractures, but also to derive information about the fluids present in them.

CONCLUSIONS

Extensive discontinuities, such as joints and fractures, have effects on the strength, compliance and hydraulic conductivity of rock masses that are important in geotechnical mining, and petroleum engineering. Whereas the effects of small cracks on seismic wave transmission through rock have been studied extensively, little has been done to analyze the seismic properties of extensive discontinuities. By treating discontinuities as a boundary condition at an interface between two half spaces, across which seismic stresses are continuous but seismic particle displacements and velocities are discontinuous, analytical expressions for the conversion, reflection and transmission of seismic waves at these discontinuities have been derived. Three separate rheologies have been assumed to relate seismic stresses to the discontinuities in seismic displacement and velocity; elastic, Kelvin and Maxwell. The elastic rheology results in frequency dependent reflection and transmission coefficients, as well as a frequency dependent group time delay, but no energy is lost. The Kelvin and Maxwell rheologies yield behavior in which viscous effects dominate when the viscous term is large and small, respectively. Observations of the effects of dry natural fractures in the field and in the laboratory show high frequency attenuation and changes in travel times in accord with predictions for a displacement discontinuity. Fluid filled joints or fractures exhibit behaviors that can be modeled using elastic and viscous components.

Reflection, conversion, transmission and group delay occur at the discontinuity. In principle, the measurement of frequency spectra and travel times of reflected, converted or transmitted waves allow values to be determined for the specific stiffness and viscosities of joints and fractures to be determined, providing information about their mechanical and hydraulic behavior.

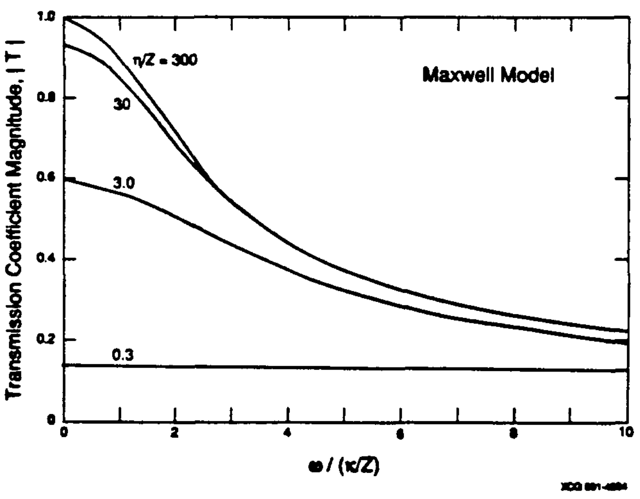

Fig. 5. Magnitude of the transmission coefficient as predicted by the Maxwell model as a function of normalized frequency for a range of normalized specific viscosities.

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REFERENCES

Bandis, S. C., Lumsolm, A. C., and Barton, N. R. 1983. Fundamentals of rock joint deformation. International Journal of Rock Mechanics and Mining Sciences and Geomechanics Abstracts. 20(6):249-268.

Biot, M. A. 1956. Theory of propagation of elastic waves in a fluid-saturated porous solid, II. Higher frequency range. Journal of the Acoustical Society of America. 28(2):179-191.

Goodman, R. E. 1976. Methods of Geological Engineering. West Publishing Company, St. Paul:170-173.

Hudson, J. A. 1981. Wave speeds and attenuation of elastic waves in material containing cracks. Geophysical Journal of the Royal Astronomical Society. 64(1):133-150.

King, M. S., Myer, L. R., and Rezowalli, J. J. 1986. Experimental studies of elastic-wave propagation in a columnar-jointed rock mass. Geophysical Prospecting. 34(8):1185-1199.

Majer, E. L., Peterson, J. E., Karasaki, K., Myer, L. R., Long, J. C. S., Martel, S., Blumling, P. and Von Voris, S. 1990. Results of fracture research investigation. NDC-14, LBL-27913, Lawrence Berkeley Laboratory, Berkeley, California.

Mavko, G. M. and Nur, A. 1979. Wave attenuation in partially saturated rocks. Geophysics. 44(2):161-178.

Pyrak, L. J. 1988. Seismic visibility of fractures. Ph.D dissertation, Department of Materials Science and Mineral Engineering, University of California, Berkeley, California.

Pyrak-Nolte, L. J., Myer, L. R., Cook, N. G. W., and Witherspoon, P. A. 1987. Hydraulic and mechanical properties of natural fractures in low permeability rock. Proceedings of 6th International Congress of Rock Mechanics, Montreal. I:225-232.

Pyrak-Nolte, L. J., Myer, L. R., and Cook, N. G. W. 1990. Transmission of seismic waves across single natural fractures. LBL-26616, Lawrence Berkeley Laboratory, Berkeley, California, accepted for publication in Journal of Geophysical Research.

Schoenberg, M. 1980. Elastic wave behavior across linear slip interfaces. J. of the Acoustical Society of America. 60(5):1516-152.

Walsh, J. B. 1966. Seismic wave attenuation in rock due to friction. Journal of Geophysical Research. 71(10):2591-2599.