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STUDIES IN NONLINEAR PROBLEMS OF ENERGY

Progress Report

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Brief review of progress on DOEFG02-87ER25027

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We continue our successful research program on Nonlinear Problems of Energy, with emphasis on combustion and flame propagation. A total of 150 papers associated with the grant has appeared in the literature, and our efforts have twice been recognized by DOE's Basic Science Division for Top Accomplishment.

In our research program we concentrate on modeling, analysis and computation of combustion phenomena, with particular emphasis on the transition from laminar to turbulent combustion. Thus we investigate the nonlinear dynamics and pattern formation at the successive stages of transition. We describe the stability of combustion waves, and transitions to waves exhibiting progressively higher degrees of spatio-temporal complexity. Combustion waves are characterized by large activation energies, so that chemical reactions are significant only in thin layers, termed reaction zones. In the limit of infinite activation energy, the zones shrink to moving surfaces, termed fronts, which must be found during the course of the analysis, so that the problems are moving free boundary problems. The analytical studies are carried out for the limiting case with fronts, while the numerical studies are carried out for the case of finite, though large, activation energy. Accurate resolution of the solution in the reaction zone(s) is essential, otherwise false predictions of dynamical behavior are possible. Since the the reaction zones move, and their location is not known a-priori, we have developed adaptive pseudo-spectral methods, which have proven to be very useful for the accurate, efficient computation of solutions of combustion, and other, problems.

Our approach is based on a synergetic combination of analytical and computational methods so that the results obtained are greater than the mere sum of results obtainable by each method separately. Our numerical computations build on and extend the information obtained analytically. Furthermore, the solutions obtained analytically serve as benchmarks for testing the accuracy of the solutions determined computationally. Finally, ideas from analysis (specifically, singular perturbation theory) have induced new approaches to computations, which we have developed and implemented. Conversely, our computational results suggest new analysis to be considered. Among the interesting results recently found, was a description of the development of spatio-temporal chaos in combustion. One of our goals is the extension of our adaptive pseudo-spectral methods to adaptive domain decomposition methods. We have begun to develop such methods for problems with multiple reaction zones, corresponding to problems with more complex, and therefore more realistic, chemistry to describe reactions. These methods offer the attractive possibility for highly parallel computation.

CUMULATIVE LIST OF PUBLICATIONS ON D.O.E. Grant

"On inhibiting runaway in catalytic reactors," SIAM J. Appl. Math. 35(2) (1978), pp. 307-314 (with D. S. Cohen).

"Propagation of a pulsating reaction front in solid fuel combustion," SIAM J. Appl. Math. 35(3) (1978), pp. 465-478 (with G. I. Sivashinsky).

"Reactive diffusive systems with Arrhenius kinetics: Multiple solutions, ignition and extinction," SIAM J. Appl. Math. 36(2) (1979), pp. 373-389 (with A. K. Kapila).

"The exit problem: A new approach to diffusion across potential barriers," SIAM J. Appl. Math. 36(3) (1979), pp. 604-623 (with Z. Schuss).

"On oscillatory necking in polymers," Quart. Appl. Math. 37(1) (1979), pp. 23-34 (with G. I. Sivashinsky).

"Acceleration effects on the stability of flame propagation," SIAM J. Appl. Math. 37(3) (1979), pp. 669-685 (with G. I. Sivashinsky).

"An asymptotic derivation of two models in flame theory associated with the constant density approximation," SIAM J. Appl. Math. 37(3) (1979), pp. 686-699 (with G. I. Sivashinsky).

"Secondary states of rectangular plates," SIAM J. Appl. Math. 38(1) (1980), pp. 38-51 (with L. J. Putnick and E. L. Reiss).

"Singular Perturbations, Stochastic Differential Equations and Applications," in Singular Perturbations and Asymptotics, Ed. R. E. Meyer and S. V. Parter, pp. 109-147, Academic Press, N.Y. (1980).

"Reactive diffusive systems with Arrhenius Kinetics: Pecularities of the spherical geometry," SIAM J. Appl. Math. 38(3) (1980), pp. 382-401 (with A. K. Kapila and J. Vega).

"A nonlinear theory of cellular flames," SIAM J. Appl. Math. 38(3) (1980), pp. 489-504 (with L. J. Putnick and G. I. Sivashinsky).

"Reactive diffusive systems with Arrhenius kinetics: The Robin problem," SIAM J. Appl. Math. 39(3) (1980), pp. 391-400 (with A. K. Kapila).

"Propagation of a pulsating flame front in a gaseous combustible mixture," SIAM J. Appl. Math. 39(2) (1980), pp. 290-300 (with D. O. Olagunju).

"On the stability of non-adiabatic flames," SIAM J. Appl. Math. 40(2) (1981), pp. 255-260 (with G. I. Sivashinksy).

"Eigenvalues of the Fokker-Planck operator and the approach to equilibrium for diffusions in potential fields," SIAM J. Appl. Math. 40(2) (1981), pp. 242-254 (with Z. Schuss).

"Pulsations in a burner-stabilized premixed plane flame," SIAM J. Appl. Math. 40(3) (1981), pp. 551-562 (with D. O. Olagunju).

"Secondary states of vibrating plates," J. Acoust. Soc. Amer. 69 (1981), pp. 1682-1687 (with L. J. Putnick and E. L. Reiss).

"Kramers' Diffusion Problem and Diffusion Across Characteristic Boundaries," in Theory and Applications of Singular Perturbations, Conf. Proceedings, Oberwolfach, 1981, pp. 318-345, Ed. W. Eckhaus and E. M. deJager, Springer Lecture Notes in Mathematics, No. 942 (with Z. Schuss).

"Spatial structures in plasmas with metastable states as bifurcation phenomena," Phys. Rev. 26A (1982), pp. 539-555 (M. F. Huerta and J. F. Magnan).

"Steady and pulsating modes of sequential flame propagation," Comb. Sci. and Tech. 27 (1982), pp. 192-213 (with S. B. Margolis).

"Traveling waves along the front of a pulsating flame," SIAM J. Appl. Math. 42(3) (1982), pp. 486-501 (with D. O. Olagunju).

"Diffusion across characteristic boundaries," SIAM J. Appl. Math. 42(4) (1982), pp. 822-834 (with Z. Schuss).

"A singular perturbation approach to Kramers' diffusion problem," SIAM J. Appl. Math. 42(4) (1982), pp. 835-849 (with Z. Schuss and E. Ben-Jacob).

"A new model in flame theory," SIAM J. Appl. Math. 42(4) (1982), pp. 850-867 (with A. van Harten).

"On the birth of isolas," SIAM J. Appl. Math. 42(5) (1982), pp. 956-963 (with D. Dellwo, H. B. Keller and E. L. Reiss).

"Flame propagation with multiple fuels," SIAM J. Appl. Math. 42(5) (1982), pp. 982-1003 (with S. B. Margolis).

"Spinning waves in gaseous combustion," SIAM J. Appl. Math. 42(5) (1982), pp. 1138-1156 (with D. O. Olagunju).

"Flame propagation with a sequential reaction mechanism," SIAM J. Appl. Math. 42(6) (1982), pp. 1175-1182 (with S. B. Margolis).

"Flames as gasdynamic discontinuities," J. Fluid Mech. 124 (1982), pp. 239-259 (with M. Matalon).

"The lifetime of oscillatory steady states," Phys. Rev. A26 (1982), pp. 2805-2816 (with E. Ben-Jacob, D. J. Bergman and Z. Schuss).

"The steady burning of a solid particle," SIAM J. Appl. Math. 42 (1982), p. 787 (with M. Matalon).

"Stability and bifurcation in a modulated Burgers system," Quart. Appl. Math. 39 (1982), pp. 467-477 (W. E. Olmstead and S. H. Davis).

"Radiation conditions for waveguide problems," SIAM J. on Sci. and Stat. Computing 3 (1982), pp. 318-326 (G. A. Kriegsmann).

"Exploiting the limiting amplitude principle to numerically solve scattering problems," Wave Motion 4 (1982), pp. 371-380 (G. A. Kriegsmann).

"A stochastic model of tumor growth," Math Biosciences 61 (1982), pp. 73-100 (C. Tier and F. B. Hanson).

"Fundamental Oseen solution for the two dimensional flow of a micropolar fluid," Int'l J. Engng. Sci. 21 (1983), pp. 423-430 (W. E. Olmstead and S. R. Majumdar).

"Harvesting under small demographic uncertainty," Springer Lectures Notes in Biomathematics 52 (1983) (C. Tier and F. B. Hanson).

"Dynamical Systems Driven by Small White Noise: Asymptotic Analysis and Applications," in Lecture Notes in Mathematics 1985 entitled Asymptotic Analysis II, edited by F. ver Hulst, Springer-Verlag (1983), pp. 2-34 (with Z. Schuss).

"Diffusion across characteristic boundaries with critical points," SIAM J. Appl. Math. 43(4) (1983), pp. 673-695 (with C. Tier and Z. Schuss).

"Ignition of a combustible half space," SIAM J. Appl. Math. 43 (1983), pp. 1-15 (W. E. Olmstead).

"End temperature control in a long rod," IFAC Control of Distributed Parameter Systems: Ed. J. P. Barbary and L. Le Letty, Pergamon Press (1983), pp. 577-581.

"Perturbed bifurcation of stationary striations in a contaminated non-uniform plasma," SIAM J. Apl. Math. 43 (1983), pp. 16-30 (J. F. Magnan and R. A. Goldstein).

"On flame stretch," Comb. Sci. and Tech. 31 (1983), pp. 169-181 (M. Matalon).

"Thermal and Shot Noise Effects on Nonlinear Oscillators," Fifth Int'l Conf. on Collective Phenomena, ed. J. Lebowitz, New York Academy of Sciences 410 (1983), pp. 323-338 (with E. Ben-Jacob, D. J. Bergman and Z. Schuss).

"Flames in fluids: Their interaction and stability," Comb. Sci. and Tech. 34 (1983), pp. 295-316 (with M. Matalon).

"Gas phase transient diffusion in droplet vaporization and combustion," Comb. and Flame 50 (1983), pp. 219-299 (M. Matalon and C. K. Law).

"Nonlinear stability and bifurcation in the transition from laminar to turbulent flame propagation," Comb. Sci. and Tech. 34 (1983), pp. 45-77 (with S. B. Margolis); also in Proceedings of Int'l Workshop on Gas Flame Structure, pp. 89-137, July 1983, ed. V. K. Baev, Novosibrisk, U.S.S.R.

"Singular Perturbations of Epidemic Models Involving a Threshold," in Lecture Notes in Mathematics #985 entitled Asymptotic Analysis, edited by F. ver Hulst, Sjpringer-Verlag (1983), pp. 400-412 (with J. Grasman).

"On the lifetime of a metastable state at low noise," Phys. Lett. 95A(5) (1983), pp. 673-695 (with Z. Schuss).

"Transitions from the equilibrium state of a hysteretic Josephson junction induced by self-generated shot noise," Phys. Lett. 99A(6,7) (1983), pp. 343-347 (with E. Ben-Jacob, D. J. Bergman and Z. Schuss).

"Shot noise effect on the nonzero voltage state of the hysteretic Josephson junction," Appl. Phys. Lett. 42(12) (1983), pp. 1045-1047 (with E. Ben-Jacob, D. J. Bergman, Y. Imry and Z. Schuss).

"Thermal activation from the fluxoid and the voltage states of DC-SQUIDS," J. Appl. Phys. 5(11) (1983), pp. 6533-6542 (with E. Ben-Jacob, D. J. Bergman, Y. Imry and Z. Schuss).

"An asymptotic theory of deflagrations and detonations. Part 1. The steady solutions," SIAM J. Appl. Math. 43(3) (1983), pp. 491-519 (with A. K. Kapila and A. van Harten).

"Propagating Flames and Their Stability," in Chemical Instabilities, edited by G. Nicolis and F. Baras, Proceedings of a NATO Advanced Research Workshop, U. Texas, March 1983, pp. 136-146 (with M. Matalon).

"Secondary Bifurcation in Flame Propagation," in Chemical Instabilities, edited by G. Nicolis and F. Baras, Proceedings of a NATO Advanced Research Workshop, U. Texas, March 1983, pp. 146-158 (with T. Èrneux).

"On the stability of plane and curved flames," SIAM J. Appl. Math. 44(2) (1984), pp. 327-343 (with M. Matalon).

"Quasi-periodic waves along a pulsating propagation front in a reaction-diffusion systems," SIAM J. on Appl. Math. 44(3) (1984), pp. 536-544 (with T. Erneux).

"Uniform expansion of the transition rate in Kramers' problem," J. Statis. Phys. 35(3,4) (1984), pp. 443-456 (with Z. Schuss and C. Tier).

"Asymptotic solution of the Kramers Moyal equation and first passage times for Markov jump processes," Phys. Rev. A29(6) (1984), pp. 3359-3369 (with Z. Schuss, C. Tier, M. Mangel and C. Knessl).

"Acoustic coupling of flames," SIAM J. Appl. Math. 44(5) (1984), pp. 982-995 (with A. K. Kapila and A. Van Harten).

"Solution of Kramers Moyal equations for problems in chemical physics," J. Chem. Phys. 81(3) (1984), pp. 1285-1293 (with Z. Schuss, C. Tier, M. Mangel and C. Knessl).

"Controlled transition to cellular flame fronts," Proceedings A.I.Ch.E. Annual Meeting 1984 (T. Erneux and M. Matalon).

"Stationary harmonic and pulsed operations of an optically bistable laser with saturable absorber I," Phys. Rev. 30A (1984), pp. 1393-1901 (P. Mandel and T. Erneux).

"Stationary harmonic and pulsed operations of an optically bistable laser with saturable absorber II," Phys. Rev. 30A (1984), pp. 1902-1909 (T. Erneux and P. Mandel).

"Bifurcation of pulsating and spinning reaction fronts on condensed two-phase combustion," Comb. Sci. and Tech. 43 (1985), pp. 127-165 (with S. B. Margolis, H. G. Kaper and G. K. Leaf).

"Singular Bifurcation in Reaction Diffusion Systems," in Modelling of Patterns in Space and Time, Springer Lecture Notes in Biomathematics #55 edited by J. Murray and W. Jager. Proc. of a SonderForschungsbereich Workshop, Heidelberg, July 1983, pp. 67-72 (with T. Erneux and E. L. Reiss).

"Flame propagation in channels: Secondary bifurcation to quasi-periodic pulsations," SIAM J. Appl. Math. 45 (1985), pp. 93-129 (with S. B. Margolis).

"Frequency fluctuations in noisy oscillators," SIAM J. Appl. Math. 45(5) (1985), pp. 843-854 (with Z. Schuss and C. Tier).

"Nonlinear control in optical bistability," IEEE J. of Quantum Electr. QE-21 (1985), pp. 1352-1355 (P. Mandel and T. Erneux).

"Analytic Studies of a Laser with a Saturable Absorber," in Optical Instabilities, Cambridge Studies in Mod. Optics 4 (1986), pp. 99-110 (T. Erneux).

"Imperfect bifurcation with a slowly varying control parameter," SIAM J. Appl. Math. 46 (1986), pp. 1-16 (T. Erneux and P. Mandel).

"First Passage Times for Processes Governed by Master Equations," in Fluctuations and Sensitivity in Nonequilibrium Systems," Springer Proc. in Physics #1, edited by W. Horsthemke and D. K. Kondepudi, Proc. of an Int'l Conf., U. of Texas, March 1984, pp. 19-36 (with Z. Schuss, C. Knessl, C. Tier and M. Mangel).

"Noise Induced Transitions in Nulti-Stable Systems," in Fluctuations and Sensitivity in Nonequilibrium Systems," Springer Proc. in Physics #1, edited by W. Horsthemke and D. K. Kondepudi, Proc. of an Int'l Conf., U. of Texas, March 1984, pp. 79-94 (with E. Ben-Jacob, D. J. Bergman and Z. Schuss).

"The stability of plane flames attached to a flameholder," Comb. Sci. and Tech. 43 (1985), pp. 271-286 (with H. G. Kaper, G. K. Leaf and M. Matalon).

"An asymptotic theory of large deviations for Markov jump processes," SIAM J. Appl. Math. 45(4) (1985), pp. 1006-1028 (with C. Knessl, Z. Schuss and C. Tier).

"On the non-Markovian theory of activated rate processes in the small friction limit," J. Chem. Phys. 83(2) (1985), pp. 597-600 (with M. Dygas and Z. Schuss).

"A singular perturbation approach to non-Markovian escape rate problems," SIAM J. Appl. Math. 46(2) (1986), pp. 265-298.
"A singular perturbation approach to first passage time for Markov jump processes," J. Statis. Phys. 42(1/2) (1986), pp. 169-184 (with C. Knessl, Z. Schuss and C. Tier).

"interaction of pulsating and spinning waves in condensed phase combustion," SIAM J. Appl. Math. 46(5) (1986), pp. 801-843 (with M. Booty and S. B. Margolis).

"On the stability of the porous plug burner flame," Comb. Sci. and Tech. 47 (1986), pp. 93-101 (with H. Kaper and G. Leaf).

"A singular perturbation approach to non-Markovian escape rate problems with state dependent friction," J. Chem. Phys. 84(7) (1986), pp. 3731-3738 (with M. Dygas and Z. Schuss).

"Lifetimes of nonequilibrium dissipative steady states in SQUID 85 - Superconducting Quantum Interference Devices and Their Applications," edited by H. D. Hahbohm and H. Lubbig, Proceedings of the Third International Conference on Superconducting Quantum Devices, W. Berlin, June 1985, pp. 219-242, W. de Gruyter, Berling 1985 (with E. Ben-Jacob, D. Bergman and Z. Schuss).

"Master equation approach to shot noise in Josephson junctions," Phys. Rev. B34(3) (1986), pp 1572-1581 (with E. Ben-Jacob, D. Bergman and Z. Schuss).

"Displacement effect of a flame in a stagnation point flow," Phys. Fluids 29(7) (1986), pp. 2172-2180 (E. Eteng, G. S. S> Ludford and M. Matalon).

bifurcation with memory," SIAM J. Appl. Math. 46 (1986), pp. 171-188.

"Boundary behavior of diffusion approximations to Markov jump processes," J. Stat. Phys. 45 No. 1/2 (1986), pp. 245-266 (with C. Knessl, Z. Schuss and C. Tier).

"Fronts, relaxation oscillations and period doubling in solid fuel combustion," J. Comp. Phys. 71 (1987), pp. 147-168 (with A. Bayliss).

"Dynamics of nonadiabatic premixed flames in a gravitational field," SIAM J. Appl. Math. 47(1) (1987), pp. 544-555 (with W. E. Olmstead, H. G. Kaper and G. K. Leaf).

"Ignition of a combustible solid with reactant consumption," SIAM J. Appl. Math. 47 (1987), pp. 332-342 (D. G. Lasseigne and W. E. Olmstead).

"Hydrodynamic and diffusion effects on the stability of spherically expanding flames," Comb. and Flame 67 (1987), pp. 77-90 (J. Bechtold and M. Matalon).

"Interaction of pulsating and spinning waves in non-adiabatic flame propagation," SIAM J. Appl. Math. 47(6) (1987), pp. 1241-1286 (with M. Booty and S. B. Margolis).

"A variational approach to nonlinear singularly perturbed boundary value problems," Studies in Applied Mathematics 77 (1987), pp. 61-88 (with W. L. Kath and C. Knessl).

"Spherically expanding flames," ASME/JSME 1987 Conf. Proceedings, pp. 95-101 (with M. Matalon and J. Bechtold).

"Flame propagation in a rotating gas," Comb. Sci. and Tech. 57 (1-3) (1988), pp. 37-53 (G. I. Sivashinsky, Z. Rakib, M. Matalon and S. Sohrab).

"On nonadiabatic condensed phase combustion," Comb. Sci. and Tech. 53(4-6) (1987), pp. 289-314 (with W. E. Olmstead, H. G. Kaper and G. K. Leaf).

"Adaptive pseudo-spectral computation of cellular flames stabilized by a point source," Appl. Math. Lett. 1 (1988), pp. 19-24 (with A. Bayliss and M. Minkoff).

"Colored noise in dynamical systems," SIAM J. Appl. Math. 48(2) (1988), pp. 425-441 (with M. M. Kłosek-Dygas and Z. Schuss).

"New modes of quasi-periodic combustion near a degenerate Hopf bifurcation point," SIAM J. Appl. Math. 48(4) (1988), pp. 828-853 (with S. B. Margolis).

"Stochastic stability of nonlinear oscillators," SIAM J. Appl. Math. 48(6) (1988), pp. 1115-1127 (with M. M. Kłosek-Dygas and Z. Schuss).

"Bistable Cellular Flames," pp. 108-115 in Proceedings of a Symposium to Honor C. C. Lin, MIT, Cambridge, MA, June 22-24, 1987, World Scientific Publishing, ed. D. J. Benney, S. H. Shu and C. Yuan, (1988) (with A. Bayliss and M. Minkoff).

"A nonlinear wave equation in nonadiabatic flame propagation," SIAM J. Appl. Math. 48(3) (1988), pp. 519-535 (with M. R. Booty and M. Matalon).

"Dynamics of nearly extinguished nonadiabatic premixed flames in a gravitational field," SIAM J. Appl. Math. 48(6) (1988), pp. 1054-1063 (with H. G. Kaper, G. K. Leaf and W. E. Olmstead).

"Does reaction rate curvature play a role in the diffusion theory of multidimensional activated rate processes," J. Chem. Phys. 88(8) (1988), pp. 4765-4771 (with A. Nitzan and Z. Schuss).

"First passage time approach to stochastic stability of nonlinear oscillators," Physics Letters A130 (1988), pp. 11-18. (with M. Kłosek-Dygas and Z. Schuss).

"Uniform asymptotic expansions in dynamical systems driven by colored noise," Phys. Rev. A38(5) (1988), pp. 2605-2613 (with M. Kłosek-Dygas and Z. Schuss).

"The stability of weakly stretched flames," in Mathematical Modeling in Combustion Science edited by J. D. Buckmaster and T. Takeno, Lecture Notes in Physics, Vol. 299, Springer-Verlag (1988), pp. 131-143. (M. Matalon).

"Propagation and extinction of a flame in a stagnation point flow," Combustion and Flame 73 (1988), pp. 303-313 (M. Matalon and Y. D. Kim).

"A regularized K-S equation describing the formation of cellular flames," Physics Letters A133(1,2) (1988) (M. Matalon and P. Rosenau).

"An adaptive-pseudo-spectral method for reaction diffusion problems," J. Comp. Phys. 81 (1989), pp. 421-443 (with A. Bayliss, D. Gottlieb and M. Minkoff).

"Diffusion theory for multi-dimensional activated rate processes: The role of anisotropy," J. Chem. Phys. 90(2) (1989), pp. 1141-1148 (with B. M. Hoffman, M. Kłosek-Dygas, A. Nitzan, M. Ratner and Z. Schuss).

"Colored noise in activated rate processes," J. Stat. Phys. 54(5/6) (1989), pp. 1309-1320 (with M. Kłosek-Dygas and Z. Schuss).

"Period doubling gained, period doubling lost," SIAM J. Appl. Math. 49(4) (1989), pp. 1047-1063 (with A. Bayliss and M Minkoff).

"Cascading cellular flames," SIAM J. Appl. Math. 49(5) (1989), pp. 1421-1432 (with A. Bayliss and M. Minkoff).

"Bifurcation and pattern formation in combustion," in Ordinary and Partial Differential Equations Vol. II, pp. 1-51, Proceedings Tenth Dundee Conference on Differential Equations, 1988, ed. B. D. Sleeman and R. J. Jarvis, Pitman Research Notes in Mathematics #216 (with A. Bayliss and M. Minkoff).

"First order dynamics driven by rapid Markovian jumps," SIAM J. Appl. Math. 49(6) (1989), pp. 1811-1833 (with M. Kłosek-Dygas and Z. Schuss).

"Linear stability analysis of cylindrical flames," Quarterly Appl. Math. 47(4) (1989), pp. 691-704 (with M. Garbey, H. G. Kaper and G. K. Leaf).

"Numerical computation of bifurcation phenomena and pattern formation in combustion," pp. 187-198 in Numerical Combustion, Eds. A. Dervieux and B. Larrouturou, Proc. Third Int'l Conference on Numerical Combustion, Antibes-Juan les Pins, May 1989, Springer-Verlag Lectures Notes in Physics 351 (with A. Bayliss and M. Minkoff).

"Two routes to chaos in condensed phase combustion," SIAM J. Appl. Math. 50(2) (1990), pp. 437-459 (with A. Bayliss).

"A direct approach to the exit problem," SIAM J. Appl. Math. 50(2) (1990), pp. 595-627 (with T. Naeh, M. M. Klosek and Z. Schuss).

"Nonlinear analysis of condensed phase surface combustion," European J. Appl. Math. 1(1) (1990), pp. 73-89 (with M. Garbey, G. K. Leaf and H. G. Kaper).

"Condensed phase combustion with a merged sequential reaction mechanism," Quarterly J. Mech. and Appl. Math. 43(2) (1990), pp. 223-249 (with M. R. Booty and J. B. Holt).

"Spinning cellular flames," Appl. Math. Lett. 3 (1990), pp. 75-79 (with A. Bayliss).

"Uniform solution of Kramers' problem by a direct approach," in Lectures in Applied Mathematics 27, Mathematics of Random Media, Ed. W. E. Kohler and B. S. White, (1991), pp. 241-267 (with T. Naeh, M. M. Klosek and Z. Schuss).

"Asymptotic methods for Markov jump processes," in Lectures in Applied Mathematics 27, Mathematics of Random Media, Ed. W. E. Kohler and B. S. White, (1991), pp. 215-240 (with Z. Schuss and C. Tier).

"New modes of quasi-periodic burning in combustion synthesis," pp. 73-82 in Combustion and Plasma Synthesis of High Temperature Materials, Ed. Z. A. Munir and J. B. Holt, VCH Publ. (1990) (with S. B. Margolis and M. R. Booty).

"Modeling and numerical computation of a nonsteady SHS process," pp. 61-72 in Combustion and Plasma Synthesis of High Temperature Materials, Ed. Z. A. Munir and J. B. Holt, VCH Publ. (1990) (with A. Bayliss).

"A two dimensional adaptive pseudo spectral method," J. Comp. Phys. 91(1) (1990), pp. 174-196 (with A. Bayliss and R. Kuske).

"Unsteady solid flames," in Flame Structure, Ed. O. P. Korobeinichev, (Proc. Third Int'l. Seminar on Flame Structure, Alma-ata, U.S.S.R. - 1990) Nauka (1991) (with A. Bayliss).

"Bifurcation, pattern formation and chaos in combustion," in Dynamical Issues in Combustion, Ed. P. Fife, A. Linan and F. A. Williams, Springer-Verlag (1991) (with A. Bayliss).

"Polyhedral Flames," SIAM J. Appl. Math. 51(1) (1991), pp. 73-89 (with D. O. Olagunju).

"Burner-stabilized cellular flames," Quarterly Appl. Math. 48 (4) (1990), pp. 645-664 (with D. O. Olagunju).

"Using MAPLE for the analysis of bifurcation phenomena in condensed phase surface combustion," J. Symbolic Computation 12(1) (1991), pp. 89-113, with M. Garbey, G. K. Leaf and H. G. Kaper).

"Modes of Burning in Filtration Combustion," European J. Appl. Math. 2(1) (1991), pp. 17-41 (with M. R. Booty).

"Quasi-periodic waves and the transfer of stability in condensed phase surface combustion," SIAM J. Appl. Math. 52(2) (1992), pp. 384-395 (with M. Garbey, G. K. Leaf and H. G. Kaper).

"The Kramers problem in the turnover regime: The role of the stochastic separatrix," Berichte der Bunsen-Gesellschaft for Physikalische Chemie 95(3) (1991), pp. 31-37 (with M. M. Klosek and Z. Schuss).

"Bifurcation, pattern formation and transition to chaos in combustion", in Bifurcation and Chaos: Analysis, Algorithms, Applications, pp. 37-52, Ed. R. Seydel, F.W. Schneider, T. Kupper and H. Troger, Birkhauser Verlag, Basel, 1991 (with A. Bayliss).

"Reply to comment on diffusion theory of multidimensional activated rate processes: the role of anisotropy", J. Chem. Phys. 95(2), (1991) pp. 1425-1426 (with M.M. Klosek, B.M. Hoffman, A. Nitzan, M. Ratner and Z. Schuss)

"On the stability of counter flow filtration combustion", Comb. Sci. and Tech. 80 (1991), pp. 231-264 (with M.R. Booty)

"Pulsating and chaotic dynamics near the extinction limit", accepted for publication, Comb. Sci. Tech. (with A. Bayliss and G. K. Leaf).

"Coupled nonlocal complex Ginzburg-Landau equations in gasless combustion", Physica D54 (1992), pp. 203-219 (with V. Volpert)

"Coupled complex Ginzburg-Landau type equations in gaseous combustion", accepted for publication, Stability and Applied Analysis of Continuous Media (with D.O. Olagunju)

"Mathematical modeling of SHS processes", accepted for publication, Proc. Int'l Conf. on SHS, Alma-Ata, USSR (with K. G. Shkadinsky and V. A. Volpert).

"Stability of plane wave solutions of complex Ginzburg-Landau equations", accepted for publication Quart. Appl. Math. (with V. A. Volpert).

"Nonlocal amplitude equations in reaction diffusion systems", accepted for publication, Random and Computational Dynamics 1, 1992 (with V. A. Volpert).

"Nonlinear dynamics of cellular flames", SIAM J. Appl. Math. 52(2) (1992), pp. 390-415 (with A. Bayliss).

"Combustion synthesis of a porous layer", accepted for publication Comb. Sci. and Tech. (with K. Shkadinsky, G. Shkadinskaya and V. Volpert).

"Self-compaction or expansion in combustion synthesis of porous layers", accepted for publication Comb. Sci. and Tech. (with K. Shkadinsky, G. Shkadinskaya and V. Volpert).

"Two-front traveling waves in filtration combustion", accepted for publication Comb. Sci. and Tech. (with K. Shkadinsky, G. Shkadinskaya and V. Volpert).

"Slowly varying filtration combustion waves", accepted for publication, Euro. J. Appl. Math. (with M. Booty).

CUMULATIVE SUMMARY OF PUBLICATIONS ON D.O.E. Grant

On inhibiting runaway in catalytic reactors - A nonlinear stability analysis is employed to account for the observation that under certain conditions reactions on catalyst pellets can pass transiently stably into a region which would correspond to instability in the steady state. One consequence of our analysis is a possible control mechanics which inhibits temperature runaway by extending the stable operating characteristics desirable in modern reactors.

Propagation of a pulsating reaction front in solid fuel combustion - We consider a system of reaction diffusion equations which describes gasless combustion of condensed systems. To analytically describe recent experimental results, we show that a solution exhibiting a periodically pulsating, propagating reaction front arises as a Hopf bifurcation from a solution describing a uniformly propagating front. The bifurcation parameter is the product of a nondimensional activation energy and a factor which is a measure of the difference between the nondimensionalized temperatures of unburned propellant and the combustion products. We show that the uniformly propagating plane front is stable for parameter values below the critical value. Above the critical value the plane front becomes unstable and perturbations of the system evolve to the bifurcated state, i.e. to the pulsating propagating state. In our nonlinear analysis we calculate the amplitude, frequency and velocity of the propagating pulsating front. In addition we demonstrate analytically that the mean velocity of the oscillatory front is less than the velocity of the uniformly propagating plane front.

Reactive-diffusive systems with Arrhenius kinetics: multiple solutions, ignition and extinction - The steady reactive-diffusive problem for a non-isothermal porous pellet with first-order exothermic Arrhenius kinetics is studied. In the large activation energy limit, asymptotic solutions are derived for the cylindrical and slab geometries.

The exit problem: A new approach to diffusion across potential barriers - We consider the problem of a Brownian particle confined in a potential well of forces, which escapes the potential barrier as the result of white noise forces acting on it. We compute the expected exit time of the particle from the well as well as the probability distribution of the exit points.

On oscillatory necking in polymers - The phenomenon of oscillatory necking in the stretching of polyethylene films is described. We propose an extension of a model of Barenblatt for isothermic necking in polymers, and we show that oscillatory necking may rise, for example, in polymers for which the effect of diffusion of stresses exceeds that of diffusion of oriented material.

Acceleration effects on the stability of flame propagation - We consider the effect of acceleration on the diffusional thermal instability of a plane flame front in a premixed gaseous mixture. We show that the diffusional thermal instability is inhibited (enhanced) if the acceleration in the direction of the cold unburned gas is sufficiently high (low). Thus we are able to explain the phenomenon that for some mixtures, cellular fronts are observed only for upward propagating flames, while downward propagating flames remain stable.

An asymptotic derivation of two models in flame theory associated with the constant density approximation - We derive two simplified models in flame theory from the general equations of combustion theory.

Secondary states of rectangular plates - The secondary buckling of rectangular elastic plates is studied as a problem of secondary bifurcation.

Singular perturbations, stochastic differential equations and applications - This paper contains the text of an invited address at the Advanced Seminar on Singular Perturbations and Asymptotics held at the Mathematics Research Center of the University of Wisconsin in May 1980. It appears as a chapter in the book entitled Singular Perturbations and Asymptotics, eds. R. E. Meyer and S. V. Parter, Academic press, New York, 1980.

Reactive-diffusive systems with Arrhenius kinetics: - Peculiarities of the spherical geometry - The steady reactive-diffusive problem for a nonisothermal permeable pellet with first-order Arrhenius kinetics is studied. The solutions exhibit multiplicity, and it is shown that a suitable choice of parameters can lead to an arbitrarily large number of solutions.

A nonlinear theory of cellular flames - We employ a model derived by Matkowsky and Sivashinsky to consider the stability of a uniformly curved propagating flame front. We show that the uniform front becomes unstable if the Lewis number L of the limiting reaction component is less than some critical number L_c, and if the radius of the front is greater than a critical value R_c. When the critical values are exceeded, perturbations of the uniform front evolve to a cellular front which bifurcates supercritically from the uniform front.

Reactive-diffusive systems with Arrhenius kinetics: The Robin problem - For a permeable catalyst pellet, the reactive-diffusive problem with first-order Arrhenius kinetics is studied. Steady solutions under Robin boundary conditions are derived for the cylindrical geometry in the limit of large activation energy. The response of the system exhibits three-fold and five-fold multiplicities as well as closed loops.

Propagation of a pulsating flame front in a gaseous combustible mixture - We employ a diffusional-thermal model derived by Matkowsky and Sivashinsky for chemically reacting gases, to show the existence of a pulsating flame front in a premixed combustible gas governed by a one-step Arrhenius reaction. We show that the pulsating front arises as a time-periodic bifurcation from a uniformly propagating plane flame front when the Lewis number L exceeds a critical value L_c. For $L > L_c$, the plane front becomes unstable and perturbations of the system evolve to the pulsating state.

On the stability of non-adiabatic flames - A linear stability analysis is carried out for the cellular instability of a nonadiabatic downward-propagating premixed flame. It is shown that if the molecular weight of the deficient reactant is sufficiently small, then an increase in heat loss may leads to destabilization of an adiabatically stable flame (i.e. a flame which is stable in the absence of heat losses).

Eigenvalues of the Fokker-Planck operator and the approach to equilibrium for diffusions in potential fields - We consider the motion of a Brownian particle in an infinite potential field. The rate of approach to equilibrium is determined by the second eigenvalue of the stationary Fokker-Planck operator. The inverse of this eigenvalue is the expected time for the potential well. The height of the largest potential barrier is termed the activation energy, and the eigenvalue is computed asymptotically for large activation energies. Applications to the calculation of chemical reaction rates and ionic conductance in crystals are given.

Pulsations in a burner-stabilized premixed plane flame - We employ a model for an edge-cooled flat flame burner to obtain expressions for the flame speed, flame temperature, standoff distance as well as the quenching distance for a plane flame front. For a given standoff distance there is a low-temperature as well as a high-temperature solution. We show by a linear stability analysis of the plane front that the high temperature solution is unstable when the Lewis number if sufficiently large and the inflow velocity sufficiently less than the adiabatic flame speed. We also show that this instability is the type that will lead to a bifurcating time-periodic solution describing a pulsating flame.

Secondary states of vibrating plates - A perturbation method is used to obtain a new class of periodic motions for the nonlinear vibrations of rectangular elastic plates. The dynamic von Karman plate theory is used in the analysis. Periodic solutions bifurcate at the natural frequencies of free vibration of the linearized plate theory. The new solutions bifurcate from these periodic solutions. Thus they are states of secondary bifurcation.

Kramers' diffusion problem and diffusion across characteristic boundaries - in "Theory and Applications of Singular Perturbations" Conf. Proceedings, Oberwolfach, 1981, pp. 318-345, ed. W. Eckhaus and E. M. DeJager, Springer Lecture Notes in Mathematics #942, Springer Verlag.

Spatial structures in plasmas with metastable states as bifurcation phenomena - The formation of orderly, steady, spatial structures in rf-heated plasmas that have metastable states is studied as the development of an instability of the solution of the reaction-diffusion equations for the plasma species. Plasma-species (electrons, ions, and metastable atoms) source functions are given in terms of the electron energy distribution in strong electric fields and of the various collision cross sections. Methods of two-time expansions and bifurcation theory are used to calculate the nonuniform states that bifurcate from the unstable uniform steady state for straight and toroidal tubes. The sinusoidal density variations give the discharge the appearance of glowing balls of gas. This pattern formation is analogous to the morphogenesis found in certain chemical reactions and biological processes.

Steady and pulsating modes of sequential flame propagation - Steady and pulsating modes of flame propagation through a premixed combustible mixture are studied for the case in which the flame is characterized by the sequential production and depletion of a significant intermediate species. We employ the method of matched asymptotic expansions to derive a model valid for large activation energies, and show that the pulsating solution is the result of a supercritical Hopf bifurcation from the steadily propagating solution (which becomes unstable). Through a nonlinear bifurcation analysis, we calculate the pulsation amplitude and other characteristics of the flame along the bifurcated branch. It is shown that the average thickness of the pulsating flame, by which we mean the average effective separation distance between production and depletion of the intermediate species, is greater than that predicted by a steady-state theory. In addition, we find that the mean propagation speed is less than that of the steadily propagating solution, but that the instantaneous peak concentration of the intermediate species is a constant equal to its steady-state value.

Travelling waves along the front of a pulsating flame - A diffusional-thermal model describing the combustion of a premixed gas is considered. We show that a uniformly propagating plane flame is unstable to two-dimensional disturbances when the Lewis number L exceed a critical value L_c. We employ a nonlinear analysis to show that for $L > L_c$ two types of solutions bifurcate from the uniformly propagating plane flame. A linear stability analysis of the bifurcated states shows that the travelling wave solutions are stable and the pulsating cellular solutions are unstable. Our analysis also shows that the average speeds of the pulsating solutions are less than that of the uniformly propagating plane flame.

Diffusion across characteristic boundaries - We consider the motion of a particle acted on by the deterministic force vector b(x(t)) and perturbed by random forces of white noise type. Such a particle will leave any bounded domain Ω in finite time. We consider the case where b is such that $\partial\Omega$ consists of a trajectory or trajectories of the system $\dot{\underline{x}} = b(x(t))$. Thus we consider the cases of an unstable limit cycle, a closed characteristic boundary with critical points, and a center. We observe that these problems are such that b is not derivable from a potential. For each problem we derive expressions for (i) the mean first passage time to $\partial\Omega$, and (ii) the probability distribution of exit points on $\partial\Omega$. Our method is to employ the Ito calculus to characterize the quantities (i) and (ii) as solutions of singularly perturbed elliptic boundary value problems, and then to derive asymptotic representations of the solutions of those problems. The results obtained are new and are of importance in a variety of applications including the estimation of jump times (due to noise) from stable periodic solutions to other stable solutions of the deterministic dynamical system.

A singular perturbation approach to Kramers' diffusion problem - We consider Kramers' diffusion problem, which seeks to calculate the rate of escape of a particle from one potential well over a barrier, to another presumably deeper, and therefore more stable well. Though Kramers introduced the problem as a model for chemical reactions, it applies to numerous rate processes including atomic migration and ionic conductivity in crystals, and transitions due to noise between stable states of dynamical systems with multi-stable states, to name but a few. We relate the rate of escape to the first passage time from the domain of attraction of the stable point corresponding to the first well. The first passage time is then characterized via the Ito calculus, as a solution of an elliptic partial differential equation of singular perturbation type. Finally this equation is solved asymptotically by methods previously developed by the authors. We obtain some new results on the rate of escape, and in addition discuss the validity of the various results derived by Kramers.

A new model in flame theory - In a recent paper Matkowsky and Sivashinsky employed asympototic methods to derive a simplified model in flame theory, from the general equations of combustion. The model was derived under the assumptions of (i) large activation energy, (ii) closeness to similarity, and (iii) weak thermal expansion, and the resulting model was associated with the constant density approximation. In this paper, assumption (iii) is relaxed somewhat and a more general temperature dependence of the thermal conductivity and diffusivity is assumed. The new model now exhibits non-constant density effects. A number of solutions of the model, representing various types of flames are presented, and their stability analyzed.

On the birth of isolas - Isolas are isolated, closed curves of solution branches of nonlinear problems. They have been observed to occur in the buckling of elastic shells, the equilibrium states of chemical reactors, and other problems. In this paper, we present a theory to analytically describe the structure of a class of isolas. Specifically, we consider isolas that shrink to a point as a parameter k of the problem approaches a critical value k_0. The point is referred to as an isola center. Equations that characterize the isola center are given. Then solutions are constructed in a neighborhood of the isola center, by a perturbation expansion in a small parameter, that is proportional to $(k-k_0)^{\alpha}$, with α appropriately chosen. The theory is applied to a problem in chemical reactor theory.

Flame propagation with multiple fuels - The steady propagation of a flame through a premixed combustible mixture is studied for the case in which the mixture consists of two distinct fuels. The overall chemical reaction mechanism is represented by $A + \nu_A O \rightarrow \mu_A P$, $B + \nu_B \rightarrow \mu_B Q$, where A and B denote the fuels, O is the oxidizer, P and Q are the corresponding products, and ν_A, μ_A and ν_B, μ_B are stoichiometric coefficients. We employ the method of matched asymptotic expansions to derive a solution for large activation energies, in which case both chemical reactions are confined to a thin layer. The (small) separation distance H between the points where the two reactions go to completion and the propagation velocity U are determined as functions of such quantities as Lewis numbers, activation energies, Damkohler numbers, and heat release fractions. The formula for H represents a relative measure of the flame thickness in terms of standard parameters, while that for U determines the role of each reaction in the overall flame speed.

Spinning waves in gaseous combustion - We consider a diffusional-thermal model describing the combustion of a premixed gas in a cylindrical domain. We show that a uniformly propagating plane flame is unstable when the Lewis number L exceeds a critical value L_c. We employ a nonlinear analysis to show that for $L > L_c$ various solutions bifurcate from the uniformly propagating plane flame. The bifurcated solutions describe pulsating flames whose spatial distribution corresponds to combinations of axial, radial and tangential modes. The tangential modes are of two types, corresponding to standing waves on, and to travelling waves, propagating in both clockwise and counter-clockwise directions around, the cylinder. The latter describes spinning waves in which a hot spot(s) (local temperature maximum) propagates in a helical motion around the axis of the cylinder. Both single and multi-headed spins are obtained.

Flame propagation with a sequential reaction mechanism - The steady propagation of a flame through a premixed combustible mixture is studied for the case in which the flame is characterized by the production and depletion of a significant intermediate species. The overall chemical reaction mechanism is of the sequential type and is represented by $A + \nu_A O \rightarrow \mu_I I$, $I + \nu_I O \rightarrow \mu_p P$, where A is the fuel, O is the oxidizer, I is the intermediate species, P is the product, and ν_A, μ_I, μ_p are stoichiometric coefficients. We employ the method of matched asymptotic expansions to derive a solution for large activation energies, in which case both chemical reactions are confined to a thin layer. The (small) separation distance H between the points where the two reactions go to completion and the propagation velocity U play the role of eigenvalues and are determined as functions of standard flame parameters. The formula for H also represents the effective spatial distance measure of the flame thickness. It may also be interpreted as a measure of the maximum concentration of the intermediate species.

Flames as gasdynamic discontinuities - Early treatments of flames as gasdynamic discontinuities in a fluid flow, are based on several hypotheses and/or phenomenological assumptions. The simplest and earliest of such analyses, by Landau and by Darrieus prescribed the flame speed to be constant. Thus, in their analysis, they ignored the structure of the flame, i.e., the details of chemical reactions, and transport processes. Employing this model to study the stability of a plane flame, they concluded that plane flames are unconditionally unstable. Yet plane flames are observed in the laboratory. To overcome this difficulty, others have attempted to improve on this model, generally through phenomenological assumptions to replace the assumption of constant velocity. In the present work we take flame structure into account and derive an equation for the propagation of the discontinuity surface for arbitrary flame shapes in general fluid flows. The structure of the flame is considered to consist of a boundary layer in which the chemical reactions occur, located inside another boudnary layer in which transport processes dominate. We employ the method of matched asymptotic expansions to obtain an equation for the evolution of the shape and location of the flame front. Matching the boundary layer solutions to the outer gasdynamic flow, we derive the appropriate jump conditions across the front. We also derive an equation for the vorticity produced in the flame, and briefly discuss the stability of a plane flame, obtaining corrections to the formula of Landau and Darrieus.

The lifetime of oscillatory steady states - We introduce a method to derive expressions for the disbribution p of large fluctuations about a stable non-equilibrium steady state, and for the transition rate from that state into another stable state. Our method is based on a WKB type expansion. The expression for p has a similar form to the Boltzmann distribution, with the energy replaced by a function W, which is the solution of a Hamilton-Jacobi type equation. For the case of small dissipation, a simple analytical approximation to W, in terms of an action increment is derived. Our results are employed to predict various measurable quantities in physical systems. Specifically we consider the problems of the physical pendulum, the shunted Josephson junction, and the transport of charge density wave excitations.

The steady burning of a solid particle - We consider the quasi-steady burning of a carbon particle which undergoes gasification at its surface by chemical reactions, followed by a homogeneous reaction in the gas phase. the burning rate M is found as a function of the gas phase Damkohler number D_g for the whole range $0 < d_g < \infty$. The monotonic $M(D_g)$ curve, obtained for relatively very hot or very cold particles, describes the gradual transition from frozen abrupt and the $M(D_g)$ curve is either S-shaped or Z-shaped. In the former the burning is enhanced at ignition while in the latter it is slowed down; this depends on the relative importance of the two competitive surface reactions. At extinction, the reverse is true: burning is slowed down in the case of an S curve and is enhanced in the case of a Z curve.

Stability and bifurcation in a modulated Burgers system - The stability of the null state for a nonlinear Burgers system is examined. The results include (i) an energy estimate for global stability for states involving arbitrary modulation in time, and (ii) analysis of the bifurcation from the null state for slow modulations. For the slow modulations it is determined that the amplitude $A(\tau)$ of the bifurcated disturbance velocity satisfies a Landau-type equation with time-dependent growth rate $\Theta(\tau)$. Particular attention is given to periodic and quasiperiodic modulations of the system, which lead to analogous behavior in $\Theta(\tau)$. For each of these oscillatory-type modulations, it is found that $A^2(\tau)$ has the same long-time mean value as the unmodulated case, implying no alteration of the final mean kinetic energy. Applications to various fluid-dynamical phenomena are discussed.

Radiation conditions for wave guide problems - An incident mode propagates down a two-dimensional wave guide until it strikes a localized obstruction which creates reflected and transmitted waves. The numerical determination of these waves is difficult because the classical radiation condition does not apply for an infinite wave guide. In this note we derive a sequence of "localized radiation conditions" which can be applied a few wavelengths away from the scattering object. These conditions allow us to numerically solve the Helmholtz equation on a finite domain.

Exploiting the limiting amplitude principle to numerically solve scattering problems - A numerical method for solving reduced wave equations is presented. The technique is basically a relaxation scheme which exploits the limiting amplitude principle. A modified radiation condition at 'infinity' is also given. The method is tested on two model problems: the scattering of plane shallow water waves off shoals and the scattering of plane acoustic waves off a sound-soft cylinder imbedded between two homogeneous but different half spaces. The numerical solutions exhibit correct refractive and diffractive effects at moderate frequencies.

A stochastic model of tumor growth - A stochastic model for tumor growth is derived as a diffusion approximation of a continuous time, density dependent branching process, with a Gomperts growth law as a deterministic part. For the diffusion process, the conditional probabilities of extinction, reaching a size C, and doubling, are computed along with the expected time of these events. The results are given in terms of integrals which are evaluated by numerical methods that account for the logarithimic singularity introduced by the Gomperts growth law. When the variance of the branching process is small compared to the deterministic term simplified asymptotic expressions are giver using methods that are modified for the Gompertsian logarithm. The results are used to find the probability of implant take in limiting - dilution assay experiments and the probability of a tumor becoming detectable in carcinogenesis.

Fundamental oseen solutions for the 2-dimensional flow of a micropolar fluid - The steady, incompressible flow of a micropolar fluid in 2 dimensions is considered. The Oseen linearization of the convective operator is introduced, and the associated problem for the fundamental solution is formulated. Solution of the fundamental problem is obtained in explicit form under a certain restriction on the physical parameters of the problem. Utilization of the fundamental solution in the investigation of general flow problems is discussed.

Harvesting under small demographic uncertainty - The constant effort harvesting of a renewable biological resource is considered. The single year class population grows logistically in the absence of harvesting and is perturbed by small demographic stochasticity. The asymptotic behavior of the expected extinction times and expected harvest yields as well as its coefficient of variation, are computed as functions of harvesting effort.

Dynamical systems driven by small white noise: Asymptotic analysis and applications - in Lecture Notes in Mathematics #985 entitled Asymptotic Analysis II, edited by F. ver Hulst, Springer Verlag 1983, pp. 2-23.

Diffusion across characteristic boundaries with critical points - We consider the problems of the effect of small white noise perturbations on a deterministic dynamical system in the plane with (i) an asymptotically stable equilibrium point or limit cycle and (ii) an equilibrium point surrounded by closed trajectories. The mean exit time and the distribution of exit points for each problem is determined by solving singularly perturbed elliptic boundary value problems in domains with closed characteristic boundaries with critical points. Uniformly valid asymptotic solutions are constructed for each of the problems. For the asymptotically stable equilibrium point, the method of matched asymptotic expansions with the integral condition of Matkowsky and Schuss is employed. A method of averaging combined with boundary layer analysis is used for the problem of an equilibrium point surrounded by closed trajectories. The influence on the solutions, of the critical points on the boundary, is exhibited and explained. An application to the physical pendulum is given. Finally, our results are shown to be in close agreement with simulations.

Ignition of a combustible half-space - A half-space of combustible material is subjected to an arbitrary energy flux at the boundary where convection heat loss is also allowed. An asymptotic analysis of the temperature growth reveals two conditions necessary for ignition to occur. Cases of both large and order unity Lewis number are shown to lead to a nonlinear integral equation governing the thermal runaway. Some global and asymptotic properties of the integral equation are obtained.

End-temperature control in a long rod - The problem considered is that of the temperature in a semi-infinite rod with the control being applied in the form of heat flux at the near end. The objective is to keep the actual end-temperature close to some desired value over a specified time interval without excessive use of the heat flux control. The optimal control is taken to be that which minimizes a given performance index of quadratic form. It is shown that a necessary and sufficient condition for minimization of the performance index is that the optimal control satisfy a certain Fredholm integral equation of the second kind. The existence of a solution to this integral equation and its various properties are determined.

Perturbed bifurcation of stationary striations in a contaminated, nonuniform plasma - A cylindrical, weakly ionized and collision dominated neon plasma can be described by a system of nonlinear, parabolic reaction-diffusion equations for the electron and metastable atom axial densities. The equations exhibit a bifurcation from a uniform to a striated state at a critical length of the plasma column. The sharp transition between states predicted by the theory is in contrast with the smooth transition observed in experiments. We apply the theory of singular perturbation of bifurcations to show that small inhomogeneities in the plasma, such as those caused by nonuniform heating and contamination, are sufficient to qualitatively explain the experimental results. We observe that a steady, axial magnetic field in the plasma can also produce a smooth transition.

On flame stretch - A general invariant expression is derived for the stretch experienced by a flame due either to its motion or to the nonuniform flow of gas through it. This expression is given in terms of the local fluid velocity and the shape of the flame front. Specific examples in which the flame stretch takes a simplified form are discussed. Some remarks are made regarding the relation between the three distinct properties of flames: stretch, speed, and temperature.

Thermal and shot noise effects on nonlinear oscillators - The effect of thermal noise on the dynamic behavior of physical systems has been a subject of continuing interest for many years. For example, thermal fluctuations about stable equilibrium states of physical systems are well-known to be described by the Boltzmann distribution. Furthermore, systems in which multiple stable states can coexist exhibit transitions due to noise between those states. The transition rate from a stable equilibrium state, which is inversely proportional to the lifetime of that state, has also been discussed in the literature. That rate has been shown to be one of the form $\Omega \exp(-\Delta E/T)$, where E denotes the activation energy, i.e., the height of the potential barrier that must be overcome in order to escape from that state, and T denotes the noise level. The dependence of the attempt frequency, Ω, on the parameters of the problem varies with the particular problem under consideration.

Of equal interest are questions of fluctuations about, and transitions from, stable nonequilibrium steady states. Indeed, a considerable effort has gone into calculating the distribution of fluctuations about such states. The distribution of fluctuations may be characterized by the contours of constant probability density.

In a previous publication, we showed that for the prototypical problem of a linearly damped forced physical pendulum at low temperature, the contours of constant probability density are equivalent to the family of steady-state phase space trajectories of the deterministic equations of motion parametrized by the dissipation coefficient. In the present paper, we generalize those results to include nonlinear oscillators as well as state dependent (multiplicative) noise.

Flames in fluids: their interaction and stability - Viewed on a hydrodynamical scale, a flame may be considered as a surface of discontinuity, separating burned from unburned gas. Unlike earlier treatments, the present study accounts for flame structure, i.e., accounts for the details of chemical reactions and transport processes, and their interaction with the fluid flow. A model, including the effects of flame structure, is derived in coordinate invariant form. It consists of the fluid equations, to be solved on either side of the flame, an evolution equation describing the deformation of the front, and jump conditions for the fluid variables across the front. The model describes the dynamics of flame fronts including their stability. In particular, we study the stability of both plane and curved flames, and discuss the effect on stability of flame front curvature, heat release, Lewis number and Prandtl number.

Gas-phase transient diffusion in droplet vaporization and combustion - By suppressing the relaxation process of fuel vapor accumulation through the use of d^2-law results as the initial conditions, the present study isolates gas-phase transient diffusion as the only transient process during droplet vaporization and combustion, and thereby successfully identifies its influence on the bulk droplet gasification characteristics. The cases of pure vaporization and flame sheet combustion are analytically solved using perturbation methods and the matched asymptotic expansion technique in the limit of the small gas-to-liquid density ratio. Results demonstrate that transient diffusion enhances the vaporization and burning rates, reduces the flame front standoff ratio, and elevates the flame temperature. However, contrary to predictions of previous studies which have inadvertently included fuel vapor accumulation, these transient diffusion effects are very small so that gas-phase quasi-steadiness is indeed an adequate and useful assumption for the modeling of subcritical droplet combustion.

Nonlinear stability and bifurcation in the transition from laminar to turbulent flame propagation - We review recent analytical results in the theory of transition from laminar to turbulent premixed flame propagation. We exploit the fact that the overall activation energy is large to formally derive dynamical flame sheet models, which are then used to predict instability thresholds as function of the various parameters in the problem, at which steps in the transition occur. Employing perturbation techniques, we then describe bifurcations from a steady, planar flame to both pulsating and cellular modes of propagation. These nonsteady, nonplanar propagation modes represent intermediate stages in the evolution from laminar to turbulent combustion.

Singular perturbations of epidemic models involving a threshold - This paper deals with mathematical model of an epidemic with a small number of initial infectives I_0. The time development of the epidemic, satisfying an integro-differential equation, is approximated with singular perturbation techniques. The asymptotic result for $I_0 \to 0$ shows that when the number of infectives exceeds a fixed small value (independent of I_0) the time course of the epidemic is fixated; the time needed to pass this value is of the order $O(-\log I_0)$.

On the lifetime of a metastable state at low noise - The mean lifetime of a metastable state of a dynamical system driven by small noise is calculated. The vector field of the dynamical system, which need not be derivable from a potential, is assumed to have vanishing normal component on the boundary of the domain of attraction of the metastable state.

Shot noise effect on the nonzero voltage state of the hysteretic Josephson junction - We consider the combined effect of shot and thermal noise on (i) the stationary distribution of fluctuations about the nonzero voltage state of an underdamped hysteretic Josephson junction, and (ii) the noise induced transition rate from the nonzero voltage to the zero voltage state. We find that the distribution is nonsymmetric in that the positive voltage fluctuations are more probable than the negative ones. We also find that the transition rates are larger than those due to thermal noise alone.

<u>Transitions from the equilibrium state of a hysteretic Josephson junction induced by self-generated shot noise</u> - We postulate the existence of self-generated normal current shot noise due to the long-lived voltage fluctuations in a hysteretic Josephson junction. The resulting low temperature transition rates out of the (zero voltage) superconducting state are much larger than those arising from Josephson noise alone. Excellent agreement with experiments is then achieved at all temperatures, removing the need to invoke macroscopic quantum tunneling.

<u>Thermal activation from the fluxoid and the voltage states of DC SQUIDS</u> - The probability density of thermal fluctuations about different types of nonequilibrium steady states of a DC SQUID are evaluated by generalizing a technique used before for the fluctuations of a single Josephson junction. Probability densities obtained for both "running" and "beating" modes are used to calculate thermal activation rates as well as the various branches of the I-V characteristic. The results are compared with the experiments of Voss et al. and good agreement is found.

<u>An asymptotic theory of deflagrations and detonations I. The steady solutions</u> - Combustion waves propagating through a reactive gas are studied in the plane, one-dimensional geometry. On a length scale large compared to the diffusion length, the waves are treated as exothermic discontinuities in an ideal, nonreactive gas. An asymptotic theory is developed which yields the steady structure of the waves in simple, analytical form. The theory, based on limits of large activation energy and small heat release, treats all possible deflagrations and detonations.

<u>Propagating flames and their stability</u> - Viewed on a hydrodynamic scale, a flame may be considered as a surface of discontinuity, separating burned from unburned gas. Unlike earlier treatments, which ignored the flame structure, the present study accounts for the interaction of the fluid flow with the transport processes and chemical reactions occurring inside the thin flame zone. Thus we derive, rather than prescribe, jump conditions across the flame front and an equation for the flame speed. The model, derived in coordinate invariant form, describes the dynamics of flame fronts including their stability. Particular attention is focused on the stability of curved flames, which reveals some characteristics that do not exist in the corresponding analysis of plane flames. Due to the stabilizing effect of curvature, disturbances of circular flames grow more slowly than those for plane flames. As in the case of plane flames, when the mass diffusivity of the deficient reactant component is sufficiently smaller than thermal diffusivity, curved flames can be stabilized. Finally, in contrast to plane flames, the effect of viscosity on curved flames is comparable to that of diffusion and is destabilizing. This dependence decreases with increasing radius of curvature and disappears entirely for plane flames.

Secondary bifurcation in flame propagation - A great deal of work has been done in attempts to explain the transition from laminar to turbulent fluid flow via successive instabilities, as a critical parameter, e. g. the Reynolds number, is varied. The successive instabilities are associated with a sequence of bifurcations, each generating more complex spatial and temporal patterns. Recently, similar attempts have been made to explain the transition from laminar to turbulent flame propagation via a succession of bifurcations and their accompanying instabilities. In this sequence, the steadily propagating planar flame loses stability to successively more complex modes of propagation such as cellular, pulsating or multi-periodic pulsating flames.

In this paper, we present two recent results of our work in this area. First, we consider the transition from periodic pulsating to quasi-periodic pulsating flames. Then, we consider the transition from a stationary cellular flame to a pulsating cellular flame. In each case, we characterize the transition as a secondary (or higher order) bifurcation).

On the stability of plane and curved flames - Theoretical studies on flame stability have generally been based on one of two approaches: (i) the hydrodynamic model which accounts for thermal expansion due to combustion but ignores flame structure, and (ii) the diffusional-thermal model which considers flame structure in a prescribed constant density flow thus ignoring thermal expansion. The present study is based on a model we derived, in which both thermal expansion and flame structure are accounted for. We find that although plane flames are unstable to disturbances of very large wavelengths, as predicted by the hydrodynamical model, there exists a critical wavelength below which plane flames may be stable, when diffusional-thermal effects are incorporated. The boundary between stability and instability is determined by the Lewis number Le, representing the ratio of thermal to mass diffusivities. If Le exceeds a critical value Le*, the plane flame is stable. Otherwise it is unstable and the flame takes on a cellular shape. In the case that thermal expansion is small, the stability boundary reduces to that obtained from the diffusional-thermal model. We determine the effect of flame stretch, thermal expansion, Lewis number and Prandtl number, on stability.

Quasi-periodic waves along a pulsating propagating front in a reaction-diffusion system - We consider a system of reaction-diffusion equations for which there exists a solution with a uniformly propagating front, and from which solutions describing a pulsating propagating from with periodic travelling and standing waves along the front, bifurcate supercritically. We construct a pulsating propagating solution branch, which quasi-periodic travelling waves along its front, which connects the periodic travelling and standing wave branches. Thus the quasi-periodic branch arises as a secondary bifurcation from the uniformly propagating solution. Our construction involves a perturbation analysis in the neighborhood of a certain degenerate point in parameter space, which we identify. The stability of the various branches is investigated.

Uniform expansion of the transition rate in Kramers' problem - Kramers' model of diffusion over potential barriers, e.g., chemical reactions, baseed on the noise activated escape of a particle from a potential well, is considered. Kramers derived escape rates valid for intermediate and large damping and in a separate analysis, for small damping. In the small damping limit, Kramers' intermediate result reduces to the transition state rate which does not agree with the small damping result. A new escape rate is derived that is uniformly valid for all values of the damping coefficient. The new rate reduces to Kramers' results in the appropriate limits and, in particular connects Kramers' intermediate and small damping results.

Asymptotic solution of the Kramers-Moyal equation and first-passage times for Markov jump processes - We calculate the activation rates of metastable states of general one-dimensional Markov jump processes by calculating mean first-passage times. We employ methods of singular perturbation theory to derive expressions for these rates, utilizing the full Kramers-Moyal expansions for the forward and backward operators in the master equation. We discuss various boundary conditions for the first-passage-time problem, and present some examples. We also discuss the validity of various diffusion approximations to the master equation, and their limitations.

Acoustic coupling of flames - In the limits of large activation energy and small Mach number, the full equations of reactive gas dynamics are reduced to a simpler set which is appropriate for studying acoustic interaction with slender flames. The model is used to study the interaction of a plane, steady flame with a normally incident acoustic wave. Explicit analytical expressions are obtained for the reflection and transmission coefficients, and, in two limiting cases, for the acoustically induced disturbance in the flame speed.

Solution of Kramers-Moyal equations for problems in chemical physics - We derive asymptotic solutions of Kramers-Moyal equations (KME's) that arise from master equations (ME's) for stochastic processes. We consider both one step processes, in which the system jumps from x to $x + \epsilon$ or $x - \epsilon$ with given probabilities, and general transitions, in which the system moves from x to $x + \epsilon\xi$, where ξ is a random variable with a given probability distribution. Our method exploits the smallness of a parameter ϵ, typically the ratio of the jump size to the system size. We employ the full KME to derive asymptotic expansions for the stationary density of fluctuations, as well as for the mean lifetime of stable equilibria. Thus we treat fluctuations or arbitrary size, including large fluctuations. In addition, we present a criterion for the validity of diffusion approximations to master equations. We show that diffusion theory can not always be used to study large deviations. When diffusion theory is valid our results reduce to those of diffusion theory. Examples from macroscopic chemical kinetics and the calculation of chemical reaction rates ("Kramers" models) are discussed).

Controlled transition to cellular flame fronts - We proposed a nonlinear control mechanism to reduce flame instabilities. An instability associated with the formation of cellular flames is known to occur when the mass diffusivity of the deficient reactant in a combustible mixture is sufficiently larger than the heat diffusivity of the mixture. The transition from the smooth front to the cellular structure can be analyzed by deriving an evolution equation for flame front disturbances. This nonlinear equation depends on the position R of the flame, which is a function of time controlled by the underlying prescribed flow. We consider both linear and time-periodic functions R(t) and show that the transition to cellular structures may be either considerably delayed or eliminated.

Stationary, harmonic and pulsed operations of an optically bistable laser with saturable absorber I. - We study the semiclassical equations for a laser with a saturable absorber in the mean-field limit, assuming homogeneously broadened two-level atoms, for a set of parameters where the system displays optical bistability and time-periodic solutions. In the first part the bifurcation diagram for stationary and periodic solutions is obtained by numerical integration. Two different classes of stable periodic solutions arise: small-amplitude solutions and passive Q-switching. We observe hysteresis domains involving up to three solutions (stationary and/or periodic). We also discuss the validity of some standard approximations and show that even in the absence of detuning the phases play an important role. We also discuss the influence of the initial conditions whose symmetry properties induce important modifications of the bifurcation diagram. In the second part we introduce an alternative adiabatic elimination scheme which allows us to construct the small-amplitude periodic solutions over nearly their whole range of existence. We then study these solutiosn near the Hopf bifurcation from which they emerge and derive analytic conditions for their stability. When they are stable, we also give the conditions under which a secondary Hopf bifurcation will occur, leading to quasiperiodic solutions.

Stationary, harmonic, and pulsed operations of an optically bistable laser with saturable absorber II. - In the preceding paper we have analyzed the bifurcation diagram of the steady and time-periodic solutions of the lasers with saturable absorbers (LSA) equations. However, a study of the experimental results presented in the literature indicates that, in general, the control parameter is a slowly varying function of time. In this second paper we analyze the influence of this dependence on the bifurcation diagram of the LSA. We show that the stability changes of slowly varying steady-state solutions do not correspond to their bifurcation or limit points in the case where all parameters are constant. In particular, we show that the zero-intensity state can be stabilized during a certain interval of time and that this stabilization can be controlled by the initial value of the time-dependent bifurcation parameter.

Bifurcation of pulsating and spinning reaction fronts in condensed two-phase combustion - We employ a nonlinear stability analysis to describe the bifurcation of pulsating and spinning modes of combustion in condensed media. We adopt the two-stage model of Margolis (1983) in which the modified non-dimensional activation energy of the reaction is large, but finite, and in which the limiting component of the mixture melts during the reaction process, as characterized by a nondimensional melting parameter M. We identify several types of nonsteady solution branches which bifurcate from the steady planar solution and show that they are supercritical and stable for a range $M > 0$, though they are subcritical and unstable for $M = 0$.

Singular bifurcation in reaction-diffusion systems - In Modelling of Patterns in Space and Time, Springer Lecture Notes in Biomathematics, edited by J. Murray and W. Jaeger (with T. Erneux and E. L. Reiss).

Flame propagation in channels: Secondary bifurcation to quasi-periodic pulsations - We consider premixed flame propagation in long rectangular channels. A steadily propagating planar flame is stable for Lewis numbers less than a critical value. For Lewis numbers exceeding this critical value a sequence of primary bifurcation states, corresponding to time-periodic pulsating cellular flames, emanates from the steadily propagating solution. We analyze the problem in a neighborhood of a multiple primary bifurcation point. By varying the channel dimensions, we split the multiple bifurcation point and show that a stable quasi-periodic pulsating flame can arise as a secondary bifurcation from one of the primary bifurcation states. We also exhibit the phenomenon of mode-jumping, in which there is an exchange of stability between two primary states.

Frequency fluctuations in noisy oscillators - We study frequency ard period fluctuations in a nonlinear oscillator driven by Gaussian white noise. We define the random period as the random time between two consecutive zero crossing by the random phase plane trajectory, and the random frequency as the number of such zero crossings per unit time. These quantities are shown to be related by renewal theory. We find asymptotic expressions for the means and variances of the random period and random frequency, for small damping and small noise.

Nonlinear control in optical bistability - We study the influence of a small periodic modulation of the input field amplitude in dispersive optical stability. When the system is initially near one of the two limit points in a stable state, the addition of a small periodic modulation may either stabilize or destabilize the system. We prove that distabilization occurs as a result of critical slowing down when the modulation frequency is decreased.

Analytic studies of a laser with a saturable absorber - The principal purpose of this paper is to summarize recent analytic studies of the two-level Laser Saturable Absorber model. The paper is divided in two parts. In the first part, we analyze the possible bifurcations from a steady state to time-periodic solutions. In particular, we shall emphasize the difference between the distinct branches of time-periodic solutions. In the second part, we investigate the effects of slowly-varying control parameters. This new class of bifurcation problems is motivated by recent experiments using time-dependent parameters.

Imperfect bifurcation with a slowly-varying control parameter - We consider a general class of imperfect bifurcation problems described by the following first order nonlinear differential equation: $\dot{y} = ky^p + \lambda(t)y + \delta$, where $k = 1$ or -1 and $p = 2$ or 3 are fixed quantities. The solution depends on the values of the "imperfection" parameter $\delta (0 < \delta << 1)$ and the time-dependent control parameter $\lambda(t) = \lambda_0 + \epsilon t$ ($\lambda_0 < 0$ and $0 < \epsilon << 1$). If $\delta = \epsilon = 0$, this equation admits at $\lambda = 0$ a bifurcation from the basic state $y = 0$ to nonzero steady states. In the first part of the paper, we analyze the perturbation of the bifurcation solutions produced both by the small imperfection ($\delta \neq 0$) and the slow variation of $\lambda (\epsilon \neq 0)$. We show that $\lambda = 0$ does not correspond to the transition between the two branches of slowly-varying steady states. This transition appears at a larger value of $\lambda = \lambda_1$. Provided that δ is sufficiently small compared to ϵ, λ_1 is an $O(1)$ quantity which only depends on λ_0, i.e. the initial position of $\lambda(t)$.

Our analysis is motivated by problems appearing in laser physics. In the second part of the paper, we show how the semiclassical equations for the simple laser and the laser with a saturable absorber can be reduced to this simple first-order nonlinear equation. We then discuss the practical interests of our results.

First passage times for processes governed by master equations - We calculate the activation rates of metastable states of processes governed by Master Equation by calculating mean first passage times. We employ methods of singular perturbation theory to derive expressions for these rates, utilizing the full Kramers-Moyal expansions for the forward and backward operators in the Master Equation. In addition we discuss the validity of various diffusion approximations to the Master Equation, showing that such approximations are not valid in general.

Noise induced transitions in multi-stable systems - The damped physical pendulum driven by constant torque serves as a model for many physical systems (e.g., the motion of an ion in a crystal subject to a uniform electrostatic field, the point Josephson junction driven by constant current, charge density states and a stable non-equilibrium state. In the presence of a random driving force of thermal or shot noise type there are transitions between the stable states of the pendulum. We calculate the steady state distribution of fluctuations about the stable states and the transition rates between them. For the point Josephson junction at very low temperatures we postulate the existence of "self-generated" shot noise and obtain transition rates which agree with the experimental results of Voss and Webb. This paper summarizes the work of Ben-Jacob, Bergman, Imry, Knessl, Matkowsky and Schuss.

The stability of plane flames attached to a flameholder - A model is derived from the full equations governing combustion, in which a flame, located near a flameholder, is viewed as a three-dimensional surface of discontinuity in the flow field. Jump conditions for the fluid variables, as well as an expression for the flame speed, are obtained from an asymptotic analysis of the detailed structure of the flame. The model is applied to investigate the linear stability of steady plane flames in the presence of heat loss to the flameholder. Stable flames exist only if the inflow velocity is sufficiently lower than the adiabatic flame speed, and then only in a certain regime of the parameters: Lewis number, heat release, and heat loss.

An asymptotic theory of large deviations for Markov jump processes - We present new asymptotic methods for the analysis of Markov jump processes. The methods, based on the WKB and other singular perturbation techniques, are applied directly to the Kolmogorov equations and not to approximate equations that come e.g. from diffusion approximations. For the homogeneous process, we construct approximations to the stationary probability density function and the mean first passage time from a given domain. Examples involving a random walk and a problem in queueing theory are presented to illusrate our methods. For a class of time inhomogeneous processes, we construct long time approximations to the transition probability density function and the probability of large deviations from a stable state. The law of Large Numbers is obtained as a special case.

On the non-Markovian theory of activated rate processes in the small friction limit - The motion of a particle in a one-dimensional potential well under the effect of thermal noise and damping plays an important role in modelling many physical and chemical phenomena. The problem of interest is to determine the escape rate from the well. The generalized Langevin equation, describing this motion, is examined in the asymptotic limit of small friction. A limiting backward Kolmogorov operator in the energy variable is derived by averaging on constant energy contours, and then employed to calculate explicit analytical expressions for the activation rate.

A singular perturbation approach to non-Markovian escape rate problems - We employ singular perturbation methods to examine the generalized Langevin equation which describes the dynamics of a Brownian particle in an arbitrary potential force field, acted on by a fluctuating force describing collisions between the Brownian particle and lighter particles comprising a thermal bath. In contrast to models in which the collisions occur instaneously, and the dynamics are modeled by a Langevin stochastic equation, we consider the situation in which the collisions do not occur instantaneously, so that the process is no longer a Markov process and the generalized Langevin equation must be employed. We compute expressions for the mean exit time of the Brownian particle from the potential well in which it is confined.

A singular perturbation approach to first passage times for Markov jump processes - We introduce singular perturbation methods for construcing asymptotic approximations to the mean first passage time for Markov jump processes. Our methods are applied directly to the integral equation for the mean first passage time and do not involve the use of diffusion approximations. An absorbing interval condition is used to properly account for the possible jumps of the process over the boundary which leads to a Wiener-Hopf problem in the neighborhood of the boundary. A model of unimolecular dissociation is considered to illustrate our methods.

Interaction of pulsating and spinning waves in condensed phase combustion - We employ a nonlinear stability analysis in the neighborhood of a multiple bifurcation point to describe the interaction of pulsating and spinning modes of condensed phase combustion. Such phenomena occur in the synthesis of refractory materials. In particular, we consider the propagation of combustion waves in a long thermally insulated cylindrical sample and show that steady, planar combustion is stable for a modified activation energy/melting parameter less than a critical value. Above this critical value primary bifurcation states, corresponding to time-periodic pulsating and spinning modes of combustion emanate from the steadily propagating solution. By varying the sample radius, we split a multiple bifurcation point to obtain bifurcation diagrams which exhibit secondary, tertiary, and quaternary branching to various types of quasi-periodic combustion waves.

On the stability of the porous plug burner flame - The results of a linear stability analysis of a premixed flame attached to a porous plub burner are presented. A dissertation relation derived by Buckmaster (SIAM J. Appl. Math. 43 (1983) 1335-1349) is used to define the neutral stability curve in the wave number vs. Lewis number plane. The nature of this curve is analyzed as the characteristic parameters of the burner and the standoff distance of the flame are varied. It is shown that the neutral stability curve may have multiple branches where pulsating flames become unstable. Depending on the parameter values, the burner may have a stabilizing or destabilizing effect.

A singular perturbation approach to non-Markovian escape rate problems with state dependent friction -- We employ singular perturbation methods to examine the generalized Langevin equation (GLE) with state and memory dependent friction. The GLE describes the dynamics of a Brownian particle in an arbitrary potential field, acted on by a fluctuation force characterizing the collisions between the particle and the thermal bath. We consider the situation in which the collisions are not restricted to occur instantaneously and additionally these interactions are nonlinear. This leads to a non-Markovian description of the dynamics of the particle. We compute explicit analytical expressions for the mean exit time τ of the Brownian particle from the potential well in which it is confined. Thus we derive expressions for the activation rate k from the well, which is inversely proportional to t. In the examples considered, the effect of the state dependent friction is to increase τ and thus to lower k.

Lifetimes of non-equilibrium dissipative steady states - Noise induced transitions of the hysteretic Josephson junction from the zero voltage state into the non-zero voltage state have been thoroughly investigated in the literature, in a wide temperature range, including the very low temperature range. This problem was studied in the context of the classical activation process of a particle in a potential well. Of equal interest is the problem of noise induced transitions in the opposite direction from the non-zero voltage state into the zero voltage state. The purpose of this paper is to summarize our recent analytical results on the latter problem of transition. In contrast to the experimentally well studied transition from the zero voltage state, the transitions in the opposite direction were not well studied experimentally and have not been compared to theoretical predictions. It is our aim to encourage experimentalists to carry out measurements of this transition process by providing a concise summary of our theoretical results.

We describe the transitions from the non-zero voltage stage of a single Josephson junction due to the thermal noise. We give the transition rate as a function of the junction's parameters. We calculate the lifetime of the zero-voltage step and the first harmonic step of a junction with induced microwave radiation. Finally, we calculate the lifetime of the non-zero voltage state at low temperatures where shot noise effects dominate those of thermal noise. In this range of temperatures we use the master equation to describe the junction's noisy dynamics.

Master equation approach to shot noise in Josephson junctions - We model the normal resistance of a hysteretic Josephson junction by writing a master equation to describe the individual quasiparticle tunneling. We solve the master equation by a WKB method near the zero-voltage state and near the nonzero-voltage state. We find that near the zero-voltage state the solution is given by the Boltzmann distribution with second-order corrections while near the nonzero-voltage state we obtain a nonsymmetric, non-Boltzmann distribution of voltage fluctuations, similar to the results obtained in a previous discussion based on the Fokker-Planck equation.

Displacement effect of a flame in a stagnation-point flow - The interaction of a premixed flame with the flow near the front stagnation point of a body has rightly received considerable attention in combustion theory. Virtually all the investigations so far have been concerned with high strain rates, when the flame lies inside the boundary layer and can be extinguished. By contrast, we consider moderate strain rates, when the flame stands clear of the boundary layer and pushes the incident flow away from the body. The object is to determine this displacement effect, which is much more pronounced than that of the boundary layer.

Bifurcation with memory - A model equation containing a memory integral is posed. The extent of the memory, the relaxation time λ, controls the bifurcation behavior as the control parameter R is increased. Small (large) λ gives steady (periodic) bifurcation. There is a double eigenvalue at $\lambda = \lambda_1$, separating purely steady ($\lambda < \lambda_1$) from combined steady/T-periodic ($\lambda > \lambda_1$) states with $T \to \infty$ as $\lambda \to \lambda_1^+$. Analysis leads to the co-existence of stable steady/periodic states, and as R is increased, the periodic states give way to the steady states. Numerical solutions show that this behavior persists away from $\lambda = \lambda_1$.

Boundary behavior of diffusion approximations to Markov jump processes - We show that diffusion approximations, including modified diffusion approximations, can be problematic since the proper choice of local boundary conditions (if any exist) is not obvious. For a class of Markov processes in one dimension, we show that to leading order it is proper to use a diffusion (Fokker-Planck) approximation to compute mean exit times with a simple absorbing boundary condition. However, this is only true for the leading term in the asymptotic expansion of the mean exit time. Higher order correction terms do not, in general, satisfy simple absorbing boundary conditions. In addition, the diffusion approximation for the calculation of mean exit times is shown to break down as the initial point approaches the boundary, and leads to an increasing relative error. By introducing a boundary layer, we show how to correct the diffusion approximation to obtain a uniform approximation of the mean exit time. We illustrate these considerations with a number of examples, including a jump process which leads to Kramers' diffusion model. This example represents an extension to a multivariate process.

Fronts, relaxation oscillations and period doubling in solid fuel combustion - We consider a reaction-diffusion system which models the gasless combustion of a solid material. The system exhibits oscillating fronts, whose nature varies as a function of the parameters of the problem. The behavior of the solution along the bifurcation branches is studied numerically, by an adaptive Chebychev pseudo-spectral method in which the coordinate system is adapted to follow the sharp oscillations of the front. As the bifurcation parameter is increased through a primary bifurcation point, the solution exhibits a transition from a steadily propagating front to a sinusoidally oscillating front. This front develops into a relaxation oscillation whose peaks become progressively sharper and steeper. As a secondary bifurcation point is exceeded, a period doubling bifurcation occurs.

Dynamics of nonadiabatic premixed flames in a gravitational field - A nonlinear stability analysis of a plane flame subject to gravitational forces and small volumetric heat loss is presented. A nonlinear partial differential equation describing the evolution and structure of the flame is derived. It is shown that the effects of volumetric heat loss can be summarized in a scale factor multiplying the strength of the gravitational force field. The scale factor is a function of the heat loss coefficient.

Ignition of a combustible solid with reactant consumption - The effects of excessive reactant consumption on the ignition of a combustible solid are introduced through a revised scaling of the heat release constant. Large activation energy asymptotics then yields a new one-parameter integral equation governing the temperature evolution near ignition. Analysis of the integral equation reveals a critical value of the parameter which distinguishes between the cases of ignition and non-ignition.

Hydrodynamic and diffusion effects on the stability of spherically expanding flames - We example the stability of an outwardly propagating spherical flame accounting for both hydrodynamic and thermo-diffusive effects. For Lewis numbers less than a critical value Le* < 1 disturbances of the flame front grow during the initial phase of propagation i.e. when the radius is comparable to the flame thickness. However, for Le > Le*, the flame which is stable to thermo-diffusive effects becomes unstable only after a critical size is reached. This instability is hydrodynamic in nature and is caused by the thermal expansion of the gas. Viscous effects were found to play a secondary role to diffusion for these freely propagating flames. In this study we provide an expression for the determination of the critical size, or a critical Peclet number, which depends on the thermal expansion coefficient and on the Lewis number. The explicit dependence on all the relevant physico-chemical parameters enables us to compare our results with experimental data.

Interaction of pulsating and spinning waves in nonadiabatic flame propagation - We consider nonadiabatic premixed flame propagation in a long cylindrical channel. A steadily propagating planar flame exists for heat losses below a critical value. It is stable provided that the Lewis number and the volumetric heat loss coefficient are sufficiently small. At critical values of these parameters, bifurcated states, corresponding to time-periodic pulsating cellular flames, emanate from the steadily propagating solution. We analyse the problem in a neighborhood of a multiple primary bifurcation point. By varying the radius of the channel, we split the multiple bifurcation point and show that various types of stable periodic and quasi-periodic pulsating flames can arise as secondary, tertiary and quaternary bifurcations. Our analysis describes several types of spinning and pulsating flame propagation which have been experimentally observed in nonadiabatic flames, and also describes additional quasi-periodic modes of burning which have yet to be documented experimentally.

A variational approach to nonlinear singularly perturbed boundary value problems - The application of the techniques of Matched Asymptotic Expansions (M.A.E.) to singularly perturbed boundary value problems has provided useful answers to both mathematical and physical questions. There are however problems for which M.A.E. apparently fails. An example of this type, given by Carrier and Pearson, is

$$\epsilon u''(x;\epsilon) + u^2(x;\epsilon) = 1 \quad ;$$

$$u(-1;\epsilon) = u(1;\epsilon) = 0 \quad , \quad 0 < \epsilon << 1 \quad .$$

Grasman and Matkowsky have shown that a relatively straightforward way to resol the indeterminancy in the boundary layer resonance problem is to use the variational formulation of the problem. Their idea was to use the family of solutions determined by M.A.E. (which were determined up to an arbitrary constant) as the class of trial functions for the variational principle, and then to pick the correct value of the constant by finding the value that makes the variation stationary. Because of the simplicity of this approach they were also able to generalize it to partial differential equations.

The purpose of this paper is to show that the use of the variational formulation also quite easily resolve the position of the interior layers for the nonlinear problem above as well as for related problems. With this aproach we will also be able to see clearly how the number of solutions changes as the boundary conditions are varied. In addition, we will see that a careful examination of the nonlinear problem using an iterative procedure will lend some insight into why the variational formulation works.

Spherically expanding flames - A theoretical study of outwardly expanding flames originating from a point ignition source is presented. Spherically symmetric solutions are discussed and tested for stability. Flames which are stable to thermo-diffusive effects are found to become unstable after reaching a critical size. This instability which is hydrodynamic in nature is a result of the interaction of the flame with the hydrodynamic disturbances that it generates via thermal expansion. In this study, we present a linear stability analysis and determine the dependence of the critical radius on the relevant physicochemical parameters.

Flame propagation in a rotating gas - In this paper we consider the influence of centrifugal and Coriolis accelerations on the form and stability of a flame propagating in a rotating channel. We detect a discrete sequence of angular velocities, near which the flame propagation velocity undergoes an unlimited amplification. It is shown that sufficiently rapid rotation may suppress cellular instability in the flame. We construct a self-similar solution that describes a flame propagating in a rotating gas in free space.

On nonadiabatic condensed phase combustion - We analyze the effects of melting and volumetric heat losses on the propagation of a reaction front in condensed phase combustion. Considering both homogeneous and heterogeneous models for the reaction rate, we calculate the propagation velocity for steady, planar burning as a function of the parameters in the problem. In particular, we show that this quantity is a multi-varied function of the heat loss parameter. We interpret the critical value of this parameter at which the propagation velocity has a vertical tangent, and which varies with the melting parameter, as an extinction limit beyond which a steady, planar combustion wave cannot sustain itself. We also present a model for nonsteady, nonplanar burning and consider the linear stability of the steady, planar solution. As in the adiabatic case, this basic solution is unstable to pulsating disturbances for sufficiently large values of a modified activation energy parameter. We show that the effects of heat loss, as well as melting, are destabilizing in the sense that the neutral stability boundary becomes more accessible when these phenomena are taken into account.

Adaptive pseudo-spectral computation of cellular flames stabilized by a point source - In this note we describe the computation of steady cellular flames for the problem of a flame stabilized by a point source of fuel in two dimensions. We consider the diffusional thermal model of combustion, which consists of a reaction, diffusion, convection system for temperature and concentration. In this problem, the convective term models a given fluid velocity field, corresponding to a point source of strength $2\pi\kappa$.

This problem was studied analytically in the limit of large activation energy N and Lewis number L, measuring the ratio of thermal conductivity to reactant diffusivity, close to one. That analysis showed that there exists a basic solution consisting of a circular flame front of radius $R = \kappa$, with angle-independent temperature and concentration profiles. It was further shown that this solution was stable for L exceeding a critical value $L_c < 1$. For $L < L_c$, the basic solution was stable (unstable) according as R was less (greater) than a critical value R_c. Finally, for $L < L_c$ and R slightly above R_c, the evolution of an angular perturbation to a steady solution in the form of a sinusoidally cellular pattern was described.

The analysis is a bifurcation analysis with R as the bifurcation parameter. It is valid only locally, in the immediate neighborhood of the bifurcation point. It is sometimes referred to as a weakly nonlinear theory. In order to determine the more global behavior of solutions along bifurcation branches, i.e., more into the fully nonlinear regimes, it is necessary to compute the solutions numerically. These computations build on and extend the weakly nonlinear analytical results. In this note we employ L as the bifurcation parameter and penetrate more deeply into the nonlinear regime by lowering L.

Colored noise in dynamical systems - We consider the problem of escape of a particle activated by small amplitude Gaussian colored noise, from a potential well. We consider various ranges of the parameters representing the bandwidth of the noise, its spectral height, and the dissipation coefficient. We employ singular perturbation methods to derive explicit analytical approximations for the stationary density of fluctuations about the deterministically meta-stable state at the bottom of the well, and for the mean first passage time to overcome the potential barrier and escape the well. The latter leads to a formula for the escape (activation) rate. Among other results we find that the mean first passage time is exponentially larger than in the white noise case.

New modes of quasi-periodic combustion near a degenerate Hopf bifurcation point - Steady, planar propagation of a condensed phase reaction front is unstable to disturbances corresponding to pulsating and spinning waves for sufficiently large values of a parameter related to the activation energy. We consider the nonlinear evolution equations for the amplitudes of the pulsating and spinning waves in a neighborhood of a double eigenvalue of the problem linearized about the steady, planar solution. In particular, near a degenerate Hopf bifurcation point, we describe closed branches of solutions which represent new quasi-periodic modes of combustion.

Stochastic stability of nonlinear oscillations - The stochastic stability of a nonlinear oscillator parametrically excited by a stationary Markov process is considered. The stochastic stability problem in terms of a mean first passage time is formulated. Specifically, if the mean first passage time of the energy E of the oscillator to a given energy level is finite, then the oscillator is unstable. A method of averaging is used to derive a Fokker-Planck equation in the energy variable. The stability criterion depends on the nature of the boundary points $E = 0$ and $E = \infty$, and is expressed in terms of a Feller type criterion. Our stability condition is derived for various types of nonlinearities, including Coulomb friction. In contrast, we observe that the standard stability criterion, in terms of Lyapunov exponents, is inconclusive for this type of problem.

Bistable cellular flames - We consider the problem of a flame stabilized by a point source of fuel in two dimensions. We employ the diffusional thermal model which consists of a reaction, diffusion, convection system of equations for the temperature and concentration of the limiting component of the reaction. In this model, the effect that a given fluid field has on the transport of temperature and concentration is considered, while changes in the underlying flow, due to thermal expansion, are not taken into account. The given fluid velocity field in our problem corresponds to a point source of fueld of strength $2\pi\kappa$.

In addition to κ, the parameters of the problem are the non-dimensional activation energy N, the ratio σ of the unburned temperature T_u to the burned temperature T_b, and the Lewis number L which is the ratio of thermal conductivity to reactant diffusivity. This problem was studied analytically in the limit of $N(1-\sigma) \to \infty$ and $L \approx 1$. That analysis showed that there exists a basic steady solution consisting of a circular flame front of radius $R = \kappa$ with axisymmetric temperature and concentration profiles. It was further shown that this solution was stable for all κ, if $1 > L > L_c$, where the critical Lewis number $L_c = 1-2/N(1-\sigma)$ was predicted by the theory. For $L < L_c$ it was shown that there exists a critical value κ_c such that for $\kappa < \kappa_c$ the basic solution was stable while for $\kappa > \kappa_c$ the basic solution was unstable to angular perturbations. In addition, for κ near κ_c, the perturbations were shown to evolve to steady, stable, small-amplitude cellular solutions, which bifurcate supercritically from the basic solution. The cellular solutions were shown to exhibit crests and troughs, with the temperature at the troughs higher than at the crests, as is the case in experimentally observed cellular flames.

The analysis, which employs κ as a bifurcation parameter, is a local analysis valid only in the neighborhood of κ_c. In this paper we extend this analysis into the more fully nonlinear regime. Our numerical results show that as κ increases from κ_c the system exhibits a multistable behavior with at least two distinct stable, steady cellular solutions coexisting for the same parameter values. Each such stable solution has its own domain of attraction, so that the resultant steady solution depends on the initial conditions employed.

In Proceedings of a Symposium to honor C. C. Lin, M.I.T., Cambridge, MA, June 1987.

A nonlinear wave equation in nonadiabatic flame propagation - We derive a nonlinear wave equation from the diffusional thermal model of gaseous combustion to describe the evolution of a flame front. The equation arises as a long wave theory, for values of the volumetric heat loss in a neighborhood of the extinction point (beyond which planar uniformly propagating flames cease to exist), and for Lewis numbers near the critical value beyond which uniformly propagating planar flames lose stability via a degenerate Hopf bifurcation. Analysis of the equation suggests the possibility of a singularity developing in finite time.

Dynamics of nearly extinguished non-adiabatic premixed flames in a gravitational field - A nonlinear stability analysis of a uniformly propagating plane flame subject to gravitational forces and small volumetric heat loss is presented. A nonlinear partial differential equation describing the evolution and structure of the flame near the extinction point (i.e. the point beyond which a uniformly propagating plane flame cannot be sustained) is derived. It is shown that this equation admits nontrivial solutions beyond the extinction point. The solutions represent steady wrinkled flames, which are spatially periodic in the direction transverse to the direction of propagation of the flame.

Does reaction path curvature play an role in the diffusion theory of multi-dimensional activated rate processes? - The two-dimensional Kramers' barrier crossing problem in the overdamped (diffusion) limit is investigated with particular attention given to possible effects of the geometry of the potential surface on the rate. Previous work ascribes corrections to the two-dimensional Kramers' formula to curvature of the reaction path. In contrast, we find that these corrections are due to the anharmonicity of the potential surface at the saddle and may become appreciable for small window frequency, i.e. flat potential surface at the saddle in the direction perpendicular to the reaction path. A general formalism to calculate such corrections is described.

First order dynamics driven by rapid Markovian jumps - We consider the effect of rapid random jumps on the behavior of deterministically stable dynamics. The random driving force (noise), is taken to be a state dependent Markov jump process with either finite or infinite state space. While the jump rate is high ($O(1/\epsilon)$), the jump size is neither small nor large, but rather $O(1)$. For a particle which is deterministically confined to a potential well, we compute the stationary joint probability density function of the state and noise variables, the probability distribution of the exit points, and the mean first passage time from the well. In contrast to the case of white noise, where these quantities depend on a barrier height determined solely by the potential, here they depend on an effective barrier height determined by both the potential and the noise process. Our method is to introduce the small parameter ϵ, where $1/\epsilon$ is a measure of the rapid jump rate, and to employ singular perturbation methods to solve the forward and backward master equations, for the above mentioned quantities.

A first passage time approach to stochastic stability of nonlinear oscillators - We consider the stochastic stability of parametrically excited nonlinear noisy oscillators. We formulate the stochastic stability problem in terms of first passage times. Specifically we calculate the probability that the energy remains bounded by a preassigned level E_c, for all time. The stability criterion is then expressed in terms of a Feller-type condition. We show that if the criterion is satisfied, the probability that the first passage time $\tilde{\tau}$ from E to E_c is finite, approaches zero as E approaches zero, so that the oscillator is stochastically stable. If the criterion is not satisfied, $\tilde{\tau}$ is finite with probability one, so that the oscillator is stochastically unstable. If $\tilde{\tau}$ is finite, we also calculate the mean first passage time to E_c. Our stability condition is derived for various types of nonlinearities, including Coulomb friction. In contrast, we observe that the standard stability criterion, in terms of Lyapunov exponents, is inconclusive for this type of problem.

An adaptive pseudo-spectral method for reaction diffusion problems - We consider the spectral interpolation error for both Chebyshev pseudo-spectral and Galerkin approximations. We develop a family of functions $I_r(u)$, with the property that the maximum norm of the error is bounded by $I_r(u)/J^r$, where r is an integer and J is the degree of the polynomial approximation. These functionals are used in an adaptive procedure whereby the problem is dynamically transformed to minimize $I_r(u)$. The number of collocation points is then chosen to maintain a prescribed error bound. The method is illustrated by various examples from combustion problems in one and two dimensions.

Diffusion theory of multidimensional activated rate processes: the role of anisotropy - We consider an anisotropic multidimensional barrier crossing problem, in the Smoluchowski (diffusion) limit. The anisotropy arises from either or both the shape of the potential energy surface and anisotropic diffusion. In such situations, the separatrix, which separates reactant and product regions of attraction, does not coincide with the ridge of the potential surface, which separates reactant and product wells, thus giving rise to a complicated time evolution. In the asymptotically long time limit, the time evolution is governed by crossing the separatrix and is exponential with a rate which may be obtained as a generalization of Kramers' theory to the anisotropic situation. In contrast, in long, though not asymptotically long times, the time evolution is dominated by repeated crossings of the ridge, and is nonexponential. Such nonexponential time evolution has been observed in many biochemical reactions, where many degrees of freedom and anisotropic diffusion processes lead to complicated dynamical behavior. Our model provides a simple prototype of such situations.

Uniform asymptotic expansions in dynamical systems driven by colored noise - We construct asymptotic expansions of the (quasi) stationary probability density function and of the mean first-passage time over a potential barrier, for bistable systems driven by weak wideband Gaussian colored noise, when the intensity ϵ and the correlation time τ of the noise are both small. Previous analyses have led to a variety of often contradictory results and to considerable confusion, which stem from the fact that the problem depends on two small parameters. This results in different expansions, with different ranges of validity, depending on the relative magnitudes of ϵ and τ. In contrast, we derive expansions that are uniformly valid throughout the entire parameter range of interest. In addition, we identify restrictions on the ranges of validity, in terms of the total power output ϵ/τ of the noise, of previously derived expansions. We show that only if the power output ϵ/τ becomes infinite can previously derived expansions be valid. Our results, when specialized to this case, reduce to expansions previously derived. Outside the restricted range, i.e., for finite or vanishingly small power outputs, our expansions contain terms which are new, and which may in fact dominate previously computed terms. In contrast to the use of one-dimensional diffusional approximations previously employed, our approach is based on the exact two-dimensional Fokker-Planck equation. Singular perturbation techniques, previously developed by the authors, are employed to systematically derive the asymptotic expansions.

Period doubling gained, period doubling lost - Pulsating solutions to a model of gasless condensed phase combustion are studied numerically, as a function of a parameter μ which is proportional to the non-dimensional activation energy. Due to the exothermic reaction, a combustion wave propagates into the fresh fuel mixture. Below a critical value μ_1 the wave propagates at a uniform velocity. For $\mu > \mu_1$, periodic pulsations occur, of period $T = T(\mu)$. That is, the velocity of the combustion wave increases and decreases periodically. The pulsations are sinusoidal for μ near μ_1, and then develop into relaxation oscillations as μ is increased. A transition to a doubly periodic (period 2T) solution branch is found at a value $\mu_2 > \mu_1$. Stable doubly periodic solutions can no longer be computed beyond a third critical point $\mu_3 > \mu_2$. For $\mu > \mu_3$, stable T periodic solutions are again found. There is an interval of bistability ($\mu^* < \mu < \mu_3$, with $\mu_2 < \mu^* < \mu_3$) in which both singly and doubly periodic solutions stably coexist, each with its own domain of attraction.

Colored noise in activated rate processes - We consider bistable systems driven by stationary wideband Gaussian colored noise. We construct uniform asymptotic expansions of the stationary probability density function and of the activation rate, for small intensity ϵ and short correlation time τ of the noise. We find that for different values of the total power output ϵ/τ of the noise, different terms in the asymptotic expansions become dominant. For $\tau \ll \epsilon$ we recover previously derived results, while for $\tau = O(\epsilon)$ and $\epsilon \ll \tau$ new results are obtained.

In Proceedings of a Conference on External Noise in Nonlinear Dissipative Systems, Los Alamos, New Mexico March 1988.

<u>Bifurcation and pattern formation in combustion</u> - We employ a combination of analytical and numerical methods to determine the behavior of solutions of combustion problems. Specifically we consider highly nonlinear time dependent systems of partial differential equations which model the behavior of both solid and gaseous fuel combustion. In gaseous fuel combustion we are particularly interested in the transition from laminar to turbulent combustion, including a description of the intermediate stages of this transition. These stages often occur as a sequence of bifurcations, as critical parameters of the problem are varied, with each successive step exhibiting more and more complex spatial and temporal behavior, often leading to spatial and temporal pattern formation. The solutions frequently exhibit very steep gradients, in both time and space, thus naturally calling for adaptive gridding techniques. We have developed an adaptive pseudo-spectral method which is both very accurate and very efficient. Our algorithm allows us to describe the solution on bifurcation branches, well beyond the region where analytical methods work well. We have however taken advantage of the analytical results that we first obtain, to aid us in choosing appropriate paramter values and initial conditions for the numerical computations. In addition the analytical results serve as benchmarks for our computations. The computations reveal new and interesting behavior, not otherwise obtainable. To illustrate our results we discuss two problems, involving solid and gaseous fuel combustion respectively.

In Proceedings of Ninth International Conference on Ordinary and Partial Differential Equations, Dundee, Scotland, July 1988.

<u>Cascading cellular flames</u> - The problem of a flame stabilized by a line source of fuel of strength $2\pi\kappa$ is solved numerically. As κ is varied, first a bifurcation from an axisymmetric solution to a stationary cellular solution is found. Increasing κ further, a sequence of transitions to stationary cellular solutions of increasing mode number is found. Each transition is accompanied by a region of bistability where two stable stationary cellular solutions coexist, each with its own domain of attraction. Evidence is presented to show that the modal transitions occur via subcritical bifurcations.

<u>New modes of quasi-periodic burning in combustion synthesis</u> - In combustion synthesis, the steady, planar burning of a propagating reaction front is unstable to time-periodic modes of burning for sufficiently large values of the activation energy and the melting temperature. At critical values of the radius of a long cylindrical sample, two sets of modes, which correspond to pulsating and spinning combustion waves, are neutrally stable simultaneously. A nonlinear stability analysis in the neighborhood of such a radius then leads to the prediction of additional stable solution branches that correspond to new quasi-periodic modes of gasless combustion.

In Proceedings of International Symposium on Combustion and Plasma Synthesis of High Temperature Materials, San Francisco, CA, Oct. 1988.

Modeling and numerical computation of a nonsteady SHS process -Combustion synthesis, or self propagating high temperature synthesis (SHS), is a new and innovative method for the fabrication of high tech ceramic and metallic materials. In this method a sample (say a cylindrical sample) consisting of a compacted power mixture is ignited at one end. A thermal wave then propagates through the sample, converting unburned reactants to products. The SHS process is important not only because of its technological implications, but also from the point of view of basic science. Modeling and analysis of the process involves the study of complex dynamical systems. The mathematical model consists of a system of highly nonlinear partial differential equations. Studies of this system not only lead to a greater understanding of the combustion process, but the theoretical problems describing the process pose serious challenges which require the development of new mathematical methods for their resolution. Finally, the insight gained from these studies helps us to understand the behavior of the process, which is a necessary prerequisite to our ability to effectively control the process.

We consider a reaction front (so-called solid flame) propagating through a cylindrical sample. Experiments have revealed a variety of modes of propagation through the sample. In addition to the uniformly propagating planar front, there have been observations of (i) pulsating combustion, in which a planar front propagates with an oscillatory velocity, (ii) spin combustion, in which one or more hot spots (luminous points) are observed to move in a helical motion along the surface of the sample, and (iii) multiple point combustion, in which the hot spots appear, disappear, and reappear repeatedly. Finally, experiments indicate that in certain cases burning occurs throughout the sample, while in other cases burning occurs only on the surface of the sample and not in its interior.

We employ both analytical and numerical methods to study the SHS process. In particular we obtain solutions to equations modeling the process, which describe the above mentioned observed modes of propagation, as well as predict new modes of propagation, exhibiting yet more complex behavior, not yet reported experimentally.

In Proceedings of International Symposium on Combustion and Plasma Synthesis of High Temperature Materials, San Francisco, CA, Oct. 1988.

Linear stability analysis of cylindrical flames - This article is concerned with the linear stability of cylindrical flames in a velocity field generated by a line source of fuel of constant strength $2\pi\kappa$ per unit length. The mathematical model involves the equations of mass and heat transfer in the regions on either side of the flame sheet and a set of jump conditions across the flame sheet. It admits a basic solution representing a stationary flame front in the shape of a circular cylinder at a radial distance κ from the line source. The circular front loses stability if either (i) the Lewis number of the reaction-limiting component is less than some critical value less than 1 and κ is greater than a critical value, or (ii) the Lewis number is greater than a critical value greater than 1. In the former case the circular front evolves into a steady cellular front, in the latter into a pulsating front, which may also be cellular. The WKB method is employed to derive approximations for the pulsating and cellular branches of the neutral stability curve.

Numerical computation of bifurcation phenomena and pattern formation in combustion - We develop and employ a new numerical method to study problems in both gaseous and solid fuel (condensed phase) combustion. The numerical method is an extension of the adaptive pseudo-spectral method previously introduced, in that two dimensional, non-product coordinate transformations are introduced so as to efficiently compute fronts which have a strong transverse variation.

In condensed phase combustion we consider a reaction front (so-called solid flame) propagating through a cylindrical sample. Such problems describe the process of self propagating high temperature synthesis (SHS), which is a new and innovative method for the fabrication of high tech ceramic and metallic materials. A cylindrical sample is ignited at one end and a thermal wave propagates through the sample, converting unburned reactants to solid products. We describe various modes of propagation, which have been experimentally observed as bifurcations from a basic solution corresponding to a uniformly propagating planar front. Experimental observations include the case when burning occurs throughout the sample as well as the case when burning occurs on the surface of the sample, but not in its interior. The additional modes of propagation include (i) oscillatory combustion - in which a planar front propagates with an oscillatory velocity, (ii) spinning combustion - in which one or more hot spots (luminous points) are observed to move in a helical fashion along the surface of the sample, and (iii) multiple point combustion - in which the hot spots appear, disappear, and reappear repeatedly. More specifically, a basic solution is obtained which describes a uniformly propagating planar front. By varying critical parameters of the problem, we construct additional solutions on branches which bifurcate from the basic solution. These solutions exhibit both spatial and temporal patterns, which become more and more complex as the distance from the bifurcation point is increased. Thus planar oscillatory combustion has been analyzed as a planar or axisymmetric Hopf bifurcation from the basic solution while spinning and multiple point combustion have been analyzed as traveling and standing wave patterns obtained as non-axisymmetric Hopf bifurcations from the basic solution.

Bifurcation theory is a local theory valid in a neighborhood of the bifurcation point. To study the behavior of the system in the fully nonlinear regime, far from the bifurcation point, numerical computations are employed. These computations reveal new and interesting behavior, not previously predicted by analysis. As an example, in planar condensed phase combustion, a nearly sinusoidal (in time) solution is computed for parameters near the bifurcation point. As the parameters are varied away from the bifurcation point, extremely severe relaxation oscillations develop and a period doubling secondary bifurcation occurs. On this branch the relaxation oscillations become yet sharper. Beyond a certain point the period doubled branch becomes unstable and stability returns to the singly periodic branch. We also consider non-planar pulsating patterns.

In Proceedings of Third International Conference on Numerical Combustion, Antibes, France, May 1989.

Nonlinear analysis of condensed-phase surface combustion - This article is concerned with the structure and stability properties of a combustion front that propagates in the axial direction along the surface of a cylindrical solid fuel element. The fuel consists of a mixture of two finely ground metallic powders, which combine upon ignition in a one-step chemical reaction. The reaction is accompanied by a melting process, which in turn enhances the reaction rate. The combustion products are in the solid state. The reaction zone, inside which the melting occurs, is modeled as a front that propagates along the surface of the cylinder. The different modes of propagation that have been observed experimentally (such as single- and multiheaded spin combustion and multiple-point combustion) are explained as the results of bifurcations from a uniformly propagating plane circular front. The stability properties of the various modes are discussed.

The stability of weakly stretched flames - In this study, the stability of plane stretched flames, more specifically plane flames in straining fields, has been examined. It is shown that flame stretch stabilizes long wavelength disturbances and so can suppress the hydrodynamic instability. If in addition, the mixture is deficient in the reactant which is also the weakly diffusing component, and hence the Lewis number is greater than unity, thermal effects will stabilize the short wavelength disturbances. Thus sufficiently strong stretch can render a flame absolutely stable. The instability, which first appears by reducing the strain rate from the critical value, is in the form of longitudinal cells with ridges in the direction of stretch. By reducing the strain rate further a cellular structure will probably emerge.

Propagation and extinction of a flame in a stagnation point flow - The interaction of a premixed flame with the flow near the front stagnation point of a body has been reconsidered in this paper for the case of moderate strain rates when the flame stands clear of the body. We provide a detailed solution which includes the determination of the flow field as affected by the burning, the flame standoff distance and the displacement of the incident flow. In particular we determine the standoff distance at which extinction takes place when the Lewis number associated with the deficient component of the mixture is sufficiently large. Unlike most previous studies this extinction curve is based on an analysis which fully accounts for thermal expansion.

A regularized K-S equation describing the formation of cellular flames - In this note we derive an evolution equation that models the spontaneous instability of a flame front which, although formally correct to the same order as the Kuramoto-Sivashinsky equation, contains a meaningful modification coming into play primarily in regions of large gradients. The approach used is a regularization procedure that modifies the conventional expansion procedure which fails whenever the gradients become large; the new equation is believed to better describe the formation of patterns on the flame front.

Two Routes to Chaos in Condensed Phase Combustion - The equations governing two models of gasless combustion which exhibit pulsating solutions are numerically solved. The models differ in that one allows for melting of the solid fuel, while the other does not. While both models undergo a Hopf bifurcation from a solution propagating with a constant velocity to one propagating with a pulsating (T-periodic) velocity when parameters related to the activation energy exceed a critical value, the subsequent behavior differs markedly. Numerically both models exhibit a period doubling transition to a 2T solution when the bifurcation parameter for each model is further increased. For the model without melting, a sequence of additional period doublings occurs, after which apparently chaotic solutions are found. For the modeling with melting, it is found that the 2T solution returns to the T-periodic solution branch. Then two additional windows of 2T behavior are found. After the last such window, the solution no longer returns to the T-periodic solution branch, but rather exhibits intermittency, with long laminar regions interrupted by randomly occurring bursts. Further increasing the bifurcation parameter leads to shorter laminar regions, with the bursts occurring more frequently. Increasing the bifurcation parameter yet further leads to apparently fully chaotic solutions. The numerical results demonstrate two mechanisms for chaotic dynamics in gasless combustion.

A Direct Approach to the Exit Problem - This paper considers the problem of exit for a dynamical system driven by small white noise, from the domain of attraction of a stable state. A direct singular perturbation analysis of the forward equation is presented, based on Kramers' approach, in which the solution to the stationary Fokker-Planck equation is constructed, for a process with absorption at the boundary and a source at the attractor. In this formulation the boundary and matching conditions fully determine the uniform expansion of the solution, without resorting to "external" selection criteria for the expansion coefficients, such as variational principles or the Lagrange identity, as in our previus theory. The exit density and the mean first passage time to the boundary are calculated from the solution of the stationary Fokker-Planck equation as the probability current density and as the inverse of the total flux on the boundary, respectively. As an application, a uniform expansion is constructed for the escape rate in Kramers' problem of activated escape from a potential well for the full range of the dissipation parameter.

Condensed Phase Combustion with a Merged Sequential Reaction Mechanism - A model is presented for the propagation of an exothermic chemical reaction through a condensed combustible mixture where the reaction is characterized by the sequential production and depletion of a significant intermediate speciesl. The effects of melting of the initial deficient component and intermediate species are also included. Under the assumptions of large activation energies and both steps of the reaction occurring at nearly the same temperature, together with other constraints on the effective heat released during each stage of the combustion process, an asymptotic approximation for the speed of a uniformly propagating planar merged reaction front is given. A time-dependent asymptotic model is also derived under the same assumptions This is used to determine conditions for the loss of stability of the uniformly propagating planar-front solution via Hopf bifurcation to a pulsating propagating-front solution in which the velocity of propagation varies periodically.

Spinning Cellular Flames - The problem of a flame stabilized by a line source of fuel of strength $2\pi\kappa$ is solved numerically for values of the Lewis number L < 1. In this parameter regime analytical studies as well as previous numerical studies have found stationary solutions corresponding to both axisymmetric and cellular flames. For sufficiently small values of L, we have computed time periodic pulsating cellular solutions in the form of traveling waves.

Uniform Solution of Kramers' Problem by a Direct Approach - We consider the problem of exit for a dynamical system driven by small white noise, from the domain of attraction of a stable state. We present a direct singular perturbation analysis of the forward equation, based on Kramers' approach, in which the solution to the stationary Fokker-Planck equation is constructed, for a process with absorption at the boundary and a source at the attractor. The exit density and the mean first passage time to the boundary are calculated from the solution of the stationary Fokker-Planck equation as the probability current density and as the inverse of the total flux on the boundary, respectively. As an application we construct a uniform expansion for the escape rate in Kramers' activated escape problem in the full range of the dissipation parameter.

Asymptotic Methods for Markov Jump Processes - We introduce singular perturbation techniques for constructing asymptotic approximations to the mean first passage time and the probability density function for Markov jump processes. Our methods are applied directly to the equations governing these quantities and do not involve the use of diffusion approximations. For the mean first passage time problem, an interval condition is used to account for possible jumps of the process over the boundary. The WKB method is used to construct the probability density function. Examples from chemical physics, conservation biology and the theory of large deviations are used to illustrate our methods. The properchoice of boundary conditions for diffusion approximations to the mean first passage time is given.

A Two Dimensional Adaptive Pseudo-Spectral Method - We develop a two dimensional adaptive pseudo-spectral procedure which is capable of improving the approximation of functions which are rapidly varying in two dimensions. The method is based on introducing two dimensional coordinate transformations chosen to minimize certain functionals of the solution to be approximated. The method is illustrated by numerical computation of the solutions to a system of reaction diffusion equations modelling the gasless combustion of a solid fuel. Spatio-temporal patterns are computed as a parameter μ, related to the activation energy, is increased above a critical value μ_c. The spatial patterns are characterized by a very rapid variation in the direction of the axis of the cylinder, together with a standing wave pattern in the direction of the azimuthal angle ψ. For small values of $\mu - \mu_c$ the solutions exhibit a nearly sinusoidal dependence in both time and ψ. As μ is increased further relaxation oscillations in both time and ψ occur. Beyond a critical value of μ stable time-periodic solutions are no longer found and the solution exhibits a quasi-periodic time dependence.

Unsteady Solid Flames - Combustion synthesis, or self propagating high temperature synthesis (SHS), is a new and innovate method for the fabrication of high temperature materials. A sample (say a cylindrical sample) consisting of a compacted powder mixture is ignited at one end. A thermal wave then propagates through the sample, converting unburned reactants to products. This process was pioneered by Merzhanov and colleagues in the U.S.S.R. and has been extensively investigated by J. B. Holt and colleagues in the U.S., as well as by others throughout the world.

In this paper we consider a reaction front (solid flame) propagating through a cylindrical sample. Experiments have revealed a variety of modes of propagation. In addition to the uniformly propagating planar front, there have been observations of (i) pulsating combustion, in which a planar front propagates with an oscillatory velocity, (ii) spin combustion, in which one or more hot spots (luminous points) are observed to move in a helical motion along the surface of the sample, and (iii) multiple point combustion, in which the hot spots appear, disappear, and reappear repeatedly. Finally, experiments indicate that burning may occur either throughout the sample, or only on the surface and not in its interior.

We employ both analytical and numerical methods to study the SHS process. In particular we obtain solutions to equations modeling the process, which describe the above mentioned observed modes of propagation, as well as predict new modes of propagation, exhibiting yet more complex behavior, not yet reported experimentally.

Bifurcation, Pattern Formation and Chaos in Combustion - Problems in gaseous combustion and in gasless condensed phase combustion are studied both analytically and numerically. In gaseous combustion we consider the problem of a flame stabilized on a line source of fuel. We find both stationary and pulsating axisymmetric solutions as well as stationary and pulsating cellular solutions. The pulsating cellular solutions take the form of either traveling waves or standing waves. Transitions between these patterns occur as parameters related to the curvature of the flame front and the Lewis number are varied. In gasless condensed phase combustion both planar and nonplanar problems are studied. For planar condensed phase combustion we consider two models: (i) accounts for melting and (ii) does not. Both models are shown to exhibit a transition from uniformly to pulsating propagating combustion when a parameter related to the activation energy is increased. Upon further increasing this parameter both models undergo a transition to chaos: (i) by intermittency and (ii) by a period doubling sequence. In nonplanar condensed phase combustion the nonlinear development of a branch of standing wave solutions is studied and is shown to lead to relaxation oscillations and subsequently to a transition to quasi-periodicity.

Polyhedral Flames - A diffusional-thermal model for a premixed burner flame is considered. We examine bifurcations from a stationary planar flame when a double eigenvalue is split into two closely spaced simple eigenvalues. Our analysis shows successive bifurcations from the stationary planar flame to a stationary non-planar axisymmetric flame; then to a stationary non-planar η-faced polyhedral flame and finally to a time periodic (i.e. rotating) polyhedral flame.

Burner-stabilized Cellular Flames - We consider a model that governs the behavior of a premixed flame anchored on a flat burner. We show that a steady planar flame is stable for Lewis numbers L in the interval $L_1 < L < L_u$ where $L_1 < 1$ and $L_u < 1$ and unstable otherwise.

We derive and analyze a Ginzburg-Landau equation for this problem in a neighborhood of the critical Lewis number L_1. We find that a family of spatially periodic solutions which correspond to stationary cellular flames, bifurcates from the basic solution. Subcritical bifurcations are found to be unstable. In the supercritical case there is a range of wavenumbers satisfying the Eckhaus criterion, within which these solutions are stable.

We also find spatially periodic solutions modulated by slowly varying amplitudes. The amplitude may be stationary spatially periodic, stationary solitary or a travelling wave.

Using MAPLE for the Analysis of Bifurcation Phenomena in Condensed-Phase Surface Combustion - This article describes the use of the symbolic manipulation language MAPLE for the analysis of bifurcation phenomena in condensed-phase combustion. The physical problem concerns the structure and stability properties of a combustion front that propagates in the axial direction along the surface of a cylindrical solid fuel element. Experimental observations suggest that the front may propagate in a number of different ways; the objective of the investigation is to describe these different modes of propagation. The analysis involves the study of a set of nonlinear partial differential equations which describe the structure and evolution of the combustion front. Because the location of the front is unknown and must be found as part of the solution, the problem is a free boundary value problem. The purpose of this article is to show how symbolic manipulation languages like MAPLE can be combined effectively with analysis and numerical computations for this type of investigation.

Modes of Burning in Filtration Combustion - We describe and analyze a model of filtration combustion, in which a gas is forced at high pressure into a porous solid matrix so that after ignition and under favorable conditions a combustion wave can propagate through the medium. We consider the case of counter flow, where the gas is forced into the reaction zone through the unreacted part of the porous solid. Relations are derived for the steady state propagation of a planr combustion wave or front in the limit of high activation energy, from which the propagation speed, reaction temperature, and reacted mass fraction of the solid product can be found in terms of the mass flux of the injected gas, the gas pressure and mass flux on exit from the front, and other physico-chemical parameters describing the system. Two distinct modes of combustion are discussed, corresponding to the reaction being driven to completion by exhaustion of either the gaseous or the solid component, these being referred to as the gas deficient and solid deficient modes of burning respectively. For both homogeneous and heterogeneous forms of the reaction rate we find that there is a critical inlet mass flux for the incoming gas below which steady state solutions no longer exist and that there are parameter values for which multiple steady states occur.

Quasi-periodic Waves and the Transfer of Stability in Condensed Phase Surface Combustion - This article is concerned with the structure and stability properties of a combustion front propagating in the axial direction along the outer surface of a cylindrical solid fuel element. A nonlinear analysis is given that explains the occurrence of spinning and standing wave patterns as bifurcations from a uniformly propagating plane circular front. Particular attention is given to a stability transfer mechanism between standing and spinning waves by means of quasi-periodic waves.

The Kramers Problem in the Turnover Regime; The Role of the Stochastic Separatrix - We consider the problem of activated escape of a Brownian particle from a potential well. We find the stochastic separatrix S (the locus of starting points of the phase space trajectories which have equal probabilities of ending up inside or outside the well) for (i) the extremely anisotropic overdamped motion of a two-dimensional Brownian particle in a bistable potential, and (ii) the damped and underdamped motion of a one-dimensional Brownian particle in a single metastable state. The significance of S is that (1) it defines the reactant and product wells in a natural though not necessarily intuitive way, and (2) it reduces the calculation of the escape rate to the solution of the stationary Fokker-Planck equation inside S, with absorbing boundary conditions on S. Finally, employing this approach we derive an expression for the Kramers escape rate which bridges uniformly between Kramers weak damping regime and transition state theory.

Bifurcation, Pattern Formation and Transition to Chaos in Combustion - We study problems in the dynamics and pattern formation of both solid and gaseous fuel combustion. Our goal is to understand the basic mechanisms affecting the combustion process, which is a necessary prerequisite to controlling it. A related goal is to describe the spatio-temporal patterns that are observed experimentally as well as to predict new dynamical behavior, as yet unobserved. Thus we wish to determine the mechanisms of successive transitions to increasingly complex patterns of dynamical behavior, as parameters of the problem are varied. The problems are formulated as initial boundary-value problems for systems of highly nonlinear partial differential equations modeling the relevant combustion process.

We employ both analytical and numerical approaches in our investigations, as well as the interaction of the two approaches. In combustion problem, the activation energies of the chemical reactions that occur, are typically large. As a result, the spatial region in which the chemical reactions occur to a significant extent, are thin, layer type regions termed reaction zones. In the limit of infinite activation energy, the reaction zone shrinks to a (in general moving) surface termed a flame front (flame sheet model). In this case, the distributed Arrhenius reaction rates become localized reaction rates on the surface, given by a surface δ function whose strength is calculated by the method of matched asymptotic expansions. The analytical investigations are carried out for the case of infinite activation energy, while the numerical investigations are carried out for finite, though large, activation energies. A high resolution of the reaction zones is necessary to accurately describe the structure and dynamics of the solution. Since the location of the reaction zone is not know a-priori, but in fact moves during the course of combustion, in a possibly oscillatory manner, the task of its accurate resolution is a challenge to numerical methods. Inaccurate resolution in the calculation can lead to numerical instabilities, and more perniciously, to spurious and incorrect predictions of dynamical behavior. To meet this challenge, we introduced an adaptive pseudo-spectral method. It has been successful in describing the experimentally observed, and analytically described patterns appearing during combustion. In addition, it has enabled us to predict new, as yet unobserved dynamical patterns.

On the Stability of Counter Flow Filtration Combustion - A model for time-dependent nonplanar propagation is derived in the limit of large activation energy, and is used to study the stability of a planar travelling wave in counter flow filtration combusiton, i.e., where an exothermic reaction front passes through a porous solid medium that reacts with a forced inflow of gas to form a posous solid, and where the direction of propagation of the front is opposite to that of the incoming gas.

The model admits two types of front propagation, (i) gas deficient propagation, for which the reaction ceases upon consumption of the gas, and (ii) solid deficient propagation, for which the reaction ceases upon complete conversion of the initial solid. Multiple steadily propagating solutions can occur. With a heterogeneous form of the reaction rate, it is found that gas deficient propagation is stable (unstable) when, for example, the activation energy is sufficiently low (high), and that solid deficient propagation is always unstable unless the reation rate is independent of the concentration of the gas. In this limit, solid deficient propagation is also stable (unstable) when the activation energy is sufficiently low (high). Sufficiently large and small mass flux of the gas entering the system can also destabilze the steadily propagating front.

Pulsating and Chaotic Dynamics Near the Extinction Limit - We numerically solve the problem of a premixed flame in an annular region, in which the combustible mixture is fed in through the inner cylinder and combustion products are removed through the outer cylinder and through the sides. We employ a model which accounts for the full coupling between fluid and transport effects and chemical reactions. The model accounts for heat loss through both the outer and inner cylindrical walls. We find that as heat loss is increased or as the inflow velocity is decreased, a transition from stationary to pulsating combustion occurs prior to extinction. For certain parameter ranges a transition to apparently chaotic dynamics occurs via a period doubling cascade.

Coupled Nonlocal Complex Ginzburg-Landau Equations in Gasless Combustion - We consider the evolution of a gasless combustion front. We derive coupled complex Ginzburg-Landau type equations for the amplitudes of waves along the front as functions of slow temporal and spatial variables. The equations are written in charcertisic variables and involve averaged terms which reflect the fact that in the slowest time scale, the effect on one wave, of a second wave traveling with the group velocity in the opposite direction on the intermediate time scale, enters only through its average. Solutions of the amplitude equations in the form of traveling, standing, and quasiperiodic waves are found, and regions of stability for these solutions are determined. In particular we find that the traveling and quasiperiodic (including standing) waves are not stable simultaneously. Finally, we observe that the stability analysis for coupled complex Ginzburg-Landau equations with averaged terms differs from that for coupled complex Ginzburg-Landau equations with the averages removed.

Coupled Complex Ginzburg-Landau Type Equations in Gaseous Combustion - Linear stability analysis of a uniformly propagating planar nonadiabatic premixed flame shows that above a critical value of the Lewis number, the flame loses stability to a continuum of modes. Employing a multiscale analysis we derive a pair of coupled complex Ginzburg-Landau type equations which govern the spatio-temporal evolution of the unstable modes. The equations are nonlocal in that certain terms are averaged. For a range of values of the heat loss parameter H, we find stable traveling waves which bifurcate from the planar uniformly propagating flame. As H increases towards the quenching limit $H_c = 1/(2e)$, traveling waves lose stability and chaotic behavior is possible. We also find quasiperiodic and standing wave solutions but they are unstable.

Mathematical Modeling of SHS Processes - We review a number of results previously obtained by mathematical modeling of SHS processes, and present two new results concerning (i) traveling waves in filtration combustion, having a two front structure and (ii) a nonlinear stability analysis in gasless combustion.

Stability of Plane Wave Solutions of Complex Ginzburg-Landau Equations - We consider the stability of plane wave solutions of both single and coupled complex Ginzburg - Landau equations, and determine stability domains in the space of coefficients of the equations.

Nonlocal Amplitude Equations in Reaction Diffusion Systems - We consider reaction diffusion systems haveing traveling wave solutions which lose stability via Hopf bifurcation, when a parameter of the system exceeds a critical value. Taking into account the continuous band of wave numbers of perturbations in the instability region, we derive coupled complex nonlocal Ginzburg-Landau equations for the amplitudes of waves along the front, as functions of slow temporal and spatial variables. The equations are written in characteristic variables and involve averaged terms which reflect the facts that (i) the basic solution is not spatially homogeneous and interacts with the waves to produce averaged terms, and (ii) in the slowest time scale, the effect on one wave, of a second wave traveling with the group velocity in the opposite direction on the intermediate time scale, enters only through its average. Solutions of the amplitude equations in the form of traveling, standing and quasiperiodic waves are found, and regions of stability for these solutions are determined. We observe that the stability analysis for coupled complex Ginzburg-Landau equations with averaged terms differs from that for coupled complex Ginzburg-Landau equations with the averages removed.

Nonlinear Dynamics of Cellular Flames - The problem of a flame stabilized by a line source of fuel in the cellular regime, i.e., $L < 1$, where L is the Lewis number, is solved numerically. It is found that as L is decreased, transitions from stationary axisymmetric to stationary cellular to nonstationary cellular flames occur. The nonstationary cellular flames can exhibit both periodic and quasi-periodic dynamics. In particular, as L is decreased successive transition from stationary axisymmetric solutions, to stationary four-cell solutions, to spinning four-cell solutions are computed. The spinning four cell solutions are very solwly traveling waves that arise due to an inifinite period, symmetry breaking bifurcation, in which the reflection symmetry of the stationary four-cell solution is broken. Near the transition point, the traveling wave solution branch is unstable and perturbations evolve to either a stationary five-cell or a nonstationary mixed-mode solution exhibiting apparently quasi-periodic dynamics. If L is further decreased beyond a critical value L_s, the traveling wave solution branch becomes stable. Beyond another critical value L_u, the traveling wave branch loses stability to a branch of mixed mode, apparently quasi-periodic, solutions that appear to arise due to the interaction of unstable three- and four-cell traveling wave solutions.

Combustion Synthesis of a Porous Layer - We consider a model of filtration combustion in a thin porous layer immersed in a bath of gaseous oxidizer, which provides for exchange of the oxidizer between the pores and the bath. We employ approximate analytical metoods to derive explicit analytical expressions for various quantities associated with uniformly propagating solutions, including the combustion temperature, the propagation velocity, and the final depth of conversion, as well as results about the structure of the solutions. We also employ numerical computation to find profiles for the reaction rate, temperature, pressure, density and depth of conversion, as well as to determine the stability of the solutions as a function of various parameters. We identify a new pulsating instability associated with gas mass transfer between the pores and the bath, and describe the mechanism of pulsations.

Self Compaction or Expansion in Combustion Synthesis of Porous Layers - We propose a mathematical model for the combustion of porous deformable condensed materials, which we use to describe the deformation of the high temperature products induced by the pressure difference of the gas outside and inside the sample, in the absence of any external forces. The deformation occurs as a result of pore compaction (expansion), resulting in a more (less) dense product material. To describe the evolution of porosity we derive an equation which allows us to define a characteristic time of deformation t_d. If t_d is sufficiently smaller than the characteristic time of combustion t_r, the deformation process is sufficiently fast to compensate for pressure gradients, so that pressure is equalized almost instantaneously, and filtration is suppressed. If $t_d > t_r$, deformation occurs solely in the product, and does not affect the propagation velocity. We determine various characteristics of a uniformly propagating combustion wave, and the materials produced by it, such as the propagation velocity, combustion temperature, final depth of conversion and final porosity of the product, as a function of the thermophysical parameters of the system. We also identify a regime of pulsating propagation, in which case the final porosity of the products is periodic in space. We show that both the uniformly propagating wave and the pulsating propagating wave solutions are stable, each corresponding to its own parameter regime. We also find that the deformation process can affect stability. In particular, the effect of viscosity is found to be stabilizing.

Two-Front Traveling Waves in Filtration Combustion - This paper considers a model of filtration combustion in a porous layer immersed in a bath of gaseous oxidizer at constant pressure p_o, which provides for exchange of the oxidizer between the pores, which have initial pressure p_o, and the bath. Such models describe the process of combustion synthesis of high-temperature materials. The case is considered when the exchange rate b_o between the pores and the bath, and p_o are both small, while the filtration coefficient b along the porous layer is large. Thus the model describes the process of combustion synthesis under low-pressure conditions. In this paper uniformly propagating solutions having two thin reaction zones in which the reaction rate is appreciable, spearated by an extended intermediate zone in which the reaction proceeds very slowly, are found. Approximate analytical methods are used to derive explicit analytical expressions for various quantities of interest, including the combustion temperature and the propagation velocity, and to describe the structure of the solution. Also, profiles for the reaction rate, temperature, pressure, density and depth of conversion are numerically computed, and the two-front structure is shown to be stable for the parameter values considered.

Slowly Varying Filtration Combustion Waves - We describe the slow evolution of the wave speed and reaction temperature in a model of filtration combustion. In the counterflow configuration of the process, a porous solid matrix is converted to a porous solid product by injecting an oxidizing gas at high pressure into one end of a fresh sample of the solid while igniting it at the other end. The solid and gas react exothermically at high activation energy, and, under favourable conditions, a self-sustaining combustion wave travels along the sample, converting reactants to product. Since the reaction rate depends on the gas pressure p in the pores, small gradients in p cause variations in the conditions of combustion, which, in turn, cause inhomogeneities in the physical properties of the product. We determine the slow evolution of the wave speed, the reaction temperature, and the mass flux of the gas downstream of the reaction zone. In the absence of a pressure gradient, there is a branch of steadily propagating solutions which has a fold. For planar disturbances on the slow time scale, we show that the middle part of the branch is unstable, with the change of stability occuring at the turning points of the branch. When the pressure gradient is non-zero, there are no steadily propagating solutions and the wave continually evolves. Conditions on the state of the gas at the inlet are described such that the variation in the wave speed and reaction temperature throughout the process can be minimized.

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