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ABSTRACT

The investigation of data falsification and/or diversion is of major concern in nuclear materials accounting procedures used in international safeguards. In this paper, two procedures, denoted by (D,MUF) and LR (Likelihood Ratio), are discussed and compared when testing the hypothesis that neither diversion nor falsification has taken place versus the one-sided alternative that at least one of these parameters is positive. Critical regions and detection probabilities are given for both tests. It is shown that the LR method outperforms (D,MUF) when diversion and falsification take place.

1. INTRODUCTION

The following is an excerpt from IAEA document INFCIRC/153, item 29.

> "...provides for the use of material accountancy as a safeguards measure of fundamental importance... ."

Materials accountancy is utilized by IAEA inspectors to review materials balance results obtained by a plant operator to investigate whether any diversion and/or falsification has taken place.

In case of diversion, the plant operator is likely to falsify his data in a manner that would make his materials balance appear valid. To "verify" the operator's results, the inspector must independently obtain observations on the same material measured by the operator. The success of uncovering any diversion and/or falsification relies upon a statistical evaluation of both operator and inspector data. In any analysis, it must be recognized that the operator has many strategies available that could conceal diversion, while the inspector has no *a priori* knowledge about the actual situation.

Many different statistical tests could be developed to test for diversion and/or falsification. It is the purpose of this paper to compare a test called (D,MUF), devised by Avenhaus and Beedgen [1], with the classical likelihood ratio (LR) test, which has been applied to this problem by Shipley [2] and Goldman, et al. [3]. The hypothesis of zero falsification and zero loss is tested against the one-sided alternative that loss and/or falsification have taken place. The comparison involves a study of critical regions, detection probabilities, and decision procedures.

2. (D,MUF) TEST

Consider the materials balance model

$$MUF_I = d + e_I \quad ; \quad MUF_O = d - f + e_O \quad ,$$

where O and I denote operator and inspector, respectively, d and f denote amounts of material diverted and falsified, and e_O and e_I are independently, normally distributed random variables having zero mean and known variances σ_O^2 and σ_I^2, respectively.

The hypothesis of concern, i.e., zero falsification and zero loss, can be written and treated in many different ways. One such way is given by the (D,MUF) approach, where

$$D = MUF_O - MUF_I \quad \text{and} \quad MUF = MUF_O \quad .$$

2.1. (D, MUF) Hypotheses

$$H_O^{'}: E(MUF) = 0 \quad ; \quad E(-D) = 0 \quad .$$

$$H_a^{'}: E(MUF) = d - f \geq 0 \quad ; \quad E(-D) = f \geq 0 \quad ; \quad d > 0 \quad .$$

A rectangular acceptance region for $H_O^{'}$ is depicted in Figure 1. An expression for thresholds s_1 and s_2 in terms of -D and MUF is given in Section 4. The decision to accept $H_O^{'}$ is made when $-D \leq s_1$ and $MUF \leq s_2$; otherwise $H_a^{'}$ is accepted.

3. LIKELIHOOD RATIO TEST

The LR test may be used to test H_0' by using a technique suggested by Kudo [5]. The (MUF_I,D) statistics are simplest to apply in the LR framework; consequently, a hypothesis expressing the expected value of $(MUF_I,-D)$ is given. The alternative hypothesis (H_a'') gives a rejection region different from H_a' and is consistent with Kudo's development.

4.1. MUF_I,D Hypotheses

$$H_0'': E(MUF_I) = d = 0 \quad ; \; E(-D) = f = 0 \quad .$$

$$H_a'': E(MUF_I) = d \geq 0 \quad ; \; E(-D) = f \geq 0 \quad ; \; d + f > 0 \quad .$$

The test statistic is given by

$$\bar{\chi}^2 = (MUF_I,-D)\Lambda^{-1}(MUF_I,-D)' - \min_{\substack{d\geq 0\\ f\geq 0}} [(MUF_I - d,-D - f)\Lambda^{-1}(MUF_I - d,-D - f)'] \quad ;$$

where

$$\Lambda = \begin{bmatrix} \sigma_I^2 & \sigma_I^2 \\ \sigma_I^2 & \sigma_O^2 + \sigma_I^2 \end{bmatrix} \quad .$$

$H_O''(H_O')$ is accepted if $\chi^2 \leq c^2$, where

c^2 is obtained from

$$1 - \alpha = P_{H_O''}\{\bar{\chi}^2 \geq c^2\} = 1 - \Phi(c) + \left[\frac{[\pi - \cos^{-1}\rho]}{2\pi}\right] e^{-c^2/2} \quad ,$$

$$\Phi(c) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{c} e^{-t^2/2}\, dt \quad \text{and} \quad \rho = \frac{\sigma_I}{\sqrt{\sigma_I^2 + \sigma_O^2}} \quad ,$$

where α denotes the false-alarm probability, $P_{H_O''}$ denotes the probability under H_O'', and c is a positive number.

5. ACCEPTANCE REGIONS AND DETECTION PROBABILITIES

Figures 2a,b,c depict critical regions for both LR and (D,MUF) tests under H_O', where $\alpha = 0.05$ and $(\sigma_O,\sigma_I) = (1,1)$, (1,2), and (1,4).

The (D,MUF) acceptance region is determined by $MUF_O \leq s_2$ and $MUF_I \leq s_1$, where s_1 and s_2 are expressed by

$$0.95 = P_{H_O'}\{-D \leq s_1, MUF \leq s_2\} \quad .$$

Complete details are given by Beedgen and Hafer [4].

The LR acceptance region is determined from the disjoint parts given by

(1) $MUF_O \geq -c$ for $MUF_I \leq 0$, $MUF_O \leq 0$;

$$(2) \quad \frac{MUF_I^2}{k^2} + MUF_O^2 \leq c^2 \quad ; \quad \text{for} \begin{cases} -c \leq MUF_O \leq 0 \ , \quad 0 \leq MUF_I \leq k\sqrt{c^2 - MUF_O^2} \ ; \\ 0 \leq MUF_O \leq MUF_I \ , \quad 0 \leq MUF_I \leq ck/\sqrt{1 + k^2} \ ; \\ \text{and} \\ 0 \leq MUF_O \leq \sqrt{(c^2k^2 - MUF_I^2)/k^2} \ , \\ ck/\sqrt{1 + k^2} \leq MUF_I \leq ck \ . \end{cases}$$

$$(3) \quad MUF_I + k^2 MUF_O \leq ck\sqrt{1 + k^2} \ \text{for} \begin{cases} 0 \leq MUF_I \leq ck\sqrt{1 + k^2}, \\ MUF_I \leq MUF_O < (ck\sqrt{1 + k^2} - MUF_I)/k^2; \\ \text{and} \\ -\infty < MUF_I \leq 0, \\ 0 \leq MUF_O \leq (ck\sqrt{1 + k^2} - MUF_I)/k^2. \end{cases}$$

where c = 2.183, 2.216, and 2.229 for $k = \sigma_I/\sigma_O = 1, 2,$ and 4, respectively. An analytical solution for c was used in the computation of $\bar{\chi}^2$.

Examples

Consider the following two hypothetical cases of operator and inspector measurements that were obtained for quantities of nuclear materials.

Case 1:

$\sigma_O = \sigma_I = 1.0$ kg , $\alpha = 0.05$, $MUF_O = 1.0$ kg , and

$MUF_I = 3.0$ kg .

It may be seen from Fig. 2a that the (D,MUF) test would accept H_O, whereas the LR test would accept H_a, i.e., the LR test would state that falsification and/or loss has taken place and the (D,MUF) test would disagree.

Case 2:

$\sigma_O = \sigma_I = 1.0$ kg , $\alpha = 0.05$, $MUF_O = 3.0$ kg , and

$MUF_I = -3.0$ kg .

Using Fig. 2a, the (D,MUF) test would reject H_O but the LR test would accept H_O. This example would indicate an estimated loss of -3 kg, i.e., an excess! The operator would have an estimated falsification of 6 kg. This event is of possible concern but probably less so than the previous example.

Detection probabilities for positive values of d and f ($f < d$) are given in Table I for $(\sigma_O,\sigma_I) = (1,1)$, (1,2), and (1,4). The LR gives far better protection than (D,MUF) in this region.

6. CONCLUSIONS AND SUMMARY

Results of this paper clearly indicate the likelihood ratio procedure should be considered in the analysis of operator-inspector differences. For example, the LR approach gave larger detection probabilities than (D,MUF) in the region $f \leq d$ under the hypothesis of zero diversion and zero falsification. It appears that LR would give even higher detection probabilities if the H_a were expressed as $f \leq d$, a matter for future investigation.

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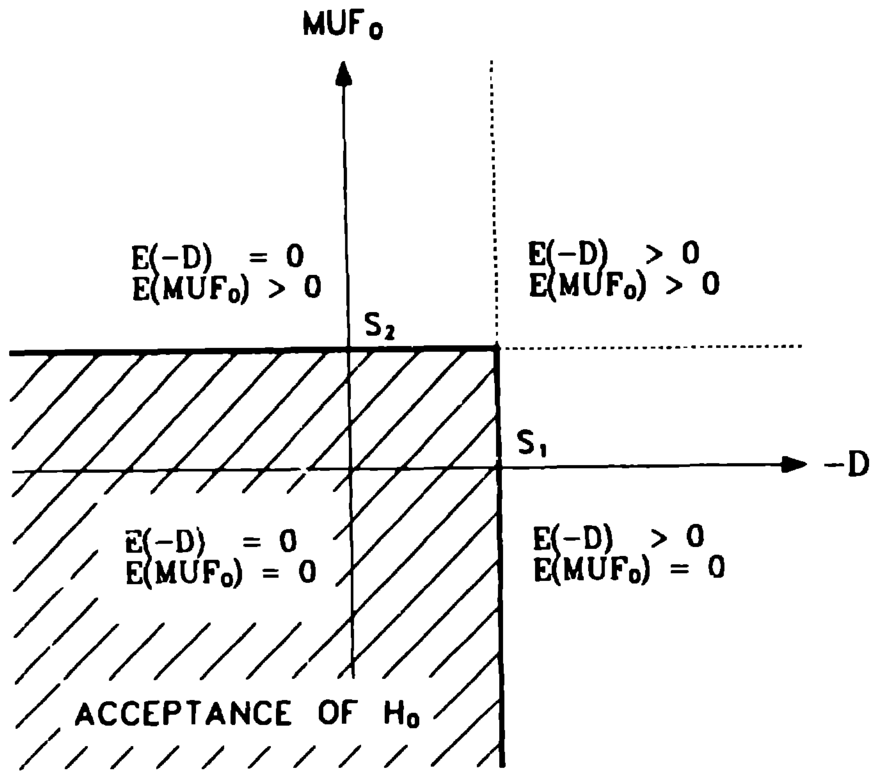

FIG. 1. Acceptance and rejection regions for the (D,MUF) test.

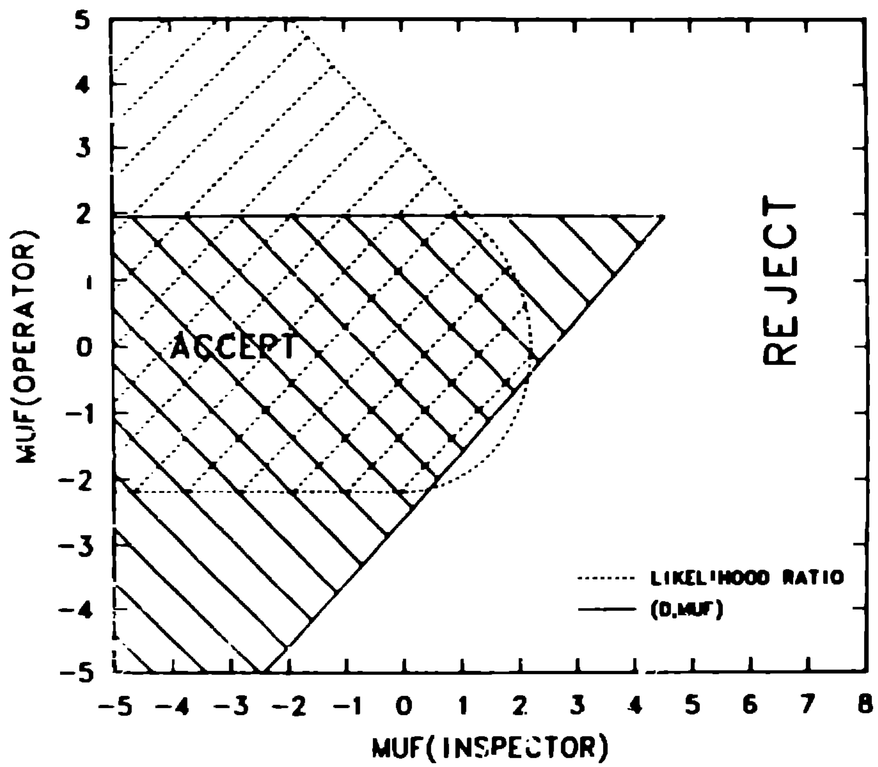

FIG. 2a. Comparison of critical regions between likelihood ratio and (D,MUF) tests for $\sigma_O = \sigma_I = 1.0$.

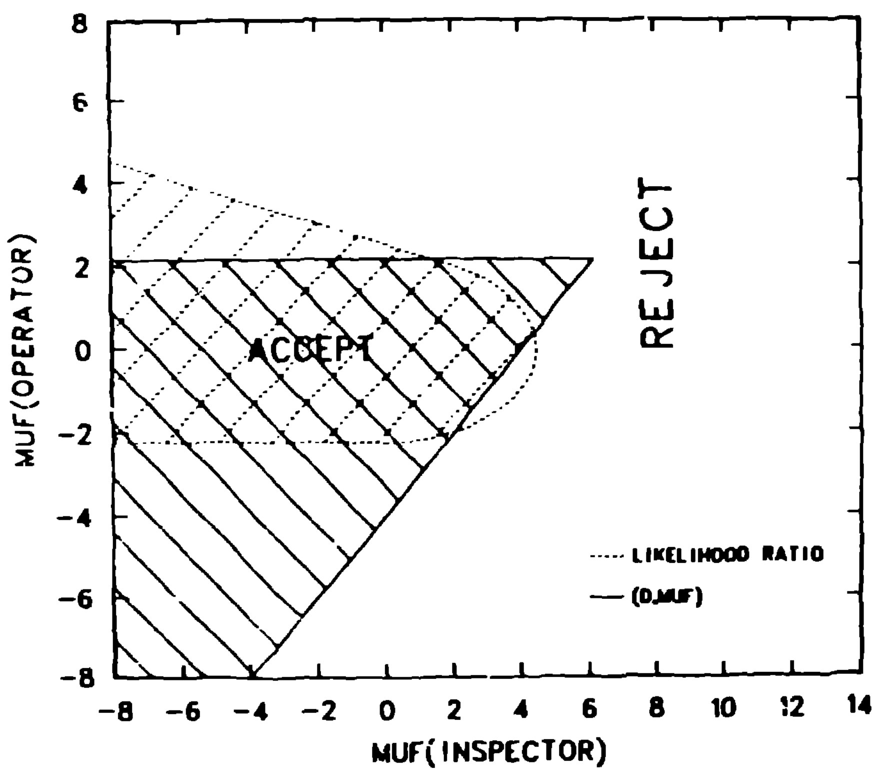

FIG. 2b. Comparison of critical regions between likelihood ratio and (D,MUF) for $\sigma_O = 1.0$, $\sigma_I = 2.0$.

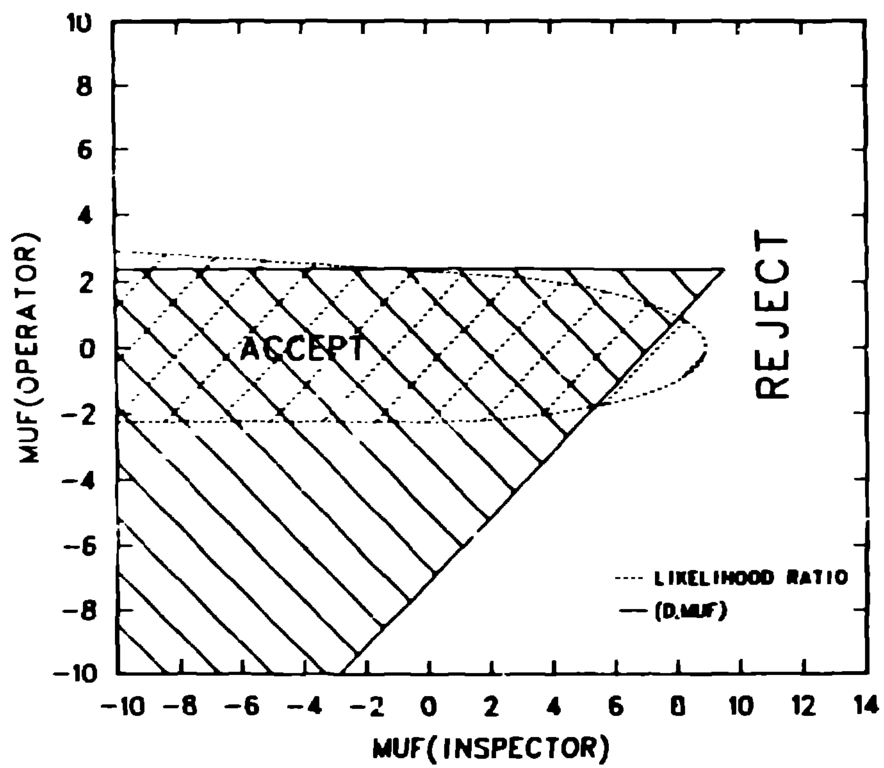

FIG. 2c. Comparison of critical regions between likelihood ratio and (D,MUF) for $\sigma_O = 1.0$, $\sigma_I = 4.0$.

TABLE 1: Detection Probabilities for (D,MUF) and Maximum Likelihood Ratio (LR) Methods $\sigma_O = 1$, $\sigma_I = k$

		k = 1		k = 2		k = 4	
d	f	(D,MUF)	LR	(D,MUF)	LR	(D,MUF)	LR
0	0	0.05	0.05	0.05	0.05	0.05	0.05
1	0	0.18	0.27	0.15	0.18	0.12	0.16
1	1	0.14	0.19	0.10	0.09	0.08	0.07
2	0	0.52	0.78	0.45	0.56	0.38	0.48
2	1	0.22	0.61	0.19	0.29	0.14	0.19
2	2	0.34	0.53	0.19	0.18	0.11	0.09
3	0	0.85	0.99	0.81	0.89	0.74	0.83
3	1	0.53	0.94	0.47	0.68	0.39	0.53
3	2	0.38	0.88	0.26	0.43	0.18	0.24
3	3	0.62	0.85	0.33	0.33	0.16	0.13
4	0	0.98	1.00	0.97	0.99	0.95	0.98
4	1	0.86	1.00	0.81	0.94	0.75	0.86
4	2	0.58	0.99	0.50	0.79	0.41	0.58
4	3	0.63	0.98	0.38	0.60	0.22	0.29
4	4	0.84	0.97	0.50	0.51	0.23	0.18
5	0	1.00	1.00	1.00	1.00	1.00	1.00
5	1	0.98	1.00	0.97	1.00	0.95	0.98
5	2	0.86	1.00	0.82	1.00	0.75	0.88
5	3	0.70	1.00	0.56	0.89	0.44	0.63
5	4	0.84	1.00	0.53	0.75	0.28	0.36
5	5	0.96	1.00	0.68	0.69	0.31	0.25
6	0	1.00	1.00	1.00	1.00	1.00	1.00
6	1	1.00	1.00	1.00	1.00	1.00	1.00
6	2	0.98	1.00	0.97	1.00	0.95	0.99
6	3	0.88	1.00	0.83	0.99	0.76	0.90
6	4	0.86	1.00	0.65	0.94	0.47	0.68
6	5	0.96	1.00	0.69	0.87	0.35	0.43
6	6	0.99	1.00	0.82	0.84	0.40	0.32