~~CESAR-90~~/03

AN ALTERNATIVE METHOD TO SOLVING THE KINEMATICS OF A REDUNDANT ROBOT*

Philippe Belmans and Jean-Christophe Culioli
Center for Engineering Systems Advanced Research
Oak Ridge National Laboratory
P.O. Box 2008
Building 6025, MS-6364
Oak Ridge, TN 37831-6364, USA

CONF-900608--10

DE90 006892

Invited Paper: American Nuclear Society, 1990 Annual Meeting, Nashville, Tennessee, June 10–14, 1990

* Research sponsored by the Engineering Research Program of the Office of Basic Energy Sciences, of the U.S. Department of Energy, under contract No. DE-AC05-84OR21400 with Martin Marietta Energy Systems, Inc.

An Alternative Method to Solving the Kinematics of a Redundant Robot

Philippe Belmans and Jean-Christophe Culioli

Center for Engineering Systems Advanced Research
Oak Ridge National Laboratory
P.O. Box 2008
Building 6025, MS-6364
Oak Ridge, TN 37831-6364

1 Introduction

The state occupied by an m degrees of freedom robot is determined by an articular joint position-vector q of R^m, but usually the specification of a task by its task-vector X takes place in the operational space R^n. Controlling the kinematics of a robot requires to find q as a function of X. In most of the cases, it is impossible to obtain this relationship analytically and globally. The problem is solved by linearization of the geometric model of the robot $X = F(q)$ in the neighborhood of the point q, introducing the $(n \times m)$ Jacobian matrix:

$$\Delta X = J(q)\Delta q; \Delta q \in R^m; \Delta X \in R^n \tag{1}$$

where ΔX is the variation of task-vector, Δq is the corresponding small variation of the articular position. For a fixed q and for a specified ΔX, the set of solutions for vector Δq is an affine space of dimension $m - n$, which we shall denote by $\mathcal{E}$.

In the case of redundant arms ($m > n$), the Jacobian matrix is not square and cannot be inverted. One method for determining the Δq of least norm that satisfies equation(1) [1] leads to the computation of the Moore-Penrose pseudo-inverse of J [2], namely $J^T(JJ^T)^{-1}$.

Nevertheless, the numerical computation of the pseudo-inverse is often very ill conditioned (small determinants, large differences between various elements, etc) [3]. Furthermore, the Singular Value Decomposition technique is too slow for a real-time implementation and raises difficulties in numeric computation [4].

Some other algorithms avoid the computation of the Moore-Penrose pseudo-inverse and lead to well-behaved computations (e.g. Gradient Projection Method to a 7-d.o.f. arm [5], combinatorial approach [6]). However, the solution obtained is not necessarily the least norm one.

2 The Proposed Algorithm

We suggest an algorithmwhich exploits the linear properties of the coordinate transformation in the neighborhood of the point q expressed by the Jacobian matrix J and its $s = C_m^n$ sub-matrices $J_1, J_2, \ldots, J_k, \ldots, J_s$ of dimension $n \times n$. Specifically, a submatrice J_k is formed by blocking columns $i_1, i_2, \ldots, i_{m-n}$ in J. For each invertible J_k, we compute the n-dimensional vector Δq_k such that:

$$\Delta q_k = J_k^{-1} \Delta X, (\Delta q_k \in R^n, k \in \{1, \ldots, s\}). \tag{2}$$

Without changing its name, we can rewrite this vector Δq_k as an m-dimensional vector in the articular space, setting the $(m-n)$ complementary components of positions $i_1, i_2, \ldots, i_{m-n}$ to 0 in the corresponding m-dimensional vector Δq_k.

We then consider a maximal set of p independent vectors $\Delta q_k, k = 1, \ldots, p$ and introduce E, the affine space of dimension $dim(E) = p - 1$ spanned by this family of vectors:

$$E = \left\{ \Delta Q \in R^m \mid \Delta Q(t_1, \ldots, t_p) = t_1 \Delta q_1 + \cdots + t_p \Delta q_p; \sum_{k=1}^{p} t_k = 1 \right\}. \tag{3}$$

E is a subspace of the affine space $\mathcal{E}$ (of dimension $m - n$). We show that, if $p - 1 = m - n$, the algorithm gives the least norm vector according to the parameterization (3)

$$\Delta Q = \sum_{k=1}^{p} t_k^* \Delta q_k. \tag{4}$$

where:

$$t^* = (t_1, \ldots, t_p)^T = \frac{G^{-1} e}{e^T G^{-1} e} \tag{5}$$

and where G is the Gramian $p \times p$ symmetric and invertible matrix such as $g(i,j) = < \Delta q_i, \Delta q_j >$ and e is the p-dimensional vector$(1, 1, \ldots, 1, 1)^T$.

In conclusion, $(m-n+1)$ independent solutions of (1) suffice to compute the least norm ΔQ. Moreover, the advantage of this algorithm over the Pseudo-Inverse Algorithm is that the square submatrices can be tested for ill-conditioning or singularity.

3 Simulation and Conclusion

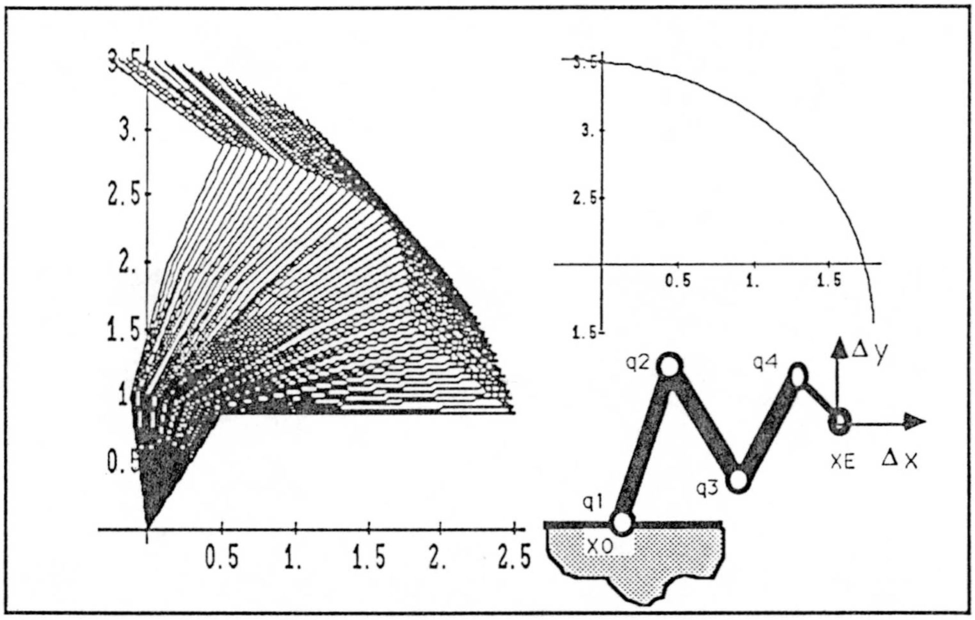

Figure 1: Arm and the end effector positions obtained by simulation.

We have applied the algorithm to the planar arm illustrated in the Fig.1, using the symbolic manipulation environment Mathematica [7] on Macintosh II. More detail of the simulation can be found in [8]. We specify $\Delta X = (\Delta x, \Delta y)$ the translational position of the robot tip at point X_E. Since we have 4 degrees of freedom and the task space is 2-dimensional, we need $(4-2+1) = 3$

solutions Δq_k. Fig.1 illustrates the motion of the arm when the end-effector follows a circular trajectory containing fifty elementary displacements ΔX.

This scheme can be efficiently parallelized and contains well-behaved matrices for numerical processing with a computer. The future work that we envision is the generalization of the algorithm to other criteria.

References

[1] A. Liegeois, " Automatic Supervisory Control of the Configuration and Behavior of Multibody Mechanisms", IEEE Trans. Syst Man, Cybern., Vol. SMC-7, N. 12, 1977. pp 868-871.

[2] R. Penrose, "A Generalized Inverse for Matrices", Proc. Cambridge Phil. Soc., Vol 51, 1955. pp 406-413.

[3] V. Klema and A. Laub, "The Singular Value Decomposition : its Computation and Some Applications", IEEE Trans. on Automatic Control, Vol. AC-25, N. 2, 1980. pp164-176.

[4] A. Maciejewski and C. Klein, "Obstacle Avoidance for Kinematically Redundant Manipulators in Dynamically Varying Environments", International Journal of Robotics Research, Vol. 4, N. 3, Fall 1985. pp109-117.

[5] R. Dubey, J. Euler and S. Babcock, "An Efficient Gradient Projection Optimization Scheme for a Seven-Degree-of-Freedom Redundant Robot with Sperical Wrist", IEEE Int. Conf. on Robotics and Automation, Philadelphia, PA, USA, April 24-29,1988. pp 28-36.

[6] S. Hirose and S. Ma, "Redundancy Decomposition Control for Multi-Joint Manipulator", Proc. of the IEEE Int. Conf. on Robotics and Automation, Scottsdale, AZ, USA, May 14-19, 1989. pp 119-124.

[7] S. Wolfram, "Mathematica - A System for Doing Mathematics by Computer", Addison-Wesley Publishing Company, August 1988.

[8] P. Belmans and J.C. Culioli, "A New Approach to Solve the Kinematics Resolution of a Redundant Robot", technical memo ORNL/TM-11435 CESAR-89/53, Oak Ridge National Laboratory, TN, USA, February 1990.