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The SU(3)-Nambu-Jona-Lasinio Soliton in the Collective Quantization Formulation

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The SU(3)-Nambu–Jona-Lasinio soliton in the collective quantization formulation

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ABSTRACT

On grounds of a semibosonized Nambu–Jona-Lasinio model, which has $SU(3)_R \otimes SU(3)_L$-symmetry in the chiral limit, mass splittings for spin 1/2 and spin 3/2 baryons are studied in the presence of an explicit chiral symmetry breaking strange quark mass. To this aim these strangeness carrying baryons are understood as $SU(3)$-rotational excitations of an $SU(2)$-embedded soliton solution. Therefore, within the framework of collective quantization, the fermion determinant with the strange quark mass is expanded up to the second order in the flavor rotation velocity and up to the first order in this quark mass. Besides the strange and non-strange moments of inertia, which have some counterparts within the Skyrme model, some so-called anomalous moments of inertia are obtained. These can be related to the imaginary part of the effective Euclidian action and contain among others the anomalous baryon current. This is shown in a gradient expansion up to the first non-vanishing order. Together with the Σ-commutator these are the solitonic ingredients of the collective hamiltonian, which is then diagonalized by means of strict perturbation theory in the strange quark mass and by the Yabu-Ando method. Both methods yield very good results for the masses of the spin 1/2 and 3/2 baryons. The former one reproduces some interesting mass formulas of Gell-Mann Okubo and of Guadagnini and the latter one is able to describe the mass splittings up to a few MeV.

1. INTRODUCTION

Quantumchromodynamics (QCD) is assumed to be the theory of the strong interactions though it could be proved only in the high energy regime thanks to the asymptotic freedom [1]. For the low energy regime, which is responsible for the properties of mesons and baryons, many phenomenological models, which should mimic at least some of the QCD features, were developped [2-7]. Especially the Nambu-Jona-Lasinio (NJL) [4] model, and closely related, the chiral quark model of *Dyakonov* and *Petrov* [8] gained a lot of interest, because it show the important feature of spontaneous breaking of chiral symmetry and they can be regarded as some long-wavelength expansion of QCD [8-12]. This breaking of the chiral symmetry proceeds in this model as a result of the fermion-loop contribution to the effective potential and is therefore of dynamical origin.

For the symmetry group $SU(2)_L \otimes SU(2)_R$ this model has been extensively studied in the past and it led to a satisfactory description of mesons [13] and baryons [14-18]. However it suffered from non-renormalizability and initially there was the problem of how to choose a suitable regularization scheme. If it is 'renormalized' in the fermion loop approximation the model suffers from a vacuum instability paradox [19], which can be avoided and furthermore it could be shown [20, 21], that the regularized theory is then rather insensitive to the special regularization scheme.
In the approximation where the meson fields are classical the model allows for solitonic solutions which are selfconsistent fields of the equation of motion. Because these configurations have no good spin and isospin, one has to evoke a collective quantization [22] in order to get the splitting of nucleon and delta. This has been done in the past theoretically [23] and numerically [15, 18] in the $SU(2)$-sector.

In the present case of $SU(3)_R \otimes SU(3)_L$-symmetry an embedding of the $SU(2)$-soliton in the isospin subgroup of $SU(3)$ [24] was frequently used within the Skyrme model [25-30] in the chiral quark model [31] and in chiral bag models [32]. The embedding rather than a full solitonic $SU(3)$-calculation ensures that only special triality zero representations survive, which correspond to those in nature, where the right hypercharge is restricted to one. Then one can treat the symmetry breaking either perturbatively in the current quark mass or one diagonalizes the collective hamiltonian exactly to all orders of the current masses. The latter treatment was proposed by Yabu-Ando because the perturbation theory leads to an unsatisfactory description for the baryons within the Skyrme model. On the other hand, the Yabu-Ando approach seems to interpolate between the perturbative treatment and the treatment proposed by *Callan* and *Klebanov* [33]. These authors described the strange baryons as bound states of the corresponding heavy mesons in the presence of a $SU(2)$-field configuration. So the rotational Ansatz is given up, but the results are still unsatisfactory. Altogether the various attempts within the $SU(3)$-Skyrme model with scalar mesons were not very successful and only the consideration of vector mesons improved the picture [34, 35].

In this paper we will show that the $SU(3)$-rotated Nambu-Jona-Lasinio model with scalar couplings is able to describe the masses of the spin 1/2 and 3/2 baryons by a trivial embedding of the $SU(2)$-soliton in $SU(3)$. The basic procedure consists in expanding the fermion-determinant perturbatively in terms of the strange current mass. This and the fact, that the NJL-model has valence quarks, provides a formalism different from that of the Skyrme model. The final results are good and show that the approach is quite satisfactory. Some preliminary results have already been published in [36]. Calculations in a similar formalism can be found in [37].

The procedure in the paper is as follows. In Sect.2 we review the $SU(3)$-NJL model and the corresponding vacuum in the perturbative treatment of the strange quark mass. Then we outline the rotational Ansatz in Sect.3, which leads to a rotational Lagrangian after expanding the fermion determinant in terms of the rotational velocity and the symmetry breaking. Therefore we take care to have the symmetry breaking parameters within the fermion determinant. The ingredients of the rotational Lagrangian, namely the anomalous and non-anomalous moments of inertia, are described in Sect.4. Using well-known quantization prescriptions, we derive in Sect. 5 the collective hamiltonian for the strict perturbative treatment. Then Sect.6 deals with the Yabu-Ando collective hamiltonian, in which the group generators are expressed in terms of collective coordinates. After discussing the numerical results in Sect.7 we summarize and discuss our results in Sect.8.

2. THE SU(3)-NAMBU–JONA-LASINIO MODEL

The $SU(3)_R \otimes SU(3)_L$-invariant NJL-Lagrangian with scalar and pseudoscalar coupling terms reads:

$$\mathcal{L}_{NJL} = \bar{q}(x)(i\partial\!\!\!/ - m)q(x) - \frac{G}{2}\left[(\bar{q}(x)\lambda^a q(x))^2 + (\bar{q}(x)i\gamma_5\lambda^a q(x))^2\right] \tag{1}$$

where $m = \mathrm{diag}(m_u, m_d, m_s) = m_1 \mathbf{1} + m_2\lambda_3 + m_3\lambda_8$ is the quark mass matrix and λ_a are the usual Gell-Mann matrices with $\lambda^0 = \sqrt{(\frac{2}{3})}\mathbf{1}$. In the following we neglect isospin breaking and set $(m_u - m_d)/2 = m_2 = 0$. Using the path-integral bosonization of Eguchi [38], we arrive at

$$\mathcal{L}_{NJL} = \bar{q}(x)(i\partial\!\!\!/ - m)q(x) - g\bar{q}(x)(\sigma^a\lambda^a + i\gamma_5\pi^a\lambda^a)q(x) + \frac{\mu^2}{2}(\sigma^a\sigma^a + \pi^a\pi^a) \tag{2}$$

with $G = \frac{g^2}{\mu^2}$. One should note that we have chosen to take the current quark masses $m_0 = \frac{1}{2}(m_u + m_d)$ and m_s in the quark terms and *not* in a local bosonic term proportional to σ^0 and σ^8. For a pure $SU(2)$-soliton it does not matter very much whether one treats $m_0 \neq 0$ in the fermion determinant or as an explicit boson field term, because m_0 is only a few MeV [39]. For m_s, which is around $150MeV$, it matters a lot and we have to treat it in the fermion determinant. Only in this way we retain the dynamical content of the current masses, which will play an important role for the anomalous sector as we will see in the following.
After integrating out the quarks the effective Euclidean action reads

$$S_{eff} = -\mathrm{Sp}\,\log\left(i\partial\!\!\!/ - m - g(\sigma^a\lambda^a + i\gamma_5\pi^a\lambda^a)\right) + \frac{\mu^2}{2}(\sigma^a\sigma^a + \pi^a\pi^a). \tag{3}$$

Because this expression contains divergent terms, which can be seen e.g. in a gradient expansion, it has to be regularized in a proper way. In Ref. 20 it was shown, that the solitonic observables are not very sensitive to the regularization scheme, so that we choose here for convenience the proper-time regularization [40]. In principle one has to apply this regularization from the very beginning but we will drop it here only for clarity. In the vacuum sector chiral symmetry is in the chiral limit spontaneously broken down by the non-vanishing vacuum expectation value (VEV) of σ_0. Due to the strange quark mass, which breaks flavor $SU(3)$, there is in general an additional non-vanishing σ_8. Both are determined from the effective potential

$$V_{eff} = -\mathrm{Tr}\int \frac{d^4k}{(2\pi)^4} \log(\not{k} + m + g(\sigma_a\lambda_a + i\gamma_5\pi_a\lambda_a)) + \frac{\mu^2}{2}(\sigma_a^2 + \pi_a^2) \tag{4}$$

by requiring a stationary phase condition

$$\left.\frac{dV_{eff}}{d\sigma_0}\right|_{vac} = \left.\frac{dV_{eff}}{d\sigma_8}\right|_{vac} = 0. \tag{5}$$

This leads after some decoupling of the scalar fields to

$$\mu^2 = 8N_c g^2 I_1(M_u) + \frac{\mu^2 m_0}{M_u} \tag{6}$$

$$\mu^2 = 8N_c g^2 I_1(M_s) + \frac{\mu^2 m_s}{M_s} \tag{7}$$

where we set $M_u = \sqrt{\frac{2}{3}}g\sigma_{0v} + m_1 + \frac{1}{\sqrt{3}}(g\sigma_{8v} + m_3)$ and $M_s = \sqrt{\frac{2}{3}}g\sigma_{0v} + m_1 - \frac{2}{\sqrt{3}}(g\sigma_{8v} + m_3)$ and

$$I_1(M_i) = \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 + M_i^2} \tag{8}$$

In order to derive the meson masses from the effective potential, we have to rescale the fields from the normalization of the two-point function, which contains the kinetic energy for the mesons in leading order. We find $Z_\pi = 4N_c g^2 I_2(M_u, M_u, q^2 = -m_\pi^2)$ and $Z_K = 4N_c g^2 I_2(M_u, M_s, q^2 = -m_K^2)$ where we have denoted

$$I_2(M_i, M_j, q^2) = \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 + M_i^2}\frac{1}{(k+q)^2 + M_j^2} \tag{9}$$

The new fields $\pi_a' = Z_\pi^{1/2}\pi_a, a = 1,2,3$ and $\pi_a' = Z_K^{1/2}\pi_a, a = 4,5,6,7$ then have the correct normalization, such that the masses can be obtained from the effective action via

$$\frac{1}{Z_\pi}\frac{1}{\delta^4(0)}\left.\frac{\delta^2 S_{eff}}{\delta\pi_a(p)\delta\pi_a(-p)}\right|_{p^2=-m_\pi^2} = (p^2 + m_\pi^2)\Big|_{p^2=-m_\pi^2}, \quad a = 1,2,3 \tag{10}$$

$$\frac{1}{Z_K}\frac{1}{\delta^4(0)}\left.\frac{\delta^2 S_{eff}}{\delta\pi_a(p)\delta\pi_a(-p)}\right|_{p^2=-m_K^2} = (p^2 + m_K^2)\Big|_{p^2=-m_K^2}, \quad a = 4,5,6,7 \tag{11}$$

This gives

$$m_\pi^2 = \frac{1}{Z_\pi}\frac{\mu^2 m_0}{M_u} \tag{12}$$

$$m_K^2 = \frac{1}{Z_K}\frac{\mu^2}{2}\left(\frac{m_0}{M_u} + \frac{m_s}{M_s}\right) + (M_u - M_s)^2 \tag{13}$$

It remains to fix the weak decay constants f_π and f_K by the matrix elements of the axial current A_μ for the pion and kaon decay. From the general form, e.g. $< 0 \mid A_\mu^a \mid \pi^a(p) >= -ip_\mu f_\pi \pi^a(p)$, a=1,2,3, we deduce

$$f_\pi = \frac{M_u}{g} Z_\pi^{1/2}, \quad f_K = \frac{M_s + M_u}{2g} Z_K^{1/2} \tag{14}$$

For consistency with the baryon sector we determine the vacuum sector also in perturbation with respect to the current masses. This gives in leading order for the meson mass ratio

$$\frac{m_K^2}{m_\pi^2} = \frac{m_s + m_0}{2m_0}. \tag{15}$$

Using $f_\pi = 93MeV$, $m_\pi = 139MeV$ and $m_K = 496MeV$ as input parameters, we require furthermore $Z_\pi = 1$ which is eqivalent to imposing the PCAC relation. From (14) we deduce $\sigma_{0v} = f_\pi$ and from (6, 12) follows the value of μ^2 and m_0. In this approximation is $f_K = f_\pi$ and $M_u = M_s = M$ and m_s follows from (15). Within the proper time regularization the $I_n(M)$ integrals are replaced by

$$I_n(M) = \frac{4}{16\pi^2}\frac{1}{\Gamma(3-n)}\int \frac{d\tau}{\tau}\phi(\tau)e^{-\tau M^2}. \tag{16}$$

where $\phi(\tau)$ is a suitable damping function. We choose

$$\phi(t) = c\,\theta(t - 1/\Lambda_1^2) + (1-c)\theta(t - 1/\Lambda_2^2).$$

which contains Λ_1, Λ_2 and c as free parameters. With imposing the strange quark mass of $m_s = 150MeV$, eq. (15) gives $m_0 = 6.13MeV$. For a given Λ_2 the Λ_1 and c are determined from the $Z_\pi = 1$-condition

$$4N_c M^2 I_2(M) = f_\pi^2 \tag{17}$$

and from (6). Then we choose the ratio $\Lambda_2/M = 3.4$ which is very close to the lowest possible value to fulfill eqs. (17, 6) (see Tab.1) simultaneously. We want to stress however that the soliton itself do not to depend on this choice very much. We find for the condensates of the vacuum

$$< uu >_0^{1/3} = < dd >_0^{1/3} = < ss >_0^{1/3} = -\left(\frac{m_\pi^2 f_\pi^2}{m_0}\right)^{1/3} = -238MeV \tag{18}$$

and the only free parameter in our model is the constituent quark mass $M = M_u = M_d = M_s$.

In order to get the classical soliton solution we evaluate the effective action for time-independent fields and obtain [14, 11] in the chiral limit:

$$M_{cl} = N_c\left(E_{val}(U) + E_{sea}(U)\right) \tag{19}$$

where E_{val} is the bound state level which comes from the upper Dirac continuum and E_{sea} is the sum over all states of the upper and lower continuum. Within the proper time regularization one obtains

$$E_{sea}(U) = \frac{1}{4\sqrt{\pi}} \int_0^{\infty} \frac{d\tau}{\tau^{3/2}} \phi(\tau) \sum_n \left(e^{-E_n^2 \tau} - e^{E_n^{(0)2} \tau} \right) \tag{20}$$

where $E_n^{(0)}$ are the energies of the free hamiltonian. This subtraction procedure corresponds to the infinite vacuum energy, which is otherwise eliminated by normal ordering. Requiring that the field U in eq. (19) is a minimum of the energy leads to a set of equations for the chiral field, which can be solved iteratively. The results are presented in Tab.2 together with the isoscalar quadratic radius as functions of the constituent quark mass.

3. The collective quantization

In order to quantize the soliton solution we will consider fluctuations in the symmetry modes (collective coordinates) of the hamiltonian. Therefore we embed the $SU(2)$-soliton, which is also a classical solution in $SU(3)$, in the isospin subgroup of $SU(3)$ [24]. We obtain for the effective action

$$S_{eff} = -\mathrm{Sp}\,\log(i\partial\!\!\!/ - m - MU_2) + \frac{1}{4}\mu^2 f_\pi^2 \int d^4x \mathrm{tr}(U_2^+(x)U_2(x)) \tag{21}$$

with

$$U_2 = \begin{pmatrix} U_0 & 0 \\ 0 & 1 \end{pmatrix} \tag{22}$$

and $U_0 = (\sigma_{(2)} + i\gamma_5 \pi\tau)/f_\pi$, where the $SU(2)$-field $\sigma_{(2)} = \sigma_0\sqrt{\frac{2}{3}} + \frac{1}{\sqrt{3}}\sigma_8$ is then a linear combination of σ_0 and σ_8 and these fields themselves are constraint by $\sigma_0\sqrt{\frac{2}{3}} - \frac{2}{\sqrt{3}}\sigma_8 = f_\pi$. In the following we will assume the system to have a hedgehog form and to obey the chiral circle $\sigma_{(2)}^2 + \pi^2 = f_\pi^2$ because only then the stability of the soliton in guaranteed [42]. Since the fermion determinant without symmetry breaking leads to another equivalent solution after substituting $U_2(\mathbf{x}) \rightarrow AU_2(\mathbf{x})A^+$, where A is a $3 \otimes 3$ rotation matrix, the collective quantization then proceeds by extending A to a function of time $A(t)$ [22]. This is the new collective variable which after the quantization provides us with the proper $SU(3)$ quantum numbers of the baryons. On the chiral circle the local mesonic mass term vanishes and hence the rotated effective Euclidean action becomes[1] :

$$S_{eff} = -N_c T \int \frac{d\omega}{2\pi} \mathrm{Sp}\,\log(i\omega + H - i\gamma_4 A^+(t) m A(t) + A^+(t)\dot{A}(t)) \tag{23}$$

where the hermitian hamiltonian H is given by

$$H = -i\gamma_4(i\partial_i\gamma_i + MU_2(x)) \tag{24}$$

[1] For the general structure we refer the reader to Ref. 22.

Note that the effective action itself is not hermitian, such that in the subsequent expansion in $A^+(t)\dot{A}(t)$ we can expect contributions from the imaginary part of the Euclidean action. We assume then that in the Maurer Cartan form [13]

$$A^+\dot{A} = \dot{q}_\alpha A^+\partial_\alpha A = \frac{i}{2}\dot{q}_\alpha C_\alpha{}^A\lambda_A = \frac{i}{2}\Omega_A\lambda_A \tag{25}$$

where the q_α are the coordinates of $SU(3)$ and the $C_\alpha{}^A$ are the vielbeins, the 8 angular velocities $\Omega_A = -i\mathrm{Tr}(A^+\dot{A}\lambda_A)$ are time-independent[2].

Now we expand the effective action (23) in Ω_A and m_s, which will be considered as small parameters. Up to the second order in Ω_A and zeroth order in m_s we obtain

$$L^{rot} = \frac{1}{2}I_{AB}\Omega_A\Omega_B - \frac{N_c}{2\sqrt{3}}\Omega_8 B(U) \tag{26}$$

with $B(U)$ indicating the baryon number of the system with field configuration U. The I_{AB} is the $SU(3)$ tensor of the moment of inertia

$$I_{AB} = \frac{N_c}{4}\int\frac{d\omega}{2\pi}\mathrm{Tr}\left[\frac{1}{i\omega+H}\lambda_A\frac{1}{i\omega+H}\lambda_B\right]. \tag{27}$$

After choosing the sector of the model with baryon number $B=1$ and applying the proper time regularization scheme, it splits into a *valence* part

$$I_{AB}^{val} = \frac{N_c}{2}\sum_{n\neq val}\frac{<n\mid\lambda_A\mid val><val\mid\lambda_B\mid n>}{E_n - E_{val}} \tag{28}$$

and a sea part

$$I_{AB}^{sea} = \frac{N_c}{4}\sum_{m\neq n}\frac{<m\mid\lambda_A\mid n><n\mid\lambda_B\mid m>}{E_m+E_n}\mathcal{R}_I(E_n,E_m) \tag{29}$$

with

$$\mathcal{R}_I(E_n,E_m) = -\frac{1}{2\sqrt{\pi}}\int_0^\infty\frac{dt}{\sqrt{t}}\phi(t)\left[E_n e^{-tE_n^2} + E_m e^{-tE_m^2} + \frac{e^{-tE_n^2}-e^{-tE_m^2}}{t(E_n-E_m)}\right] \tag{30}$$

and where $\mid n>$ and E_n are the eigenfunctions and eigenvalues of the Hamilton operator H of eq. (24). Because of the embedding (22) we have in fact

$$I_{AB} = \begin{cases} I_1\delta_{AB} & \text{for A,B=1,2,3} \\ I_2\delta_{AB} & \text{for A,B=4,5,6,7} \\ 0 & \text{for A,B=8} \end{cases} \tag{31}$$

In the first order in Ω_A only the Ω_8-term contributes, which is due to the hedgehog structure and trivial embedding of the $SU(2)$-group. The appearance of the Ω_8-term in eq. (26) is dependent on the baryon number of the system and therefore entirely due to the discrete valence level in the

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[2] In the same way we could define $\dot{A}A^+$ with some vielbein E_α^A.

spectrum of the hamiltonian. Such a term can be derived within the Skyrme model only by adding the Wess-Zumino term [25]. The rotational Lagrangian simplifies with (31) to

$$L^{rot} = \frac{1}{2} I_1 \sum_{A=1}^{3} \Omega_A \Omega_A + \frac{1}{2} I_2 \sum_{A=4}^{7} \Omega_A \Omega_A - \frac{N_c}{2\sqrt{3}} \Omega_8 B(U). \tag{32}$$

Apart from these terms there are corrections from the strange quark mass, which are of the order $m_s N_c^1$ and $m_s N_c^0$, corresponding to terms proportional to m_s and $m_s \Omega_A$. Let us consider first the terms of the order $m_s N_c^1$. We find

$$L^{(1)} = i N_c \int \frac{d\omega}{2\pi} \mathrm{Sp} \left[\frac{1}{i\omega + H} A^+ m \gamma_4 A \right]. \tag{33}$$

Since the mass matrix m can be written as $m = m_1 \mathbf{1} + m_3 \lambda_8$ and the unit matrix $\mathbf{1}$ produces only a constant shift for all members of the multiplet, we consider only the part with $m_3 = -(m_s/\sqrt{3})$ and set $m_0 = 0$. With the relation $\lambda_A \mathrm{Tr}(A^+ \lambda_8 A \lambda_A) = 2 A^+ \lambda_8 A$ one directly notices that again only λ_8 gives a non-varishing contribution. We recall that we have for the Σ-commutator the relation

$$\Sigma = m_0 \frac{\partial E(m)}{\partial m} \bigg|_{m=0} = \Sigma_{val} + \Sigma_{sea}$$

where the valence

$$\Sigma_{val} = m_0 N_c < val \mid \gamma_0 \mid val > \tag{34}$$

and the sea contribution

$$\Sigma_{sea} = m_0 \frac{N_c}{2} \sum_n < n \mid \gamma_0 \mid n > \mathrm{sign}(E_n) \mathcal{R}_\Sigma(E_n) \tag{35}$$

are again sums over single particle energies E_n. The regularization function is given by

$$\mathcal{R}_\Sigma(E_n) = \frac{1}{\sqrt{\pi}} \int_0^\infty \frac{d\tau}{\sqrt{\tau}} e^{-\tau} \phi(\frac{\tau}{E_n^2}). \tag{36}$$

Then one can express $L^{(1)}$ in terms of Σ and obtains

$$L^{(1)} = -\frac{1}{2} \gamma \left(1 - D_{88}^{(8)}(A) \right) \tag{37}$$

where $\gamma = \frac{4}{3} \frac{m_s}{m_u + m_d} \Sigma$ and and $D_{AB}^{(8)}(A) = \frac{1}{2} \mathrm{Tr}(\lambda_A A \lambda_B A^+)$ is a $SU(3)$-Wigner function.

Now we have to consider terms of the order $m_s N_c^0$. We obtain

$$L^{(2)} = i N_c \int \frac{d\omega}{2\pi} \mathrm{Sp} \left[\frac{1}{i\omega + H} A^+ m \gamma_4 A \frac{1}{i\omega + H} A^+ \dot{A} \right]. \tag{38}$$

Analog to $L^{(1)}$ this can be simplified to

$$L^{(2)} = -\frac{2 m_s}{\sqrt{3}} K_{AB} D_{8A}^{(8)}(A) \Omega_B. \tag{39}$$

where we have denoted by K_{AB} the tensor of the anomalous moments of inertia, given by

$$K_{AB} = \frac{N_c}{4}\int \frac{d\omega}{2\pi}\mathrm{tr}\left[\frac{1}{i\omega+H}\lambda_A\frac{1}{i\omega+H}\gamma_4\lambda_B\right]. \tag{40}$$

This reduces in a compact notation to

$$K_{AB} = \frac{N_c}{4}\sum_{m,n}\frac{<m\mid\lambda_A\mid n><n\mid\gamma_4\lambda_B\mid m>}{\mid E_m\mid+\mid E_n\mid}\left[\frac{1-\mathrm{sign}(E_n-\mu_F)\mathrm{sign}(E_m-\mu_F)}{2}\right] \tag{41}$$

It is the full baryon number one contribution, provided that the chemical potential μ_F [44] is above E_{val}. In the same way like I_{AB} it can be split into a *sea* and a *valence* contribution, such that the explicit appearance of μ_F drops out.

These K_{AB} moments need no regularization, because they originate from the imaginary part of the effective Euclidean action and so they are finite. In the same way like eq. (31) we find

$$K_{AB} = \begin{cases} K_1\delta_{AB} & \text{for A,B=1,2,3} \\ K_2\delta_{AB} & \text{for A,B=4,5,6,7} \\ 0 & \text{for A,B=8} \end{cases} \tag{42}$$

such that $L^{(2)}$ reduces to

$$L^{(2)} = -\frac{2m_s}{\sqrt{3}}\left[K_1\sum_{A=1}^{3}D^{(8)}_{8A}(A)\Omega_A + K_2\sum_{A=4}^{7}D^{(8)}_{8A}(A)\Omega_A\right]. \tag{43}$$

Collecting the terms (32,37,43) we obtain finally the expression for the collective Lagrangian in second order in Ω_A and first order in m_s:

$$L = \frac{1}{2}I_1\sum_{A=1}^{3}\Omega_A\Omega_A + \frac{1}{2}I_2\sum_{A=4}^{7}\Omega_A\Omega_A - \frac{N_c}{2\sqrt{3}}\Omega_8 B(U) - \frac{1}{2}\gamma\left(1-D^{(8)}_{88}(A)\right)$$

$$-\frac{2m_s}{\sqrt{3}}\left[K_1\sum_{A=1}^{3}D^{(8)}_{8A}(A)\Omega_A + K_2\sum_{A=4}^{7}D^{(8)}_{8A}(A)\Omega_A\right] \tag{44}$$

In Fig.1 we show all the various moments of inertia for a reasonable range of values of the constituent quark masses.

4. Anomalous and Non-Anomalous Matrix Elements

Because the main techniques in diagonalizing the Dirac-equation and obtaining self-consistent solutions of the equations of motion are already outlined in refs. [6,41,44], we restrict ourself here in this section mainly to $SU(3)$-quantities. Therefore we outline the crucial steps used in computing the moments of inertia $I_{1,2}$ given by eq. (27) and the similar anomalous moments of inertia $K_{1,2}$ given by eq. (40).

All the moments of inertia are double sums over the basis states m and n of the Hamilton operator (24). These states can be written as $\mid G, M, (lj)>$, where G characterizes the so-called grand-spin [5, 6], M is the corresponding magnetic quantum number and (lj) denotes the orbital and the total angular momentum. Because the $\lambda^A, A = 1, 2, 3$, which are the Pauli-Spin matrices, are not diagonal with respect to the grandspin, there are some matrix-elements, which connects basis states for a certain G with $G+1$.

As already mentioned by Wakamatsu et. al. [18], the non-diagonal elements lead to mistakes, if one sums over them up blindly. The reason for that is the grandspin-dependent boundary condition $j_G(k_n D) = 0$, where the j_G are the sperical Bessel-functions, the k_n are the momenta and D is box size. This leads to the disadvantage, that the discretized set of states for grandspin G and G+1 are not exactly orthogonal, as long the box size has any finite value. So we adopt the technique to diagonalize the hamiltonian for a given grandspin twice, with a boundary condition $j_G(k_n D) = 0$ for matrix elements with the same G and $j_{G+1}(k_n D) = 0$ for matrix elements between G and G+1. In this way we have for each matrix element only states k_n involved, which come from the same boundary condition, i.e. the same basis. So we have created an orthogonal set of basis states, at least locally for every separate G.

The strange moment of inertia I_2 and the quantity K_2 are formally given by eqs. (27, 40), where the Hamilton operator H contains the $SU(2)$ chiral soliton fields in a subspace. The H can therefore be written as

$$H = H_{SU(2)} T + H_{1-SU(2)} S \tag{45}$$

where $T = diag(1, 1, 0)$ and $S = diag(0, 0, 1)$ are the projectors on the relevant subspaces and

$$H_{SU(2)} = -i\gamma_4 (i\partial_i \gamma_i + M U_2(x))$$

$$H_{1-SU(2)} = -i\gamma_4 (i\partial_i \gamma_i + M) \tag{46}$$

are the Hamilton operators of the subspaces. Therefore, in this perturbative treatment, we have equal constituent masses for strange and non-strange quarks and a translational invariant Hamilton operator for the strange direction. We can write

$$\frac{1}{i\omega + H} = \frac{1}{i\omega + H_{SU(2)}} T + \frac{1}{i\omega + H_{1-SU(2)}} S. \tag{47}$$

In this way, we see that in contrast to I_1, K_1 the strange quantities I_2, K_2 (see eqs. (29, 41)) contain matrix elements connecting $H_{SU(2)}$-eigenfunctions with $H_{1-SU(2)}$-eigenfunctions, i.e. plane waves.

The exact calculations of the moments of inertia I_i and K_i can be checked by means of a gradient expansion up to the first non-vanishing order. To this end we consider the moments of inertia for a given meson-profile in dependence of the size. Similar to the case of the Skyrme model, the leading terms in the gradient expansion are given by

$$I_1^{grad} = \frac{8\pi}{3} \int dr\, r^2 \sin^2 P(r)$$

$$I_2^{grad} = 2\pi \int dr\, r^2 (1 - \cos P(r)) . \tag{48}$$

These terms behave like R^3, where R is the typical scale of the profile, which we choose $P(r) = -\pi(1-\frac{r}{2R})$ in Fig. 2. Whereas the coincidence of the total value for I_2 with I_2^{grad} is almost perfect (see Fig.2) for large R, the slight deviation of the total I_1 from I_1^{2nd} indicates that the gradient expansion here is not an asymptotic series in $\frac{1}{MR}$ [45].

For the non-regularized K_1 and K_2, the situation is different because of their anomalous nature. The leading non-trivial term in the gradient expansion for K_1, which could give a constant contribution for large R, can be shown to vanish theoretically. The corresponding expression for K_2 is

$$K_2^{grad} = \frac{N_c}{8M} \int dx \frac{\epsilon^{0\nu\alpha\beta}}{24\pi^2} U^+\partial_\nu U \; U^+\partial_\alpha U \; U^+\partial_\beta U \; = \frac{N_c}{8} \frac{q_0}{M} \tag{49}$$

where q_0 is the topological charge of the hedgehog solution. This is the winding number n, which we choose to coincide with the baryon number $B = 1$. That K_2^{grad} is just a constant and that K_1^{grad} is vanishing with at least as R^{-1} for large R is very well confirmed by our numerical values for K_1, K_2 and can be seen in Fig.3.

5. Perturbative Treatment of the Collective Hamiltonian

So far we have written down the expansion of the rotated action (44) in angular velocity and strange quark mass without quantizing it. Therefore we write according to ref. [43, 46] for the right generators

$$R_A = -\frac{1}{2}\{\pi_\alpha, C^\alpha{}_A\} \tag{50}$$

where the canonical momenta π_α are defined by $\pi_\alpha = (\partial L/\partial \dot{q}_\alpha)$ and the vielbeins $C^\alpha{}_A$ are the inverses of the ones in eq. (25)[3]. Imposing now canonical quantization rules for both π_α and the coordinates q_α leads to the $SU(3)$-algebra for the generators R_A [43]. In terms of the angular velocities Ω_A we obtain :

$$R_A = -\frac{\partial L}{\partial \Omega_A} = \begin{cases} -(I_1\Omega_A + \frac{2m_s}{\sqrt{3}} K_1 D_{8A}), & A=1,2,3 \\ -(I_2\Omega_A + \frac{2m_s}{\sqrt{3}} K_2 D_{8A}), & A=4,...7 \\ \frac{1}{2}\sqrt{3}, & A=8. \end{cases} \tag{51}$$

Without the terms of order $O(m_s^2)$ we can write for the symmetric $(O(m_s^0))$ and the symmetry breaking $(O(m_s^1))$ part of the collective hamiltonian.

$$H^{sym}_{coll} = M_{cl} + \frac{1}{2I_2}\sum_{A=1}^{7} R_A^2 + (\frac{1}{2I_1} - \frac{1}{2I_2})\sum_{A=1}^{3} R_A^2 \tag{52}$$

$$H^{sb}_{coll} = -m_s\frac{K_2}{I_2}Y + \frac{2m_s}{\sqrt{3}}\left(\frac{K_1}{I_1} - \frac{K_2}{I_2}\right)\sum_{A=1}^{3} D^{(8)}_{8A}(A)R_A +$$

---

[3] We could introduce simultaneously left generators L_A by using $E^\alpha{}_A$ instead of $C^\alpha{}_A$, where the L_A are connected with R_A but obey commutation rules with different sign (see App.).

$$\frac{2}{3}\frac{m_s}{m_u + m_d}\Sigma\left(1 - D^{(8)}_{88}(A)\right) + \frac{N_c m_s}{3}\frac{K_2}{I_2}D^{(8)}_{88}(A). \tag{53}$$

Here we have used the relation $\sum_A D_{8A}R_A = L_A = \frac{\sqrt{3}}{2}Y$, where Y is the (left) hypercharge. Because the right hypercharge from R_8 is constraint as a consequence of the trivial embedding, it does not appear in the collective hamiltonian explicitly. One should note that the appearance of the left hypercharge Y is entirely due to the imaginary part of the effective action.

In a strict perturbation theory one uses now the wavefunctions of the collective hamiltonian H^{sym}_{coll} and considers then matrix elements between these wavefunctions and the breaking part H^{sb}_{coll}. Because the $SU(3)$-wavefunctions are themselves certain D-functions [47] and the collective hamiltonian contains at most one Wigner-function, one has to consider finally integrals over three D-functions (see App.). Then one can express the splitting between octet and decuplet and within each multiplet through two quantities. This is little bit astonishing, because we have *three* different operators $Y, D^{(8)}_{88}(A)$ and $D^{(8)}_{8A}(A)J_A$ and so one would expect that the splitting itself is determined by three quantities. However, due to a non-trivial group-theoretical property we find that the two quantities Δ and δ are sufficient. They are given by:

$$\Delta = \frac{2}{3}\frac{m_s}{m_u + m_d}\Sigma + m_s\left(2\frac{K_2}{I_2} - 3\frac{K_1}{I_1}\right) \tag{54}$$

$$\delta = m_s\frac{K_1}{I_1}. \tag{55}$$

The splitting between octet and decuplet

$$\Delta_{8-10} = \frac{3}{2I_1} \tag{56}$$

comes soley from H^{sym}_{coll}, i.e. from the difference of the Casimir operators $C_2(SU(2) = \sum_{A=1}^{3} R_A^2$ and $C_2(SU(3) = \sum_{A=1}^{8} R_A^2$ in octet and decuplet representations, and coincides precisely with the nucleon-Δ splitting within the quantization of the $SU(2)$ model. The masses relative to m_{Σ^*} can be written then as

$$\Delta m_N = -\frac{3}{10}\Delta - \delta - \Delta_{8-10} \tag{57a}$$

$$\Delta m_\Lambda = -\frac{1}{10}\Delta - \Delta_{8-10} \tag{57b}$$

$$\Delta m_\Sigma = \frac{1}{10}\Delta - \Delta_{8-10} \tag{57c}$$

$$\Delta m_\Xi = \frac{1}{5}\Delta + \delta - \Delta_{8-10} \tag{57d}$$

$$\Delta m_\Delta = -\frac{1}{8}\Delta - \delta \tag{57e}$$

$$\Delta m_{\Sigma^*} = 0 \tag{57f}$$

$$\Delta m_{\Xi^*} = \frac{1}{8}\Delta + \delta \tag{57g}$$

$$\Delta m_\Omega = \frac{1}{4}\Delta + 2\delta \tag{57h}$$

We fixed m_{Σ^*} to the experimental value, because as in the $SU(2)$-case the quantization yields always too large values for the masses and zero point corrections, as they are suggested in [48] are not calculated on the present level, but will be discussed in sect. 7.

Actually the same equations for the splittings were obtained by Guadagnini [25], who introduced an *ad hoc* term proportional to the hypercharge Y into the Skyrme model. Nevertheless he used the parameters as fitting parameters, whereas we deduced all of them from purely solitonic quantities based on actual selfconsistent NJL calculations.

One can directly derive from (57) the Gell-Mann Okubo relations [3, 49]

$$2(m_N + m_\Xi) = 3m_\Lambda + m_\Sigma$$

and

$$m_\Omega - m_{\Xi^*} = m_{\Xi^*} - m_{\Sigma^*} = m_{\Sigma^*} - m_\Delta$$

as well as the Guadagnini formula [25]

$$m_{\Xi^*} - m_{\Sigma^*} + m_N = \frac{1}{8}(11 m_\Lambda - 3 m_\Sigma). \tag{58}$$

In the same way like the expansion of the rotated action for the energy of the system, we can also determine the expectation values of the various condensates. Within a path integral formulation one can express the condensate $< q_i q_i >$ as

$$< q_i q_i > = N \int Dq D\bar{q} (\bar{q} Q_i q) e^{-\int d^4 x \mathcal{L}_{NJL}} \tag{59}$$

where Q_i is the projection for the individual quark flavor i. Written as

$$< q_i q_i > = \frac{\delta}{\delta s(x)} \mathrm{Splog}(i\omega + H - i\gamma_4 A^+(t) m A(t) + A^+(t)\dot{A}(t) - i s(x) \gamma_4 A^+(t) Q_i A(t)) \Big|_{s=0} \tag{60}$$

we can use immediatedly the expansion of the action above to obtain expressions for the non-strange

$$2 < uu > = \frac{\Sigma}{m_0}(\frac{2}{3} + \frac{1}{3} D^{(8)}_{88}(A))$$

$$+ \frac{2}{\sqrt{3}}(\frac{K_1}{I_1} - \frac{K_2}{I_2}) \sum_{A=1}^{3} D^{(8)}_{8A}(A) R_A - \frac{2}{3}\frac{K_2}{I_2} D^{(8)}_{88}(A) \frac{\sqrt{3}}{2} + \frac{K_2}{I_2} Y \tag{61}$$

and strange condensates

$$< ss > = \frac{\Sigma}{m_0}(\frac{1}{3} - \frac{1}{3} D^{(8)}_{88}(A))$$

$$- \frac{2}{\sqrt{3}}(\frac{K_1}{I_1} - \frac{K_2}{I_2}) \sum_{A=1}^{3} D^{(8)}_{8A}(A) R_A + \frac{2}{3}\frac{K_2}{I_2} D^{(8)}_{88}(A) \frac{\sqrt{3}}{2} - \frac{K_2}{I_2} Y \tag{62}$$

From that one can directly consider the so called strange content of the proton y, which is defined as:

$$y = \frac{< p \mid \bar{s}s \mid p >}{< p \mid \bar{u}u + \bar{d}d + \bar{s}s \mid p >} \tag{63}$$

6. Diagonalization of the collective hamiltonian: Yabu-Ando-Approach

Within the Skyrme model Yabu and Ando [50] investigated the influence of higher order corrections due to a finite m_s in the collective hamiltonian. This corresponds in the context of perturbation theory to take higher $SU(3)$ representations into account. They found therefore that e.g. the proton is then not a pure octet state **8** but has admixtures of $\overline{\mathbf{10}}$ and **27** and so forth. To get these corrections to all orders they suggested to diagonalize the collective hamiltonian in the basis [51] provided by all multiplets with definite Y, J and I of the unperturbed collective hamiltonian H^{sym}_{coll}. In terms of a perturbative approach in terms of powers of m_s the Yabu-Ando approach is not consistent. However in the Skyrme model it has been proven to be successful and hence we will apply it here as well.

Therefore one uses an 'Euler angle' representation for the rotation matrix A [52] and represents the right generators as differential operators. The coupled second order differential equations contains the collective Hamilton operator

$$H^{YA}_{coll} = M_{cl} + \frac{1}{2}\left(\frac{1}{I_1} - \frac{1}{I_2}\right) \sum_{A=1}^{3} R_A^2 - \frac{3}{8I_2} + \frac{1}{2I_2}\epsilon_{SB} \tag{64}$$

where ϵ_{SB} is the eigenvalue of

$$C_2(SU(3)) + I_2 \frac{1}{3}\frac{m_s}{m_u + m_d}\Sigma(1 - D_{88}) + I_2 \frac{2m_2}{\sqrt{3}}\frac{K_1}{I_1} \sum_{A=1}^{3} D_{8A}\left(2R_A + \frac{2m_2}{\sqrt{3}} K_1 D_{8A}\right) +$$

$$\frac{2m_s}{\sqrt{3}} K_2 \sum_{A=1}^{7} D_{8A}\left(2R_A + \frac{2m_s}{\sqrt{3}} K_2 D_{8A}\right) \tag{65}$$

where $C_2(SU(3)) = \sum_{a=1}^{8} R_a^2 = \sum_{a=1}^{8} L_a^2$, which is the quadratic Casimir operator of $SU(3)_{R/L}$. In contrast to the perturbative treatment, whe C_2 depend only on the lowest representations, i.e. octet or decuplet, C_2 depends now also on the symmetry breaking. This is because the collective hamiltonian is diagonalized in a basis, that contains also higher representations of the spin 1/2 and 3/2 baryons. For an explicit form of the generators see Ref. 53.

7. Total masses

Although we mentioned already that the total masses after the quantization procedure come out too large, we want to exhibit that there are some subtraction mechanism in analogy to non-relativistic many particle physics [54]. These are based on the fact, that the two-particle Casimir operators $C_2(SU(2))$ and $C_2(SU(3))$ and the momentum operator $\mathbf{P}^2$ have finite expectation values already on the mean-field level.

First if we consider the masses for the center of the octet and the decuplet, i.e. the chiral limit of the formulae above, we find

$$M_8 = M_{cl}^{SU(2)} + \frac{1}{I_2}\frac{3}{4} + \frac{1}{I_1}\frac{3}{8}, \quad M_{10} = M_{cl}^{SU(2)} + \frac{1}{I_2}\frac{3}{4} + \frac{1}{I_1}\frac{15}{8}. \tag{66}$$

So we get for $M = 391 MeV$ and $M_{cl}^{SU(2)} = M_{cl} + \Sigma = 1307 MeV$ the values $M_8 = 1602 MeV$ and $M_{10} = 1830 MeV$ which is, as it stands, $\simeq 450 MeV$ too high compared with the experimentally observed $M_8 = 1155 MeV$ and $M_{10} = 1385 MeV$. Now we take into account that already on the mean-field level, without any rotation, a non-vanishing expectation value of the Casimir operators emerges. This expectation value corresponds in the present model to the eigenvalue of the single particle Casimir operator within the valence quark states, i.e. a spherical hedgehog state with $G = 0$, $M_G = 0$ and $N_c = 3$ quarks. Therefore we need the eigenvalues of the Casimir operators $C_2(SU(2))$ and $C_2(SU(3))$ for the fundamental representation of the triplet. This yields

$$< C_2(SU(2)) >= \frac{3}{4} N_c \tag{67}$$

$$< C_2(SU(3)) >= \frac{4}{3} N_c \tag{68}$$

so that the mass

$$\Delta M^{rot} = \frac{1}{2I_2} < C_2(SU(3)) > + \frac{1}{2}(\frac{1}{I_1} - \frac{1}{I_2}) < C_2(SU(2)) >= \frac{1}{I_2}\frac{7}{8} + \frac{1}{I_1}\frac{9}{8}, \tag{69}$$

which has to be subtracted, is evaluated to $\Delta M^{rot} = 157.6 MeV$. The total masses reduce then to $M_8 = 1150 MeV$ and $M_{10} = 1383 MeV$, extremely close to the experimentally observed masses. But this picture is worsened, if one includes finally the symmetry breaking term $\frac{2}{3}\frac{m_s}{m_u + m_d}\Sigma \simeq 424 MeV$ for $m_s = 150 MeV$, $m_u + m_d = 12.2 MeV$ and $\Sigma = 51.7 MeV$.

Another quantity, which represents the fact that the soliton is not in the rest frame [48], is obtained after quantization of the translational degrees of freedom [48]. After expanding the system up to the second order in the translational velocity $\mathbf{v}$ one gets for a system at rest the correction

$$\Delta M^{transl} = \frac{< \mathbf{P}^2 >}{2M_{cl}} \tag{70}$$

where $< \mathbf{P}^2 >$ consists now of a valence and a sea contribution and which was calculated in the present model already in [18]. For $M = 391 MeV$, we found $\Delta M^{transl} = 310 MeV$, so that our total prediction for the mass of the center of the octet is

$$M_8 = M_{cl}^{SU(2)} + \frac{1}{I_2}\frac{3}{4} + \frac{1}{I_1}\frac{3}{8} + \frac{2}{3}\frac{m_s}{m_u + m_d}\Sigma - \Delta M^{rot} - \Delta M^{transl} \tag{71}$$

which gives $M_8 = 1261 MeV$. Because the splittings for the baryons itselves are almost perfect we resume that the model predictions for all total masses are only $\simeq 100 MeV$ too high compared with the experiment.

8. Numerical results

As explained in the previous sections we proceed in the following way for the final numerical calculations. The perturbative relation (15) is used to fix the relation between strange and non-strange current masses from mesonic data. We choose $m_s = 150MeV$ which yields then $m_0 = 6.1MeV$. The parameters Λ_1 and Λ_2 of the modified proper time regularization (see sect.2) are then used to reproduce $m_0 = 6.1MeV$ and so to yield a reasonable vacuum condensate of $< \bar{u}u >=< \bar{s}s >= -238MeV$. This procedure is done for $372 \leq M \leq 558MeV$ and the results are presented in Tab. 1. Tab. 2 shows then the basic results for the solitonic sector in $SU(2)$ assuming strict chiral symmetry, i.e. working with $m_\pi = 0$ and an identical $M = M_u = M_d$. The table supports the well known results on the classical mass $M_{cl} \simeq 1250MeV$, the isoscalar quadratic charge radius $< r^2 >_c^{T=0}$ and the Σ-commutator. The Σ-commutator is known to show some dependence on the regularization scheme used [20, 21]. However the present scheme seems to work well for this quantity and the Σ is in good agreement with the experimental values. This is important because the $SU(2)-\Sigma$-commutator plays a decisive role for the splittings of the spin 1/2 and 3/2 baryons.

The moments of inertia I_1, I_2, K_1 and K_2 are plotted vs. the constituent quark mass M in Fig. 1. The values show a clear decrease with increasing M. Actually in all cases we obtain $I_1 > 2I_2$, a relation, which holds in the Skyrme model as well and can be proved there even analytically. The actual values of the moments of inertia can be extracted from Tab. 3. For I_1 and I_2 the sea quark contribution, see (29), is about 30% and does not vary very much with M, for K_1 and K_2 the polarization of the dirac sea contributes only negligibly.

The strange quark content of the nucleon is is defined in (63) and can be evaluated easily. For $M = 391MeV$ and $M = 419MeV$, which are our prefered values, the strange content of the proton is 19.6% and 19.2%. These values come out from Skyrme calculations as well. Actually the Callan-Klebanov approach to symmetry breaking in the strange sector yields different values, they are all very small and only a few percent.

In the perturbative approach the splitting between the centre of the octet and the decuplet is presented in Fig.1 and is given by (56). Plotted is the deviation from the experimental value of $\Delta_{8-10}^{exp} = 230MeV$. Apparently for $M = 391MeV$ the deviation from experiment is zero. This M value is used for all perturbative calculations. For the Yabu-Ando method the splitting is not so well defined and the M is adjusted to reproduce the whole spin 1/2 and 3/2 baryon spectrum. The masses of the members of the baryon octet and decuplet can be extracted for the perturbative approach from Tab. 5 and Fig. 1. Since the semiclassical quantization yields always too large absolute masses we shift the theoretical values such that the experimental mass of the Σ^* is matched by the calculation. The experimental numbers are then given in absolute values and the theoretical numbers indicate the deviation from experiment. Apparantly for the canonical value of $m_s = 150MeV$, for which the condensates and m_0 where properly adjusted according to (6, 18), one finds an acceptable agreement of the theory with experiment. All calculated masses agree within 5% with the corresponding experimental numbers. If one increases m_s, without recalculating the moments of inertia, one obtains noticeably better agreement with the experiment with an overall deviation of less than 2% for $m_s = 200MeV$.

Especially the $\Delta_{\Sigma-\Lambda}$-splitting ($76MeV$), which is experimentally much lower than $\Delta_{N-\Lambda}$ ($178MeV$) and $\Delta_{\Sigma-\Xi}$ ($125MeV$), is qualitatively better here ($92MeV$,$152MeV$ and $106MeV$ for $m_s = 170MeV$) even than a non-perturbative (Yabu-Ando) treatment of the Skyrme model [55], which predicts $\Delta_{\Sigma-\Lambda} \geq \Delta_{\Sigma-\Xi}$ in contrast to experiment. Also the treatment of m_s as a large quantity and the giving up of $SU(3)$ as a rotational symmetry by *Callan* and *Klebanov* [33] evoked severe problems within the Skyrme model. These ordering problems like $m_{\Sigma^*} < m_\Sigma$ in ref. [33] or $m_\Xi < m_\Delta$ in ref. [56] do not appear here.

The results of the Yabu-Ando approach are given in Fig. 5 and Tab. 5. They generally agree better with the experimental values and the agreement is almost perfect if one chooses $m_s = 187MeV$. Apparantly the presence of higher representations in the basis in which the collective hamiltonian is diagonalized exactly improves the baryon masses for the Yabu-Ando method significantly. This can be understood in a way that the proton for example is not a pure octet state, but has significant admixtures of $\overline{\mathbf{10}}$ and $\mathbf{27}$, which was also found in the Skyrme model [57]. This is quite an encouraging result and tells that the model as such has certain merits. The deviation of the Yabu-Ando results from perturbative indicates, that higher powers of m_s are relevant. A direct comparison between the Yabu-Ando and the perturbative approach is made in Fig.6, where the prediction for Σ and Λ is compared for a constituent quark mass of $M = 419MeV$.

The absolute values of the masses of the octet and the decuplet can be estimated without artificial shift if one subtracts from the classical hedgehog mass the zero point energies of rotation in ordinary space and strange space and of translation, see (71). The actual values for $< \mathbf{P}^2 >$ can be taken from Ref. 18 in the chiral limit. Apparantly the deviation from the experimental mass is less $100MeV$. Hence with the above corrections, taken empirical from [18], even the absolute values of the masses of the baryon octet and decuplet in perturbation theory are well reproduced.

8. Summary and discussion

The present model provides in the solitonic sector a generalization of the $SU(2)$-Nambu-Jona-Lasinio model to $SU(3)$. The model is based on scalar and pseudoscalar quark-couplings and ignores vector mesonic degrees of freedom. The $SU(2)$ soliton with baryon number $B = 1$ is trivially embedded is $SU(3)$ guaranteeing the right dimension for the lower multiplets. In order to quantize the collective rotations a systematic expansion of the rotated $SU(3)$-effective action in the chiral limit up to second order in in angular velocity and first order in the strange current mass m_s is performed. The expansion is done in the fermion determinant. The collective hamiltonian contains then a part H^{sb}_{coll}, which is partially already known from the Skyrme model. Due to the fact that we have an explicit quark theory one obtains anomalous moments of inertia. Their origin is due to the imaginary part of the chiral fermion determinant and has therefore no counterpart in the Skyrme model. Only the chiral quark model or the introduction of vector mesons in the Skyrme model serves as a source for similar terms.

The theory is treated in two ways. First a strictly perturbative approach is performed. This results in a collective hamiltonian whose octet and decuplet states fulfill the mass formulae of Gell-Mann-Okubo and of Guadagnini. Not only the structure of the collective hamiltonian is correct but also the numerical values of the anomalous and non-anomalous moments of inertia and of the nucleon Σ-term. Thus, for $m_s = 150 MeV$, the splittings within the multiplets are reproduced in reasonable way if the constituent mass is adjusted to the octet-decuplet splitting. For the larger value of $m_s = 200 MeV$ the deviation of theory from experiment is noticeably smaller.

Second a Yabu-Ando approach is performed. It consists in diagonalizing the symmetry breaking terms linear in m_s in the basis provided by all unperturbed multiplets. By this procedure the numbers for the splittings get further improved and the deviation from the experimental numbers becomes negligible for $m_s = 187 MeV$.

The absolute values of the masses can be arranged by subtracting the zero point energies of strange and non-strange rotation and of translation. If one does this, the theoretical values of the Σ^*-mass deviates less than $100 MeV$ from experiment. Since all splittings are well reproduced in the present approach, this small deviation of the absolute mass holds generally for all members of the octet and decuplet

The strange quark content of the nucleon comes out to 20% in the present model and does not depend very much on details of the calculations.

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Appendix A.

The wave functions $\Psi(A)$ of the rotated soliton are functions of a unitary rotational matrix A, - provided that all other degrees of freedom are frozen. It is clear from $[L_a, A] = -\frac{1}{2}\lambda_a A$ that the $SU(3)$ *flavour* generators L_a act on the rotational matrix A from the *left*,

$$\left[e^{i\omega_a L_a}\Psi\right](A) = \Psi\left(e^{-i\omega_a\lambda_a/2}A\right) \tag{A.1}$$

while the *spin* operators R_a, from $[R_a, A] = -\frac{1}{2}A\lambda_a$, are the *right* generators:

$$\left[e^{i\omega_a R_a}\Psi\right](A) = \Psi\left(Ae^{-i\omega_a\lambda_a/2}\right) \tag{A.2}$$

The standard commutation relations with the usual $SU(3)$ structure constants f_{abc}

$$[L_a, L_b] = if_{abc}L_c, \quad [R_a, R_b] = -if_{abc}R_c, \quad [R_a, L_b] = 0 \tag{A.3}$$

hold, where the different sign in the commututator relation for R_a in by (A.3) can be compensated by a redefinition of the corresponding raising and lowering operators [43]. Among the eight generators R_a only three ($a = 1, 2, 3$) have the meaning of spin generators; R_8 should obey the quantization rule of eq. (51).

In principle, the commutation relations (A.3) are sufficient to construct the wave functions $\Psi(A)$ for rotating states of the soliton. They are the eigenfunctions of commuting generators T, T_3, Y from L_a and $J, J_3, Y' = N_c/3$ from R_a and can be expressed in terms of the so-called $SU(3)$ Wigner functions $D^{(n)}_{\nu\nu'}$. They a realize the representation $\{n\}$ of the $SU(3)$ group, where ν stands for the eigenvalues of T, T_3, Y and ν' for J, j_3, Y'. They fulfill :

$$\sum_{\nu'} D^{(n)}_{\nu_1\nu'}(A_1)D^{(n)}_{\nu'\nu_2}(A_2) = D^{(n)}_{\nu_1\nu_2}(A_1A_2). \tag{A.4}$$

and they are unitary matrices in ν, ν' indices:

$$\sum_{\nu'} D^{(n)}_{\nu'\nu_1}(A)\left[D^{(n)}_{\nu'\nu_2}(A)\right]^* = \delta_{\nu_1\nu_2}. \tag{A.5}$$

The normalization is

$$\int dA D^{(n)}_{\nu_1\nu_2}(A)(n)D^{(n')*}_{\nu_1'\nu_2'}(A) = \frac{\delta_{nn'}\delta_{\nu_1\nu_1'}\delta_{\nu_2\nu_2'}}{dim(n)} \tag{A.6}$$

where dA is the group Haar measure normalized to unity. These relations do not fix the phase freedom, though. Usually one imposes the De Swart conditions [47] requiring that all matrix elements of $T_1 \pm iT_2$ and $T_1 \pm iT_5$ in the given representation be non-negative. If one accepts the same agreement for the wave functions $\Psi(A)$ both in the T and J sectors, then these functions can be expressed through the D-functions in the following way:

$$\Psi^{n}_{TT_3Y,JJ_3Y'}(A) = \sqrt{dim(n)}(-1)^{Y'/2+J_3}D^{(n)}_{J,J_3,Y';T,T_3,Y}(A^{-1}). \tag{A.7}$$

Using the definitions of J and T it is simple to check that the right-hand side of this eq. has correct quantum numbers. As for the phase factor, it is fixed by the De Swart conditions. Using the unitarity of the D-functions we can rewrite this function in the following form:

$$\Psi^{n}_{TT_3Y,JJ_3Y'}(A) = \sqrt{dim(n)}(-1)^{Y'/2+J_3} D^{(n)*}_{T,T_3,Y;J,J_3,Y'}(A). \qquad (A.8)$$

Let us proceed now with the matrix elements of the operators under consideration. We need the matrix elements of $\frac{1}{2}Tr(\lambda_8 R\lambda_8 R^+)$ and of $\sum_{a=1}^{3} D^{(8)}_{8a} J_a$. The hypercharge Y is already diagonal in the representation of the D-functions. Let us note that the octet representation is unitarity-equivalent to the adjoint representation:

$$\frac{1}{2}Tr(\lambda_a R\lambda_b A^+) = \sum_{\nu,\nu'} U_{\nu' a} U^*_{\nu b} D^{(8)}_{\nu'\nu}(A). \qquad (A.9)$$

The non-zero matrix elements of $U_{\nu A}$ are:

$$U_{\Lambda 3} = U_{\Sigma^0 3} = 1,$$

$$U_{\Sigma^- 1} = -U_{\Sigma^+ 1} = -U_{p4} = -U_{n6} = -U_{\Xi^0 6} = U_{\Xi^- 4} = \frac{1}{\sqrt{2}},$$

$$U_{\Xi^0 7} = -U_{\Sigma^+ 2} = -U_{\Sigma^- 2} = -U_{\Xi^- 5} = -U_{p5} = -U_{n7} = \frac{i}{\sqrt{2}}. \qquad (A.10)$$

We use the notation of particles to label states with corresponding quantum numbers.

Now that both the wave functions and the operators are expressed in terms of the D-functions one can calculate the matrix elements using the general formula:

$$\int dA D^{(n)*}_{\nu\nu'}(A) D^{(n)_1}_{\nu_1\nu_1'}(A) D^{(n)_2}_{\nu_2\nu_2'}(A) \qquad (A.11)$$

$$= \frac{1}{dim(n)} \sum_{\mu} \begin{pmatrix} n_1 & n_2 & n_\mu \\ \nu_1 & \nu_2 & \nu \end{pmatrix} \begin{pmatrix} n_1 & n_2 & n_\mu \\ \nu_1' & \nu_2' & \nu' \end{pmatrix} \qquad (A.12)$$

where the sum goes over all occurences of the representation n_μ in the product of representations n_1 and n_2. With the eq.(A.11) at hand it is possible to use the standard tables of the $SU(3)$ Clebsch Gordan coefficients, e.g. from Ref. 17, 58.

As for the matrix elements of $\sum_{a=1}^{3} D^{(8)}_{8a} J_a$ one can calculate them using the standard matrix elements for $J_{1,2,3}$, since the De Swart conditions include the positivity of the $J_\pm = \frac{1}{\sqrt{2}}(J_1 \pm iJ_2)$ matrix elements, which is adopted as a standard one for the $SU(2)$ group. This yields

$$J_\pm \Psi^{n}_{TT_3Y,JJ_3Y'}(A) = \frac{1}{\sqrt{2}} \sqrt{(J \mp J_3)(J \pm J_3 + 1)}\, \Psi^{n}_{TT_3Y,J(J_3\pm 1)Y'}(A) \qquad (A.13)$$

which can be used together with

$$\sum_{a=1}^{3} D^{(8)}_{8a} J_a = D^{(8)}_{8\pi^0} J_3 - D^{(8)}_{8\pi^+} J_- + D^{(8)}_{8\pi^-} J_+ \qquad (A.14)$$

to determine $< \sum_{a=1}^{3} D^{(8)}_{8a} J_a >$.

M	c	Λ_1 [MeV]	Λ_2 [MeV]	$B^{1/4}$ [MeV]
372.00	0.599	276.8	1264.8	168.6
376.65	0.614	292.0	1280.6	170.0
381.30	0.629	305.2	1296.4	171.5
385.95	0.643	317.1	1312.2	172.9
390.60	0.656	327.9	1328.0	174.2
395.25	0.669	337.8	1343.8	175.6
399.90	0.681	347.1	1359.6	177.0
404.55	0.693	355.7	1375.4	178.3
409.20	0.704	363.8	1391.2	179.6
413.85	0.714	371.6	1407.0	180.9
418.50	0.724	378.5	1422.9	182.3
465.00	0.804	437.9	1581.0	195.1
558.00	0.933	520.5	1897.2	219.7

Tab.1: For reasonable constituent quark masses we show the coefficient c, the two cutoff's Λ_1, Λ_2 and the energy density B of the vacuum (bagconstant) [21], which sign ensures that we have in fact a spontaneous breaking of chiral symmetry breaking. For all constituent masses we fixed $m_0 = 6.4 MeV$ and $< qq >^{1/3} = -300 MeV$, i.e. $< uu >^{1/3} = -238 MeV$.

M	E_{level} [MeV]	M_{cl} [MeV]	Σ [MeV]	$< r^2 >$ [fm^2]
372.00	590	1259	54.2	0.58
376.65	583	1258	53.3	0.56
381.30	576	1257	53.3	0.55
385.95	569	1256	52.6	0.54
390.60	563	1255	51.7	0.53
395.25	553	1253	51.1	0.52
399.90	549	1252	50.6	0.51
404.55	536	1250	50.2	0.50
409.20	530	1249	50.1	0.49
413.85	522	1247	50.0	0.48
418.50	514	1245	49.9	0.47
465.00	438	1228	45.9	0.41
558.00	290	1190	38.3	0.33

Tab.2: For a range of constituent quark masses the table presents the energies of N_c valence quarks, the classical hedhehog energy M_{cl}, the sigma-commutator Σ and the corresponding isoscalar quadratic radius $< r^2 >$ for the SU(2)-soliton.

	M [MeV]	valence [fm]	sea [fm]	total [fm]
I_1	391.5	0.843	0.452	1.296
I_2	391.5	0.407	0.216	0.623
K_1	391.5	0.456	0.001	0.457
K_2	391.5	0.300	-0.002	0.298
I_1	418.5	0.724	0.4533	1.177
I_2	418.5	0.351	0.218	0.569
K_1	418.5	0.368	0.001	0.369
K_2	418.5	0.258	-0.002	0.255

Tab.3: The contribution of the valence and the sea part of moments of inertia for $M = 391.5 MeV$ *and* $M = 418.5 MeV$.

		T	Y	
octet	N	1/2	1	3/10
	Σ	1	0	$-1/10$
	Ξ	1/2	-1	$-1/5$
	Λ	0	0	1/10
decuplet	Δ	3/2	1	1/8
	Σ^*	1	0	0
	Ξ^*	1/2	-1	$-1/8$
	Ω	0	-2	$-1/4$

Tab.4: Values of $< rTYJY \mid D^{(8)}_{88}(R) \mid rTJY >$.

		T	Y	
octet	N	1/2	1	$-\sqrt{3}/20$
	Σ	1	0	$-3\sqrt{3}/20$
	Ξ	1/2	-1	$\sqrt{3}/5$
	Λ	0	0	$3\sqrt{3}/20$
decuplet	Δ	3/2	1	$-5\sqrt{3}/16$
	Σ^*	1	0	0
	Ξ^*	1/2	-1	$5\sqrt{3}/16$
	Ω	0	-2	$5\sqrt{3}/8$

Tab.5: Values of $< rTYJY \mid D^{(8)}_{8i}(R)J_i \mid rTJY >$.

	perturbative		Yabu-Ando		exp.
	$M = 390.6 MeV$		$M = 419 MeV$		
$m_0 = 6.1 MeV$	$m_s = 150 MeV$	$m_s = 200 MeV$	$m_s = 150 MeV$	$m_s = 187 MeV$	
	$[MeV]$	$[MeV]$	$[MeV]$	$[MeV]$	$[MeV]$
N	11.2	-17.1	36.9	-0.1	939.0
Λ	-2.6	-16.2	-4.6	-5.6	1116.0
Σ	2.6	16.2	-9.0	5.2	1193.0
Ξ	-28.9	15.9	-35.8	6.9	1318.0
Δ	18.1	13.8	37.4	4.4	1232.0
Σ^*	0.0	0.0	0.0	0.0	1385.0
Ξ^*	-48.1	-13.8	-33.2	-1.4	1533.0
Ω	-80.7	-11.5	-68.2	-6.4	1672.0

Tab.6: The deviation of the theoretical mass from the experimental value is shown for the perturbative treatment and a constituent quark mass $M = 391 MeV$ and the Yabu-Ando method and a constituent quark mass $M = 419 MeV$. The theoretical value of the Σ^*-mass is adjusted to the experimental one. For two values of the strange current quark mass, these deviations are compared with the absolute experimental mass.

TABLE CAPTIONS

1. For reasonable constituent quark masses we show the coefficient c, the two cutoff's Λ_1, Λ_2 and the energy density B of the vacuum (bagconstant) [14], which sign ensures that we have in fact a spontaneous breaking of chiral symmetry breaking. For all constituent masses we fixed $m_0 = 6.1 MeV$ and $< qq >^{1/3} = -300 MeV$, i.e. $< \bar{u}u >^{1/3} = -238 MeV$.

2. For a range of constituent quark masses the table presents the energies of N_c valence quarks,the classical hedhehog energy M_{cl}, the sigma-commutator Σ and the corresponding isoscalar quadratic radius $< r^2 >$ for the $SU(2)$-soliton.

3. For two values of the constituent quark mass the table shows the moments of inertia I_1, I_2, K_1 and K_2 with their distribution into valence and sea part.

4. Values of $< rTYJY \mid D^{(8)}_{88}(R) \mid rTJY >$.

5. Values of $< rTYJY \mid D^{(8)}_{8i}(R)J_i \mid rTJY >$.

6. The deviation of the theoretical masses to the experimental values is shown for the perturbative and the Yabu-Ando method for constituent and strange current masses, which give the best agreement with the experiment.

FIGURE CAPTIONS

1. The moments of inertia for a selfconsistent profile in dependence of the constituent quark mass is shown together with the octet-decuplet splitting in perturbation theory, which hits the experimental value for $M = 391 MeV$

2. The total contribution to I_1 and I_2 are shown for a fixed linear profile and a constituent quark mass $M = 372 MeV$ in dependence of the size R of the profile. Also shown is the leading term in the gradient expansion for both quantities. At small radii ($\simeq 0.5 fm$ the total contribution contain a pole due to the fact, that the valence level become part of the positive continuum.

3. The total contribution to K_1 and K_2 are shown for a fixed linear profile and a constituent quark mass $M = 372 MeV$ in dependence of the size R of the profile. Also shown is the leading term in the gradient expansion for K_2, which is a constant proportional to the topological charge.

4. The deviation of the theoretical mass from the experimental one is shown for the perturbative treatment and a constituent quark mass of $M = 391 MeV$.

5. The deviation of the theoretical mass from the experimental one is shown for the Yabu-Ando treatment and a constituent quark mass of $M = 419 MeV$.

6. The deviation of the theoretical mass from the experimental one is shown for the Σ and the Λ, comparing the perturbative and the Yabu-Ando method for $M = 419 MeV$.

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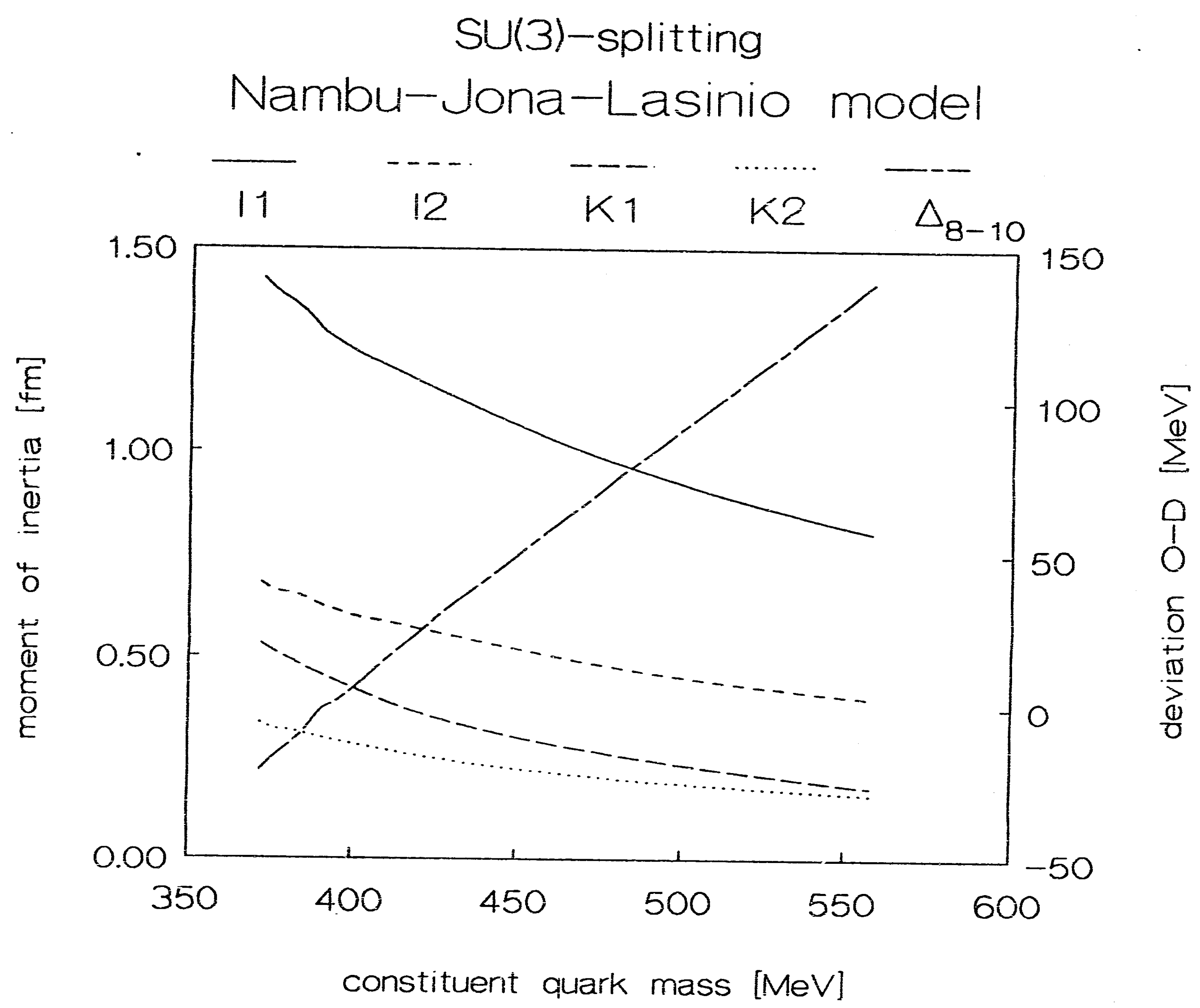
SU(3)-splitting
Nambu-Jona-Lasinio model
I1
I2
K1
K2
Δ_{8-10}
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1.00
0.50
0.00
150
100
50
0
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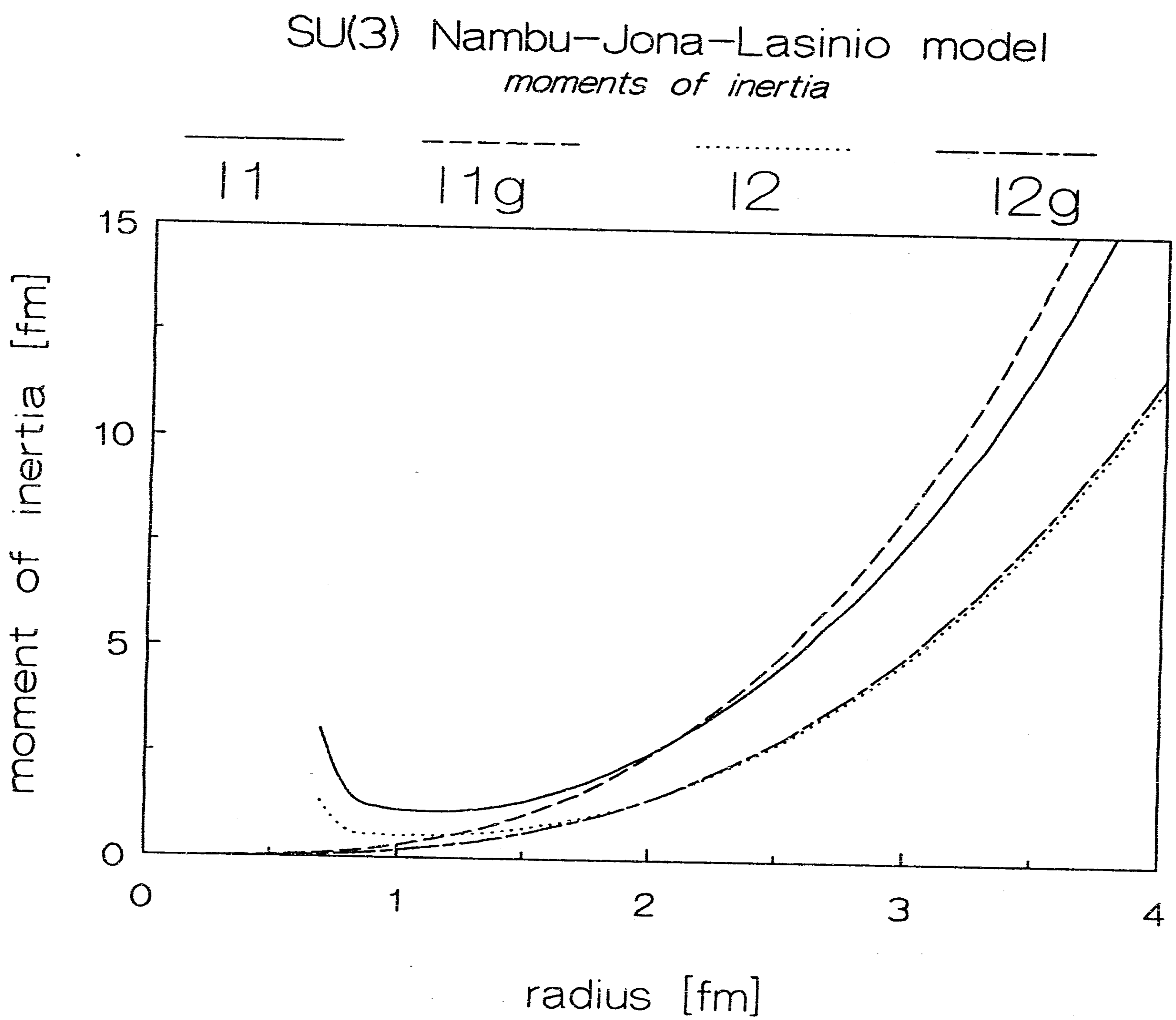
350
400
450
500
550
600
moment of inertia [fm]
deviation O-D [MeV]
constituent quark mass [MeV]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moments of inertia
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I1g
I2
I2g
moment of inertia [fm]
radius [fm]
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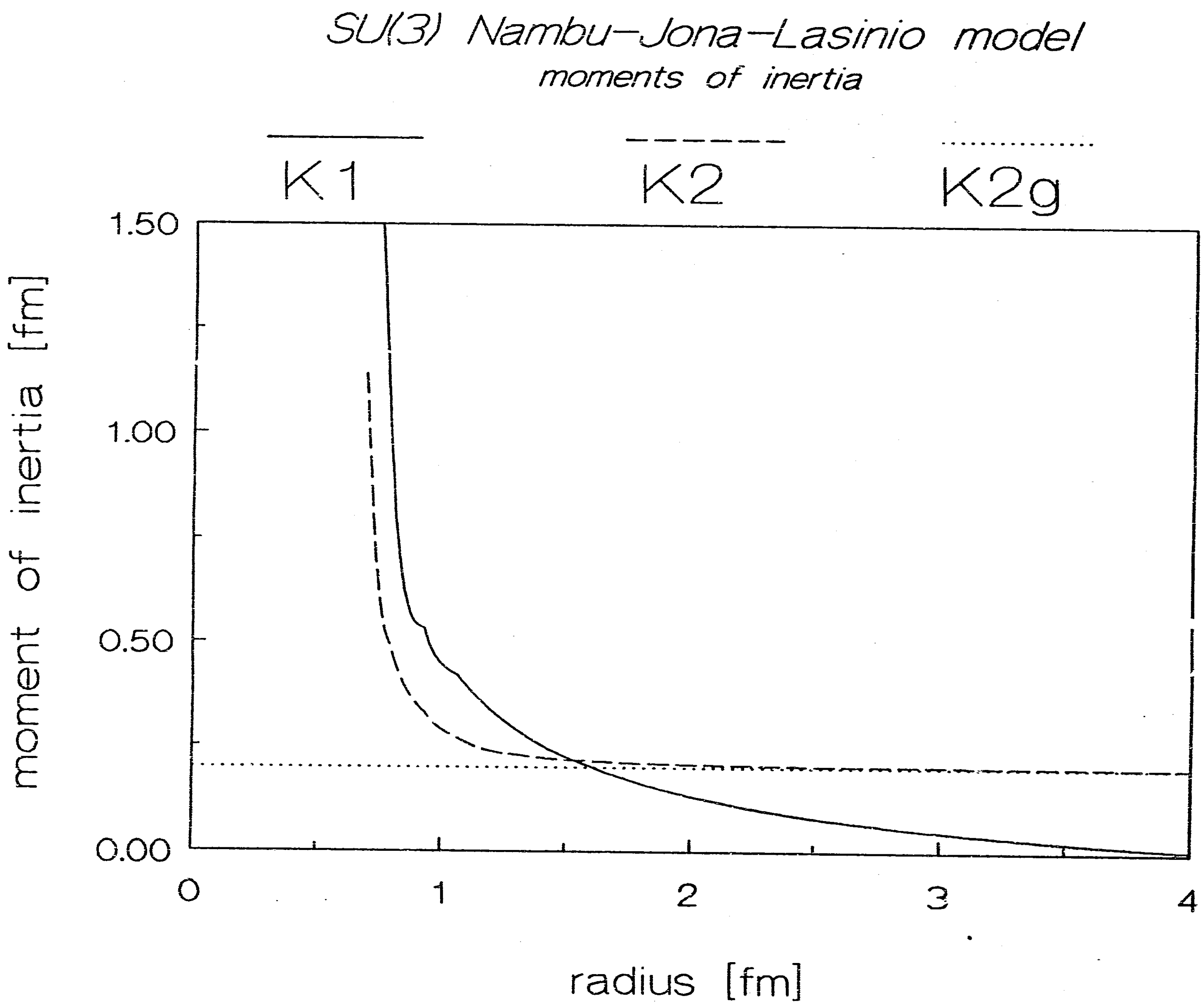
5
10
15
0
1
2
3
4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moments of inertia
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K2
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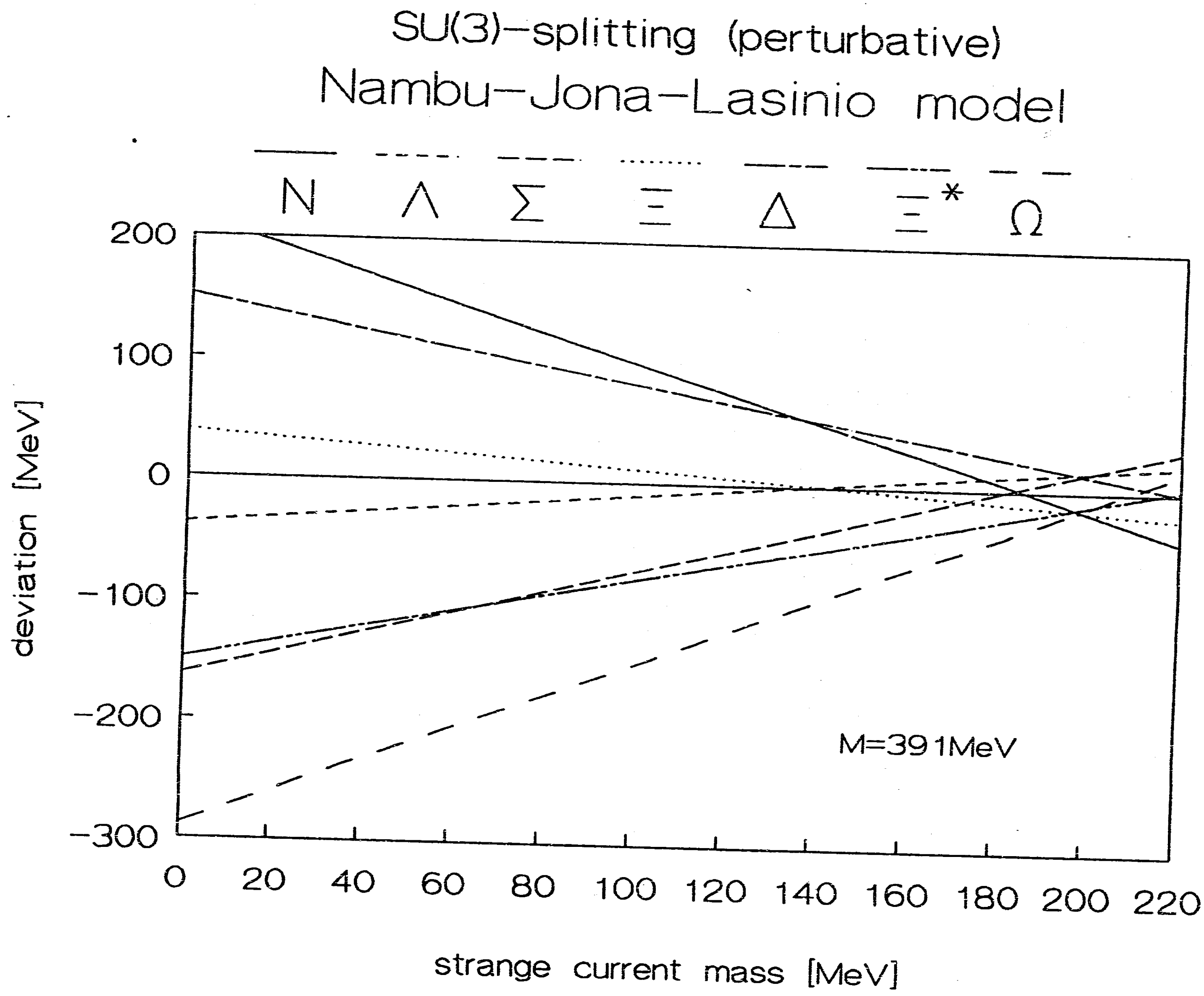
moment of inertia [fm]
0
1
2
3
4
radius [fm]


SU(3)-splitting (perturbative)
Nambu-Jona-Lasinio model
N
Λ
Σ
Ξ
Δ
Ξ*
Ω
200
100
0
-100
-200
-300
deviation [MeV]
M=391MeV
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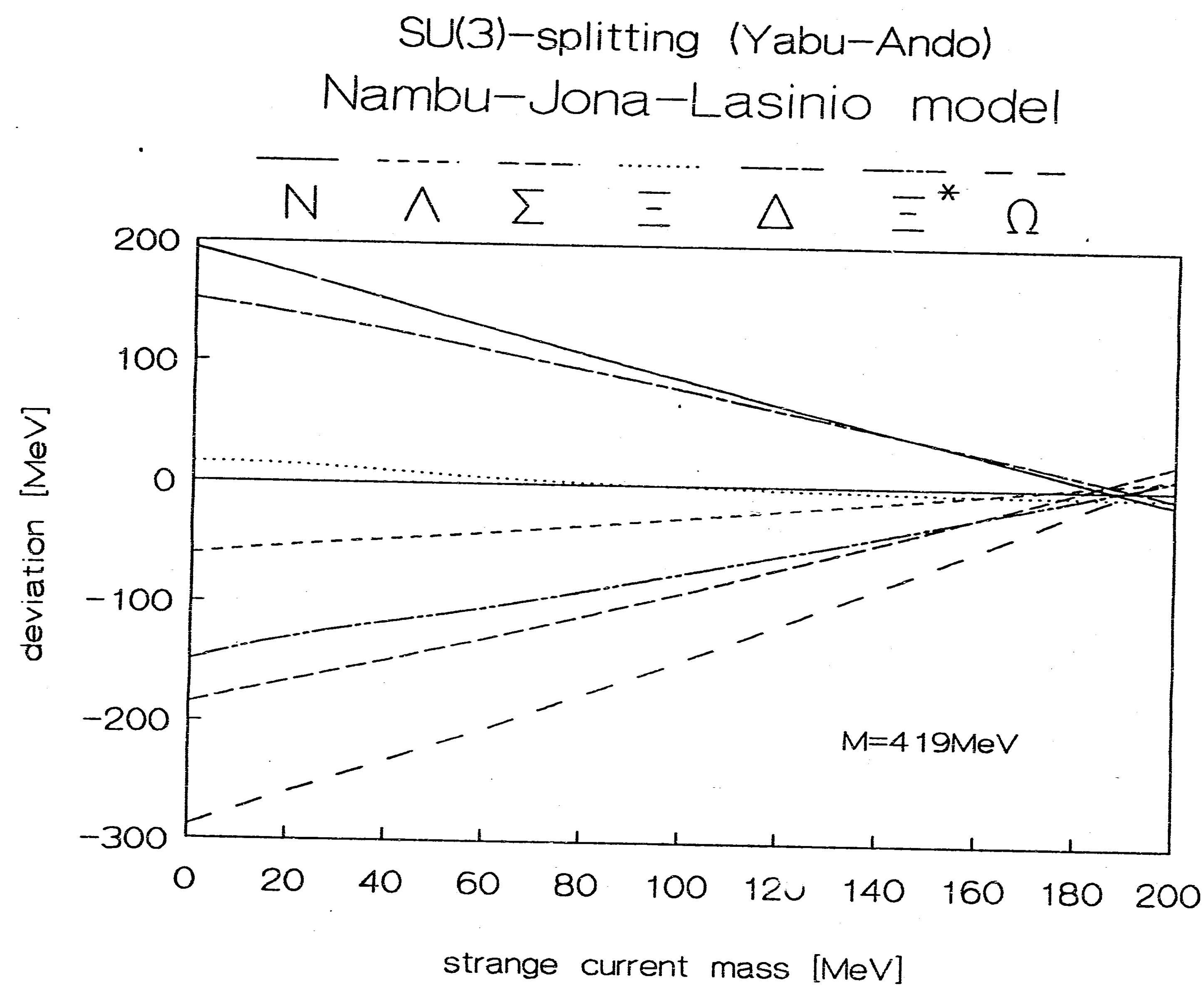
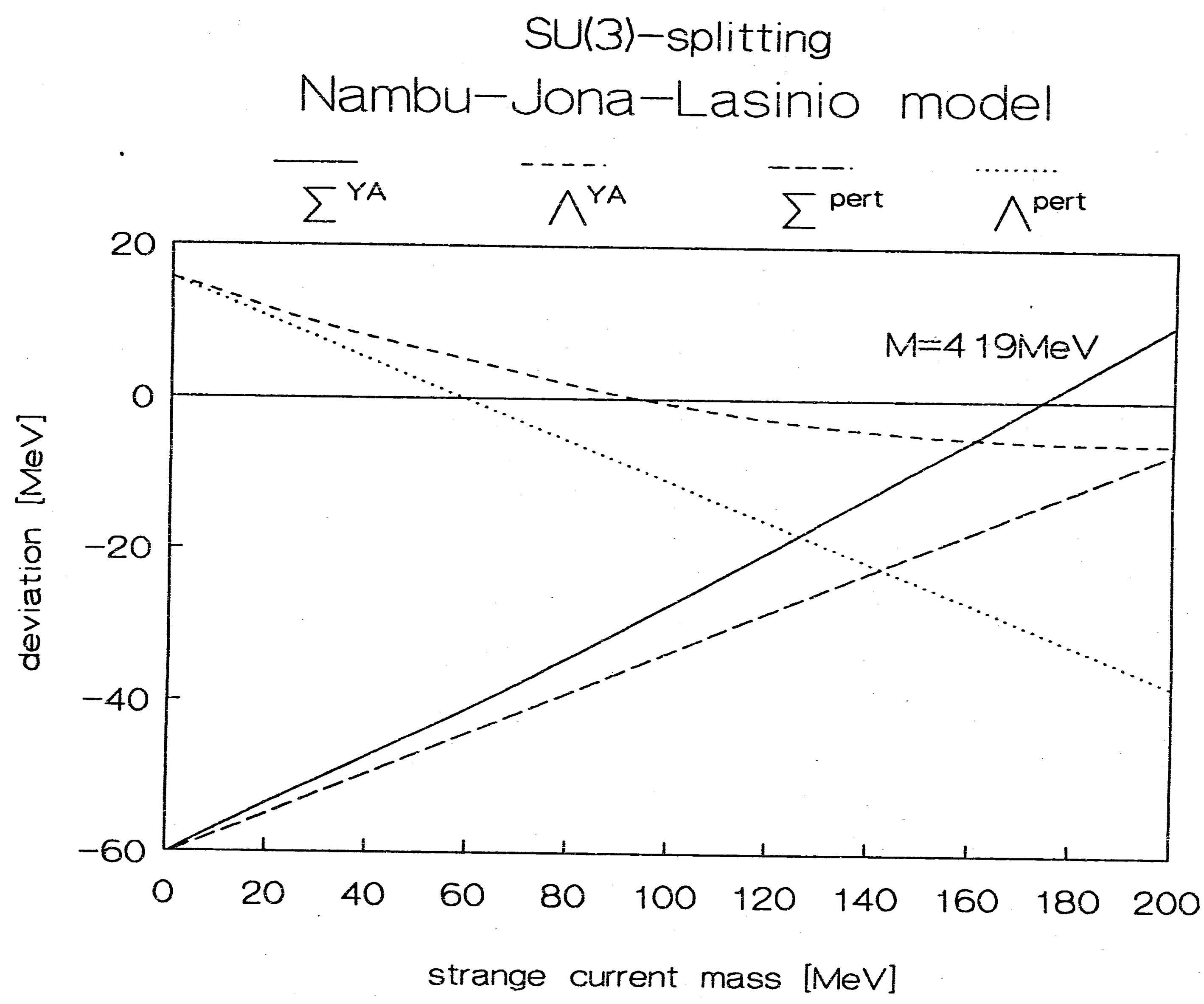
20
40
60
80
100
120
140
160
180
200
220
strange current mass [MeV]



SU(3)-splitting (Yabu-Ando)
Nambu-Jona-Lasinio model
N
Λ
Σ
Ξ
Δ
Ξ*
Ω
200
100
0
−100
−200
−300
deviation [MeV]
M=419MeV
0
20
40
60
80
100
120
140
160
180
200
strange current mass [MeV]



SU(3)-splitting
Nambu-Jona-Lasinio model
Σ^{YA}
Λ^{YA}
Σ^{pert}
Λ^{pert}
M=419MeV
20
0
-20
-40
-60
0
20
40
60
80
100
120
140
160
180
200
deviation [MeV]
strange current mass [MeV]

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