SSC--16

DE90 014107

MORE ON THE OVERLAP KNOCKOUT RESONANCES IN THE SSC

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Introduction

Steve Myers[1] has pointed out that the overlap knockout resonances are potentially harmful to the stability of two beams with different revolution periods. In this note we present a follow-up study of this effect for the SSC.

One possible reason for the two beams to have different revolution periods is operation at different energies, especially at injection. The velocity difference then translates into a difference in revolution periods. Another case is when the two beams have different horizontal orbit distortions (even when they have the same energy), which in turn makes the circumferences different. A third possible cause is when there is a circumference difference due to survey errors during construction. As pointed out in Ref. 1, these effects can in principle be compensated by adjusting the RF frequencies, but this is done at the cost of aperture and ought to be avoided if possible.

1. The model

Consider one interaction region between $s = -L$ and $s = L$. A test particle passes through this interaction region, encountering the particle bunches of the on-coming beam. The encounters occur at locations spaced by a distance $d/2$, where d is the spacing between bunches. The total number of encounters is therefore about $4L/d$. For the SSC, L is about 100 m, d is about 15 m, and the number of encounters per passage is about 26.

At each encounter, the test particle receives a kick from the on-coming beam bunch. The bunches are regarded as unperturbed by the test particle. We assume that in the interaction region the two beams are by a fixed distance r.

Let ΔC be the difference in path length of the two beams (or equivalently $\Delta C = c\Delta T$, with ΔT the difference in revolution periods) between two adjacent

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1. S. Meyers, "Overlap Knockout Resonances at the SSC," SSC-14 (1985 March).

interaction regions. As the test particle passes through the next interaction region, the encounter locations will be shifted by a distance $\Delta C/2$ relative to the locations at the previous passage.

The pattern of the encounter locations repeats after $d/\Delta C$ passages. This is equivalent to saying that this beam-beam perturbation is modulated at the frequency (in tune units) of $\Delta C/d$. Since the betatron tune is $\nu/6$ between passages (assuming 6 evenly distributed interaction regions), there is a resonance whenever

$$\nu/6 - n = k \ \Delta C/d \tag{1}$$

where n and k are integers.

If ΔC comes from operating the two beams at different energies, the value of ΔC is about 1 cm per superperiod if the lower beam energy is 1 TeV and the higher beam energy is much higher than 1 TeV. The resonances occur at the closely spaced tunes

$$\nu = 6n + 0.004 \ k \quad . \tag{2}$$

In the next section, we will present an analytic calculation of the strength of these resonances for the special case when L is an integral multiple of d/2. A numerical simulation is then given in the following section, yielding more general results. Finally, in the last section we discuss the results.

2. Analytic calculation

For each passage through the interaction region, the net accumulated kick received by the test particle can be described by an effective single kick at the center of the interaction region (s = 0). This effective kick is modulated at a tune of $6d/\Delta C$. The kick contains a time-independent component which causes a static orbit distortion which is not of interest to us. The modulation component drives the resonances.

In case L is an integral multiple of d/2, the modulation component has a simple sawtooth behavior. The modulation component of the effective kick is a net displacement at s = 0. The peak value of the sawtooth modulation of this effective displacement is

$$\Delta\hat{x}_{eff} = \pm\Theta L \tag{3}$$

where θ is the kick received by the test particle at each encounter location. We have assumed θ is a fixed value given by

$$\theta = \frac{2Nr_0}{\gamma r} \tag{4}$$

where N is the number of particles per bunch, r_0 is the classical radius of proton, and r is the constant separation between the two beams.

Near the kth sideband resonance given in Eqn. (1), the relevant part of the sawtooth modulation is its kth Fourier component. For the mth passage the effective displacement is then

$$\Delta x_{eff} = \theta L \frac{2}{\pi k} \sin(2\pi mk \frac{\Delta C}{d}) \tag{5}$$

If we now follow the test particle for M passages, the accumulated orbit distortion at s = 0 due to these beam-beam kicks is

$$\Delta x^* = \theta L \frac{2}{\pi k} \sum_{m=0}^{M} \sin(2\pi mk \frac{\Delta C}{d}) \cos(m\mu) \tag{6}$$

where μ is the betatron phase advance between interaction regions ($\mu = 2\pi\ \nu/6$).

Near the resonance given in Eqn. (1), i.e.

$$\frac{\mu}{2\pi} - n = k \frac{\Delta C}{d} + \delta \tag{7}$$

with δ << 1, Eqn. (6) gives approximately

$$\Delta x^* = \theta L \frac{2}{\pi k} \frac{\sin^2(M\pi\delta)}{2\pi\delta} \tag{8}$$

The kicks also give approximately

$$\Delta x'^* = \theta L \frac{2}{\pi k \beta^*} \frac{\sin(M\pi\delta)\cos(M\pi\delta)}{2\pi\delta} \tag{9}$$

where β^* is the beta function at s = 0. For the SSC, $\beta^* \simeq 1$ m.

From Eqns. (8) and (9), we find that the emittance growth due to the resonant driving is

$$\Delta\varepsilon = \frac{1}{\beta^*}\left[\frac{2}{\pi k}\,\theta L\,\frac{\sin(M\pi\delta)}{2\pi\delta}\right]^2 \qquad (10)$$

The emittance growth needs to be much less than the natural emittance of the beam.

In this model, the driven beam moves collectively. A feedback system can thus in principle be used to damp the emittance growth. More will be said about this in the discussion section.

3. <u>Numerical calculations</u>

A simulation program has been written to simulate the overlap knockout resonance effects. The numbers used are

beam separation $r = 5$ mm

$L = 97.5$ m

$d = 15$ m

$\theta = 0.8\times10^{-8}$ rad

number of passages $M = 10000$, or the nearest number for maximum $\Delta\varepsilon$

The distance L is chosen such that it is an integral multiple of d/2, and the analytical calculation of the previous section applies.

Figure 1 shows the emittance growth as a function of the tune ν in the neighborhood of $\nu = 84$, which is a multiple of 6. As expected, the resonances are spaced 0.004 apart. A closer inspection shows that the behavior near the resonances agrees very well with the expected (Eqn. (10)).

Figure 2 shows what happens if L is changed to 100.1 m. The sawtooth modulation no longer holds. The Fourier decomposition is then different from Eqn. (5). In particular, the higher order sidebands no longer become weaker as the resonance order increases in the simple way as Figure 1 shows. Nevertheless, the resonance strengths are basically the same.

Note that the tails of resonance are not negligible. If the tune is chosen to be in the range shown in Figures 1 and 2, for example, the emittance growth even "away from" resonances is larger than the natural beam emittance at 1 Tev (1×10^{-9}m).

4. Discussion

We have made an attempt to study quantitatively the strength of the overlap knockout resonances. We found that these resonances are closely spaced (0.004 spacing in tune units). If uncorrected, there will be intolerable emittance growth if the tune is close to a multiple of 6. By choosing the tune far enough away from 84 so that we are dealing only with very high order sidebands, however, the resonance strengths decrease algebraically. The background emittance growth "away from" resonances will at least become acceptable. In addition, the resonances are weak enough that perhaps a wide-band feedback system can cure most of the problem. The needed strength is of the order of a few gauss-meter.

We have not exhausted the study in this note. In particular, there are a few unanswered questions, as listed below:

1. We have not studied the effects of a betatron tune spread. The tune spread can in fact easily be wider than the resonance spacing of 0.004.

2. During operation, there may be a need to cross a large number of these resonances. If the resonances are caused by the different energies of the two beams, resonance crossings will occur when the lower energy beam is accelerated. This effect has also not been studied.

3. We have studied only a dipole motion of the driven beam. The higher modes are also driven by the overlap knockout mechanism. Although the strengths will be much weaker, they cannot be easily damped by feedback.

We would like to thank Steve Myers and Jack Peterson for useful discussions.

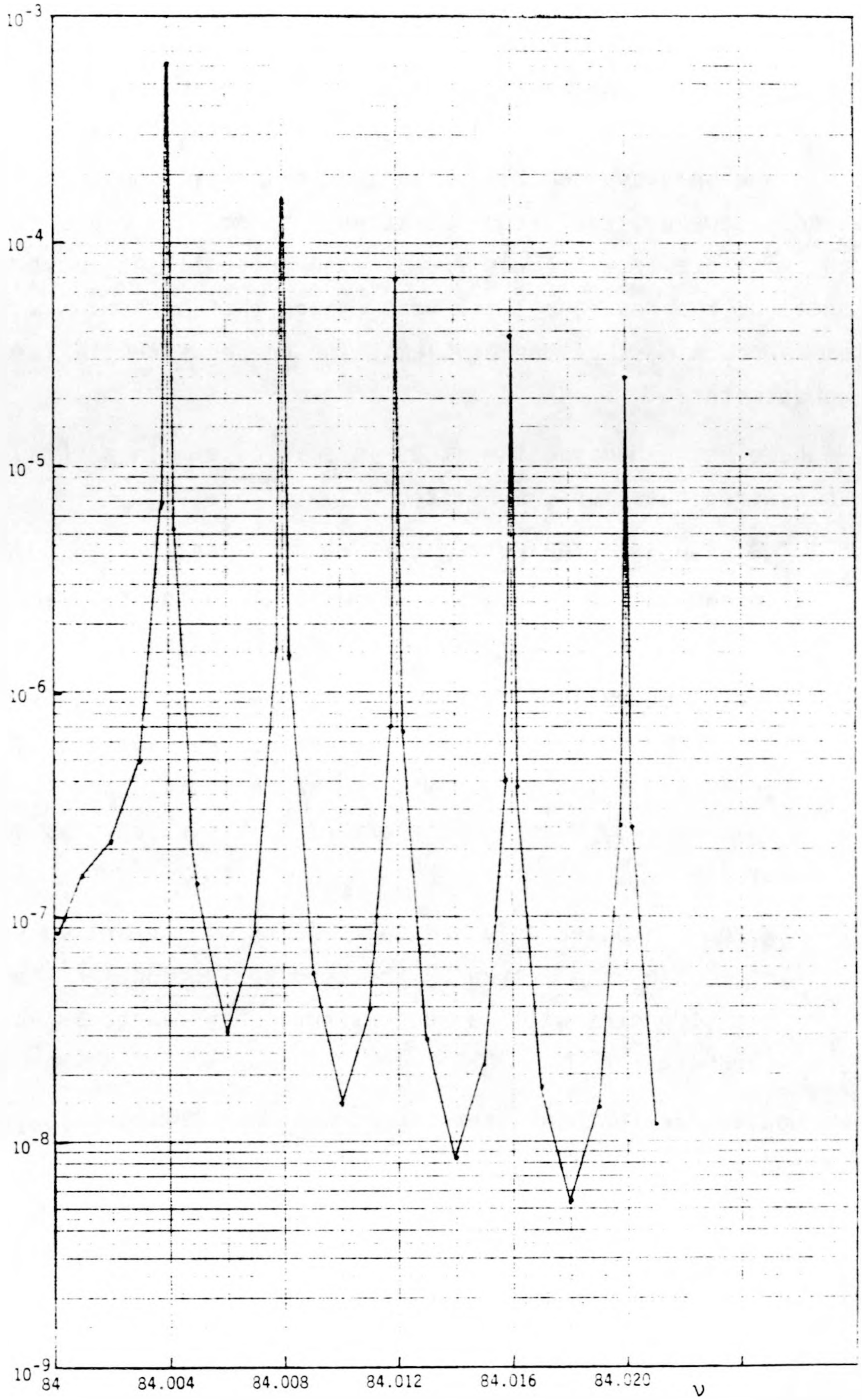

Fig. 1. The effective emittance growth due to overlap knockout resonance effects as a function of tune (ν) near ν = 84, for L = 97.4 m (= 13 d/2).

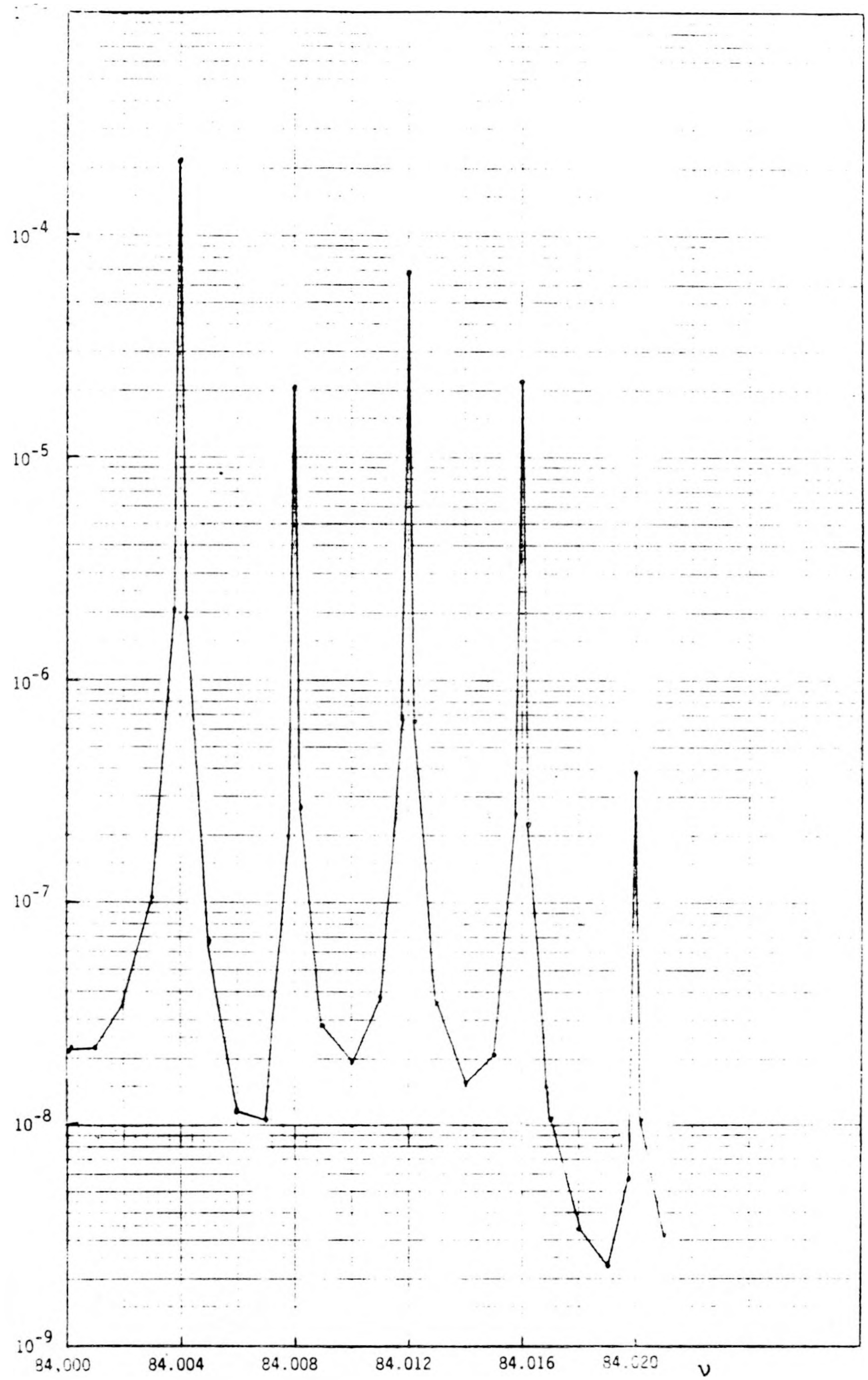

Fig. 2. The effective emittance growth due to overlap knockout resonace effects as a function of tune (ν) near ν = 84, for L = 100.1 m (= 13.35 d/2).