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RESEARCH IN ACCELERATOR PHYSICS (THEORY)

Final Technical Report

May 15, 1989 - May 14, 1990

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## I. INTRODUCTION

This is the final technical report of the grant DE-FG05-87ER40374 covering the third year of the program, from 05/15/89 to 05/14/90.

Since a progress report DOE/ER/40374-36, which describes our activities during the period 05/15/89 - 12/31/89, has already been made available, this report will emphasize the activities during the second half of the project year. Our plan for the fourth year is discussed fully in the renewal proposal which has been submitted to the DOE in January. Subsequently, the continuation has been approved for one year, from 05/15/90 to 05/14/91, for the research in accelerator physics.

The major difference in the proposal for the next year from the previous ones is the separation of two subgroups, the nonlinear dynamics group headed by Bambi Hu and the accelerator physics group. Since this is the final year for the nonlinear dynamics group, Bambi Hu has provided a report covering the entire grant period, from 08/15/87 to 05/14/90.

During the past year, we have maintained close collaborative relations with the Texas Accelerator Center (TAC) and Fermi National Accelerator Laboratory. In addition, we have started a new collaborative effort with the SSC Laboratory and the Argonne National Laboratory.

Through TAC, Houston Advanced Research Center (HARC) has allowed us to use its supercomputer NEC SX-2 free of charge, and this has been invaluable to some of our projects. The success we have achieved in developing the effective use of SX-2 in beam tracking has led directly to the collaborative project between TAC and the SSC Collider Group. It has been established through many sample calculations that SX-2 is in most cases superior to CRAY2 in CPU time, this in addition to the much better turn-around time. We intend to extend this collaboration further when the next computer, SX-X, becomes available at HARC.

Fermilab has been particularly helpful to us in making available its booster for the extended period of beam studies in order to understand the emittance growth at low energies. Results of the beam study have been reported in an EXP note of Fermilab (DOE/ER/40374-37). The project has been supported partially by the SSC Laboratory since it is relevant to the optimum design of its injector system.

One graduate student, who has been stationed at Fermilab since September 1987, has finished the final beam study using Tevatron. The study is to explore the possibility of operating a storage ring such as Tevatron near integer tune values. Findings from this have been reported by him at the APS meeting in April.

During this reporting period, we have initiated a new collaboration with the Advanced Photon Source project at Argonne National Laboratory. A graduate student will be stationed there to participate in the research and development of various insertion devices such wiggler and undulators and their impact on the beam dynamics in synchrotrons.

We have made available for distribution the following reports during the past project year. Some of these have already been mentioned in the previous progress report DOE/ER/40374-36.

- DOE/ER/40374-26 B. Hu, J. Shi, and S.Y. Kim, "Recurrence of KAM Tori in Smooth Twist Maps", submitted to *Physica D*.
- 40374-27 S. Machida, "Tracking Study of a Low Energy Proton Synchrotron", *Particle Accelerators*, 28, 131 (1990).
- 40374-28 N.K. Mahale and S. Ohnuma, "Beam-Beam Interaction and Pacman Effects in the SSC with Momentum Oscillation", *Particle Accelerators*, 27, 175 (1990).
- 40374-29 D. Raparia, "HESQ, a Low Energy Beam Transport", *Proceedings of the Fifth International Symposium on Production and Neutralization of Negative Ions and Beams*, to be published, (October 28 - November 4, 1989, Brookhaven National Lab.).
- 40374-30 S. Ohnuma and D. Raparia (with others), "Test of a Compact 750 keV H<sup>-</sup> Preinjector", *Proceedings of the Fifth International Symposium on Production and Neutralization of Negative Ions and Beams*, to be published.
- 40374-31 S.Y. Kim and B. Hu, "Scaling Patterns of Period Doubling in Four Dimensions", *Phys. Rev. A41* (1990).
- 40374-32 N.K. Mahale and S. Ohnuma (with others), "Beam Transport in the Crystal X-Ray Accelerator", *Particle Accelerators*, 32, 235 (1990).
- 40374-33 B. Hu, J. Shi, and S.Y. Kim, "Power Spectra of Higher Period Multiplications in Area-Preserving Maps", *Phys. Lett. A140*, 158 (1989).

DOE/ER/40374-34      G. Wang, B. Hu, and S. Chen, "Arnold Diffusion in a Four-Dimensional Standard Map", submitted to Phys. Lett.

40374-35      B. Hu, J. Shi, and S.Y. Kim, "Resurrection of KAM Tori", submitted to Phys. Lett.

40374-36      Progress Report, May 15, 1989 - December 31, 1989.

40374-37      S. Machida, "Study of Space Charge Effects in the Fermilab Booster", Fermilab Accelerator Division note EXP-170, April 2, 1990.

40374-38      M. Li and S. Ohnuma, "Two-Parameter Sorting of Dipoles in Large Synchrotrons", EPAC90 - 2nd European Particle Accelerator Conference, Nice, France, June 12-16, 1990.

40374-39      S. Machida, "Space Charge Effects in Low Energy Synchrotrons", EPAC 90, 2nd European Particle Accelerator Conference, Nice, France, June 12-16, 1990.

40374-40      Final Technical Report, May 15, 1989 - May 14, 1990.

Members of the group and their participation in the grant programs during the past twelve months are:

S. Ohnuma (P.I.)	Professor	50%	2 months, summer
B. Hu	Professor	50%	2 months, summer
N.K. Mahale*	Research Associate	100%	5/15 - 8/31/89
L-K. Chen*	Research Associate	100%	5/15 - 7/04/89
S. Machida	Research Associate	100%	5/15 - 5/31/89
		80%	6/01 - 5/14/90
Y-X. Huang	Research Associate	100%	10/02/89 - 5/14/90
G-R. Wang	Research Associate	100%	5/15 - 9/18/89
		40%	9/19/89 - 1/18/90
Y-C. Chae	Graduate Student	50%	1/15 - 5/14/90
M-Y. Li	Graduate Student	50%	5/15/89 - 5/14/90
D. Raparia*	Graduate Student	50%	5/15 - 5/31/89
		38%	6/01/89 - 2/28/90
J-C. Shi	Graduate Student	50%	5/15/89 - 5/14/90
P-L. Zhang	Graduate Student	50%	Supported by Fermilab.

\* Now at the SSC Laboratory.

Deepak Raparia is the first graduate student from our group to complete all the requirements for Ph. D. Two more students are expected to finish within this year.

## II. MAIN RESULTS FROM RESEARCH ACTIVITIES IN ACCELERATOR PHYSICS

From the beginning of the project, the group has been divided into two subgroups: nonlinear dynamics group (Bambi Hu, group leader) and accelerator physics group (Sho Ohnuma, P.I.). Since this is the final year for them, the activities during the past three project years of the nonlinear group are summarized in the attached report.

Altogether six topics have been studied during the past project year with varying degrees of accomplishments.

### 1. Beam-Beam Interaction and Pacman Effects in the SSC with Momentum Oscillation (Mahale, Ohnuma)

In order to find the combined effects of beam-beam interaction (head-on and long-range) and random nonlinear multipoles in dipole magnets, the transverse oscillations of "regular" as well as "pacman" particles are tracked for 256 momentum oscillation periods (corresponding to 135K turns) in the proposed SSC. Results obtained in our study do not show any obvious reduction in dynamic or linear apertures for pacman particles when compared with regular particles for  $(\Delta p/p)=0$ . At the same time, there are some indications of possible sudden or gradual increases in the oscillation amplitude, for pacman as well as regular particles, when the amplitude of momentum oscillation is as large as  $3\sigma$ .

It has become obvious to us that, in order to increase the number of turns by an order of magnitude to the level of a few millions, it is necessary to use a more efficient tracking code and to utilize a faster supercomputer. Fortunately, a new code SSCTRK has been developed by D. Ritson for the SSC which is much more efficient than TEAPOT we have been using so far. We are now in the process of adding the beam-beam interaction into this code (a feature absent at the moment). As for a faster supercomputer, A. Sato of TAC has done, in collaboration with K. Kauffmann of the SSC, an extensive comparative study of two supercomputers, CRAY2 used by the SSC and NEC SX-2 available at HARC/TAC with the following results:

number of turns tracked:	10,000
number of different random sets of magnet errors:	1 to 64 (M)
number of particles:	1 to 64 (N)

CPU time in seconds

	CRAY2 assembler	SX-2 inline	SX-2 assembler
(M, N) = (8, 4)	256.8	284.7	168.5
(16,2)	266.4	296.6	183.1
(1,64)	387.6	305.9	174.0
(8, 8)	391.8	309.3	234.9
(16,4)	399.0	316.3	182.8
(32,2)	420.0	337.6	199.9
(64,1)	476.4	379.0	234.6

A new model, SX-3, of the NEC supercomputer should become available at HARC in the spring of 1991 and we anticipate a further improvement in the CPU time.

2. Two-Parameter Sorting of Dipoles in Large Synchrotrons (Li, Ohnuma)

A relatively simple procedure for finding the optimum arrangement of a given set of dipoles in a large synchrotrons has been studied when normal and skew sextupole components in bending magnets are the dominant factors in the aperture reduction. The analytical figure-of-merit (F.M.) used previously in the presence of normal sextupole component alone has been extended to include the contribution from skew component as well. This F.M. has been used to weed out several different sorting schemes which are not very promising. For a model lattice composed of 576 dipoles in six superperiods, the dynamic aperture for 2,000 turns and the linear aperture corresponding to a 5% "smear" parameter have been found from numerical trackings for several unsorted arrangements and for a "one-parameter" and a "two-parameter" sorted arrangement. In general, results indicate a significant improvement in aperture, both dynamic and linear, when two parameters (normal and skew sextupole components) are taken into account.

We intend to apply the sorting scheme for the proposed high energy booster (HEB) of the SSC injector complex.

3. Self-Consistent Treatment of Space Charge Effects in Low Energy Synchrotrons (Machida)

Space charge effects in low energy proton synchrotrons on the emittance dilution have been exhaustively studied using a simulation code that has been developed specifically for supercomputers. The total CPU time exceeded 1,000 hours. The tracking of a few thousand macroparticles representing a beam is completely self-consistent, thereby making it possible to follow the charge distribution as

well as the beam emittance as a function of time. A proper substitution of the kick to represent the continuous space charge force has been carefully considered.

The simulation has been performed for a model lattice with and without half-integer resonances. With the resonances, a limit in the phase space density has been observed but the limiting value is not directly related to the traditional Laslett tune shift for individual particles. Modified tunes of some particles can be within the resonance width. In the lattice without half-integer resonances, the growth in the rms emittance is associated with higher order resonances driven by the space charge itself and the periodicity of the lattice plays an important role under this condition. Furthermore, the tail of the distribution is affected dominantly and the density at the core remains almost unchanged. It should be stressed that the tracking of a few thousand particles is essential for the detailed study of the changing distribution (core and tail, separately).

The simulation code we have developed should be extremely valuable for the study of space charge effect, for example, in the low energy booster (LEB) of the SSC injector complex and in the RHIC at BNL.

#### 4. New Acceleration Scheme in RFQ; Low Energy Beam Transport HESQ (Raparia)

Raparia has completed this task during the project year (1989-90) and earned Ph.D. from the University of Houston in May 1990. He is currently a member of the SSC Laboratory, working in the injector group. The following is from the abstract of his doctoral dissertation.

"We have studied the beam dynamics of the low energy beam transport (LEBT) and radio-frequency quadrupole (RFQ) linac and propose a new design for both. To avoid neutralization of the H<sup>-</sup> beam due to the background gas, we suggest a new device, helical electrostatic quadrupole (HESQ) for the LEBT. The HESQ provides stronger first-order focusing in contrast to weak second-order focusing einzel lenses. The HESQ also provides an approximately axial symmetric beam which is necessary for the RFQ matching. Being a spatially continuous transport channel, the HESQ provides stronger focusing than an alternating gradient. We will present an analytical formalism for such a channel and results of the particle-in-cell simulation with space charge. We also propose a new design, based on current limits, for the RFQ to increase its output energy. This design results in approximately one-third shorter RFQ with the same transmission and the same beam quality. The higher accelerating field will make the matching to the drift tube linac easier. Emittance growth studies show that, for a given RFQ, there is a minimum output emittance no matter how small the input emittance is."

## 5. Improved Beam Dynamics Treatment for Compact Synchrotrons (Huang)

An entirely new tracking code has been developed by Huang in which exact equations of motion are numerically integrated without any approximation. It also calculates various integrals (in exact forms) which are necessary for the estimate of damping time and beam emittance. The code has been tested for linear cases and for analytically specified edge field of dipole magnets. Huang is now working to use field values from measurements or from magnet codes such as POISSON (two-dimensional) and MAGNUS (three-dimensional).

We will collaborate with TAC personnel to study the expected performance of a 90° superferric dipole which is being built at TAC for a possible compact synchrotron. The emphasis of our work will be on developing efficient iterative procedures involving magnet designers, magnet builders, field measurement and beam dynamics calculation.

## 6. Analysis of E778 Data on Coupled Resonances by Sextupoles (Li)

Li has completed the analysis of E778 data on coupled sextupole resonances in collaboration with Mike Syphers and Don Edwards of the SSC Laboratory. He has also worked with Leo Michelotti of Fermilab. He has demonstrated that, with a small adjustment in the Tevatron linear lattice parameters, measured results are consistent with the lowest-order analytical prediction based on the action-angle formalism. At present, he is working on the analysis of five islands observed in horizontal phase space. For this it is necessary to extend the perturbation to the third order in sextupole strength and to introduce a small amount of asymmetry in the Tevatron sextupole configuration. This work will be a part of Li's doctoral dissertation.

In addition to six topics discussed above, our group (Peilei Zhang) has continued the collaboration with Fermilab personnel (Rol Johnson and others) in the study of possible Tevatron operation near integer tune values. The latest beam study was conducted in March with the following conclusions:

1. There are no fundamental reasons to avoid tunes near an integer for hadron colliders.
2. The large resonance-free area in tune space near integers can accommodate large tune spreads arising from the beam-beam interaction.

3. Practical problems such as the control of closed orbit and power supply noise must be solved for a stable operation.
4. Future beam studies will require additional filtering of the power supplies at low frequencies.

For his doctoral dissertation, Zhang will include results from tracking calculation to support his experimental findings.

### III. OTHER ACTIVITIES

#### a) DOE Site Visit for Review of the Project

Eugene Colton of DOE and Leo Michelotti of Fermilab visited TAC for the review of our research activities on December 1, 1989. Seven members of the group have presented reports on various activities and their status. Visitors toured the facilities at TAC and HARC.

#### b) Foreign Trips

Four foreign trips made during the past year have already been reported in the Progress Report (DOE/ER/40374-36). They are:

1. S. Ohnuma, Third Advanced ICFA Beam Dynamics Workshop at Novosibirsk, May 29 - June 3, 1989.
2. S. Ohnuma and S. Machida, XIVth International Conference on High Energy Accelerators at Tsukuba, Japan, August 1989.
3. B. Hu, 17th International Conference on Thermodynamics and Statistical Mechanics at Rio de Janeiro, Brazil, July 31 - August 4, 1989.

The support for these trips from the grant was \$2,542.

#### c) Review Committee

S. Ohnuma served as a member of the RHIC Magnet Review Committee in February 1990 at Brookhaven National Laboratory.

**Research in Accelerator Physics (Theory):  
Nonlinear Dynamics\***

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\*Final report to the U. S. Department of Energy for grant DE-FG05-87ER40374

During the grant period 1987-1990 we have conducted research on three main topics of Hamiltonian dynamics: period doubling and period  $n$ -tupling; recurrence of KAM tori; and Arnold diffusion. A summary of our work is given below.

## **I. Period doubling and period $n$ -tupling in Hamiltonian systems**

### **1. The fine structure of period doubling in Hamiltonian systems with two and three degrees of freedom.**

To describe the fine structure of period doubling, we have generalized Feigenbaum's simple scaling law to a multiple scaling law. This multiple scaling law is applied to period doubling in Hamiltonian systems with two and three degrees of freedom, modelled respectively by two- and four-dimensional symplectic maps. The detailed fine structure of period doubling, described by new universal exponents, is revealed.

### **2. Power and singularity spectra of period $n$ -tupling in Hamiltonian systems with two degrees of freedom**

We have studied the power spectra of higher period  $n$ -tuplings for  $n = 3, 4, 5$  and  $6$  in area-preserving maps. The ratio of the successive average heights of the peaks of the spectrum is found to approach a universal scaling limit that increases with  $n$ .

We have also studied the singularity spectrum  $f(\alpha)$  and the generalized dimension  $D(q)$  of period  $n$ -tupling ( $n = 2, 3, 4$ ) in area-

preserving maps. It is found that  $f(\alpha)$  behaves quite differently as  $n$  is changed, and the generalized dimension  $D(q)$  increases for all  $q$  as  $n$  increases. The global scaling behavior of conservative systems is different from that of dissipative systems. Moreover, for conservative systems, the global scaling behavior seems to depend on dimensionality.

### **3. Scaling patterns of period doubling in Hamiltonian systems with three degrees of freedom**

We study period- $M$  ( $M = 1$  and  $2$ ) scaling behaviors of period doubling in a symmetric four-dimensional volume-preserving map. The four fundamental noncoordinate scaling factors are obtained by a direct numerical study and a renormalization analysis. We find that there are three kinds of period-1 scaling behaviors and one kind of period-2 scaling behavior. Furthermore, we conjecture that all the scaling behaviors of period doubling could be classified by the route sequence.

## **II. Recurrence of KAM tori**

The Kolmogorov-Arnold-Moser (KAM) theorem was a landmark in the history of mechanics -- in a sense, it ushered in the modern era of mechanics. However, as the original proof of the KAM theorem goes, its validity is restricted to an extremely small perturbation. An important contribution was made by Greene, whose pioneering work can be viewed as an extension of the KAM theorem from the weak- to the strong-coupling regime. In particular, his famous residue criterion provides a precise

method to determine the transition point at which the last KAM torus disappears.

However, one outstanding defect in these remarkable studies is that most of the result obtained so far are based on the standard map. It is therefore not clear how general the conclusions are beyond the standard map. Greene's criterion, albeit it has worked amazingly well, is still in want of a proof.

To test the generality of the standard map, we have studied the behavior of KAM tori in a class of smooth twist maps in which the nonlinear function is endowed with a varying degree of inflection  $z$ . Many novel features have been discovered. Reappearance of KAM tori has been observed for  $z > 3$ . An "inverse residue criterion" is proposed to determine the reappearance point. We have also studied the scaling behavior at the disappearance and reappearance points. The scaling exponents are found to vary with  $z$  for  $2 < z < 3$ , but are independent of  $z$  for  $z \geq 3$ . In this sense  $z = 3$  plays a role quite similar to that of the upper critical dimension in phase transitions.

### III. Arnold diffusion

It is a special property of a Hamiltonian system with two degrees of freedom that two dimensional KAM tori partition the three-dimensional energy manifold. This is no longer the case for systems with three or more degrees of freedom. KAM tori no longer isolate the resonance layers, which

now intersect and form a connected web (the Arnold web) dense in the action space. Conservation of energy no longer forbids stochastic motion of the action along the resonance layers and diffusion occurs: this is known as Arnold diffusion.

Although at present the study of Arnold diffusion is mainly of theoretical interest, its potential relevance to practical problems should not be overlooked. For example, in the design of particle accelerators, the lifetime of a charged beam is determined by the rate at which the amplitudes of the transverse oscillations diffuse. The diffusion can take place either by resonance or Arnold diffusion. Resonance can sometimes be avoided by controlling the frequency. Arnold diffusion, on the other hand, is generic and always present in systems with three or more degrees of freedom. Although the rate of Arnold diffusion is typically very small, its cumulative effect on the lifetime of a beam may not be negligible in view of the huge number of revolutions a particle executes in an accelerator.

To study Arnold diffusion, we model a three-degree-of-freedom Hamiltonian system by a four-dimensional symplectic map which is a coupled version of two standard maps. We employ the stochastic pump model in which the original three-degree-of-freedom system is decomposed sequentially into two two-degree-of-freedom systems. The coupling between two degrees of freedom generates stochasticity which serves as a "pump" for diffusion. After this motion is solved, we then substitute it into the Hamiltonian describing the coupling between the other two degrees of freedom. We have computed the diffusion rate as a function of the coupling constant, the nonlinearity parameter, and the time.

Theoretical predictions agree well with numerical results; however, for large parameter values, deviation begins to emerge.

#### IV. Publications

1. "Multiple Scaling and the Fine Structure of Period Doubling,"  
J. M. Mao and B. Hu  
*Int. J. Mod. Phys. B* **2**, 65 (1988). DOE/ER/40374-3
2. "Transitions to Chaos in Higher Dimensions,"  
B. Hu and J.M. Mao,  
*Directions in Chaos*, Vol. 1, p. 206, ed. B. L. Hao (World Scientific, Singapore, 1988). DOE/ER/40374-2
3. "Singularity Spectrum for Period  $n$ -Tupling in Area-Preserving Maps,"  
S. Y. Kim and B. Hu  
*Phys. Rev. A* **38**, 1534 (1988). DOE/ER/40374-9
4. "Power Spectra of Higher Period Multiplings in Area-Preserving Maps,"  
B. Hu , J. Shi and S. Y. Kim,  
*Phys. Lett. A* **140**, 158 (1989). DOE/ER/40374-33
5. "Scaling Patterns of Period Doubling in Four Dimensions,"  
S. Y. Kim and B. Hu,  
*Phys. Rev. A* **41**, (1990). DOE/ER/40374-31
6. "Resurrection of KAM Tori,"  
B. Hu, J. Shi and S. Y. Kim,  
*Phys. Lett.* (submitted). DOE/ER/40374-35
7. "Recurrence of KAM Tori in Smooth Twist Maps,"  
B. Hu, J. Shi and S. Y. Kim,  
*Physica D* (submitted). DOE/ER/40374-26
8. "Arnold Diffusion in a Four-Dimensional Standard Map,"  
G. R. Wang, B. Hu and S. G. Chen,  
*Phys. Lett.* (submitted). DOE/ER/40374-34