

Mode Analysis and Ward Identities for
Perturbative Quantum Gravity in de Sitter Space

N. C. Tsamis*

Department of Physics
University of Crete
Iraklion, Crete 71409
GREECE

DOE/ER/40272--167
DE92 040503

and

R. P. Woodard**

Department of Physics
University of Florida
Gainesville, FL 32611
UNITED STATES

ABSTRACT: We study linearized gravitons on the D-dimensional open submanifold spanned by de Sitter conformal coordinates. The physical modes are found in the same way as for flat space by imposing exact gauge conditions on the invariant field equations and then exploiting the residual gauge freedom of solutions. The resulting polarization tensors have vanishing zero components and are transverse and traceless, just as in flat space. We also show that vacua exist such that the ghost and graviton propagators obey the Ward identity relating them.

MASTER

Received by OSTI

* E-mail: TSAMIS @ IESL.FORTH.GR
** E-mail: WOODARD @ UFHEPA.PHYS.UFL.EDU

SEP 11 1992

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

Quantum gravity has been little considered in the search for an explanation of the smallness of the cosmological constant. The reasons for this seem to be the theory's well known ultraviolet problems and the widespread belief that de Sitter space, the natural background for $\Lambda > 0$, is such a strange environment that we can never hope to understand physics on it. This is a great pity because the far infrared sector of quantum gravity is in many ways the natural place to look for a resolution to what is, after all, a gravitational problem.

We shall elsewhere establish that Einstein's theory can be used reliably in the far infrared; here we seek to dispel the notion that it is significantly more difficult to understand linearized gravitons on a de Sitter background than in flat space. We make this point by solving the theory, *in toto*, using the same methods that are employed in flat space. (See, for example, chapter 10 of ref. [1].) The analysis is so simple that we have been able to carry it out generally in D dimensions. The result is simple too: in conformal coordinates and with a suitably rescaled field variable the modes can be written as plane waves characterized by their spatial $(D - 1)$ -momenta and by polarization tensors which are identical to the polarization tensors of flat space. The only complicating feature is a slightly different time dependence. In four dimensional flat space the plane wave solutions are oscillatory; in four dimensional de Sitter space this oscillatory function acquires a time dependent prefactor and a phase:

$$e^{\pm i k x^0} \rightarrow \left(1 \mp \frac{i}{k u}\right) \exp\left[\mp i k \left(u - \frac{1}{H}\right)\right] \quad (1)$$

where u is our time variable and $k \equiv \|\vec{k}\|$ is the Euclidean norm of the 3-momenta. (In D dimensions the time dependence is proportional to $\sqrt{k u}$ times a $\nu = \frac{D-1}{2}$ Hankel function.) As a bonus we apply our technology to determine all possible vacua for which the ghost and graviton propagators obey the Ward identity relating them.

This paper is based on a previous one [2] whose notational conventions we shall follow.

The invariant Lagrangian is:

$$\mathcal{L}_{\text{inv}} = \frac{1}{\kappa^2} \left[R - (D-2)(D-1)H^2 \right] \sqrt{-g} \quad (2)$$

where the Hubble constant is $H^2 = \frac{1}{D-1}\Lambda$, our metric has spacelike signature and R is the Ricci scalar formed from $R^\alpha_{\beta\mu\nu} = \Gamma^\alpha_{\nu\beta,\mu} + \Gamma^\alpha_{\mu\rho} \Gamma^\rho_{\nu\beta} - (\mu \leftrightarrow \nu)$. Perturbation theory derives from the expansion:

$$g_{\mu\nu} = \hat{g}_{\mu\nu} + \kappa h_{\mu\nu} \quad (3)$$

where $\hat{g}_{\mu\nu}$ is an exact solution. We shall work in the open conformal coordinate system where the background metric is:

$$\hat{g}_{\mu\nu} = \frac{1}{(Hu)^2} \eta_{\mu\nu} \equiv \Omega^2 \eta_{\mu\nu} \quad (4)$$

A peculiarity of this system is that while the spatial coordinate, \vec{x} , can take any value in $(D-1)$ -dimensional Euclidean space the time coordinate, u , runs only from zero to infinity. It is also inverted with respect to physical time; that is, the far future is obtained by letting u approach zero while the far past is probed by taking u to $+\infty$. The flat space limit is obtained by substituting $u = \frac{1}{H} - x^0$ and taking H to zero while holding the flat space time x^0 fixed. Although the conformal coordinate system covers only half of the full de Sitter manifold it is complete in the sense that nothing leaks into or out of the submanifold; surfaces of constant u are Cauchy surfaces. An important advantage of restricting physics to this submanifold is that one avoids the linearization instability which has frustrated all previous attempts to formulate quantum gravity on de Sitter space [2].

Although interactions are most easily described using the pseudo-graviton field, $\psi_{\mu\nu} \equiv \Omega^{-2} h_{\mu\nu}$, a slightly different rescaling gives the simplest formulation of the free theory:

$$\chi_{\mu\nu} \equiv \Omega^{\frac{D}{2}-3} h_{\mu\nu} \quad (5)$$

The indices on $h_{\mu\nu}$ are raised and lowered with $\hat{g}_{\mu\nu}$ but those of both $\psi_{\mu\nu}$ and $\chi_{\mu\nu}$ are raised and lowered with the Minkowski metric. It is of course completely trivial to convert

the mode solutions, Green's functions or propagators from one of these fields to any other. As an example we consider the pseudo-graviton propagator, $i[\rho\sigma\Delta^{\alpha\beta}](x, x')$, in the gauge that will shortly be used. Up to a real term which depends upon the vacuum — and about which we shall have more to say later — we have shown [2] that the four dimensional limit of this object is $\frac{H^2}{8\pi^2}$ times:

$$-\ln[H^2(x - x')^2 + i\epsilon] [2\bar{\delta}_\rho^{(\alpha}\bar{\delta}_\sigma^{\beta)} - 2\bar{\eta}_{\rho\sigma}\bar{\eta}^{\alpha\beta}] + \frac{2u'u}{(x - x')^2 + i\epsilon} [2\delta_\rho^{(\alpha}\delta_\sigma^{\beta)} - \eta_{\rho\sigma}\eta^{\alpha\beta}] \quad (6)$$

(Our notation is that $(x - x')^2 \equiv -(u - u')^2 + \|\vec{x} - \vec{x}'\|^2$, parenthesized indices are symmetrized and a bar over a standard tensor such as $\eta_{\rho\sigma}$ or δ_ρ^α indicates the suppression of its zero components.) To obtain the propagator for $\chi_{\mu\nu}$ in four dimensions we just multiply by a conformal factor for each of the two fields in the expectation value:

$$i[\rho\sigma\Delta_\chi^{\alpha\beta}](x, x') = \Omega^{\frac{D}{2}-1}(x)\Omega^{\frac{D}{2}-1}(x')i[\rho\sigma\Delta^{\alpha\beta}](x, x') \quad (7a)$$

$$\xrightarrow{D=4} -\frac{1}{8\pi^2uu'}\ln[H^2(x - x')^2 + i\epsilon] [2\bar{\delta}_\rho^{(\alpha}\bar{\delta}_\sigma^{\beta)} - 2\bar{\eta}_{\rho\sigma}\bar{\eta}^{\alpha\beta}] + \frac{1}{4\pi^2}\frac{1}{(x - x')^2 + i\epsilon} [2\delta_\rho^{(\alpha}\delta_\sigma^{\beta)} - \eta_{\rho\sigma}\eta^{\alpha\beta}] + (\text{vac. dep.}) \quad (7b)$$

It is worth drawing attention to the remarkable simplicity of these results and their close relation to the flat space limit.

In terms of the field $\chi_{\mu\nu}$ the quadratic part of the invariant Lagrangian is:

$$\begin{aligned} \mathcal{L}_{\text{inv}}^2 = & \frac{1}{2}\chi^{\rho\sigma,\mu}\chi_{\mu\sigma,\rho} - \frac{1}{2}\chi^{\mu\nu}_{,\mu}\chi_{,\nu} + \frac{1}{4}\chi^{\cdot\mu}_{,\mu} - \frac{1}{4}\chi^{\rho\sigma,\mu}\chi_{\rho\sigma,\mu} \\ & + \left(\frac{D-2}{2}\right)\left\{-\chi^{\mu\rho,\sigma}\chi_{\rho\sigma}\phi_{,\mu} - \frac{1}{2}\chi^{\mu\nu}\chi_{,\mu}\phi_{,\nu} + \frac{1}{2}\chi^{\mu\nu}_{,\mu}\phi_{,\nu} + \frac{1}{4}\left(-\chi^2 + \chi^{\rho\sigma}\chi_{\rho\sigma}\right)^{\mu}\phi_{,\mu}\right\} \\ & + \left(\frac{D-2}{2}\right)^2\left\{\frac{1}{2}\chi^{\mu\rho}\chi^\nu_{,\rho}\phi_{,\mu}\phi_{,\nu} + \frac{1}{2}\chi^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + \frac{1}{4}\chi^2\phi^{\mu}\phi_{,\mu} - \frac{1}{4}\chi^{\rho\sigma}\chi_{\rho\sigma}\phi^{\mu}\phi_{,\mu}\right\} \end{aligned} \quad (8)$$

where $\phi \equiv \ln(\Omega)$. The associated action is invariant under a linearized transformation characterized by a parameter $\epsilon_\mu(u, \vec{x})$:

$$\delta\chi_{\mu\nu} = -2\epsilon_{(\mu,\nu)} + (D-2)\epsilon_{(\mu}\phi_{,\nu)} - 2\eta_{\mu\nu}\epsilon^\rho\phi_{,\rho} \quad (9)$$

The simplest gauge fixing functional seems to be $-\frac{1}{2}\eta^{\mu\nu}F_\mu F_\nu$ where:

$$F_\mu \equiv \chi_{,\mu}^\rho - \frac{1}{2}\chi_{,\rho}^\mu + (\frac{D-2}{2})\chi_\mu^\rho \phi_{,\rho} + (\frac{D-2}{4})\chi_\rho^\rho \phi_{,\mu} \quad (10)$$

With some partial integrations the quadratic gauge fixed action can be written as $\mathcal{L}_{GF}^2 = \frac{1}{2}\chi^{\mu\nu}\mathcal{D}_{\mu\nu}^{\rho\sigma}\chi_{\rho\sigma}$, where we define the kinetic operator:

$$\begin{aligned} \mathcal{D}_{\mu\nu}^{\rho\sigma} = & \left[\frac{1}{2}\bar{\delta}_\mu^{(\rho}\bar{\delta}_\nu^{\sigma)} - \frac{1}{4}\eta_{\mu\nu}\eta^{\rho\sigma} - \frac{1}{2(D-3)}\delta_\mu^0\delta_\nu^0\delta_0^\rho\delta_0^\sigma \right] \mathcal{D}_A \\ & + \delta_{(\mu}^0\bar{\delta}_{\nu)}^{(\rho}\delta_0^{\sigma)}\mathcal{D}_B + \frac{1}{2}(\frac{D-2}{D-3})\delta_\mu^0\delta_\nu^0\delta_0^\rho\delta_0^\sigma\mathcal{D}_C \end{aligned} \quad (11)$$

in terms of the following differential operators:

$$\mathcal{D}_A \equiv \left[\partial^2 + (\frac{D-2}{2})(\frac{D}{2})\frac{1}{u^2} \right] \quad (12a)$$

$$\mathcal{D}_B \equiv \left[\partial^2 + (\frac{D-4}{2})(\frac{D-2}{2})\frac{1}{u^2} \right] \quad (12b)$$

$$\mathcal{D}_C \equiv \left[\partial^2 + (\frac{D-6}{2})(\frac{D-4}{2})\frac{1}{u^2} \right] \quad (12c)$$

Since the gauge fixed field equations can be written as the invariant ones plus terms which vanish with F_μ :

$$0 = \mathcal{D}_{\mu\nu}^{\rho\sigma}\chi_{\rho\sigma} \quad (13a)$$

$$= \frac{\delta S_{inv}^2}{\delta \chi^{\mu\nu}} + F_{(\mu,\nu)} - \frac{1}{2}\eta_{\mu\nu}F_{,\rho}^\rho - (\frac{D-2}{2})F_{(\mu}\phi_{,\nu)} - (\frac{D-2}{4})\eta_{\mu\nu}F^\rho\phi_{,\rho} \quad (13b)$$

we see that imposing the invariant field equations in $F_\mu = 0$ gauge is the same as solving the gauge fixed equation (13a) and then imposing $F_\mu = 0$ as a supplementary condition.

It should be obvious from the preceding discussion that we wish to study solutions to $\mathcal{D}_{A,B,C} = 0$. The spatial plane wave solutions turn out to have the form:

$$\chi_a(u, \vec{x}; \vec{k}) = \sqrt{\frac{1}{2}\pi ku} H_{\frac{D-1}{2}}^{(1)}(ku) \exp\left[i\vec{k} \cdot \vec{x} - i\frac{k}{H} + i\frac{D-4}{4}\pi\right] \quad (14a)$$

$$\chi_b(u, \vec{x}; \vec{k}) = \sqrt{\frac{1}{2}\pi ku} H_{\frac{D-3}{2}}^{(1)}(ku) \exp\left[i\vec{k} \cdot \vec{x} - i\frac{k}{H} + i\frac{D-2}{4}\pi\right] \quad (14b)$$

$$\chi_c(u, \vec{x}; \vec{k}) = \sqrt{\frac{1}{2}\pi ku} H_{\frac{D-5}{2}}^{(1)}(ku) \exp\left[i\vec{k} \cdot \vec{x} - i\frac{k}{H} + i\frac{D-4}{4}\pi\right] \quad (14c)$$

The normalizations are such that each of these goes to $\exp[-ikx^0 + i\vec{k} \cdot \vec{x}]$ in the flat space limit. In four dimensions these formulae reduce to elementary functions:

$$\chi_a(u, \vec{x}; \vec{k}) \xrightarrow{D=4} \left(1 + \frac{i}{ku}\right) \exp\left[ik(u - \frac{1}{H}) + i\vec{k} \cdot \vec{x}\right] \quad (15a)$$

$$\chi_{b,c}(u, \vec{x}; \vec{k}) \xrightarrow{D=4} \exp\left[ik(u - \frac{1}{H}) + i\vec{k} \cdot \vec{x}\right] \quad (15b)$$

Although the various modes differ for $H \neq 0$ it is simple to convert them into one another using the standard recursion relations that all Bessel functions obey:

$$\left[\partial_u \pm \left(\frac{D-2}{2}\right) \frac{1}{u}\right] \chi_{a,b} = ik \chi_{b,a} \quad (16a)$$

$$\left[\partial_u \pm \left(\frac{D-4}{2}\right) \frac{1}{u}\right] \chi_{b,c} = ik \chi_{c,b} \quad (16b)$$

It turns out that one of these “raising” or “lowering” operators is always present whenever modes of different types appear in the same equation.

A convenient reexpression of the field $\chi_{\rho\sigma}$ is as follows:

$$\chi_{\rho\sigma} = \left(\delta_\rho^i \delta_\sigma^j - \frac{1}{D-3} \bar{\eta}_{\rho\sigma} \bar{\eta}^{ij}\right) \epsilon_a^{ij} + 2\delta_{(\rho}^0 \delta_{\sigma)}^i \epsilon_b^i + \left(\delta_\rho^0 \delta_\sigma^0 + \frac{1}{D-3} \bar{\eta}_{\rho\sigma}\right) \epsilon_c \quad (17)$$

(Note that we make no distinction between contravariant and covariant spatial indices i , j , etc.) Substituting (17) in (13a) we obtain:

$$\frac{1}{2} \left(\delta_\mu^i \delta_\nu^j - \frac{1}{D-3} \delta_\mu^0 \delta_\nu^0 \eta^{ij}\right) \mathcal{D}_A \epsilon_a^{ij} + \delta_{(\mu}^0 \delta_{\nu)}^i \mathcal{D}_B \epsilon_b^i + \frac{1}{2} \left(\frac{D-2}{D-3}\right) \delta_\mu^0 \delta_\nu^0 \mathcal{D}_C \epsilon_c = 0 \quad (18)$$

Although the tensor factor of the A term has zero components, the vanishing of the spatial components is sufficient to enforce $\mathcal{D}_A \epsilon_a^{ij} = 0$. It follows that $\mathcal{D}_B \epsilon_b^i = 0 = \mathcal{D}_C \epsilon_c$. The most general solution to (13a) is obtained by superposing plane waves:

$$\epsilon_a^{ij}(u, \vec{x}) = \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \left\{ A^{ij}(\vec{k}) \chi_a(u, \vec{x}; \vec{k}) + \text{c.c.} \right\} \quad (19a)$$

$$\epsilon_b^i(u, \vec{x}) = \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \left\{ B^i(\vec{k}) \chi_b(u, \vec{x}; \vec{k}) + \text{c.c.} \right\} \quad (19b)$$

$$\epsilon_c(u, \vec{x}) = \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \left\{ C(\vec{k}) \chi_c(u, \vec{x}; \vec{k}) + \text{c.c.} \right\} \quad (19c)$$

We must now enforce as a supplementary condition the vanishing of:

$$F_\mu = \delta_\mu^i \left\{ \epsilon_a^{ij} - \left[\partial_u - \left(\frac{D-2}{2} \right) \frac{1}{u} \right] \epsilon_b^i \right\} + \delta_\mu^0 \left\{ \frac{1}{D-3} \left[\partial_u + \left(\frac{D-2}{2} \right) \frac{1}{u} \right] \epsilon_a^{ii} + \epsilon_b^i, i - \left(\frac{D-2}{D-3} \right) \left[\partial_u - \left(\frac{D-4}{2} \right) \frac{1}{u} \right] \epsilon_c \right\} \quad (20)$$

These equations can be satisfied, just as in flat space, by determining the temporal polarizations in terms of the purely spatial ones:

$$B^i(\vec{k}) = A^{ij}(\vec{k}) \hat{k}_j \quad (21a)$$

$$C(\vec{k}) = \frac{1}{D-2} A^{ii}(\vec{k}) + \left(\frac{D-3}{D-2} \right) A^{ij}(\vec{k}) \hat{k}_i \hat{k}_j \quad (21b)$$

where $\hat{k}^i \equiv k^i/k$ is the momentum direction vector.

The final step is to exploit the residual gauge freedom. Since the invariant field equations are unchanged by any transformation of the form (9) the only restriction on e_μ comes from preserving the supplementary condition, $F_\mu = 0$. The variation of this condition is:

$$\delta F_\mu = -\delta_\mu^i \mathcal{D}_A e_i - \delta_\mu^0 \mathcal{D}_B e_0 \quad (22)$$

And so we find the most general residual symmetry:

$$e_i(u, \vec{x}) = \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \left\{ a_i(\vec{k}) \chi_a(u, \vec{x}; \vec{k}) + \text{c.c.} \right\} \quad (23a)$$

$$e_0(u, \vec{x}) = \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \left\{ b(\vec{k}) \chi_b(u, \vec{x}; \vec{k}) + \text{c.c.} \right\} \quad (23b)$$

Such a transformation induces the following change on the polarizations:

$$\delta A_{ij}(\vec{k}) = -2i k_{(i} a_{j)}(\vec{k}) + \eta_{ij} [ik_\ell a_\ell(\vec{k}) - ik b(\vec{k})] \quad (24a)$$

$$\delta B_i(\vec{k}) = -ik_i b(\vec{k}) - ik a_i(\vec{k}) \quad (24b)$$

$$\delta C(\vec{k}) = -2i k b(\vec{k}) \quad (24c)$$

We can suppress the temporal polarizations entirely with the following choice:

$$a_i(\vec{k}) = \frac{i}{2k} \left[-2A_{ij}(\vec{k}) \hat{k}_j + \frac{1}{D-2} \hat{k}_i A_{jj}(\vec{k}) + \left(\frac{D-3}{D-2}\right) \hat{k}_i A_{j\ell}(\vec{k}) \hat{k}_j \hat{k}_\ell \right] \quad (25a)$$

$$b(\vec{k}) = \frac{i}{2k} \left[-\frac{1}{D-2} A_{ii}(\vec{k}) - \left(\frac{D-3}{D-2}\right) A_{ij}(\vec{k}) \hat{k}_i \hat{k}_j \right] \quad (25b)$$

Note that once we have enforced $B'_i = C' = 0$ no further transformations are possible. It follows that an irreducible characterization of all physical linearized solutions is given by the transformed polarization:

$$A'_{ij} = A_{ij} - 2\hat{k}_{(i} A_{j)\ell} \hat{k}_\ell + \hat{k}_i \hat{k}_j A_{\ell m} \hat{k}_\ell \hat{k}_m - \frac{1}{D-2} (\eta_{ij} - \hat{k}_i \hat{k}_j) (A_{\ell\ell} - A_{\ell m} \hat{k}_\ell \hat{k}_m) \quad (26)$$

This is manifestly transverse and traceless, just as in flat space.

We emphasize that this is an invariant result even though it has been derived in a special gauge using a conformally rescaled field variable. Our plane waves obey the invariant field equations; one can check from (24a) that the transverse-traceless polarizations are gauge invariant; and we have just proven that all other solutions can be gauged to zero. Note that the general coordinate invariance of our solution set does not imply the existence of a de Sitter invariant vacuum. These are logically distinct things, even as they are for Minkowski gravitons. The general coordinate transformations we have considered correspond to parameters $e_\mu(u, \vec{x})$ which fall off for large $\|\vec{x}\|$, whereas an element of the de Sitter group cannot fall off because it affects points everywhere. For this theory no normalizable states exist which are de Sitter invariant [2]. Strange as this may seem, Allen and Folacci have shown that the same thing happens for a massless, minimally coupled scalar in de Sitter space [3]. Note that in neither theory does the absence of a de Sitter invariant state prevent the *background metric* from being de Sitter. Indeed, the metric is not even dynamical in the scalar model.

We can now understand the roles played by the various types of modes. The transverse-traceless A modes represent dynamical gravitons. From (15a) it is apparent that they

behave badly in the infrared singular region of $ku \approx 0$. The B and C modes behave better here; in four dimensions they act like massless fields on flat space. These modes represent constrained variables; in the absence of linearized matter sources they are zero. We have shown previously that their good infrared behavior is crucial in allowing the classical theory to respond correctly to localized distributions of positive energy density [2]. The appearance of different types of graviton modes is therefore essential to the correspondence limit of quantum gravity on a de Sitter background.

One should not take the longest wave lengths too seriously. It is very doubtful that any real process could establish a uniform de Sitter background throughout infinite space. Even in such a background the longest wave length modes can not be excited by localized causal processes. On the other hand, the problem at small u can be accessed causally and so is undoubtedly real. Note that of the observed quanta, gravitons must dominate this regime. The other particles are either irrelevant by virtue of possessing nonzero mass, or else they act like massless fields in flat space by virtue of their conformal invariance. One can see from the logarithm in (7b) that gravitons are *more* infrared singular than massless fields in flat space.

We move now to the BRS formulation of the gauge fixed theory. At the linearized level this is described by the Lagrangian:

$$\mathcal{L}_{\text{brs}}^2 = \mathcal{L}_{\text{inv}}^2 - \frac{1}{2} \eta^{\mu\nu} F_\mu F_\nu + \bar{\theta}^\mu \left[\bar{\delta}_\mu^\rho \mathcal{D}_A + \delta_\mu^0 \delta_0^\rho \mathcal{D}_B \right] \theta_\rho \quad (27)$$

(Note that the antighost $\bar{\theta}^0$ is not zero despite the bar; our convention that bars suppress the zero components applies only to standard tensors such as $\eta_{\mu\nu}$ and the derivative operator.) Only $\chi_{\rho\sigma}$ and $\bar{\theta}^\mu$ change under the linearized BRS transformation:

$$\delta_{\text{brs}} \chi_{\mu\nu} = \left[2\theta_{(\mu,\nu)} - (D-2)\theta_{(\mu} \phi_{,\nu)} + 2\eta_{\mu\nu} \theta^\rho \phi_{,\rho} \right] \delta\zeta \quad (28a)$$

$$\delta_{\text{brs}} \bar{\theta}^\mu = -F^\mu \delta\zeta \quad (28b)$$

where $\delta\zeta$ is the usual constant anti-commuting C-number.

By computing the retarded Green's function and then analytically continuing we have shown [2] that the χ propagator has the form:

$$i[\rho\sigma\Delta_\chi^{\alpha\beta}](x, x') \equiv \left\langle \text{vac} \left| T \left[\chi_{\rho\sigma}(x) \chi^{\alpha\beta}(x') \right] \right| \text{vac} \right\rangle_{\mathcal{L}_{\text{brs}}^2} \quad (29a)$$

$$= i\Delta_a(x, x') [\rho\sigma T_A^{\alpha\beta}] + i\Delta_b(x, x') [\rho\sigma T_B^{\alpha\beta}] + i\Delta_c(x, x') [\rho\sigma T_C^{\alpha\beta}] \quad (29b)$$

where we make the following definitions for the three constant tensor factors:

$$[\rho\sigma T_A^{\alpha\beta}] \equiv \left[2\bar{\delta}_\rho^{(\alpha} \bar{\delta}_\sigma^{\beta)} - \frac{2}{D-3} \bar{\eta}_{\rho\sigma} \bar{\eta}^{\alpha\beta} \right] \quad (30a)$$

$$[\rho\sigma T_B^{\alpha\beta}] \equiv 4\delta_{(\rho}^0 \bar{\delta}_{\sigma)}^{(\alpha} \delta_0^{\beta)} \quad (30b)$$

$$[\rho\sigma T_C^{\alpha\beta}] \equiv 2\left(\frac{D-2}{D-3}\right) \left[\delta_\rho^0 \delta_\sigma^0 + \frac{1}{D-2} \eta_{\rho\sigma} \right] \left[\delta_0^\alpha \delta_0^\beta + \frac{1}{D-2} \eta^{\alpha\beta} \right] \quad (30c)$$

The three scalar functions which multiply them can only be determined up to real terms which depend upon the choice of vacuum. These terms are consequently real, analytic and satisfy the appropriate homogeneous equation, $\mathcal{D}_{A,B,C} = 0$; they are also necessarily symmetric in x and x' and there is the further requirement that they derive from evaluating the canonical field operators in the presence of a normalizable state. We shall leave these "R.A.H." terms arbitrary for the time being and consider what constraints BRS invariance places upon them. In odd dimensions we have no simple expression for the unambiguous portions of the three propagator functions but for $D = 2d$ we find:

$$i\Delta_a(x, x') = \frac{1}{4\pi} \left[\frac{1}{4\pi uu'} \right]^{d-1} \left\{ \sum_{k=1}^{d-1} \frac{(2d-k-2)!}{(d-k-1)! k} \left[\frac{4uu'}{(x-x')^2 + i\epsilon} \right]^k \right. \\ \left. - \frac{(2d-2)!}{(d-1)!} \ln \left[H^2 (x-x')^2 + i\epsilon \right] \right\} + (\text{R.A.H.})_a \quad (31a)$$

$$i\Delta_b(x, x') = \frac{1}{4\pi} \left[\frac{1}{4\pi uu'} \right]^{d-1} \sum_{k=1}^{d-1} \frac{(2d-k-3)!}{(d-k-1)!} \left[\frac{4uu'}{(x-x')^2 + i\epsilon} \right]^k + (\text{R.A.H.})_b \quad (31b)$$

and, for $D \geq 6$:

$$i\Delta_c(x, x') = \frac{1}{4\pi} \left[\frac{1}{4\pi uu'} \right]^{d-1} \sum_{k=2}^{d-1} \frac{(2d-k-4)!(k-1)}{(d-k-1)!} \left[\frac{4uu'}{(x-x')^2 + i\epsilon} \right]^k + (\text{R.A.H.})_c \quad (31c)$$

For $D = 4$ these expressions assume the simple form:

$$i\Delta_a(x, x') \xrightarrow{D=4} \frac{1}{4\pi^2} \frac{1}{(x-x')^2 + i\epsilon} - \frac{1}{8\pi^2 uu'} \ln [H^2 (x-x')^2 + i\epsilon] + (\text{R.A.H.})_a \quad (32a)$$

$$i\Delta_{b,c}(x, x') \xrightarrow{D=4} \frac{1}{4\pi^2} \frac{1}{(x-x')^2 + i\epsilon} + (\text{R.A.H.})_{b,c} \quad (32b)$$

from whence we infer expression (7b). The ghost propagator is even simpler:

$$i[\rho\Delta_{\bar{\theta}}^{\alpha}](x, x') \equiv \left\langle \text{vac} \left| T[\theta_{\rho}(x) \bar{\theta}^{\alpha}(x')] \right| \text{vac} \right\rangle_{\mathcal{L}_{\text{brs}}^2} \quad (33a)$$

$$= i\Delta_a(x, x') \bar{\delta}_{\rho}^{\alpha} + i\Delta_b(x, x') \delta_{\rho}^0 \delta_0^{\alpha} \quad (33b)$$

Unitarity clearly requires that we make the same choices for the R.A.H. parts of the functions $i\Delta_a$ and $i\Delta_b$ in the two propagators.

Slavnov-Taylor identities derive from the presumed BRS invariance of the vacuum. The one obeyed by the two propagators comes from transforming the operator $\chi_{\rho\sigma} \bar{\theta}^{\mu}$:

$$\delta_{\text{brs}} [\chi_{\rho\sigma}(x) \bar{\theta}^{\mu}(x')] = -\chi_{\rho\sigma} F^{\mu} \delta\zeta - [2\theta_{(\rho,\sigma)} - 2(D-2)\theta_{(\rho}\phi_{,\sigma)} + 2\eta_{\rho\sigma}\theta^{\alpha}\phi_{,\alpha}] \bar{\theta}^{\mu} \delta\zeta \quad (34)$$

Taking expectation values and assuming a BRS invariant vacuum we see that the following two quantities must be equal:

$$\begin{aligned} \left\langle \text{vac} \left| T[\chi_{\rho\sigma}(x) F^{\mu}(x')] \right| \text{vac} \right\rangle_{\mathcal{L}_{\text{brs}}^2} &= 2\bar{\delta}_{(\rho}^{\mu} \bar{\partial}_{\sigma)}^{\prime} i\Delta_a + 2\delta_0^{\mu} \delta_{(\rho}^0 \bar{\partial}_{\sigma)}^{\prime} i\Delta_b \\ &+ \frac{2}{D-3} \delta_0^{\mu} \bar{\eta}_{\rho\sigma} \left(\left[\partial_{u'} + \left(\frac{D-2}{2} \right) \frac{1}{u'} \right] i\Delta_a - \left[\partial_{u'} - \left(\frac{D-4}{2} \right) \frac{1}{u'} \right] i\Delta_c \right) \\ &- 2\bar{\delta}_{(\rho}^{\mu} \delta_{\sigma)}^0 \left[\partial_{u'} - \left(\frac{D-2}{2} \right) \frac{1}{u'} \right] i\Delta_b - 2\delta_0^{\mu} \delta_{\rho}^0 \delta_{\sigma}^0 \left[\partial_{u'} - \left(\frac{D-4}{2} \right) \frac{1}{u'} \right] i\Delta_c \end{aligned} \quad (35a)$$

$$\begin{aligned} - \left\langle \text{vac} \left| T \left[\left\{ 2\theta_{(\rho,\sigma)} - 2(D-2)\theta_{(\rho}\phi_{,\sigma)} + 2\eta_{\rho\sigma}\theta^{\alpha}\phi_{,\alpha} \right\} \bar{\theta}^{\mu} \right] \right| \text{vac} \right\rangle_{\mathcal{L}_{\text{brs}}^2} &= -2\bar{\delta}_{(\rho}^{\mu} \bar{\partial}_{\sigma)}^{\prime} i\Delta_a \\ - 2\delta_0^{\mu} \delta_{(\rho}^0 \bar{\partial}_{\sigma)}^{\prime} i\Delta_b - \frac{2}{D-3} \delta_0^{\mu} \bar{\eta}_{\rho\sigma} \left(\left[\partial_u - \left(\frac{D-2}{2} \right) \frac{1}{u} \right] i\Delta_b - \left[\partial_u + \left(\frac{D-4}{2} \right) \frac{1}{u} \right] i\Delta_b \right) \\ + 2\bar{\delta}_{(\rho}^{\mu} \delta_{\sigma)}^0 \left[\partial_u + \left(\frac{D-2}{2} \right) \frac{1}{u} \right] i\Delta_a + 2\delta_0^{\mu} \delta_{\rho}^0 \delta_{\sigma}^0 \left[\partial_u + \left(\frac{D-4}{2} \right) \frac{1}{u} \right] i\Delta_b \end{aligned} \quad (35b)$$

Comparison of the various tensor factors shows that equality implies the three scalar functions depend upon their spatial arguments only through the difference, $\vec{x} - \vec{x}'$, and also reflect into one another under the raising and lowering operators (16):

$$\left[\partial_{u'} + \left(\frac{D-2}{2} \right) \frac{1}{u'} \right] i\Delta_a(x, x') = - \left[\partial_u - \left(\frac{D-2}{2} \right) \frac{1}{u} \right] i\Delta_b(x, x') \quad (36a)$$

$$\left[\partial_{u'} + \left(\frac{D-4}{2} \right) \frac{1}{u'} \right] i\Delta_b(x, x') = - \left[\partial_u - \left(\frac{D-4}{2} \right) \frac{1}{u} \right] i\Delta_c(x, x') \quad (36b)$$

From (36) we see that the most general R.A.H. terms compatible with BRS invariance are parameterized by a single complex function $R(\vec{k})$:

$$\left(\text{R.A.H.} \right)_r(x, x') = \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \left\{ R(\vec{k}) \chi_r(u, \vec{x}; \vec{k}) \chi_r^*(u', \vec{x}'; \vec{k}) + \text{c.c.} \right\} \quad (37)$$

where "r" stands for a , b or c . Many choices are possible, including $\left(\text{R.A.H.} \right)_r = 0$, which corresponds to $R(\vec{k}) = 0$. An interesting excluded choice is the $O(4)$ vacuum we borrowed [2] from the analogous solution of Allen and Folacci for the massless minimally coupled scalar field [3]. (Of course there is no problem with the $O(4)$ vacuum in the scalar model.) It should also be noted that no choice can give a de Sitter invariant vacuum.

This work was partially supported by Department of Energy contract DE-FG05-86-ER40272 and by NATO Collaborative Research Grant CRG-920627.

REFERENCES

- [1] S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972).
- [2] N. C. Tsamis and R. P. Woodard, "The Structure Of Perturbative Quantum Gravity On A De Sitter Background," University of Florida preprint UFIFT-HEP-92-14, June 1992.
- [3] B. Allen and A. Folacci, *Phys. Rev. D* **35** (1987) 3771.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

END

DATE
FILMED

10/29/92

