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Department of Chemical and Petroleum Engineering

Department of Industrial Engineering  
Engineering Management and Operations Research

University of Pittsburgh  
Pittsburgh, PA 15261

COMPUTER SIMULATION OF  
COAL PREPARATION PLANTS

FINAL REPORT  
PART I - MATHEMATICAL PROCEDURES

**MASTER**

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Prepared by

Byron S. Gottfried  
Professor of Industrial Engineering, Engineering Management and  
Operations Research  
University of Pittsburgh

John W. Tierney  
Professor of Chemical and Petroleum Engineering  
University of Pittsburgh

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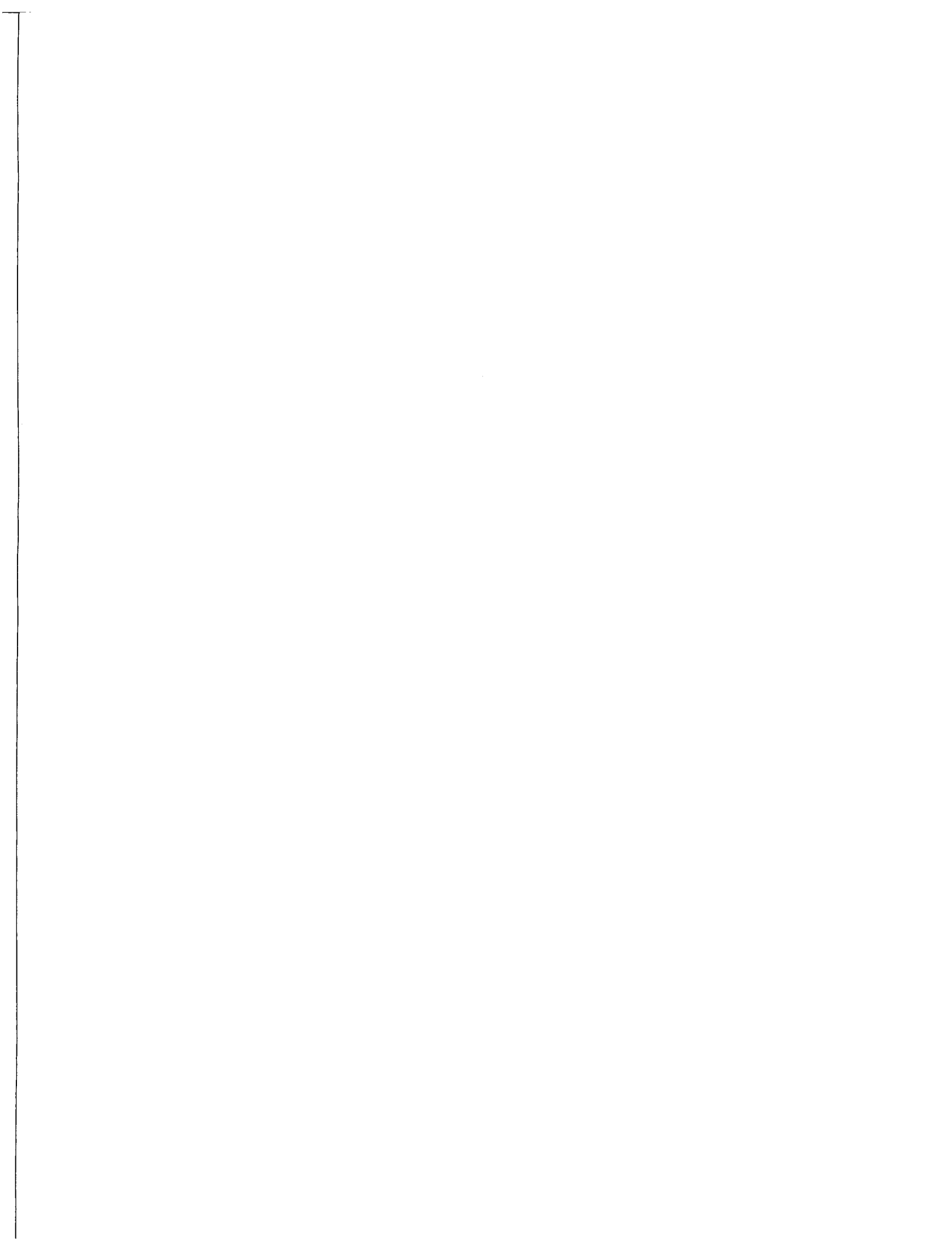
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## I. INTRODUCTION

Coal preparation, commonly known as coal washing, offers a practical and economical approach to the removal of ash and pyritic sulfur from raw coal. For example, the ash in a typical U.S. Northern Appalachian bituminous coal can typically be reduced from 14 percent to 6 percent and the total sulfur from 3 to 1-1/2 percent by existing coal preparation techniques (Cavallaro, et. al., 1976). Such beneficiation can be obtained at yields ranging from 60 to 90 percent.

A typical coal preparation plant consists primarily of various interconnected crushing, screening, classifying, washing, dewatering, drying and thickening units. Several types of washing units will be used, because each piece of washing equipment is best suited for a certain characteristic size fraction of coal. The crushers break the coal into the proper sizes, and the screens and classifying units separate the crushed coal into appropriate size fractions. The dewatering and thermal drying units reduce surface moisture to acceptable limits. These various units are interconnected in a number of ways, the choice being dependent upon the characteristics of the coal being processed and the desired degree of beneficiation.

The principal consideration in operating a coal preparation plant is to maximize the yield of clean coal while reducing the impurities to an acceptably low level. In addition, in assessing the possible interconnection of units in an existing plant, or when a new plant is being considered, it is essential that the optimum configuration be determined for a given degree of cleaning. Usually these matters are handled by some prudent combination of experience and experimentation. A computerized plant simulator, however, is an effective aid in comparing alternatives, provided a program can be

developed which is sufficiently comprehensive to simulate conditions found in industrial practice.

This report describes a comprehensive computer program that allows the user to simulate the performance of realistic coal preparation plants. The program is very flexible in the sense that it can accommodate any particular plant configuration that may be of interest. This allows the user to compare the performance of different plant configurations and to determine the impact of various modes of operation with the same configuration. In addition, the program can be used to assess the degree of cleaning obtained with different coal feeds for a given plant configuration and a given mode of operation. Thus, the user is able to consider a wide variety of alternatives through the use of this simulator.

A plant configuration may contain as many as 100 units and 200 flowstreams, though it is unlikely that an actual configuration will be this large. Thus, for all practical purposes, there are no restrictions on the number of units or flowstreams within a proposed plant configuration.

Use of the simulator requires that the user specify the appearance of the plant configuration, the plant operating conditions, and a description of the coal feed. The simulator will then determine the flowrates within the plant, and a description of each flowrate (i.e., the weight distribution, percent ash, pyritic sulfur and total sulfur, moisture, and BTU content).

The simulation program has been written in modular form using the Fortran language. Some Fortran-77 features are used, though most of the program is written in Fortran IV (Fortran-66). It can be implemented on a great many different types of computers, ranging from large scientific mainframes to IBM-type personal computers with a fixed disk. Some customization may be

required, however, to ensure compatibility with the features of Fortran available on a particular computer.

Part I of this report contains a general description of the methods used to carry out the simulation. Each of the major types of units is described separately, in addition to a description of the overall system analysis. Part II is intended as a user's manual. It contains a listing of the mainframe version of the program, instructions for its use (on both a mainframe and a microcomputer), and output for a representative sample problem. Copies of the program can be obtained in machine-readable form from:

**Coal Preparation Analysis Laboratory**

**Pittsburgh Energy Technology Center**

**U.S. Department of Energy**

**P.O. Box 10940**

**Pittsburgh, PA 15236**

Therefore, the distribution curve should be known as a function of specific gravity,  $\rho$ , as well as the clean coal and refuse products, as well as the distribution curve allows prediction of a function of specific gravity. For a given coal cleaning device, the location of the curve would be relatively stable if the curve were uniform. However, the location of the curve relative to the abscissa depends upon the control settings of the coal cleaning device. Thus, a family of distribution curves may be obtained for a given coal and a given feed. This section presents a method for obtaining these curves, and the feed curve which is independent of the control settings.

#### The Distribution Curve

The distribution curve is a plot of the weight percent of feed reporting to clean coal,  $f(\rho)$ , as a function of specific gravity,  $\rho$  (see Gatz and Tannery, 1969). Thus the yield of clean coal,  $Y$ , can be written as:

## II. WASHERS AND FROTH FLOTATION

### Washers

Most coal washing equipment makes use of a float-and-sink principle, based upon the specific gravity differences between the coal and its associated impurities. (An exception is the froth flotation process, in which the separation is dependent upon the difference in surface characteristics of the coal and its associated impurities.) The performance of these float-and-sink devices is characterized by a distribution curve (also called a partition curve), which expresses the weight percent of feed reporting to clean coal as a function of specific gravity (see Geer and Yancey, 1968). Use of the distribution curve allows prediction of the ash and pyritic sulfur content of the clean coal and refuse products, as well as the yield of clean coal. Therefore, the distribution curve should be known as accurately as possible for a given coal cleaning device. Determination of the distribution curve would be relatively simple if the curve were unique. However, the location of the curve relative to its abscissa depends upon the control settings of the coal cleaning device. Thus, a family of distribution curves may be obtained for a given vessel and a given feed. This section presents a method for combining these curves into one generalized curve which is independent of the vessel's control settings.

### The Distribution Curve

The distribution curve is a plot of the weight percent of feed reporting to clean coal,  $f(\rho)$ , as a function of specific gravity,  $\rho$  (see Geer and Yancey, 1968). Thus the yield of clean coal,  $Y$ , can be written as:

$$Y = \frac{\int_0^{\rho} f(\rho) F(\rho) d\rho}{\int_0^{\rho} F(\rho) d\rho} \quad (1)$$

where  $F(\rho)$  represents the feed rate of material having specific gravity  $\rho$  , and  $\rho_1$  represents the highest value of specific gravity of the feed material.

The specific gravity of separation,  $\rho_s$ , is defined as the value of specific gravity corresponding to a weight percent of 50 (the midpoint of the ordinate). By making appropriate physical adjustments on a coal washing device, the value of the specific gravity of separation can be increased or decreased, thus shifting the entire distribution curve to the right or left. The specific gravity of separation can therefore be thought of as a control variable.

#### Generalization of the Distribution Curve

For a given vessel and a given feed, a family of distribution curves can be combined into a single curve simply by plotting the weight percent of feed reporting to clean coal,  $F(x)$ , against the reduced gravity,  $x$ . The latter is defined as the ratio of specific gravity to the specific gravity of separation, i.e.:

$$x = \rho / \rho_s \quad (2)$$

The resulting distribution curve is independent of the specific gravity of separation. This curve is, therefore, called the generalized distribution curve.

Consider, for example, the distribution data presented by Deurbrouck and Hudy (1972) for seven different coal cleaning plants which utilize dense-medium cyclones. The distribution curves for several composite feeds are shown in Figure 1. Figure 2 shows the generalized distribution curve for the same data. Notice the manner in which the individual distribution curves, having different specific gravities of separation, are brought together into

... (have  $\rho$ ) ... the highest value of specific gravity of the feed material. The specific gravity of separation,  $\rho_s$ , is defined as the value of specific gravity corresponding to a weight percent of 50 (the midpoint of the ...). By using appropriate physical adjustments on a coal washing ... the value of the specific gravity of separation can be increased or decreased, thus shifting the entire distribution curve to the left or right. The specific gravity of separation can therefore be thought of as a control variable.

Determination of the Distribution Curve

For a given vessel and a given feed, a family of distribution curves can be obtained by changing the specific gravity of separation. The weight percent of feed reporting to clean coal,  $F(x)$ , against the reduced gravity,  $x$ , for the latter is plotted against the specific gravity of separation,  $\rho_s$ . The resulting curves are shown in Figure 1.

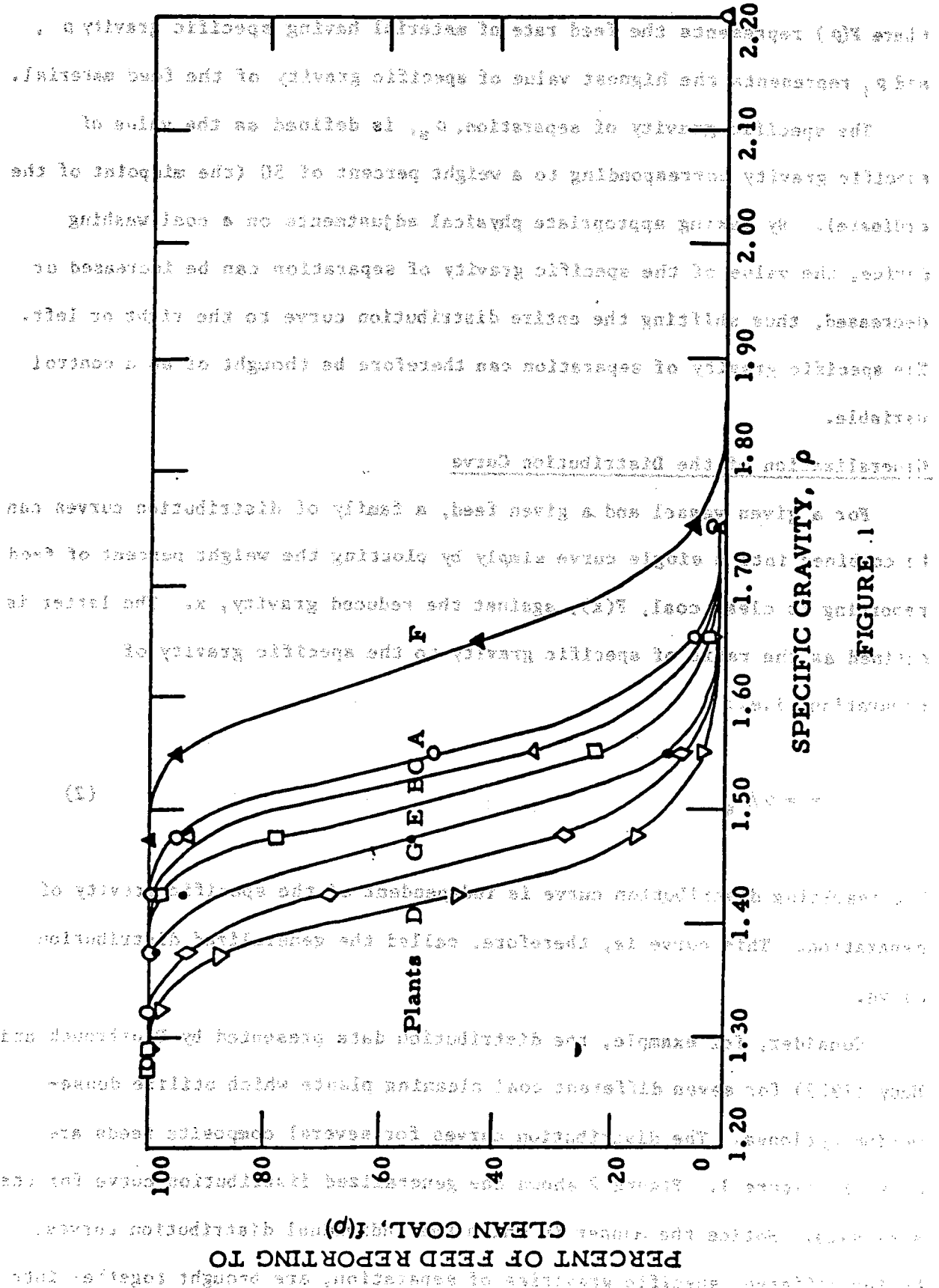


FIGURE 1

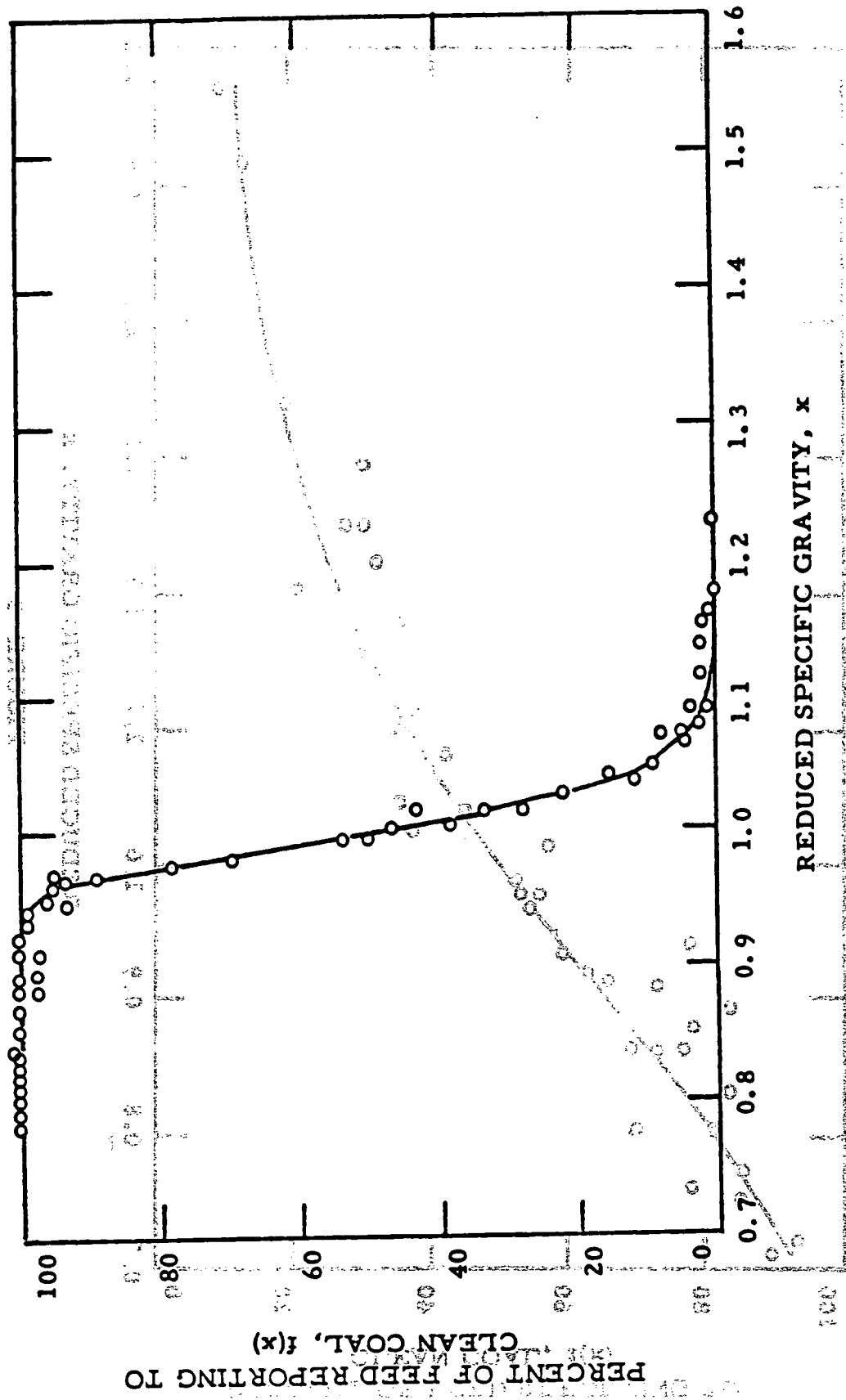


FIGURE 2

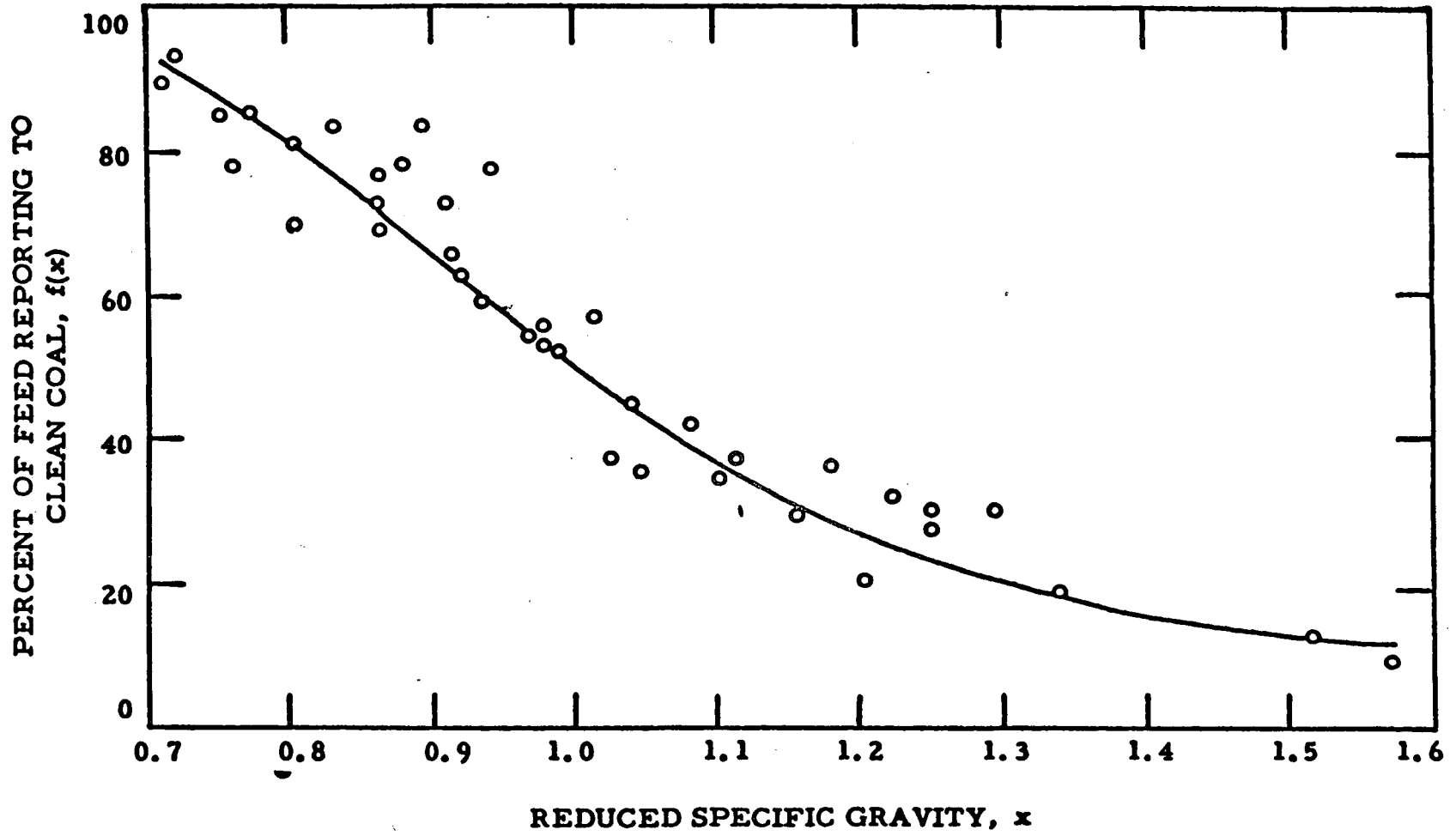


FIGURE 3

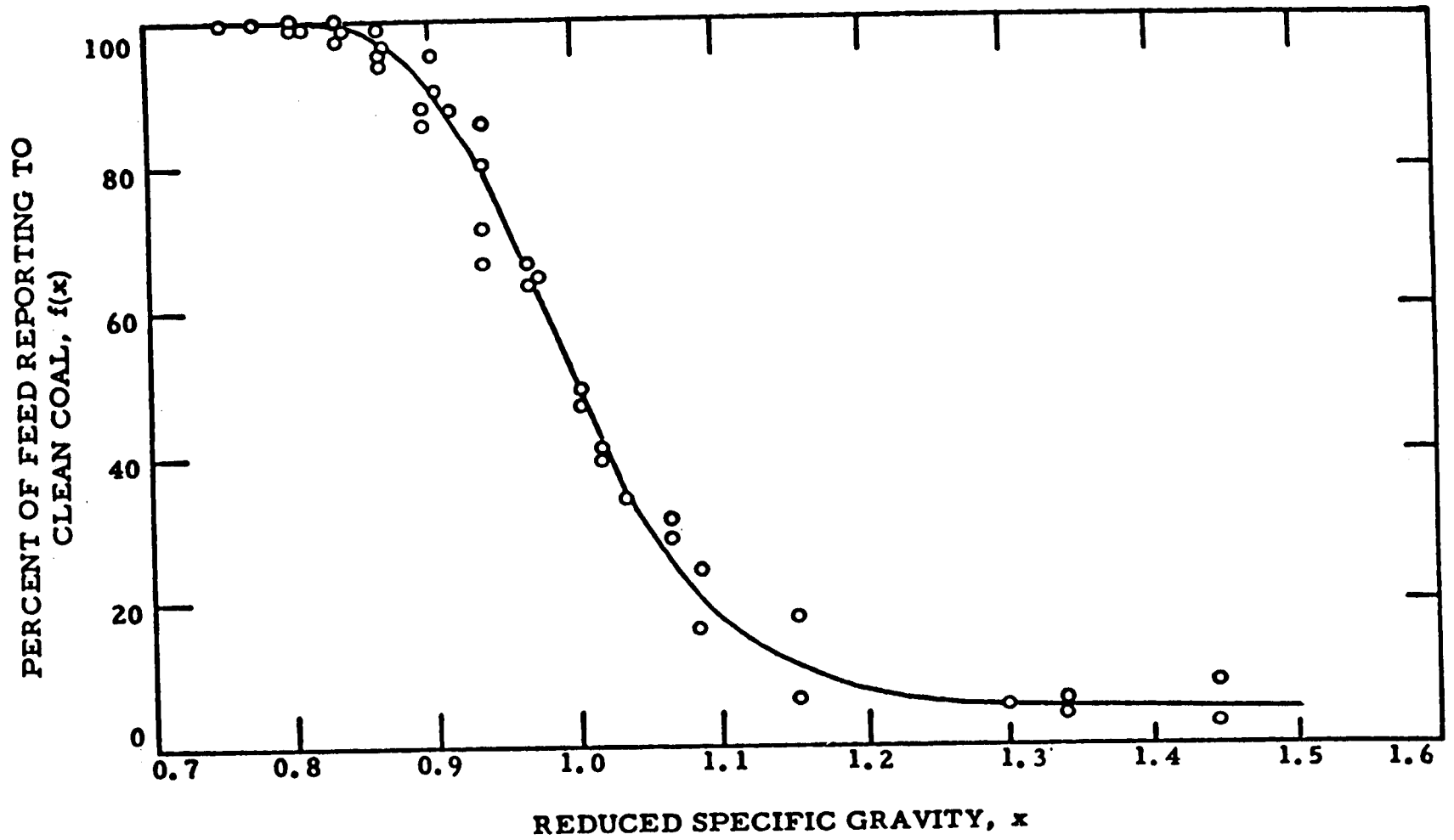


FIGURE 4

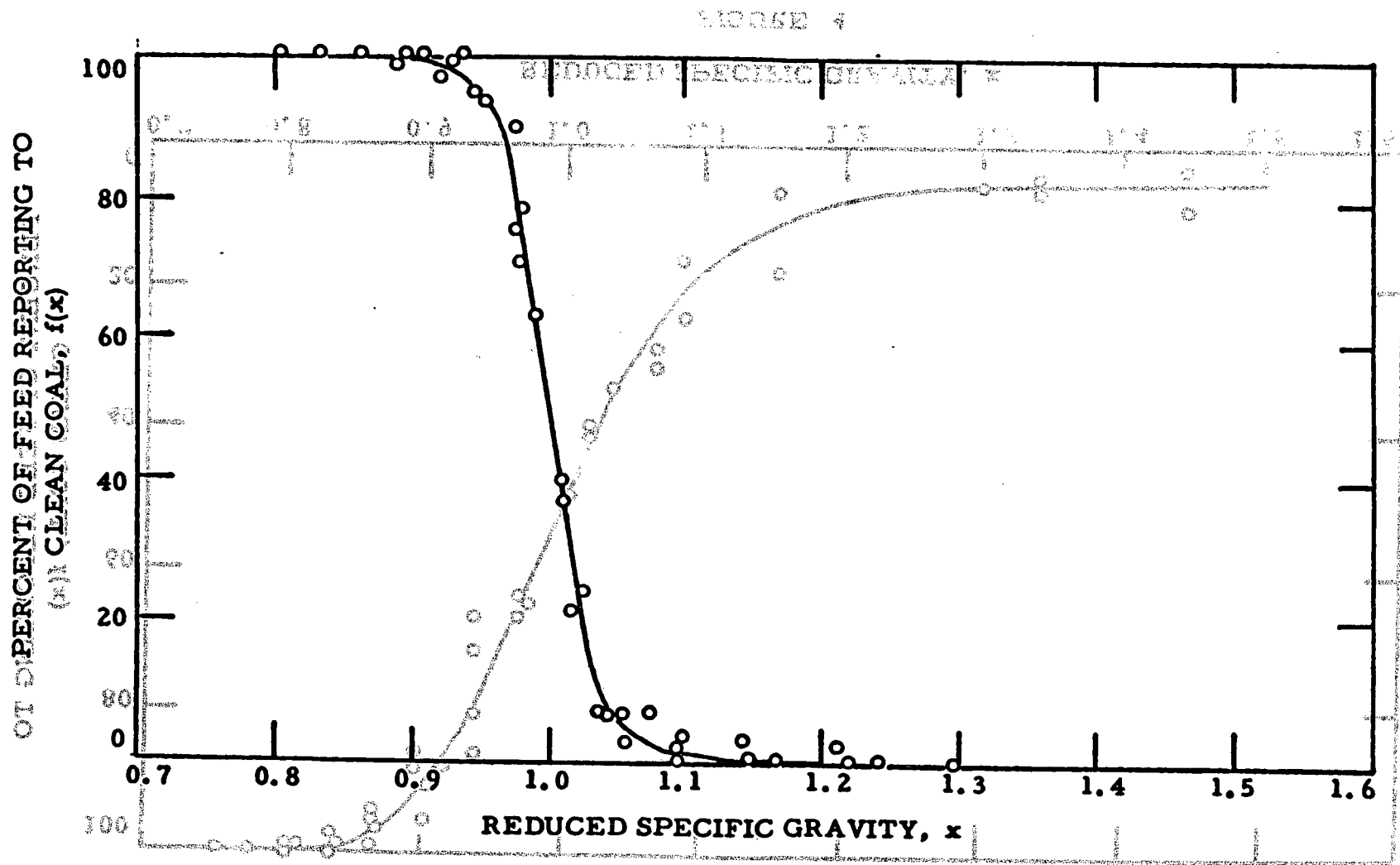


FIGURE 5

one generalized curve with little data scatter. Figures 3-5 show similar results for other commonly used coal cleaning devices with composite feeds. These curves were obtained from actual published operating data on the various vessels (Deurbrouck and Balowitch, 1963; Deurbrouck, 1974; Hudy, 1968).

A conventional distribution curve can be obtained from this generalized curve simply by multiplying the reduced specific gravity (the abscissa) by the specific gravity of separation, i.e.:

$$\rho = \rho_s x \quad (3)$$

Note, however, that the significance of the generalized distribution curve is that it can be used with any specific gravity of separation, not only the specific gravity of separation corresponding to the given data. Thus, it is possible to construct an entire family of conventional distribution curves from a single generalized distribution curve.

#### Effect of Size

When sampling an actual coal cleaning device, it is fairly common to obtain a separate set of performance data for each of several different feed size-fractions. Thus, a unique distribution curve will be obtained for each size fraction. The use of such data enables a more accurate description of a given separation than the use of a single composite distribution curve.

A different generalized distribution curve will be obtained for each size fraction. In this case, the reduced specific gravity must be defined as the ratio of specific gravity to the specific gravity of separation for that particular size fraction. Usually, however, the overall specific gravity of separation (for the entire feed composite) will be specified, rather than the specific gravity of separation for a particular size fraction. Therefore, it

will be necessary to determine a value for the specific gravity of separation for that size fraction from the overall specific gravity of separation. This can be accomplished by calculating the ratio of the specific gravities of separation, i.e.:

$$r_1 = (\rho_s)_1 / (\rho_s)_c \quad (4)$$

where  $(\rho_s)_1$  refers to the specific gravity of separation for the  $i$ th size fraction and  $(\rho_s)_c$  refers to the overall specific gravity of separation.

A conventional distribution curve for the  $i$ th size fraction can be obtained from the corresponding generalized distribution curve by transforming the abscissa from  $x$  to  $\rho$ , i.e.:

$$\rho = (\rho_s)_1 x \quad (5)$$

combining this result with equation 4 yields:

$$\rho = (\rho_s)_c r_1 x \quad (6)$$

Thus, if the overall specific gravity of separation and the proper value for  $r_1$  are known, a conventional distribution curve for the  $i$ th size fraction can be constructed.

The success of this method is implicitly dependent upon the constancy of the ratio  $r_1$  for each size fraction. The validity of this assumption is questionable, however, as indicated by the variations shown in Figure 6 for the seven coal cleaning plants studied by Deurbrouck and Hudy (1972). Similar results were obtained with other sets of distribution data. Nevertheless,

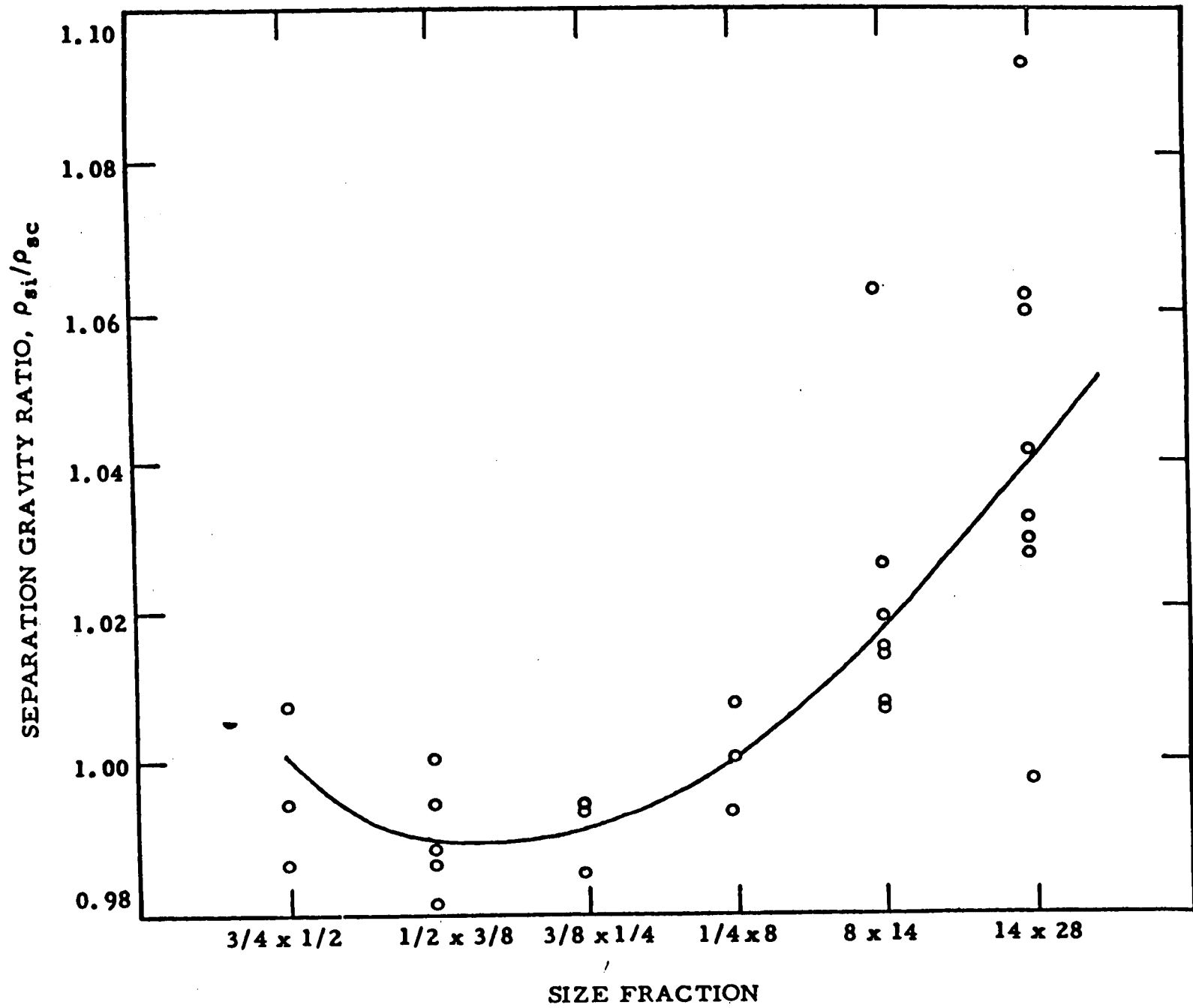


FIGURE 6

this method offers a reasonable approach to the problem, in the absence of more complete information. Additional work in this area would be desirable.

Gottfried and Jacobsen (1977) present extensive generalized distribution data, including values for  $r_1$ , in tabular form for dense-medium cyclones, hydrocyclones, concentrating tables, dense-medium vessels and Baum jigs. The generalized distribution curves shown in Figures 2-5 represent portions of this data.

#### Mathematical Representation of the Generalized Distribution Curve

Gottfried (1978) has shown that the generalized distribution curve can be expressed as a modified Weibull function of the form:

$$f(x) = 100 [f_0 + c [\exp - \{(x - x_0)^{a/b}\}]] \quad (7)$$

where  $x$  is the reduced specific gravity,  $\rho/\rho_s$  ( $\rho_s$  is the specific gravity of separation);  $f(x)$  is the percent of feed reporting to clean coal; and  $f_0$ ,  $c$ ,  $a$ ,  $b$  and  $x_0$  are constants to be determined for a given vessel and a given feed size fraction.

Of the five constants to be determined, four are independent and the fifth,  $x_0$ , can be calculated from the conditions that  $f(x) = 50$  when  $x = 1$ . When this condition is imposed on equation 7, the following expression is obtained:

$$x_0 = 1 - [b \log_e \left( \frac{c}{0.5 - f_0} \right)]^{1/a} \quad (8)$$

The values of  $f_0$  and  $c$  are determined directly from the performance data:  $f_0$  represents the minimum fraction of feed that will report to clean coal at high gravities (i.e., very large values of  $x$ );  $c$  represents the

difference between the highest and lowest fractions of feed reporting to clean coal, so that

$$c + f_0 < 1$$

For the float-sink coal cleaning devices reported herein, 100 percent of the feed reports to clean coal at low  $x$  values so that  $c$  is taken to equal the quantity  $(1-f_0)$ .

Two parameters remain to be determined in equation 7:  $a$  and  $b$ . The values for  $a$  and  $b$  are obtained through nonlinear regression techniques.

Once the values of  $f_0$ ,  $c$ ,  $a$ ,  $b$  and  $x_0$  are known, the generalized probable error corresponding to the curve can be calculated from the following equation:

$$\text{GPE} = 0.5 \left\{ \left[ b \log_e \left( \frac{c}{0.25-f_0} \right) \right]^{1/a} - \left[ b \log_e \left( \frac{c}{0.75-f_0} \right) \right]^{1/a} \right\} \quad (9)$$

Values of  $f_0$ ,  $a$ ,  $b$ ,  $c$  and  $x_0$  are included in the simulator for several size fractions and the overall feed composite for each coal cleaning device (Gottfried, 1978; Gottfried, 1981; Fallon and Gottfried, in press). Results for the Baum jig and Batac jig include primary, secondary and overall separation steps. The appropriate coefficients can be used in conjunction with equation 7 to describe the generalized distribution curve of each coal cleaning device and feed size fraction by an analytical equation.

Figure 7 shows a typical curve fit for an overall (2-stage) separation using a Batac jig with a 1/2 by 3/8-inch size fraction. Figure 8 shows another fit, for a Dynawhirpool with a 14 by 28 mesh size fraction. Notice the difficulty in fitting the upper "shoulder" of the latter curve. (The data

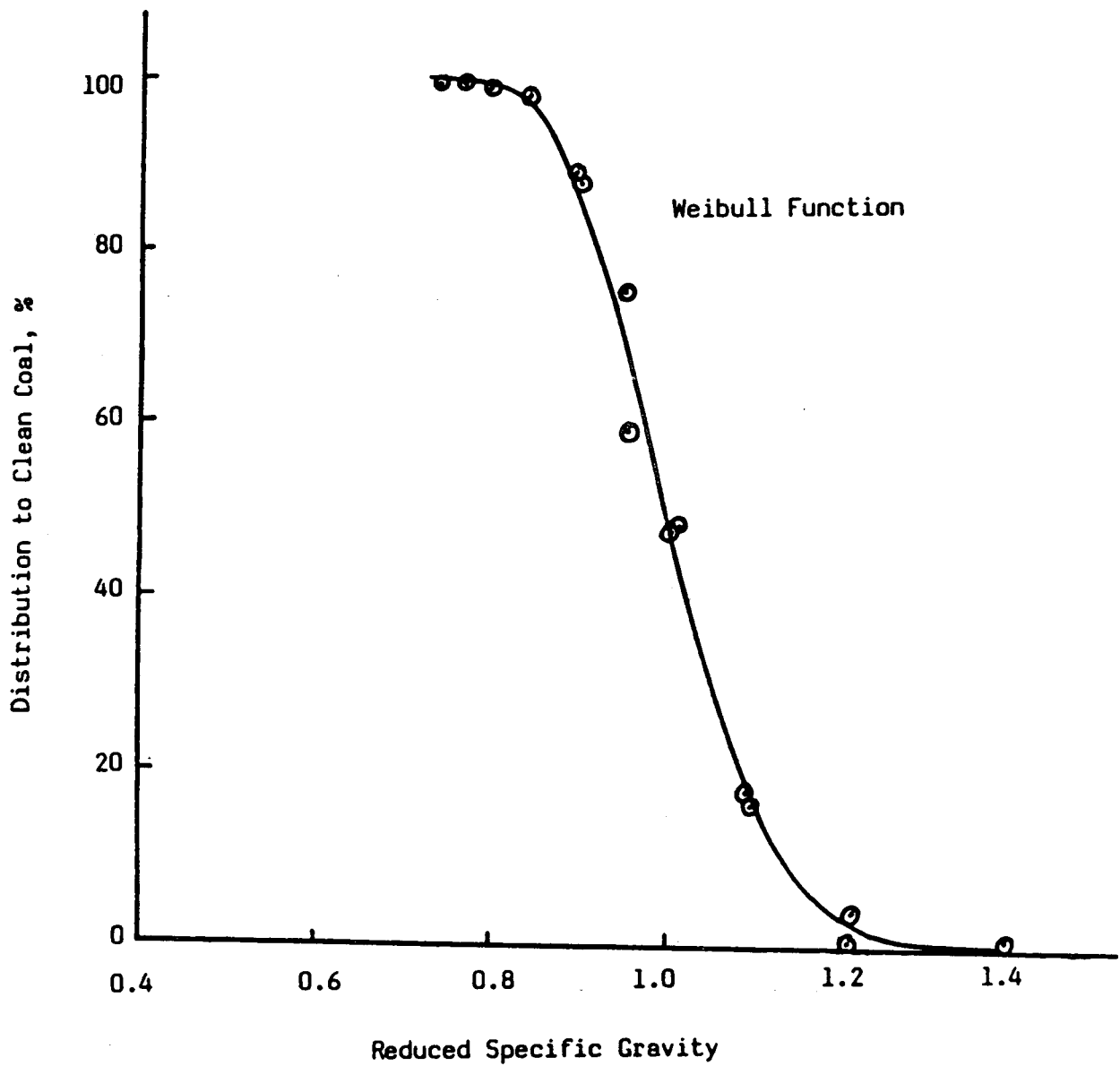


FIGURE 7

Generalized Distribution Curve for Batac Jigs (Overall)

1/2 x 3/8 inch

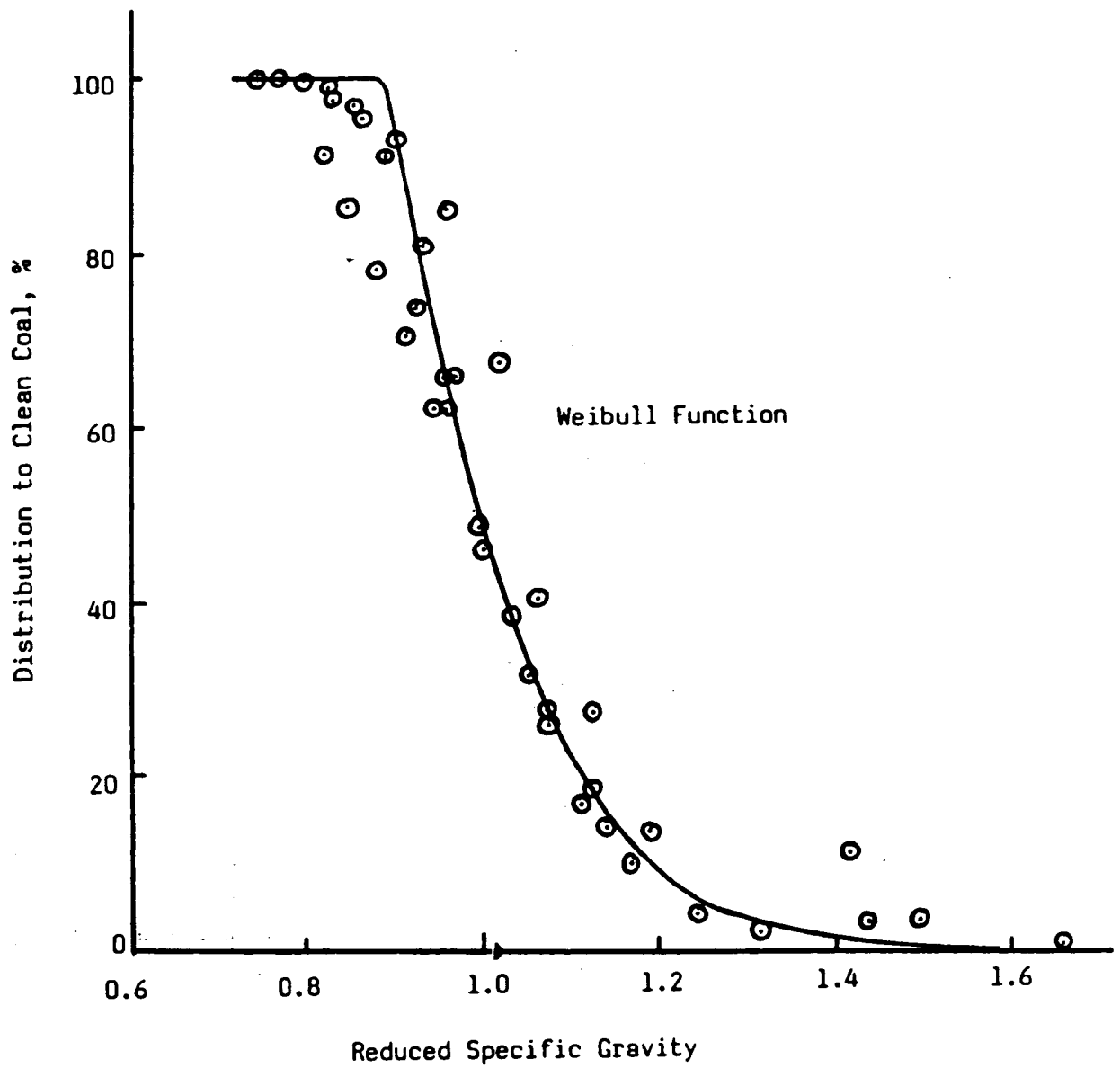


FIGURE 8  
 Generalized Distribution Curve for Dynawhirlpools  
 14 x 28 mesh

scatter further contributes to the poor fit.) These results are typical of the better fits and the poorer fits, respectively.

### Error Area

Error area is a valid measure of the sharpness of separation for those distributions curves whose ordinate values range from 100 to 0, as shown in Figure 2. For such curves, the conventional error area (EA) is defined as:

$$EA = \int_s^{\rho} [100 - f(\rho)] d\rho + \int_s^{\rho_1} f(\rho) d\rho \quad (10)$$

where  $\rho_1$  now represents the value of specific gravity where the distribution curve touches the abscissa. (The value of  $\rho_1$  will typically be between 2.2 and 2.6, though it can be as high as 5.0.) Normally the error area is determined by graphical integration of equation 10 for each distribution curve.

Similarly, a generalized error area (GEA) can be defined as:

$$GEA = \int_1^x [100 - f(x)] dx + \int_1^{x_1} f(x) dx \quad (11)$$

where  $x_1$  represents a value of reduced specific gravity where the generalized distribution curve touches the abscissa. Equation 11 can also be evaluated graphically.

Because the generalized distribution curve is independent of the specific gravity of separation, there will be only one value of GEA for a given vessel and a given feed size. The significance of the generalized error area can be seen by differentiating equation 4 and substituting into equation 10, resulting in:

$$EA = \rho_s \int_0^1 [100 - f(x)] dx + \rho_s \int_1^{x_1} f(x) dx \quad (12)$$

Comparing this expression with equation (11), it is seen that:

$$EA = \rho_s GEA \quad (13)$$

As stated above, GEA is independent of the specific gravity of separation. Hence, for a given vessel and a feed falling within a given size fraction, the error area is directly proportional to the specific gravity of separation.

#### Probable Error

In order to develop similar expressions for probable error, consider the (conventional) distribution curve as a relationship showing the dependence of specific gravity on percent of feed reporting to clean coal, i.e.:

$$\rho = \phi(f) \quad (14)$$

Then the probable error (PE) is simply expressed as:

$$PE = 0.5 [\phi(25) - \phi(75)] \quad (15)$$

The probable error can be determined directly from a given distribution curve. Note that, for a given vessel and a given feed size, the use of equation 15 results in a different value of PE for each (conventional) distribution curve. Hence, the probable error is a function of the specific gravity of separation.

If the generalized distribution curve is thought of in the same sense as the conventional distribution curve, i.e.:

$$x = \theta(f) \tag{16}$$

then the generalized probable error (GPE) is simply:

$$\text{GPE} = 0.5 [\theta(25) - \theta(75)] \tag{17}$$

Since the generalized distribution curve is independent of the specific gravity of separation, the value of GPE obtained from equation (17) will also be independent of specific gravity of separation. Hence, there will be only one value of GPE for a given vessel and a feed falling within a given size fraction.

Notice, from equations 15 and 17, that the conventional probable error is defined as one-half the difference in two values of the specific gravity ( $\phi$ ), and the generalized probable error is defined as one-half the difference in two values of the reduced specific gravity ( $x$ ). From equation 4, however, it is seen that the specific gravity is simply equal to the product of reduced specific gravity and specific gravity of separation. Therefore, the conventional probable error can be obtained simply by multiplying the generalized probable error by the specific gravity of separation, i.e.:

$$\text{PE} = \rho_s \text{GPE} \tag{18}$$

Because GPE is independent of the specific gravity of separation it is seen that, for a given vessel and a feed falling within a given size fraction, the probable error is directly proportional to the specific gravity of separation. Therefore, the probable error will increase as the specific

gravity of separation increases, indicating a less sharp separation. This behavior is observed consistently in the tabulated performance data of Gottfried and Jacobsen (1977), which include a tabulated value for GPE corresponding to each generalized distribution curve.

#### Use of the Distribution Curve

The easiest way to predict the outcome of a separation is to use a single composite distribution curve for the entire feed, i.e.:

$$\begin{aligned} F_c &= 0.01 \int_0^{\infty} f(\rho)w(\rho) d\rho \\ &= 0.01 \int_0^{\infty} f(x)w(x)dx \end{aligned} \quad (19)$$

where  $w(\rho)$  or  $w(x)$  represents the weight distribution of the feed, i.e.:

$$F = \int_0^{\infty} w(\rho) d\rho = (\rho_s)_c \int_0^{\infty} w(x) dx \quad (20)$$

[Note that the overall specific gravity of separation,  $(\rho_s)_c$ , must be specified in order to carry out this calculation.]

Such a calculation is relatively inaccurate, however, because it does not account for the manner in which the feed material is distributed within different size fractions.

A more accurate prediction can be obtained by summing the calculated recoveries for the individual constituent size fractions, i.e.:

$$\begin{aligned} F_c &= \sum_1 (F_c)_i = 0.01 \sum_1 \int_0^{\infty} f_i(\rho)w_i(\rho) d\rho \\ &= 0.01 (\rho_s)_c \sum_1 r_i \int_0^{\infty} f_i(x)w_i(x) dx \end{aligned} \quad (21)$$

As a rule, the integration is carried out numerically, using some simple procedure such as the trapezoidal rule, with either an analytical curve fit or a tabular set of distribution data.

### Size Transformations

The use of equation 21 is straightforward when the size fractions that are used to characterize the feed coincide with the size fractions corresponding to the distribution data. Usually, however, these size fractions will not be the same. This problem can be resolved by transforming the feed from its original size fractions,  $S$ , to the size fractions corresponding to the distribution data,  $\sigma$ ; the separation is then carried out in the  $\sigma$ -space, and the product stream is then transformed back to the  $S$ -space.

The method used for carrying out the transformations, based upon simple proportionality, makes use of the following two assumptions:

- (1) The material is distributed uniformly within each size fraction,
- (2) The range of sizes corresponding to the distribution data encompasses the range of sizes used to characterize the feed, i.e.,  $\sigma_0 > S_0$ , and  $\sigma_m < S_n$ . (Note that  $\sigma_0 > \sigma_1 > \sigma_2 > \dots > \sigma_m$  and  $S_0 > S_1 > S_2 > \dots > S_n$ .) The first assumption is quite reasonable provided the size fractions are small (though the question of "how small" remains unanswered). The second assumption will be valid provided the distribution data span a reasonably large range of sizes.

Twelve different situations can arise when transforming from the  $S$ -space to the  $\sigma$ -space, as outlined below. The procedure is to select and utilize an appropriate transformation equation for each size fraction in the  $\sigma$ -space, i.e.:

for each  $\Delta\sigma_j$  (where  $\Delta\sigma_j = \sigma_j - \sigma_{j-1}$ ,  $j = 1, 2, \dots, m$ ).

$$(1) \sigma_{j-1} < S_{i-1} \text{ and } \sigma_j > S_i$$


---

$$w_j(\sigma) = w_1(S) \left( \frac{\sigma_j - \sigma_{j-1}}{S_i - S_{i-1}} \right) \quad (22a)$$

(see Figure 9a)

$$(2) \sigma_{j-1} < S_{i-2} \text{ and } \sigma_j > S_i$$


---

$$w_j(\sigma) = w_{i-1}(S) \left( \frac{S_{i-1} - \sigma_{j-1}}{S_{i-1} - S_{i-2}} \right) + w_1(S) \left( \frac{\sigma_j - S_{i-1}}{S_i - S_{i-1}} \right) \quad (22b)$$

(see Figure 9b)

$$(3) \sigma_{j-1} < S_{i-k-2} \text{ and } \sigma_j > S_i$$


---

$$w_j(\sigma) = w_{i-k-1}(S) \left( \frac{S_{i-k-1} - \sigma_{j-1}}{S_{i-k-1} - S_{i-k-2}} \right) + \sum_{\ell=1}^k w_{i-\ell}(S) + w_1(S) \left( \frac{\sigma_j - S_{i-1}}{S_i - S_{i-1}} \right) \quad (22c)$$

(see Figure 9c)

$$(4) \sigma_{j-1} > S_{i-1} \text{ and } \sigma_j > S_i$$


---

$$w_j(\sigma) = w_1(S) \left( \frac{\sigma_j - S_{i-1}}{S_i - S_{i-1}} \right) \quad (22d)$$

(see Figure 9d)

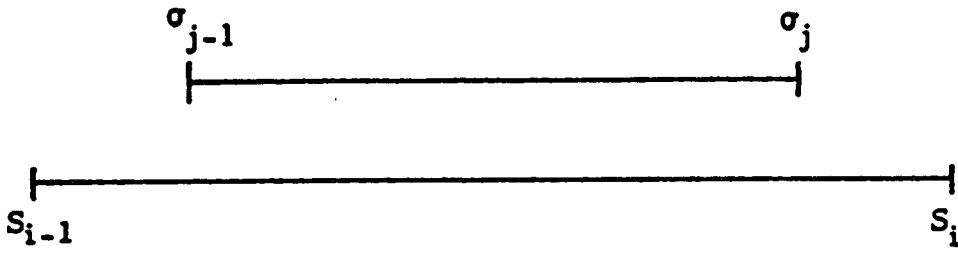


FIGURE 9 a

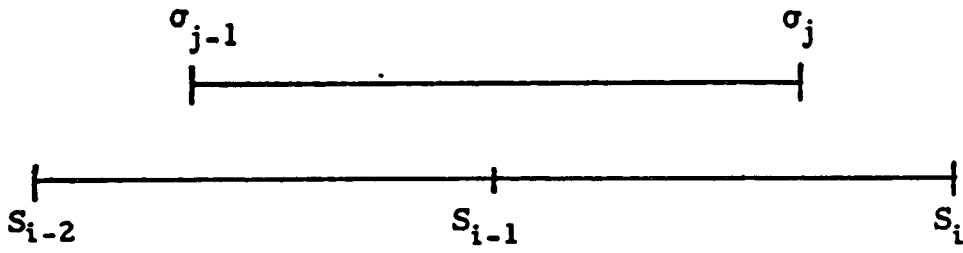


FIGURE 9 b

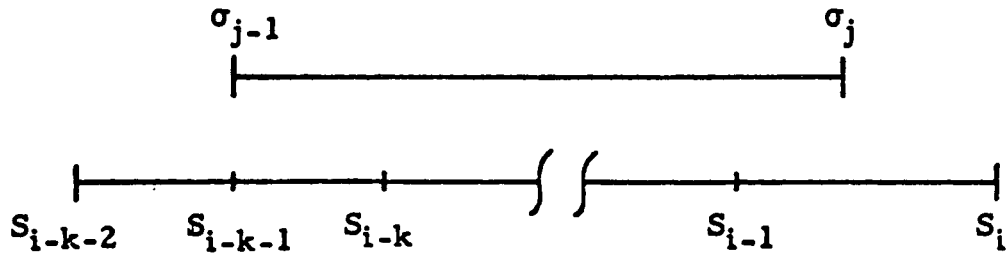


FIGURE 9 c

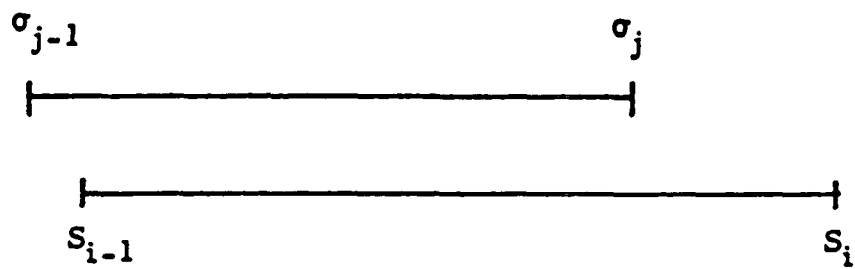


FIGURE 9d

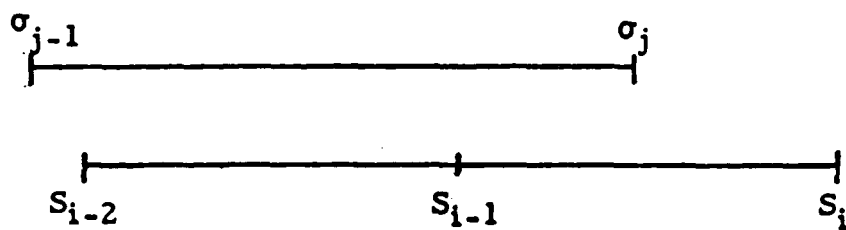


FIGURE 9e

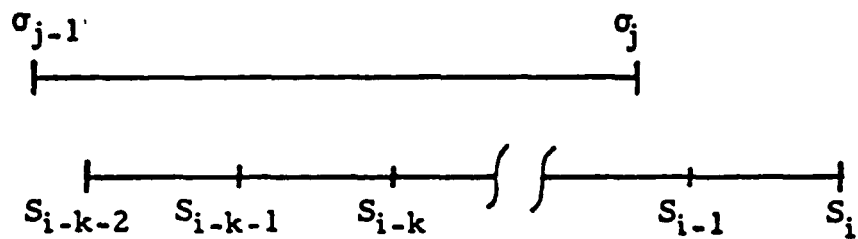


FIGURE 9f

$$(5) \sigma_{j-1} > S_{i-2} \text{ and } \sigma_j > S_i$$


---

$$w_j(\sigma) = w_{i-1}(S) + w_i(S) \left( \frac{\sigma_j - S_{i-1}}{S_i - S_{i-1}} \right) \quad (22e)$$

(see Figure 9e)

$$(6) \sigma_{j-1} > S_{i-k-2} \text{ and } \sigma_j > S_i$$


---

$$w_j(\sigma) = \sum_{l=1}^{k+1} w_{i-l}(S) + w_i(S) \left( \frac{\sigma_j - S_{i-1}}{S_i - S_{i-1}} \right) \quad (22f)$$

(see Figure 9f)

$$(7) \sigma_{j-1} < S_{i-1} \text{ and } \sigma_j < S_i$$


---

$$w_j(\sigma) = w_i(S) \left( \frac{S_i - \sigma_{j-1}}{S_i - S_{i-1}} \right) \quad (22g)$$

(see Figure 9g)

$$(8) \sigma_{j-1} < S_{i-2} \text{ and } \sigma_j < S_i$$


---

$$w_j(\sigma) = w_{i-1}(S) \left( \frac{S_{i-1} - \sigma_{j-1}}{S_{i-1} - S_{i-2}} \right) + w_i(S) \quad (22h)$$

(see Figure 9h)

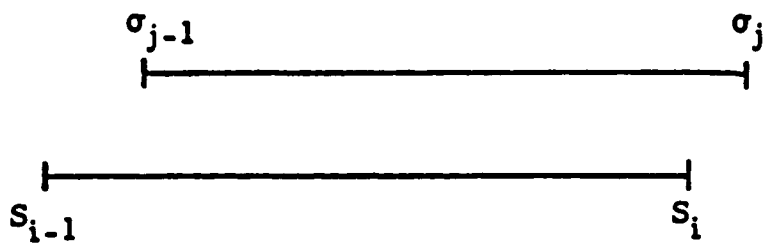


FIGURE 9g

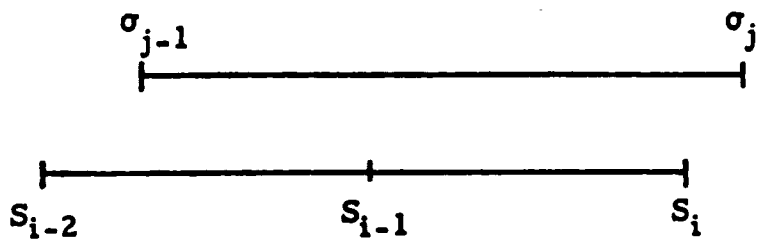


FIGURE 9h

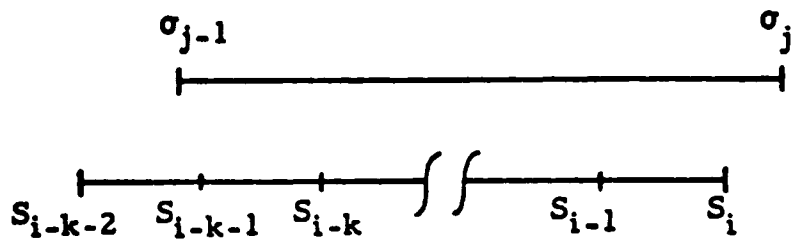


FIGURE 9i

$$(9) \sigma_{j-1} < S_{i-k-2} \text{ and } \sigma_j < S_i$$


---

$$w_j(\sigma) = w_{i-k-1}(S) \left( \frac{S_{i-k-1} - \sigma_{j-1}}{S_{i-k-1} - S_{i-k-2}} \right) + \sum_{\ell=0}^k w_{i-\ell}(S) \quad (22i)$$

(see Figure 9i)

$$(10) \sigma_{j-1} > S_{i-1} \text{ and } \sigma_j < S_i$$


---

$$w_j(\sigma) = w_i(S) \quad (22j)$$

(see Figure 9j)

$$(11) \sigma_{j-1} > S_{i-2} \text{ and } \sigma_j < S_i$$


---

$$w_j(\sigma) = w_{i-1}(S) + w_i(S) \quad (22k)$$

(see Figure 9k)

$$(12) \sigma_{j-1} > S_{i-k-2} \text{ and } \sigma_j < S_i$$


---

$$w_j(\sigma) = \sum_{\ell=0}^{k+1} w_{i-\ell}(S) \quad (22l)$$

(see Figure 9l)

Once the separation has been carried out in the  $\sigma$ -space, then the weight distribution of the products can be transformed back into the  $S$ -space by reversing the above procedure. In other words, for each  $\Delta S_i$  (where  $\Delta S_i = S_i -$

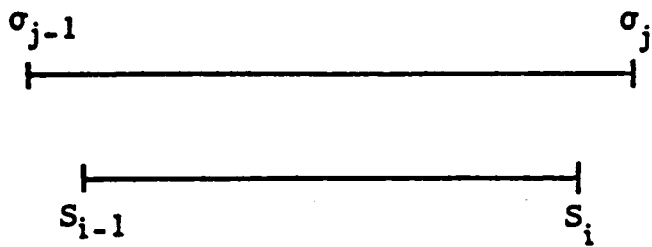


FIGURE 9j

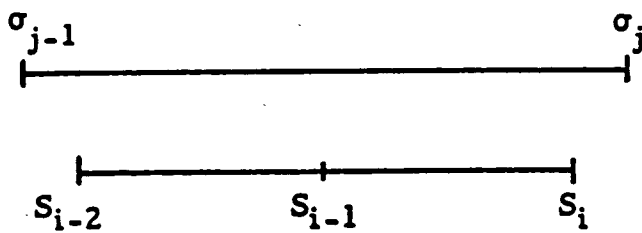


FIGURE 9k

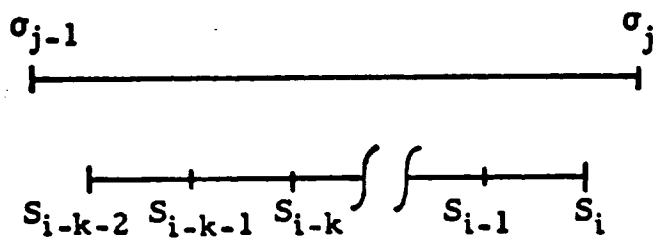


FIGURE 9l

$S_{i-1}$ ,  $i = 1, 2, \dots, n$ ), select an appropriate transformation equation, in accordance with the relationship of the boundaries of  $\Delta S_i$  with respect to the boundaries of the corresponding  $\Delta \sigma$ 's.

The use of the above transformation equations enables one to predict the outcome of a solid-solid separation where any set of arbitrary sizes is used to characterize the feed. The entire procedure is well suited for computer implementation.

When carrying out the calculations a set of distribution data is generated from the composite feed and clean coal. A value for  $\rho_{sc}$ , the overall specific gravific of separation, is then calculated from the distribution data by reverse linear interpolation. This value is compared with the specified value for  $\rho_{sc}$ . If the two values do not agree to within 0.003, then the value of  $\rho_{sc}$  that is used to carry out the calculations is adjusted and the entire procedure is repeated. Convergence is normally attained in three or four iterations.

The overall ash, sulfur, and Btu content of product streams are easily determined once the separation has been carried out. These calculations are based upon the assumption that within each size and specific gravity fraction, the direct (as contrasted with cumulative) values of ash, sulfur, and Btu content are unchanged by cleaning. (In practice, this assumption is not strictly true, but it simplifies calculations during this stage of the simulator.)

After the separation has been computed, a set of summary data is generated for each size fraction, and for the overall composite feed. The following quantities are computed.

1. Screen analysis for each flowstream (i.e., feed, clean coal, refuse and, if applicable, middlings)

2. Percent ash for each flowstream
3. Percent pyritic sulfur for each flowstream
4. Percent total sulfur for each flowstream
5. BTU content of each flowstream
6. Actual recovery, theoretical recovery, recovery efficiency, BTU recovery and ash error
7. Float in refuse, sink in clean coal, total misplaced material and near gravity  $\pm 0.1$  material
8. Specific gravity of separation, probable error, imperfection and error area
9. Distribution data, determined as the ratio of calculated clean coal to feed
10. Moisture content of the coal.

Most of these quantities are obtained by direct summation of the appropriate quantities over the specific gravity fractions. However, the theoretical recovery, ash error, specific gravity of separation and probable error are obtained by interpolation, and the error area is obtained by numerical integration (using Simpson's rule). The composite data are obtained by actually summing the computed results over all size fractions and interpolating (where necessary) between the points composited in this manner.

#### Froth Flotation

The simulator also contains a froth flotation module. The method used to carry out these calculations is less precise than that described above for other types of coal washing devices. The overall yield of clean coal is taken as the value of the cumulative weight distribution in the froth flotation feed at a specific gravity of 1.50. Similarly, the overall ash content of the

clean coal is taken as the cumulative ash of the feed at a specific gravity of 1.60. Detailed specific gravity analyses of the clean coal and refuse products are obtained by fitting a distribution curve to the feed that will produce the "1.50-yield/1.60-ash" product. This procedure is carried out iteratively, using different distribution curves, until a satisfactory fit is obtained.

The sulfur and Btu contents of the products are then obtained from the distribution curve calculations. These values may differ from the amounts present in either the 1.50 or 1.60 specific gravity feed level.

### III. DEWATERING DEVICES

Dewatering is an important operation in any coal preparation plant. Vacuum filters and centrifuges are commonly used to reduce the moisture of coal fines to acceptable levels for shipment and ultimate consumption. In each of these equipments a desaturating force--centrifugal force or a pressure difference--is used to overcome the capillary forces which are present in a solid made up of fine particles. The final moisture content depends on many factors, the most important of which are the physical characteristics of the particles themselves--their size, shape, and surface properties--and the characteristics of the equipment which determine the time the desaturating force is applied, the kind of solid matrix which is formed, as well as any changes which may take place in the size and amount of the particles themselves because of mechanical forces. Because of the large number of particles involved with randomly distributed shapes and sizes, a complete description is impossible and one can only estimate the most probable value of process variables. In designing the dewatering modules which are described here, an operating viewpoint rather than a design viewpoint is taken (Tierney et al, 1983). It is assumed that the equipments are installed, their operating parameters are known, and a complete particle size and gravity analysis of the coal to be cleaned is known. The primary objective is to calculate the properties of the products from each unit and in particular to judge the effect of any change in operating conditions on product quality. It is important, therefore, that all major variables be included in the model, but it is not critical that the operating parameters of the equipments be known precisely because the model includes certain "tuneup" variables which can be adjusted in use.

### Structure of Dewatering Modules

An important criterion used in designing all modules is that they be "generic" and not designed for any particular manufacturer's equipment. To accomplish this, fundamental physical relations are used wherever possible and parameters specific to a piece of equipment must be provided by the user. Default values are supplied by the program if necessary. When used to describe an operating plant, these values can be adjusted to tune the model. The calculations for dewatering equipment can be grouped into three parts-- changes in particle size caused by the equipment, division of particles between product streams, and the calculation of the final moisture content. Each will be treated separately in the following.

### Mechanical Degradation

The forces necessary to propel coal particles through a centrifuge will result in some degradation of large particles into smaller particles. This is, of course, undesirable, but it is unavoidable, and should be accounted for in the dewatering module for the centrifuge. Mechanical action is much less severe in the vacuum filter, and it is assumed that no degradation takes place. The only systematic study of mechanical degradation in centrifuges was made by Lyons (1951). He found that more degradation occurred with larger particle sizes than with smaller and that experimental data could be satisfactorily accounted for by assuming that the distribution of particle sizes for the newly created small particles was the same as in the feed. Using experimental degradation data obtained from five commercial centrifuges he found that a plot of fraction degraded versus average particle size yielded a smooth curve which depended on the type of centrifuge and the properties of the coal being treated. He did not investigate the effect of specific gravity differences within a size range.

It is assumed that the feed to the centrifuge is subdivided into n size ranges with size 1 being the largest size and size n being the smallest. The analysis is made by screening the feed so the upper and lower screen sizes are known for each size range. Each size range is divided into m gravity ranges with 1 being the lowest gravity and m being the highest gravity. For each fraction of the feed, the weight fraction sulfur, the weight fraction ash, and the heating value are determined and defined as follows:

$F_{i,j}$  = weight fraction of feed which is in size range *i* and gravity range *j*.

$P_{i,j}$  = weight percent pyritic sulfur in size range *i* and gravity range *j*.

$T_{i,j}$  = weight percent total sulfur in size range *i* and gravity range *j*.

$W_{i,j}$  = weight percent ash in size range *i* and gravity range *j*.

Since no information is available on the effect of gravity on degradation, we assume that the fraction degraded is independent of gravity. Further, we assume that the fraction degraded is linearly related to the average size of the particle by the relation:

$$D_i = k_d A_i; \quad 0 < D_i < 0.9 \quad (23)$$

$$D_n = 0$$

where  $D_i$  = the fraction degraded

$A_i$  = the average size, defined as the mean of the upper and lower mesh sizes for size range *i*, inches

$k_d$  = a constant determined by the centrifuge and the coal being cleaned.

The parameter  $k_d$  is unique to a particular equipment in a particular coal preparation plant and can be adjusted to match centrifuge performance. For

the centrifuges and coals reported by Lyons (1951) an average value of 2.5 was calculated and is used as the default value of  $k_d$  in the module.

The fraction of size range  $i$  which is degraded is given by  $F_{i,j} D_i$  and is assumed to be the same for all gravities in size range  $i$ . This material is distributed among all size ranges smaller than size  $i$  in proportion to the fractions in the feed to the centrifuge. The contribution to size range  $p$  ( $p > i$ ) due to degradation from size  $i$  is given by:

$$G_{p,i,j} = F_{i,j} D_i F_{p,j} / \sum_{k=j+1}^n (F_{k,j}) \quad (24)$$

The total contribution to size range  $p$  from each of the sizes larger than  $p$  is obtained by summing  $G_{p,i,j}$  from  $i = 1$  to  $p - 1$ . To this must be added the amount of undegraded material in size range  $p$ , and the result is the corrected fraction of feed in size range  $p$  after degradation.

$$F_{p,j}^* = \sum_{i=1}^{p-1} (G_{p,i,j}) + (1 - D_p) F_{p,j} \quad (25)$$

where  $F_{p,j}^*$  = the fraction of the feed which appears in size range  $p$  and gravity range  $j$  after degradation.

The ash and sulfur content of the degraded material can be calculated by modifying equation 24 to account for the pyritic sulfur, the total sulfur and the ash content of each fraction. The mass of pyritic sulfur, for example, which is in the material degraded from size  $i$  to size  $p$  per unit mass of coal in the feed is given by:

$$G_{p,i,j} P_{i,j} / 100$$

and a pyritic sulfur balance for size p gives:

$$P_{p,j}^* F_{p,j}^* = \sum_{i=1}^{p-1} (G_{p,i,j} P_{i,j}) + (1 - D_p)(F_{p,j} P_{p,j}) \quad (26)$$

where  $P_{p,j}^*$  is the percent pyritic sulfur in size range p and gravity range j after degradation.

Similar equations can be written for total sulfur and for ash:

$$T_{p,j}^* F_{p,j}^* = \sum_{i=1}^{p-1} (G_{p,i,j} T_{i,j}) + (1 - D_p)(F_{p,j} T_{p,j}) \quad (27)$$

$$W_{p,j}^* F_{p,j}^* = \sum_{i=1}^{p-1} (G_{p,i,j} W_{i,j}) + (1 - D_p)(F_{p,j} W_{p,j}) \quad (28)$$

where  $T_{p,j}^*$  = percent total sulfur in size range p and gravity range j for the degraded coal

$W_{p,j}^*$  = percent ash in size range p and gravity range j for the degraded coal

Since  $F_{p,j}^*$  is known from equation 25, the corrected sulfur and ash contents can be found from equations 26, 27 and 28.

#### Loss of Coal Fines to Water

The vacuum filter and centrifuge have two products--one is a dried coal, and the other is the water which has been removed together with some fine particles which leave with the water. The latter are referred to here as loss, although they are not a true loss since they will be recovered in subsequent operations. The amount of fines which leave with the water will be

strongly dependent on the kind of equipment being used and on the manner of operation. In the module we assume that there will be no loss of fines for size ranges larger than a critical size ( $i=c$ ), defined as the size range corresponding to the largest opening in the filter cloth or centrifuge basket. For size ranges smaller than  $c$  the amount lost to the filtrate is proportional to the amount in that size range.

$$E_{i,j} = k_i^* F_{i,j}^* \quad i > c \quad (29)$$

$$E_{i,j} = 0 \quad i < c \quad (30)$$

where  $E_{i,j}$  is the amount of material in size range  $i$  and gravity range  $j$  which reports to the water product. It is expressed as a fraction of the total coal feed to the unit. The fraction lost,  $k_i^*$  is assumed to vary linearly from 0 for sizes larger than  $c$  to  $k_f$  for the smallest size.

The fraction of feed reporting to the clean coal product is given by:

$$C_{i,j} = F_{i,j}^* - E_{i,j} \quad (31)$$

where  $C_{i,j}$  is the fraction of the size range  $i$  and gravity range  $j$  which reports to clean coal.

#### Calculation of Moisture Content

The moisture content of the products is calculated in a two-step procedure. First, the minimum moisture content is estimated assuming that the desaturation force acts for a very long time. Then this value is adjusted upward to allow for the fact that actual dewatering times are finite. Rather than make calculations in terms of moisture content, it is more convenient to

use the saturation,  $S$ , defined as the fraction of the void space occupied by water in the product. The minimum saturation,  $S_r$ , called the residual saturation, is estimated using correlations proposed by Wakeman (1979). These correlations express  $S_r$  in terms of the capillary number,  $N_c$ , a dimensionless group defined as follows:

$$N_c = \frac{e^3 d_p^2 f}{(1-\epsilon)^2 \sigma} \quad (32)$$

where  $d_p$  = Average particle diameter, ft.  
 $f$  = Desaturating force,  $lb_f/ft^2$ .  
 $\epsilon$  = Void fraction  
 $\sigma$  = Surface tension,  $lb_f/ft$ .

The desaturating force for centrifugally formed cakes is

$$f = (3.59E-7) \omega^2 B \rho_w \quad (33)$$

where  $B$  = Diameter of centrifuge basket, inches  
 $\rho_w$  = Density of water,  $lb_m/ft^3$ .  
 $\omega$  = Rotational speed of centrifuge, rpm.

For the vacuum filter

$$f = P/L \quad (34)$$

where  $P$  = pressure drop across the filter cake,  $lb_f/ft^2$ .  
 $L$  = thickness of filter cake, feet.

The pressure drop across the filter cake is less than the measured pressure drop in the filter because of the losses in piping and resistance of the filter medium. In terms of a measured or apparent pressure drop

$$P = k_p P_a$$

An average particle diameter is needed in Equation 32. For a mixture of particles of various sizes and shapes, there are several ways to estimate an average diameter. We have found that using the mean diameter based on surface area as defined below together with the equations above gives good results for filter cakes made of fine coal.

$$d_p = \frac{1}{\sum_{i=1}^n \left( \sum_{j=1}^m F_{ij}^* \right) / A_i} \quad (35)$$

The above equation gives the diameter of a particle which would have the same area per unit volume as the collection of particles does. Calculations of individual areas are made assuming that the particles are spheres. The  $A_i$  used in the equation is the arithmetic average of the mesh sizes boundaries for size range  $i$ .

The residual saturation for a filter cake is given by Wakeman (1979) as

$$S_r = 0.155 (1 + 0.031 N_c^{-.49}), \quad N_c > 10^{-4} \quad (36)$$

and for a centrifuge cake

$$S_r = 0.0524 N_c^{-.19}, \quad 10^{-5} \leq N_c < 0.14 \quad (37)$$

$$S_r = 0.0139 N_c^{-.86}, \quad 0.14 \leq N_c \leq 10 \quad (38)$$

The residual saturation found from Equations 36-38 is the lowest value possible. In practice because of limited contact time, higher saturations are found. The dewatering of a filter of centrifuge cake is a complicated process involving two-phase flow through a porous medium. Brown (1950, p. 248) presents a simplified solution which we have found useful. It is derived for pressure filtration, but we use it also for a centrifuge. He defines the saturation,  $S$ , in terms of an effective saturation,  $S_e$

$$S_e = \frac{S - S_r}{1 - 2 S_r + S S_r} \quad (39)$$

Assuming that the cake starts completely saturated, then  $S_e$  at any time,  $t$ , can be found from

$$S_e^{1-y} = 1 + \frac{2t(y-1)}{C_t((1-S_r)^2 + 1)} \quad (40)$$

where  $C_t = \frac{\mu \epsilon L^2}{Kp} \quad (41)$

$\mu$  = Viscosity of fluid, lbm/ft-s.

$K$  = Permeability, lb<sub>m</sub>-ft<sup>3</sup>/lb<sub>f</sub>-s<sup>2</sup>.

The permeability can be estimated from Brown (1950, p. 216).

$$K = \frac{32.2 \epsilon_d^3 p}{180(1-\epsilon)^2} \quad (42)$$

The time to be used in Equation 42 is the effective dewatering time. For a drum filter, it is the cycle time multiplied by the fraction of this cycle used for vacuum dewatering. For a centrifuge it is the average time the coal remains in the centrifuge. The dimensionless parameter  $y$  in equation 40 is a function of particle size  $d_p$ , and can be approximated by

$$y = 1.384 + 0.0275 \log d_p + 0.140 (\log d_p)^2 \quad (43)$$

By using equation 40 through 43,  $S_e$  can be found for a given set of operating parameters, and then Equation 39 is used to find  $S$ , the final saturation. The moisture content is

$$M = \frac{\epsilon \rho_w}{\epsilon \rho_w + (1 - \epsilon) \rho_s} \quad (44)$$

#### Specification of Parameters

The equations given above have parameters which must be established for a given plant configuration. As used in the module, they are divided into two groups--decision variables and built-in parameters. The decision variables are normally specified in the program input. If not they are assigned default values. Built-in parameters on the other hand can only be changed by modifying the FORTRAN program and then recompiling. Values assigned to decision and built-in variables are shown in Table 1. Decision variables are discussed below.

There are three decision variables for the centrifuge. They are the diameter of the centrifuge basket ( $B$ ), the thickness of the centrifuge cake at discharge ( $L$ ), and the rotational speed in rpm ( $\omega$ ). If no value for  $B$  or  $\omega$  is given in the input data to the program, then it is assumed that the

Table 1

<u>Description</u>	<u>Symbol</u>	<u>Program Name</u>	<u>Default Value</u>
<u>Decision Variables</u>			
<u>Filter</u>			
Area of Filter	$A_f$	D1	*
Cycle Time	$t_f$	D2, CYTIME	*
Cake Thickness	L	D2, THICK	1 in.
Apparent Pressure Drop		D3, PRDIFF	9.8 psi
<u>Centrifuge</u>			
Basket Diameter	B	D1	*
Thickness of Cake	L	D2, THICK	1 in.
Rotational Speed	$\omega$	D3	*
<u>Built-in Parameters</u>			
<u>Common</u>			
Porosity	$\epsilon$	PORS	0.45
Temperature		TEMP	25 <sup>o</sup>
Screen Size			
Filter		SCREEN	0.0098 in.
Basket Centrifuge		SCREEN	0.0116 in.
Solid Bowl Centrifuge		SCREEN	0.0029 in.
Maximum Loss to Liquid			
Filter	$R_f$	FLMAXF	0.30
Centrifuge	$R_f$	FLOSM	0.40
<u>Filter Parameters</u>			
Fraction of Cycle for Dewatering		FCDEW	0.333
Fraction of Apparent Pressure	$k_p$	PRFAC	0.8
<u>Centrifuge Parameters</u>			
Effective Dewatering Time		HLDTM	60 secc.
Degradation Constant	$k_d$	CKD	2.5

\*See Text for default value.

combination of basket diameter and centrifuge speed is such that the centrifugal force is 300 g's. If no value of L is specified, a default value of one inch is used.

There are also three decision variables for the filter. They are the area of the filter ( $A_f$ ); either the cycle time ( $t_f$ ) or the cake thickness (L); and the apparent pressure drop across the filter cake. The area is the total area available for filtration. It is not possible to specify both the cycle time and cake thickness because they are related by the equation

$$t_f = (5L\rho_s(1-\epsilon) A_f / (Y \sum_{i=1}^n \sum_{j=1}^m (C_{ij}))) \quad (45)$$

where Y = coal feed rate to dewatering unit, lbs/hr.

The sign of the second decision variable determines whether  $t_f$  or L is being specified (for  $t_f$  a positive value is used and for L a negative value). Default values for decision variables are as follows. If the first variable is zero, the feed rate to the filter is set to 75 lbs/hr-ft<sup>2</sup>. If the second variable is zero, the thickness is set to 1 inch. If the third is zero, the apparent pressure drop is set to 9.8 psi.

#### IV. THERMAL DRYER

In the thermal dryer module, the performance of a fluidized bed coal dryer is calculated. It is assumed that the energy source is a portion of the coal being dried and that the hot combustion gases are cooled by the addition of tempering air to reduce the temperature to a safe level for direct contact with coal particles. If another fuel is used, or if combustion gases are not tempered and contacted directly with the coal, program modifications will be needed. It is also assumed that a suitable control system is used to adjust the amount of tempering air in order to maintain a constant known temperature for the gases entering the drying chamber. The gases and coal particles in the drying chamber are well mixed and are assumed to be at a uniform temperature.

The amount of water evaporated in the dryer will depend on the feed rates of coal and air, the time the gases are in contact with the coal particles, the temperature of the gases, the degree of agitation, particle size distribution and many other factors. To properly include all these factors in the calculation would require a detailed knowledge of the dryer construction and operation--information which is not available nor is it needed since a good approximation of the performance of the dryer can be made using only material and energy balances if two additional assumptions about the dryer performance are made. The first is the moisture content of the dried coal, and the second is either the dryer temperature or the relative humidity of the gases leaving the dryer. It is assumed that the dryer can be operated in such a way that these specifications can be met.

Two limiting cases may occur in operation--the gases can become saturated with water or the coal will be completely dry. Neither of these alternatives is desirable since bone dry coal will tend to dust easily, and if the gases

are saturated, water will condense from them as they are cooled in the gas handling system. Proper specifications should avoid either of these limits. The program includes a check to make sure that the relative humidity of the exit gases is less than 100% if dryer temperature is specified. If more than 100%, an error message is generated.

### Calculation Sequence

The calculation sequence is organized to eliminate or at least minimize iterations. This is done by using 1 kg of dry coal as a starting basis. This coal is burned with the theoretical amount of air. The gaseous product consists of  $\text{CO}_2$ ,  $\text{H}_2\text{O}$ ,  $\text{SO}_2$  and  $\text{N}_2$ . The amount of each can readily be calculated if the ultimate composition of the coal is known. One of the options in the program input data is specification of percent hydrogen, nitrogen, and oxygen in the raw coal (percent sulfur is available from the washability data, and percent carbon is determined by difference). This data is used in subroutine BTUPLB to calculate the heat of combustion using Dulong's formula. The same data can be used here if it is available. If not, default values for a typical bituminous coal are provided in the program-- hydrogen, 4%, oxygen, 8%, nitrogen, 1%.

The hot combustion gases are next tempered by the addition of air. From the dryer specifications, the temperature of the tempered gases is known. An energy balance can then be used to calculate the amount of air needed to produce a combustion gas-air mixture with the desired temperature. The total amount of hot combustion gas resulting from the combustion of one kg of coal is now known, and the temperature of the gas is known. The next step depends on the option selected for condition of the gases leaving the dryer.

If the temperature of the gases leaving the dryer was specified, then the energy given up by cooling the gases from the tempering temperature to the

dryer temperature can be directly calculated. This gives the energy available for drying per kg of coal burned. In making this calculation it is assumed that heat losses and other inefficiencies result in a loss of 10% of the heat of combustion. The energy needed to dry one kg of coal can also be calculated since both the initial and final moisture contents and temperatures of the coal are known. The fraction of coal which must be burned to dry the coal can then be calculated, as well as the composition of the gases leaving the dryer. A check is made to see if the relative humidity of the gases is more than 100%. If it is, an error message is generated.

The second option for outlet gas conditions is to specify the relative humidity. In this case, it is not possible to make a direct calculation for the gas temperature. Instead, a trial and error method must be used, with the gas temperature being the variable of iteration. The first trial value is 150 F (65.6 C) and the second is 155.4 F (68.6 C). For successive iterations, a Newton-Raphson algorithm is used to calculate trial values. For each iteration, the trial temperature is used to calculate a value of relative humidity as described above for the first option. The calculated value of relative humidity is compared with the desired value, and if the difference is more than 0.1% another iteration is carried out. Usually three or four iterations are sufficient for convergence. If more than 10 iterations are required, an error message is generated.

#### Decision Variables

The decision variables which must be specified in the program input are as follows:

D1: The desired moisture content (percent) in the dried coal on a wet basis. If D1 is less than .01 in the input data, a default value of 4% is used.

D2: Temperature of the hot tempered combustion gases, Fahrenheit. If D2 is less than 100 in the input data, a default value of 1100 F is used.

D3: May be positive or negative, depending on the option chosen for the conditions of the gases leaving the dryer.

D3 positive: Temperature (Fahrenheit) of gases leaving the dryer. If D3 is less than 10, then a default value of 150 F is used.

D3 negative: Relative humidity of the exit gases--percent.

## V. SIZE REDUCTION

Modern comminution practice uses the kinetics of size reduction approach to predict the product size consist produced by a size reduction device. This approach assumes that the fracture of particles within a narrow size interval (material which is contained between two sieve sizes whose size ratio is  $\sqrt{2}$  ; e.g., 3 1/2" by 2 1/2" or 200 x 270 mesh) can be completely described in terms of two breakage functions. One of these, the primary breakage distribution function from which the primary breakage distribution values can be determined, defines the size distribution of the progeny particles that result when particles undergo primary breakage (primary breakage is defined as the ability to measure the progeny fragments before any of them are broken).

The other breakage function, the specific-rate-of-breakage function, from which the specific-rate-of-breakage values can be determined, defines the rate at which particles within a narrow size interval are selected for fracture and broken per unit time. The specific-rate-of-breakage and the primary breakage distribution values are incorporated into an appropriate mass balance relationship to produce a prediction model of a size reduction device. The rate of production of size  $i$  is

$$= b_{ij} S_j w_j W \quad (46)$$

where  $b_{ij}$  = primary breakage distribution function,

$S_j$  = specific rate of breakage function

$w_j$  = fraction of material of size  $x_j$

$W$  = is the quantity of material.

Similarly the rate of production of less-than-size  $x_i$  from breakage of larger size  $x_j$

$$= B_{ij} S_j w_j W \quad (47)$$

where  $B_{ij} = \sum_{k=1}^j b_{ik}$

The primary breakage distribution function varies with the type of material and size reduction device. However, it is usually constant for a particular material/device combination. The specific-rate-of-breakage function also varies with the type of material and size reduction device. In addition, this breakage function also varies with operating conditions.

#### Rotary Breaker

The rotary breaker is used extensively for the size reduction of run-of-mine coal. The principle is that of a tumbling autogeneous mill, except the cylindrical shell is full of uniform sized holes which allow raw coal of lesser size to fall through, while a reject stream of unbroken stronger material, enriched in ash content, leaves the end of the cylinder. The ROM coal in the rotary breaker is moved along by a series of ploughs. The level of material filling in the device is low, and decreases to essentially zero at some point along the axis. Thus, the device can be envisioned as a cylinder with a thin layer of ROM coal moving along the shell, not unlike a rotary cylindrical screen.

The holes in the shell typically range from 1 1/2" to 6", giving a raw coal top size of approximately 1/2" less than the opening. Although the rotary breaker appears to receive all the ROM coal going to the coal preparation plant, it is not uncommon to scalp the feed to a rotary breaker with an inefficient screen, in order to remove material already finer than the holes in the shell of the rotary breaker.

For a given size rotary breaker, e.g., 10' x 24', the maximum feed rate to the device increases with increasing hole size. Naturally, for a fixed hole size, the maximum feed rate increases with increasing rotary breaker dimensions. Rotary breakers are available with diameters of 9', 10', 12' and 14' and in lengths of 12' to 31'. Obviously, the top size of the feed and the amount of material in the feed less than the hole size will also impact on the capacity of a rotary breaker. A size/hole/feed rate table for one manufacturer of rotary breakers gives -

Size Breaker Dia. c/c Tires	Size Product						
	1 1/2"	2"	2 1/2"	3"	4"	5"	6"
9' x 16'				400	500	600	700
9' x 20'			400	500	600	700	800
10' x 16'			400	500	600	700	800
10' x 20'		400	500	650	800	1000	1200
10' x 24'	400	550	700	850	1000	1250	1500
12' x 22'	400	550	700	850	1000	1250	1500
12' x 27'	700	800	900	1000	1200	1450	1700
14' x 31'-6"	800	1000	1200	1400	1600	1800	2000

Since there is a holdup of material within the rotary breaker, it is a retention device, acting on the reservoir of contained material. Hence, there is a selection for fracture process, and a description of size reduction in a rotary breaker must include the specific-rate-of-breakage values as well as the primary breakage distribution values. By assuming that the velocity along the axis is constant because of the ploughs and that particles less than the whole size are removed rapidly compared to their production by breakage, a simple model can be developed by dividing the rotary breaker along its axis into a series of elements and performing a size - mass balance across the element. Applying the concepts of breakage of material in a spatial interval, a size-mass balance on the element gives --

$$(\text{rate of axial flow of size } x_1 \text{ out}) - (\text{rate of axial flow of size } x_1 \text{ in})$$

$$= (\text{rate of production of size } x_i \text{ by breakage of larger sizes}) - (\text{rate of breakage of size } x_i)$$

for all sizes larger than the hole size. In symbols, this is --

$$(w_i F + \frac{d(w_i F)}{dy}) - w_i F = \sum_{j=1}^{i-1} S_j b_{ij} w_j W dy - S_i w_i W dy, \quad x_i > \text{hole size} \quad (48)$$

where F is the feed rate

$w_i$  is the fraction of material of size  $x_i$  in the interval, dy,

W is the quantity of material in the interval, dy,

$S_i$  is the specific rate of breakage,

and  $b_{ij}$  is the primary breakage value.

For all sizes less than the hole size, the rate of production equals

rate of production of sizes smaller than the hole size from breakage of all sizes larger than the hole size

In symbols, this is --

$$- \frac{dF}{dy} = \sum_{j=1}^{i_c-1} S_j B_{i_c, j} w_j W \quad (49)$$

where  $x_{i_c}$  is the hole size interval and  $B_{i_c, j}$  is the primary breakage level distribution value. However, the size interval must be  $\sqrt{2}$  or less in order for the balance to be independent of the feed size consist.

Since the specific-rate-of-breakage values change with the filling level of the material, solving the balance is an iterative process. In symbols, the iterative solution is

$$w_i(k+1)F(k+1) = w_i(k)F(k) - \Delta y \overline{S_i w_i} + \Delta y \sum_{j=1}^{i-1} b_{ij} \overline{S_j w_j} \quad (50)$$

for all size intervals greater than the hole size (the indicator, k,

represents the spatial element). After the solution is constant, then the calculations are repeated, substituting  $w_{ij}$  for  $w_i$ , where the indicator  $i$  represents size interval and the indicator  $j$  represents specific gravity interval. The size consists from each element are combined to give the raw coal size consist.

$$p_{ij}P = w_{ij}(0) + \sum_{k=1}^m F(k)\Delta y \sum_{j=1}^{i_c-1} b_{ij} \overline{S_j w_{ij}} \quad (51)$$

The size consists of the material leaving the final element represents the reject leaving the end of the rotary breaker. By specifying the diameter and the length of the rotary breaker, the hole size and the feed washability, the rotary breaker subroutine

- . interpolates the feed size consist into  $\sqrt{2}$  size intervals
- . calculates the primary breakage distribution values
- . calculates the raw coal size consist and the reject size consist
- . interpolates the raw coal size consist and the reject size consist into the original size intervals.

### Roll Crusher

Probably the most commonly used crusher in coal preparation is the double roll crusher. The operating principle is quite simple. Two cylindrical rolls, capable of rotating on their principal axes and set parallel to each other in a horizontal plane, are driven towards each other. The coal particles are nipped between the two surfaces, causing the particles to fracture by compressing and, perhaps, shearing them.

The ability of the rolls to nip the particles is increased by adding "teeth" to the surface of the roll. The relationship among the spacing between the rolls, the diameter of the rolls, the height of the teeth and the

largest particle size that can be nipped is -

$$(\text{tooth height factor}) = \frac{(\text{roll diameter} + \text{roll spacing})}{(\text{roll diameter} + \text{max. particle size})} \quad (52)$$

The tooth height factor can range between 0.55 to 0.95, decreasing with increasing tooth height. Thus, the maximum particle size for 34" roll diameters and a 2" roll spacing is 8" while the maximum particle for 30" roll diameters and a 6" roll spacing is 36". A size reduction ratio can be defined as the maximum particle size divided by the roll spacing.

The roll crusher lends itself to staging, hence it is not unusual to find a two stage-four roll crusher, i.e., one double roll crusher mounter on top of another. In this manner, the size reduction ratio is equal to the product of the size reduction ratio for each stage. Thus, it is possible to crush 8" top size coal to 1" top size coal, a 8:1 ratio, with 16" diameter rolls.

In a crusher such as the roll crusher, a certain fraction of the feed material in a narrow size interval ( $\sqrt{2}$ ) will by-pass the breakage action of the crusher. This primary by-passing varies from 0 for particles much larger than the spacing between the rolls to 1 for particles much smaller than the spacing. The probability of fracture values have been measured for  $\sqrt{2}$  screen intervals of coals in a double roll crusher, and have been found to follow the empirical relation for each size interval,  $x_i$ , of

$$s_i = \frac{1}{1 + \left( \frac{x_i/x_g}{d_{50}/x_g} \right)^\lambda} \quad (53)$$

where  $x_g$  is the gap setting of the rolls. The value of  $\lambda$  is 6.6 for all materials tested. The value of  $d_{50}/x_g$  is a characteristic of each material. The progeny particles also are subjected to a similar by-passing phenomenon.

It differs because the progeny particles are in a physical orientation of higher selectivity for breakage. Hence, there are progeny by-passing values. These probabilities of fracture values were found to be related to the primary values by

$$s'_i = \begin{cases} s_{i-1} & , x_i > (\sqrt{2})(\text{gap size}) \\ (s_{i_g-1} - s_{i_g-2})/2 & , x_i = (\sqrt{2})(\text{gap size}) \\ s_i & , x_i < \text{gap size} \end{cases} \quad (54)$$

The third line states that sizes smaller than the gap setting have the same probability of falling through unbroken, whether present in the feed or produced by breakage of larger sizes. The first line states that the probability of fracture for progeny sizes larger than the size interval above the gap is the same as that for feed sizes in the next lower size interval. The second line states that the probability of fracture in the size interval above the gap is intermediate. Both the primary and the progeny selectivity values are a function of the spacing between the rolls and the particle size. Since the roll crusher, if operated in a nonchoking condition, is a once-through size reduction device, not a retention device, there are no specific-rate-of-breakage values.

The three quantities - the primary breakage distribution values, the primary selectivity values and the progeny selectivity values - can be combined into a set of values known as transfer values, or --

$$d_{ij} = \begin{cases} 1 - s_i & , i = j \\ (1-s'_i)[b_{ij}s_j + \sum_{\substack{\ell=j+1 \\ i>j+2}}^{i-1} b_{i\ell}s'_\ell d_{\ell j} / (1-s'_\ell)] & , i > j \end{cases} \quad (55)$$

where the subscripts represent size intervals, the largest one being 1, the smallest n. A transfer value is the fraction of material in a particular size interval ( $\sqrt{2}$ ) which breaks into another size interval ( $\sqrt{2}$ ), becoming part of the material in that size interval. Thus, by multiplying the fraction of material in each of the feed size intervals by the appropriate transfer value, and, then summing the resulting products, the fraction of material in a product size interval can be determined. In symbols, this is

$$p_{ij} = \sum_{l=1}^n d_{il} f_{lj} \quad (56)$$

where i and l indicate the size interval and j indicates the specific gravity interval. By repeating this calculation for each product size interval, the crusher product size consist is calculated. Such an approach is valid for any feed size consist only if the feed and product size intervals have a  $\sqrt{2}$  size ratio.

By specifying the roll spacing in inches and the feed washability, the crusher subroutine

- . interpolates the feed size consist into  $\sqrt{2}$  size intervals
- . calculates the primary breakage distribution values
- . calculates the primary and progeny selectivity values for the roll spacing
- . calculates the transfer values
- . calculates the crusher product size consist
- . interpolates the crusher product size consist into the original size intervals

In order to insure realism, the user must select a roll spacing such that the size reduction ratio is not greater than 6:1. In order to emulate a two stage-four roll crusher, the user should set two double roll crushers in series.

## VI. CLASSIFICATION

Classification is the process of splitting a stream of particles into a stream of coarser particles and a stream of finer particles. Ideal classification can be defined as the separation of a particle stream into a stream of only fine particles and a stream of only coarse particles. Practical classification, however, is not ideal; even particles of exactly the same size and with the same physical characteristics of shape and density will receive different classification actions within the same classifier, due to a finite width of the entering stream and edge effects and due to dispersive-mixing actions in the fluid. Some of these identical particles will be sent to the coarser particle stream while some will be sent to the fine particle stream. Even for very simple classification actions, the presence of statistical dispersion makes it difficult to obtain an ideal split of sizes. Other complicating factors arise from the influence of other variable physical properties of the feed material, such as shape and specific gravity, which also affect the separation process.

The various types of classifiers fall into two general categories: those that separate by forces of fluid dynamics and those involving screening. Fluid dynamic classification takes advantage of the difference in rates of travel of particles in a fluid due to differences in particle size. An example of a fluid dynamic classifier used in coal preparation is the classifying hydrocyclone. Screening classification takes advantage of the ability to pass through an aperture due to differences in particle size. An example of a screening type classifier used in coal preparation is the sieve bend.

Since the separations tend to be of a statistical nature, it is not surprising to find most predictive efforts concentrating on estimating the probability that particles within a narrow size interval (material which is

contained between two sieve sizes whose size ratio is  $\sqrt{2}$  ; e.g., 200 x 700 mesh) will report to the coarse stream. For classifiers, these probabilities have been termed size selectivity values. Thus

$$s_i = \frac{t_i C}{p_i(1+C)} \quad (57)$$

where  $t_i$  is the weight fraction of material in size interval  $i$  in the coarse stream,  $p_i$  is the weight fraction of material in size interval  $i$  in the feed stream and  $C$  is the mass flow ratio of coarse stream to fine stream.

Obviously, the fraction of the feed sent to the finer stream is  $1-s_i$ .

This definition of  $s_i$  assumes that the process of size selection of each size interval in the classifier is first order. Then prediction of the size split is possible simply as

$$t_i = s_i p_i \left( \frac{1+C}{C} \right) \quad (58)$$

Since the quantity  $\left( \frac{1+C}{C} \right)$  is equal to  $\sum_{j=1}^R s_j p_j$ , that is, the sum of the products of size selectivity times weight fraction of feed for all size intervals, the size split can be predicted simply by knowing all the size selectivity values and feed size consist.

If the size selectivity values can be estimated for a particular classifier operating at known conditions, then the size consists of the coarse stream can be determined by multiplying the fraction feed material in each size interval ( $\sqrt{2}$ ) by the size selectivity value for that size interval and dividing the resulting product by the sum of the products for all the size intervals. Usually, the size selectivity values can be reproduced from an equation requiring three parameters - a size modulus commonly known as the cut

size; a distribution modulus commonly called the sharpness index; an apparent by-pass value which represents material which behaves as if it by-passes the classification action. In symbols, this is --

$$s_i = (1-a)c_i + a \quad (59)$$

where  $a$  is the apparent by-pass and  $c_i$  is a two parameter (cut size and sharpness index) function. Thus if, for a particular classifying device, these three parameters can be estimated for the particular operating conditions, then the size selectivity values can be estimated and the separation calculated.

#### The Classifying Hydrocyclone

In a centrifugal device such as the hydrocyclone, the separation action is produced by the rotation of the water, which develops higher centrifugal forces. These forces tend to direct the heavier (larger) particles to the wall where they flow down to the underflow exit. At the same time, the lighter (smaller) particles are left nearer the center of the cyclone with the bulk of the water, because the drag effect (relative to inertial forces) of the water on the smaller particles causes slower movement of particles with respect to water. The major part of the water, carrying the finer particles, reverses direction at the bottom of the tapered end of the cyclone (called the apex or spigot and flows upward through a central outlet (called the vortex finder) to the cyclone overflow.

The three parameters required to define the size selectivity values must be estimated for the particular hydrocyclone geometry and operating conditions. The cut size can be estimated by following a procedure developed by Dick Arterburn of Krebs Engineering, a manufacturer of classifying

hydrocyclones. He assumed a standard geometry and then developed an equation that predicts the cut size based upon

- . the cyclone diameter (inches)
- . the % solids by volume of the feed slurry
- . the operating pressure drop (psi)
- . the specific gravity of the material,  $\rho_s$

$$\text{cut size, } \mu\text{m} = 18 \frac{(\text{cyclone dia., inches})^{0.68}}{(\Delta P, \text{psi})^{0.3} (\rho_s - 1)^{0.5} (1 - 1.9 \text{ vol. frac. solids})^{1.43}} \quad (60)$$

The distribution modulus for a classifying hydrocyclone is essentially constant. Thus, the  $c_{ij}$  values can be calculated and the fine stream composition estimated by

$$q_{ij} = \frac{(1 - c_{ij}) p_{ij}}{1 - \sum_{\text{all } l} \sum_{\text{all } j} c_{lj} p_{lj}} \quad (61)$$

where  $i$  and  $l$  indicate size interval and  $j$  indicates specific gravity intervals. The apparent by-pass can be approximated by the water split, the fraction of water in the feed slurry that reports to the underflow. The typical coal preparation classifying cyclone feed slurry is less than 15% solids by weight. The typical cyclone underflow slurry is between 40% and 50% solids by weight. The typical water split lies between 5% to 15%. By fixing the feed weight fraction solids,  $C_p$ , and the underflow weight fraction solids,  $C_T$ , the  $C$  value can be calculated as

$$C = \left[ \frac{(1 - C_p) C_T}{C_T - C_p} \right] \frac{\sum_i \sum_j c_{ij} p_{ij}}{1 - \sum_i \sum_j c_{ij} p_{ij}} \quad (62)$$

Now the coarse stream composition is estimated from the mass balance --

$$t_{ij}C = p_{ij}(1+C) - q_{ij} \quad (63)$$

Typical coal preparation classifying cyclone diameters are 8", 14" and 24". The smaller the diameter,

- . the smaller the top size of the feed particles
- . the greater the operating pressure drop should be.

The typical operating pressure drop for a particular diameter can be predicted from a simple equation, using the following table --

<u>cyclone dia., inches</u>	<u>ΔP, psi</u>
8"	30
14"	25
24"	15

Thus, by specifying the diameter in inches and the feed washability, the classifier/hydrocyclone subroutine

- . interpolates the feed size consist into  $\sqrt{2}$  size intervals
- . calculates the size selectivity values for the cyclone diameter
- . calculates the coarse stream and the fine stream size consist
- . interpolates the coarse steam and fine stream size consists into the original size intervals.

#### The Sieve Bend

The sieve bend is a member of the class of screen classifiers known as cross flow screens. The feed slurry flows across a steeply pitched stationary wedge bar screen, at right angles to the slots. The slotted surface tends to remove slices of the flowing slurry. The finer particles and the water pass

through the slots while the coarser particles overflow at the end of the screen.

The cut size of the sieve bend is much smaller than the slot size (1/2 to 2/3) and is not affected by the specific gravity of the particles. The distribution modulus for a sieve bend seems to be constant. Thus the  $c_i$  values can be calculated and the fine stream composition estimated as

$$q_{ij} = \frac{(1-c_i)p_{ij}}{1 - \sum_{\text{all } \ell} c_{\ell} \sum_{\text{all } j} p_{\ell j}} \quad (64)$$

The apparent by-pass value can be approximated by the water split. However, the water split increases with decreasing slot width, hence a 325 mesh slot width would essentially be completely blinded, resulting in no separation. To circumvent this problem, manufacturers have added vibrating devices that greatly reduce the blinding, producing a rather constant water split value of 5%. Once the water split,  $a$ , has been estimated (between 0.05 and 1), then

$$1+C = \frac{1}{(1-a)(1 - \sum_{\text{all } i} c_i \sum_{\text{all } j} p_{ij})} \quad (65)$$

Now the coarse stream composition is estimated from the mass balance --

$$t_{ij}^C = p_{ij}(1+C) - q_{ij} \quad (66)$$

Thus, by specifying the slot width in inches and the feed washability, the classifier/sieve bend subroutine

- . interpolates the feed size consist into  $\sqrt{2}$  size intervals
- . calculates the size selectivity values for the slot width, with or without vibration

- . calculates the coarse stream and the fine stream size consists
- . interpolates the coarse stream and fine stream size consists into the original size intervals

## VII. SCREENS

Size separation is commonly carried out by passing coal over a vibrating screen containing regularly sized holes through which the smaller coal particles can fall, the larger particles being retained on the surface. The vibrating action clears the openings and serves to propel the coal across the screen. The screen may be operated wet or dry, and more than one screen may be used if separation into more than two size ranges is desired. A perfect screen would allow all coal particles smaller than the holes in the screen to pass and reject all larger particles. Perfect screening does not take place because the particles may approach the openings at an angle or off center, there may be interference between two or more particles at an opening, or the particle may fail to approach an opening. Obviously particles which are much smaller than the opening are more likely to pass through than particles which are only slightly smaller than the opening. The methods used here to calculate the screen efficiencies were developed at Automated Process Surveys by Dr. A. Vaillant and are presented in more detail in Computer Simulation of Coal Preparation Plants (1978).

Let two particles of sizes  $x_1$  and  $x_2$  attempt to pass through an opening of size  $y_0$ . If it is assumed that the probability of both particles passing through the opening is the same as the probability of a single particle of size  $(x_1 + x_2)$  passing through, and if the event of a particle of size  $x_1$  passing through is independent of the event of a particle of size  $x_2$  passing through, then

$$P(x_1) \cdot P(x_2) = P(x_1 + x_2)$$

his equation can be satisfied by

$$P(x) = \exp(kx)$$

which can readily be verified by substituting  $\exp(kx_1)$  and  $\exp(kx_2)$  in the above equation. Tests by Automated Process Surveys show that the fraction of feed particles of size  $x$  that goes over the screen ( $C_s(x)$ ) is exponential over a wide range of  $x$  and furthermore that

$$C_s(x) = \exp(-A(1-x/y_0)) \quad , x < y_0 \quad (67)$$

where  $A$  is an experimental constant, and  $y_0$  is the projected screen opening. Vaillant calls the experimental constant  $A$  the separation strength of the screen. Its magnitude depends on screen type, screen opening, and production rate but it is independent of the feed size distribution.

The separation strength depends also on the location of the screen deck within the machine. Experience shows that a single deck machine has a higher separation strength than does a similar screen deck that is placed below another deck. Furthermore, the separation strength is improved by operating wet. Some reasonable values of separation strength are incorporated in the module SCREEN for wet and dry screens and for upper and lower decks.

### VIII. BLENDER AND SPLITTER

A "blender" is simply a point within the plant where two different flow streams are combined. The flowrate of blended product can be expressed as

$$P = \sum_i \sum_j (f_{ij1} + f_{ij2}) \quad (68)$$

where P represents the flowrates of the product stream, and  $f_{ij1}$  and  $f_{ij2}$  represent the flowrates of the first and second feed streams in the  $i$ th size fraction and the  $j$ th specific gravity fraction. Similarly, any attribute in the blended product can be determined as

$$A = \frac{1}{P} \sum_i \sum_j (A_{ij1} f_{ij1} + A_{ij2} f_{ij2}) \quad (69)$$

where A represents the attribute (e.g. ash, sulfur or Btu content) in the blended stream, and  $A_{ij1}$  and  $A_{ij2}$  represent the attributes of the first and second feed streams in the  $i$ th size fraction and the  $j$ th specific gravity fraction. Notice that the blender calculations do not require the specification of a control parameter.

A "splitter" is a point within the plant where a portion of a flow stream is diverted. The net effect is to create two flow streams from one flow stream, where the combined flowrates of the two new streams equal the flowrate of the original stream. The composition of the new streams will be the same as the original stream. Thus,

$$F_1(\alpha) = \sum_i \sum_j \alpha f_{ij} \quad (70)$$

and

$$F_2(\alpha) = \sum_i \sum_j (1-\alpha) f_{ij} \quad (71)$$

where  $f_{ij}$  represents the flowrate of the feed stream in the  $i$ th size fraction and the  $j$ th specific gravity fraction,  $F_1$  and  $F_2$  represent the product streams, and  $\alpha$  is a control parameter,  $0 < \alpha < 1$ .

Within the simulator, the blender and splitter calculations are carried out for each size fraction. The results are then combined to form composite exit streams.

## IX. MOISTURE AND HEAT OF COMBUSTION

### Moisture Content of Drained Coal

The moisture content of coal can be determined for each of the flowstreams that leaves an aqueous-type device, such as a dense-medium vessel, froth-flotation cell or wet screen. The moisture content that is computed is surface (pendular) moisture, which remains after all excess moisture has been allowed to drain off.

In contrast to the other flowstream-related properties (e.g., ash, sulfur, etc.), moisture content cannot be considered on the basis of individual size fractions. Rather, all of the material within the flowstream (i.e., all sizes) must be considered to obtain a single value. The reason for this is that moisture is not an intrinsic size-related characteristic, as are ash and sulfur, but is determined by the interaction of different-size particles within the flowstream. Small particles, if present, will tend to fill the gaps between the larger particles, causing the entire flowstream to retain more moisture. Thus, the overall moisture content of each flowstream will be determined by the constituent size distribution of the material within the flowstream.

The method used to estimate moisture content is similar to that used in the dewatering module except that pressure and centrifugal forces do not need to be considered, and a modification has been introduced for larger size coals. In the dewatering module the moisture content was determined by a dimensionless capillary number,  $N_c$ , defined as follows:

$$N_c = \frac{\epsilon_d^3 p_f}{(1-\epsilon)^2 \sigma} \quad (72)$$

where  $\epsilon$  = the void fraction (assumed to be 0.47)

$d_p$  = average particle diameter, ft.

$f$  = desaturating force,  $62.4 \text{ lb}_f/\text{ft}^3$

$\sigma$  = surface tension of fluid,  $\text{lb}_f/\text{ft}$ .

The desaturating force is gravity acting on the liquid in the bed of coal.

The average particle diameter used is the mean surface diameter per unit mass, which is defined as the diameter of a particle which would have the same surface area per unit of mass as the sample being considered. It can be calculated by

$$d_p = \frac{1}{\sum_{i=1}^n \left( \frac{\sum_{j=1}^m F_{ij}}{A_i} \right)} \quad (73)$$

Where  $F_{ij}$  is the mass fraction of the coal sample in size range  $i$  and gravity range  $j$ .  $A_i$  is the average diameter of a particle in size range  $i$ , taken to be the mean of the upper and lower limits for size range  $i$ .

The capillary number  $N_c$  is a measure of the capillary forces which hold water in the interstices between particles. A small value indicates large capillary forces, and a large value indicates small forces.

We have found that the correlation proposed by Wakeman (1979) for the residual saturation of filter cakes gives good results for fine coals but is less satisfactory for coarse coals where capillary forces are not so important and pendular or surface moisture controls. For these coals we use the correlation proposed by Dombrowski and Brownell (1954). The combined correlation is:

$$\begin{array}{ll}
S = 0.115 (1 + 0.031 N_c^{-0.49}) & 0.0085 > N_c \\
S = 1/(565 N_c) & 0.024 > N_c \geq 0.0085 \quad (74) \\
S = 0.076 & N_c \geq 0.024
\end{array}$$

In these equations, S is the residual saturation, defined as the fraction of the void space which is filled with water. The moisture content on a dry basis is given by

$$M = \epsilon S / ((1 - \epsilon) \rho)$$

where  $\rho$  is the specific gravity of the coal, and  $\epsilon$  is the void fraction (assumed to be 0.47). On a wet basis, the moisture content is

$$M' = M / (1 + M)$$

#### Heat of Combustion

Two methods are provided in the simulator for calculating the heat of combustion of the coal. The first uses an empirical correlation of heat of combustion and ash content.

$$H = \text{MAX}_d \quad a - b P_a + c P_a^2 \quad (75)$$

where H is the gross or higher heat of combustion (Btu/lb) of coal, and  $P_a$  is the percent ash in the coal. The constants a, b, c and d are provided by the user in the input data.

The second method uses Dulong's formula (Berkowitz, 1979) for estimating the heat of combustion

$$H^r = 81370 + 345 (P_h - (P_o + P_n - 1)/8) + 22.2 P_s \quad (76)$$

where  $H^r$  is the gross heat of combustion on an ash free basis (Btu/lb).  $P_h$ ,  $P_o$ ,  $P_n$ , and  $P_s$  are percents of hydrogen, oxygen, nitrogen, and sulfur, all on an ash-free basis.

The percent sulfur can be obtained from the washability data for the coal. The percent hydrogen, oxygen, and nitrogen must be supplied by the user as part of the input data. Percent carbon is obtained by difference.

#### Input to the Simulator

The method to be used for calculating heat of combustion is determined by the input data provided by the user. Four real numbers are required in the input data file. If the first number is greater than zero, then the empirical method (equation 75) is used. If the first number is zero or less, then Dulong's formula is used. The second, third and fourth numbers in this case will be the percent of hydrogen, oxygen, and nitrogen. If the second number, percent hydrogen is zero, then the following default values are used.

Percent hydrogen = 4.

Percent oxygen = 8.

Percent nitrogen = 1.

These values are typical for a high volatility bituminous coal (Wender, 1979).

## X. OVERALL WATER BALANCING

Many units within a preparation plant require the addition of water in order to function properly. In particular, washers, screens and classifiers often require water addition. The simulator provides for water addition to all of these units, and to blenders as well.

The quantity of water added to each unit is generally an operating parameter that is determined by the plant engineer, within certain broadly defined bounds. Therefore, the quantity of water added to a unit is treated as a decision variable within the simulator. The manner in which the water leaves each unit will vary, however, from one unit to the next. With some units the excess water and coal flow together in the same exit streams, whereas other units cause the coal and water to be largely separated from each other.

Flowstreams that consist primarily of excess water are directed to a thickener, where fine solid material is removed. The clarified water is then recirculated through the plant. Thus, the simulator must maintain a water balance for the entire plant. This is accomplished by routing the water between individual units and the thickener. The details of the calculations are described below.

### Distribution of Excess Water Leaving the Individual Units

The assumptions concerning the distribution of excess water leaving each unit depend upon the overall approach that is taken when modeling the performance of that unit. The specific assumptions vary from one type of unit to the next.

Heavy media washers (e.g., dense-medium vessel, dense-medium cyclone and sand cone) are assumed to be closed-circuit systems that include the washer and a heavy medium recovery system. Within the circuit is a dewatering device

that removes the bulk of the solid material from the exit water. (This assumption is particularly plausible for coarse coal washers, such as dense-medium vessels and sand cones.) Thus, the clean coal and refuse streams that exit the circuit are assumed to contain only "pendular" surface moisture. (These values are very low for coarse coals.) The excess water leaving the circuit is assumed to be a distinct stream containing some small, fixed amount of fine solid material. Such exit water streams are automatically routed directly to the thickener.

Every heavy-medium washer is assumed to have two distinct water streams associated with it. The first is referred to as WATER (1,IU). It represents the amount of water, in gallons per minute, that is added to the unit. this quantity is treated as a decision variable that is entered into the computer as input data for each unit. The second water stream, WATER (2, IU), represents the water that is routed from the unit (actually, from the closed circuit) to the thickener. The value of this flowstream is calculated by a water balance for the unit, i.e.,

$$\text{WATER (2,IU)} = \text{WATER (1,IU)} + W_f - (W_{cc} + W_{ref}) \quad (77)$$

where  $W_f$  = Water entering with the coal feed stream, gal/min  
 $W_{cc}$  = Water leaving with clean coal, gal/min  
 $W_{ref}$  = Water leaving with the refuse material, gal/min

The quantities  $W_f$ ,  $W_{cc}$  and  $W_{ref}$  are calculated using the special function MOISTR (see Section IX - Moisture Calculations).

For the other float-sink washers, the total amount of water leaving the device is calculated via an overall water balance; i.e.,

$$W_{tot} = \text{WATER}(1, IU) + W_f \quad (78)$$

This water is assumed to be distributed within each output stream in direct proportion to the mass of solid material within the stream. Thus,

$$\begin{aligned} W_{cc} &= W_{tot} F_{cc} / (F_{cc} + F_{mid} + F_{ref}) \\ W_{mid} &= W_{tot} F_{mid} / (F_{cc} + F_{mid} + F_{ref}) \\ W_{ref} &= W_{tot} F_{ref} / (F_{cc} + F_{mid} + F_{ref}) \end{aligned} \quad (79)$$

Each output water stream is not separated automatically into pendular moisture accompanying the solid material and excess water. Such separations must be carried out by including a dewatering device explicitly within the flowsheet.

Classifying hydrocyclones and sieve bends are handled in the same manner as the float-sink washers described in the previous paragraph.

Froth flotation cells require a different assumption. With these devices the output concentrate (clean coal) is assumed to be 25 percent solids provided sufficient water is added so that a water-balance can be maintained. Any additional water will then leave with the tailings (refuse). If there is not enough water added to the device to support the 25 percent solids assumption, then the output water is assumed to be divided between the output streams in the same ratio as the solids.

Wet screens are modeled using the assumption that the overflow material

contains only "pendular" surface moisture. All remaining water passes through the screen with the underflow material. The quantity of water passing through the screen is determined by a material balance; i.e.,

$$W_{\text{under}} = \text{WATER}(1, \text{IU}) + W_f - W_{\text{over}} \quad (80)$$

where  $W_{\text{over}}$  is determined using the MOISTR function, as described above.

Water can also be added to a blender. Since a blender has only one output stream, however, there is no problem in distributing the water between streams. The output water is determined by a material balance, i.e.,

$$W = \text{WATER}(1, \text{IU}) + W_{f1} + W_{f2} \quad (81)$$

The output water and the solid material are carried in the same stream. They must be separated by a dewatering device, elsewhere in the plant.

The manner in which water is removed from a flowstream by a dewatering device depends upon the type of dewatering device that is selected. The commonly used dewatering devices (e.g., centrifuge, vacuum filter, thermal dryer and thickener) all operate in their own unique way, each requiring a separate model. Such models, including the thickener, are described elsewhere in this report.

#### Overall Inventory of Water within the Plant

After the individual units have been simulated and the exit water from each unit has been calculated, a water balance is carried out for the entire plant. This can be written as

$$W_{\text{make up}} = \sum \text{Water}(2, \text{IU}) - W_c \quad (82)$$

Note that this water balance applies only to free water streams. It does not include moisture that accompanies the coal.  $W_c$  is the water available from the thickener. This water is available for recycle in the plant. The

difference between this water and the demand for water is the makeup which must be provided externally.

XI. GRAVITY THICKENER

A gravity thickener is used to remove fine particles (usually refuse) which are suspended in water. It serves to concentrate the fine particles into a sludge which can be disposed of efficiently and to produce a clarified water product which can be recycled to the plant or discharged from the plant. Current environmental regulations are such that as much of the water is recycled as possible.

Only conventional thickeners are simulated in this module. Lamella and deep cone thickeners are not included. The conventional thickener is a large circular tank with a provision to admit the feed at the center of the tank. The solids slowly sink to the bottom, and the water moves up and to the outer edge of the tank where it is removed. Rotating rakes move the settled sludge to a discharge well at the bottom of the tank, where it is removed by a slurry pump. The separation obtained is not perfect. There will be some very small particles in the thickener overflow, and the sludge will have a high moisture content--usually more than fifty percent. A flocculant is almost always added to the feed to coalesce the fine particles and increase settling rates.

Thickener Equations

The flow rate of feed, properties and amount of solids in the feed, cross-sectional area of the tank, and the depth of the tank determine the performance of the unit. The most important feature of the thickener is its cross-sectional area which is determined by the feed flow rate and properties of the sludge. Laboratory tests are made using the material to be thickened together with any flocculant to determine the settling rate as a function of composition. A plot of settling rate versus concentration can be made and used to determine whether there is an intermediate concentration which has a lower settling rate than the underflow. If so, then the performance of the

thickener will be determined by this "critical concentration" (see, for example, Fitch, 1979). Since settling tests of the sort needed for complete analysis are not likely to be available to the user of the simulator, a simplified procedure is used. The user specifies the underflow concentration and the cross sectional area of the tank. It is assumed that the specified underflow concentration can be achieved with the given flow rate and tank area.

A clear overflow from the thickener is desired. Since the particle size distribution of the feed to the thickener is known, and the tank area and flow rates are known, a calculation can be made to determine what material will be in the overflow. The liquid leaving the tank has an upward velocity component. Theoretically any particle which falls faster than this velocity should report to the underflow, and any particle which falls slower should report to the overflow. This does not happen, however, because of inefficiencies in separation. Plitt (1971) proposes the following equation for calculating the efficiency of separation,

$$E_c = 1 - \exp -0.693 (d/d_{50})^m \quad (83)$$

where  $E_c$  is the efficiency of separation, defined as the fraction of solids of size  $d$  which report to the underflow because of thickener action. The value  $d_{50}$  is the particle size which divides evenly between underflow and overflow, and  $m$  is an empirical constant. Based on data from 150 units he found  $m$  to vary from 1.5 to 3. From his data a value of  $m=2$  seems representative of thickeners and is used in the simulator module. The actual fraction of solids reporting to the underflow is not the same as  $E_c$  because in addition to thickener action some liquid is leaving with the underflow. After making a

correction for liquid flow and setting  $m=2$ , the observed fraction reporting to the underflow is given by

$$E = [1 - \exp -0.693 (d/d_{50})^2 ] 1-f + f \quad (84)$$

where  $f$  is the fraction of the liquid feed which reports to the underflow. If  $d_{50}$  and  $f$  are known, the distribution of solids of this size between overflow and underflow can be determined.

Particle size is not the only factor which determines the rate at which a particle will settle. The density or specific gravity of a particle is also important. The density of the particle must be taken into account in calculating  $d_{50}$ . Since the distribution of the solid feed to the thickener into specific gravity and size increments is known, a separate  $d_{50}$  for each size and gravity range can be calculated. Stokes law for the terminal velocity of a spherical particle is used.

$$v = \frac{d^2 g (\rho_s - \rho_w)}{18 \mu} \quad (85)$$

where  $\rho_s$  is the density of the solid particle,  $\rho_w$  is the density of water,  $\mu$  is the viscosity of water,  $v$  is the upward velocity of water, and  $g$  is the gravity constant.

#### Calculation Sequence

In order to make the calculation for  $d_{50}$  the upward velocity of water is needed. It is related to  $f$  by

$$v = [F (1-f)]/(\rho_w A) \quad (86)$$

where  $F$  is the rate at which water enters the thickener, and  $A$  is the cross-sectional area of the tank. However,  $f$  is not known when calculations are started because the fraction of solids leaving in the underflow is not known. Since  $f$  is very close to zero, a good initial approximation can be made by setting  $f=0$ . This is used to calculate a trial value for  $v$ , which is later checked and corrected if necessary.

Since  $d_{50}$  will be the same for all particles with the same specific gravity, the distribution between overflow and underflow is determined for each specific gravity range. Equation 84 gives the fraction of the feed going to the underflow as a function of particle size. For any size range with upper bound  $d_2$  and lower bound  $d_1$  the fraction reporting to the underflow is given by integrating equation 84 between  $d_1$  and  $d_2$ .

$$E_a = \left\{ \int_{d_1/d_{50}}^{d_2/d_{50}} E d \left( \frac{d}{d_{50}} \right) \right\} / (d_2/d_{50} - d_1/d_{50}) \quad (87)$$

The upper and lower bounds for this integration are determined by the size boundaries for the feed. The value of the integral is then multiplied by the fraction of the feed in that size and gravity range to give the fraction of the feed reporting to the underflow in the size and gravity range being considered. This procedure is then repeated for each size range and each gravity range to determine the distribution in the underflow and overflow. The integral in equation 87 is evaluated using a four point Gauss-Legendre quadrature formula (Carnahan, 1960).

The above calculations are based on an assumed value of  $v$ , the upward velocity of liquid. This can now be checked since the total solids reporting to the underflow is known, and the moisture content of the underflow is

known. The new value of  $v$  is compared to the assumed value, and calculations are repeated if the difference is more than one percent.

The sulfur and ash distributions in overflow and underflow can be readily calculated since the distribution of each size and gravity range between overflow and underflow is known.

### Overall Water Balancing

The feed to the thickener consists of any flows to the thickener shown on the flow diagram and all the excess water streams leaving any other unit. (See Section X for a discussion of how excess water is handled in each unit).

These latter streams are stored in the vector WATER (2,IU) for each unit and may not appear on the flow diagram. It is assumed that all excess water is treated in the thickener. It is for this reason that there can only be one thickener in the plant, and the products from the thickener cannot be inputs to any other unit.

The clarified water produced by the thickener is assumed to be available to satisfy water demands of other units (stored in the vector WATER(1,IU) and specified in the input data by the user). The net water demand of the plant is found by subtracting the water available from the clarifier from the water requirements. A negative demand indicates a net water production. This could result from using a very wet feed.

### Effect of Flocculant

The calculations described above for determining the solids in the thickener overflow assume that the size distribution in the solids in the thickener is the same as that in the feed, which is in error because a flocculant is almost always used, and the flocculant will definitely affect particle sizes, particularly in the very fine size range. The user can provide a flocculant correction factor in the data input to the simulator if

desired. This is the third decision variable and has a default value of 1.0. If the simulator is being used to simulate the performance of the actual unit, this factor can be adjusted based on measured performance of the actual unit.

A few comparisons of simulator predictions with published plant operating data have indicated that a value of 1.0 is reasonable. It could be that the failure to include a correction for flocculant is compensated for by the fact that the size distribution in the feed to the thickener is not known accurately for the very fine sizes which are of importance in the thickener. This is certainly the case when the size ranges selected for the coal feed to the plant do not provide for very small sizes.

## XII. ECONOMICS

The simulator includes a capability for calculating the cleaning cost per ton of coal. This information is reported three ways: per ton of feed entering the plant, per ton of (combined) clean coal leaving the plant, and per ton of clean coal + middlings leaving the plant. This capability is somewhat limited, however, because of the nature of the required input data. In particular an independent, more detailed economic analysis may be required in order to obtain the necessary input data. This type of analysis is not directly related to coal processing technology.

The simulator calculates the cleaning cost per ton of coal as

$$C = EAC / (50 * W * H) \quad (88)$$

where C = cost per ton of coal cleaned, in \$/ton

EAC = equivalent uniform annual cost, defined over some specified time horizon (e.g., 10 years)

W = flowrate of coal entering or leaving the plant (i.e., the feed rate, combined clean coal rate, or combined clean coal plus middlings), in tons/hr.

H = number of hours per week that the plant is in operation

The computation is based upon the following input information:

1. The feed rate to the plant (F, tons/hr)
2. The hours of operation per week (H)
3. The economic horizon (n, years)

4. Either

- (a) a known value for the equivalent uniform annual cost (EAC), or
- (b) an initial investment (I), an interest rate (R) and a yearly cash flow ( $YCF_i$ ,  $i = 1, 2, \dots, n$ ).

In the latter case, the equivalent uniform annual cost is calculated as

$$EAC = 0.1 * R * PW * \left[ \frac{(1 + 0.01 * R)^n}{(1 + 0.01 * R)^n - 1} \right] \quad (89)$$

where PW represents the present worth of the original investment and the yearly cash flows, determined as

$$PW = I + \sum_{i=1}^n YCF_i / (1 + 0.01 * R)^i \quad (90)$$

It is a simple matter to calculate W, i.e.

$$W = \begin{cases} F \\ CC \\ CC + MID \end{cases} \quad (91)$$

and then evaluate C using Equation 88.

Notice that this calculation requires that the yearly cash flow ( $YCF_i$ ) be known for each year within the time horizon. Some consideration was given to calculating each yearly cash flow in terms of its constituent items (e.g., labor, utilities, depreciation, taxes, etc.). This approach was abandoned, however, because of the great variability in accounting procedures that are used in different organizations.

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