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# OPEN SUPERSTRINGS AS THE THEORY OF EVERYTHING?\*

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## ABSTRACT

We discuss the first examples of one-loop finite four-dimensional superstrings. These examples can be either space-time supersymmetric or not depending on the details of the models.

## 1. INTRODUCTION

One of the most important features of all string theories is that they provide a framework for a consistent theory of quantum gravity unified with all other forces [1]. The recent interest in string theories began with the observation of anomaly cancellation in the Green-Schwarz  $SO(32)$  ten-dimensional open superstring [2]. Since then open strings have been neglected as potential theories of nature, although there is no a priori reason for this. Since the original ten-dimensional superstring was constructed there has been great progress in constructing four-dimensional closed heterotic string theories [3-6]. However, there has been no analogous progress for four-dimensional open superstrings. Part of the reason for this is sociological and part is technical. The basic technical difficulty with lower-dimensional open strings in contrast to closed strings is that there is no simple symmetry principle analogous to modular invariance. This makes the construction of lower-dimensional open superstrings less mechanical than the corresponding closed constructions constructions, but is not a fundamental obstruction. Indeed, we have succeeded in constructing the first examples of sensible four-dimensional open superstrings.

The main motivation for studying open string theories as compared to closed string theories is the hope that some of the important unsolved problems in string theories such as the dilaton problem [7], and the fact that there seems to be no fundamental way to choose between the large number of consistent lower-dimensional string models [3-6], may have a simpler resolution in open string theory. The con-

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struction of open string models is considerably more constrained [8,9,10] so that the number of viable open string vacua seems to be considerably less than that of closed strings alone. Also since open string field theory is much simpler than closed string field theory, a string field theory understanding of why a particular vacuum is chosen over others would be much simpler for the open string case.

## 2. EXAMPLE OF CONSTRUCTION

For simplicity, we only focus on the simplest example of a four-dimensional superstring, although we have developed a formalism for constructing a variety of open superstrings. For our example we take the world sheet fermion's contribution to the partition function on the torus to be

$$Z^{\text{torus}} = \frac{1}{2}(Z_{W_0}^{W_0} + Z_0^{W_0}) + \frac{1}{2}(Z_{W_0}^0 - Z_0^0), \quad (1)$$

where we are following the notation of refs. [4,8]. The upper vectors specify the space boundary conditions and the lower vectors specify the time boundary conditions of the world sheet fermions on a torus. In this example,  $W_0 = ((1/2)^{10}|(1/2)^{10})$  and  $0 = (0^{10}|0^{10})$  respectively correspond to Neveu-Schwarz and Ramond boundary conditions on all twenty left- and right-mover complex world-sheet fermions. The complete model consists of tensoring the world-sheet fermionic contributions with the bosonic contributions and integrating over the modular parameter as described in, for example, refs. [1,4].

The consistency of this closed model follows from its modular invariance properties which can be easily checked using the transformations which may be found, for example, in ref. [4]. (The standard bosonic contribution to the partition function is also modular invariant by itself.) Furthermore, the model can be truncated to a type I unoriented closed model [8], as follows from the left-right symmetry of the model.

The set of states of the closed string model which may couple to a boundary or crosscap state are the left-right-symmetric (LRS) ones, that is states which have the same left- and right-mover content [11,8,10]. Thus, as discussed in refs. [8,12] the cylinder with two boundary states is

$$\begin{aligned} Z^{BB}(\tau') &= \text{tr}_{\text{LRS}}[q^{\hat{H}_{W_0}^{\text{left}}} \bar{q}^{\hat{H}_{W_0}^{\text{right}}} \hat{P}(W_0)] + \text{tr}_{\text{LRS}}[q^{\hat{H}_0^{\text{left}}} \bar{q}^{\hat{H}_0^{\text{right}}} \hat{P}(0)] \\ &= \text{tr}[|q|^{2\hat{H}_{W_0}^{\text{left}}}] \\ &\equiv F_{U_0}^{U_0}(\tau'), \end{aligned} \quad (2)$$

where  $\tau' = \ln|q|/i\pi$  and  $U_0 = ((1/2)^{10})$  is a Neveu-Schwarz boundary condition vector of half the length of  $W_0$ ,  $P$  is a GSO projector and  $\hat{H}_{W_0}$  is the closed string hamiltonian for the world sheet fermions in the Neveu-Schwarz sector. The GSO projectors collapse because they are restricted to LRS states so that

$$\begin{aligned} \hat{P}(W_0)|_{\text{LRS}} &= \frac{1}{2}(1 + e^{2\pi i W_0 \cdot N_{W_0}})|_{\text{LRS}} = 1 \\ \hat{P}(0)|_{\text{LRS}} &= \frac{1}{2}(1 - e^{2\pi i W_0 \cdot N_0})|_{\text{LRS}} = 0. \end{aligned} \quad (3)$$

For LRS states,  $N = (N_L|N_R)$  and so  $\exp(2\pi i W_0 \cdot N)$  is unity for such states. By Jacobi transforming this, we obtain the annulus contribution to the open string partition function

$$Z^{\text{ann}}(\tau) = F_{U_0}^{U_0}(\tau) = \text{tr} [e^{2\pi i \tau \hat{H}_{U_0}^{\text{open}}}] \equiv \text{tr} [w^{\hat{H}_{U_0}^{\text{open}}}] , \quad (4)$$

where  $\tau = -1/\tau'$  and  $\hat{H}_{U_0}^{\text{open}}$  is the open string hamiltonian in the  $U_0$  sector.

Given the annulus contribution, we can then construct the möbius contribution by requiring a physically sensible projection between the annulus and möbius contributions. This is equivalent to requiring the action of the twist operator [1,8]  $\hat{\Omega}$  on open string states to satisfy  $\hat{\Omega}^2 = 1$  when acting on a state. Since the open string  $U_0$  sector, which does not have a GSO projector, contains states at both integer and half integer mass levels, the naive twist operator  $\hat{\Omega} \sim e^{\pi i \hat{H}_{U_0}}$  is not sensible. In the presence of a GSO projector which would ensure that this twist operator is well defined, consistent open string models exist only in  $D = 2, 6, 10$  as described in ref. [8,9] and not in  $D = 4$ .

The solution to this difficulty is actually quite simple: Use a GSO projector to separate the states into integer and half-integer mass levels before applying the twist operator and associate different phases with the two parts of the twist operator so as to ensure  $\hat{\Omega}^2 = 1$ . The explicit form for the twist operator is then

$$\hat{\Omega} = e^{\pi i d/24} \left[ \eta_1 \left( \frac{1 + (-1)^{\hat{N}_{U_0}}}{2} \right) e^{\pi i \hat{H}_{U_0}^{\text{open}}} - i \eta_2 \left( \frac{1 - (-1)^{\hat{N}_{U_0}}}{2} \right) e^{\pi i \hat{H}_{U_0}^{\text{open}}} \right] , \quad (5)$$

where  $\hat{N}$  is the usual fermion number operator and  $d = 10$  for a four-dimensional model. The phase  $e^{\pi i d/24}$  absorbs the phase in  $e^{\pi i \hat{H}_{U_0}^{\text{open}}}$  due to the zero-point energy of the hamiltonian  $\hat{H}_{U_0}^{\text{open}}$ . For  $\eta_1 = \eta_2$  this twist operator is equivalent to the one given by Clavelli and Shapiro 16 years ago [13]. The choice of the  $\eta_i = \pm 1$  determines the Chan-Paton gauge group representation [14] of the various mass levels. (See ref. [1] for details.) (We are allowing for an independent choice of the Chan-Paton gauge group representation at integer and half-integer mass levels because these lie in different representations of the two-dimensional Kac-Moody symmetry.) The Chan-Paton representation of the massless gauge bosons corresponds to the adjoint representation of the gauge group, so for  $\eta_2 = +1, -1$ , which controls the representation of the massless states, the gauge group will respectively be a Sp or SO group.

With this choice of twist operator the möbius contribution is

$$\begin{aligned} Z^{\text{mob}} &= e^{i\pi d/24} \left( \frac{1}{2} \eta_1 \text{tr} [w^{\hat{H}_{U_0}^{\text{open}}} (1 + (-1)^{\hat{N}_{U_0}}) \hat{\Omega}] - \frac{i}{2} \eta_2 \text{tr} [w^{\hat{H}_{U_0}^{\text{open}}} (1 - (-1)^{\hat{N}_{U_0}}) \hat{\Omega}] \right) \\ &= e^{i\pi d/24} \left( \frac{1}{2} \eta_1 \left( F_{U_0}^{U_0}(\tau + 1/2) + F_0^{U_0}(\tau + 1/2) \right) \right. \\ &\quad \left. - \frac{i}{2} \eta_2 \left( F_{U_0}^{U_0}(\tau + 1/2) - F_0^{U_0}(\tau + 1/2) \right) \right) . \end{aligned} \quad (6)$$

The Jacobi transformation properties of the möbius contributions to the partition function are given by

$$F_V^U(\tau + 1/2) = e^{4\pi i(U-U_0) \cdot (V-U_0)} e^{2\pi i(V \cdot V - d/6)} e^{\pi i(U \cdot U - d/6)} \overline{F_{V-U}^U}(\tau'/4 + 1/2), \quad (7)$$

where  $U$  and  $V$  consist of Neveu-Schwarz and Ramond open string boundary conditions and the overbar on the boundary condition vector indicated that it should be evaluated mod 1. By Jacobi transforming the möbius partition function (6) (after the usual rescaling [2]  $\tau'/4 \rightarrow \tau'$ ) we obtain the cylinder with one boundary and one crosscap ( $D = 4, d = 10$ )

$$Z^{BC} = e^{i\pi d/24} \left( \frac{1}{2} \eta_2 \left( F_{U_0}^{U_0}(\tau' + 1/2) + F_0^{U_0}(\tau' + 1/2) \right) - \frac{i}{2} \eta_1 \left( F_{U_0}^{U_0}(\tau' + 1/2) - F_0^{U_0}(\tau' + 1/2) \right) \right), \quad (8)$$

which contains the same closed string states as the cylinder with two boundary states (2).

The potential divergences of the amplitudes in eqs. (2) and (8) are determined by the leading and next to leading terms in the Taylor expansion, in  $e^{2\pi i \tau'}$ , corresponding to the tachyon and massless scalars propagating into the vacuum. In the Jacobi transformed möbius contribution (8), the relative normalization of the tachyon singularity is  $2^2 \eta_2$  while for the massless scalar it is  $2^2 \eta_1 M$ , where we are including the factor  $2^{D/2}$  arising from the bosonic [1,8]. Similarly for the Jacobi transformed annulus (2), the relative normalizations of these singularities is  $M$  for gauge groups  $\text{Sp}(M)$  or  $\text{SO}(M)$ . Thus, for cancellation of both tachyon and massless scalar divergences

$$M + \eta_2 2^2 = 0, \quad M + \eta_1 2^2 = 0, \quad (9)$$

so the gauge group is  $\text{SO}(4)$ .

Further models can be constructed by the addition of extra boundary condition basis vectors. In ref. [12] examples of open-closed string models with  $N = 4, 2, 1$  space-time supergravity were presented.

### 3. CONCLUSIONS

Much work remains to be done before open strings are as well developed as the heterotic string for phenomenological purposes. For example, the question of chirality remains a difficulty with lower-dimensional open superstring constructions, apparant progress has been made on this question within the context of the orbifold approach to string theory [15]. Additionally, there is the question of higher loop amplitudes [16] as well as the potential of using non-trivial projections on the Chan-Paton factors [17,10,15]. However, we have shown that it is possible to construct a variety of four-dimensional one-loop finite open superstring theories and are hopeful that such constructions will eventually lead to phenomenologically viable models.

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