BNL-38665
IAEA-SM-293/122
CONF-861104--8

"Neutron Multiplicity for Neutron Induced Fission of ^{235}U, ^{238}U, and ^{239}Pu as a Function of Neutron Energy"*

M.S. Zucker
N.E. Holden

Brookhaven National Laboratory
Department of Nuclear Energy
Upton, New York 11973

BNL--38665

DE87 002156

Abstract

Recent development in the theory and practice of neutron correlation ("coincidence") counting require knowledge of the higher factorial moments of the P_ν distribution (the probability that (ν) neutrons are emitted in a fission) for the case where the fission is induced by bombarding neutrons of more than thermal energies.

In contrast to the situation with spontaneous and thermal neutron induced fission, where, with a few exceptions the P_ν is reasonably well known,[2] in the fast neutron energy region, almost no information is available concerning the multiplicity beyond the average value, $\langle\nu\rangle$, even for the most important nuclides. The reason for this is the difficulty of such experiments, with consequent statistically poor and physically inconsistent results.

*Research carried out under the auspices of the U. S. Department of Energy Under Contract No. DE-AC02-76CH00016.

In this paper, previously unpublished data kindly made available from a fast neutron fission multiplicity experiment has been used to extract useable multiplicity probabilities from otherwise intractable data, using smoothing techniques supplemented by mathematical and physical plausibility arguments.

In contrast, it appears that attempts to apply systematics to deducing such parameters as P_ν to the required accuracy will not work because of particular differences between even neighboring nuclides. Thus, despite the difficulties, further experimental work is required supplemented by data evaluation, if necessary, as in the present paper.

Introduction

For many technical purposes it suffices to know only the first moment, $\langle\nu\rangle$, ("nubar"), of the neutron multiplicity probability, P_ν, that ν prompt neutrons are emitted in a fission.

Certain applications however do require knowledge of the higher moments of the P_ν distribution. A case in point and the immediate reason for the present work are several methods for the analysis of nuclear material by what amounts to an autocorrelation performed on the pulse train from a detector exposed to the unknown sample. Then it can be shown that the second factorial moment $\langle\nu(\nu-1)\rangle = \sum\nu(\nu-1)P_\nu$ is proportional to the fission rate in the sample, and that the third factorial moment $\langle\nu(\nu-1)(\nu-2)\rangle = \sum\nu(\nu-1)(\nu-2)P_\nu$ can be of use in disentangling spontaneous fission (which can be related directly to the amount of material present), from induced fission, which is only partly related with the amount of material, since it is also a function of geometry, density, and other artifacts.

Calculating the effects of induced fission in the sample, i.e., fission caused by either spontaneous fission generated neutrons or those produced by (α,n) processes, or, in the case of certain nuclear material assay systems, by neutrons from an external source bombarding the sample, clearly requires knowledge of these moments for neutron energies ranging from the thermal to the fast (fission) neutron energy region.[1]

While it has been possible to glean from the published literature much information (though never enough!) for nuclides undergoing spontaneous fission induced by thermal or tens of keV neutrons,[2] we are unaware of any published P_ν date for fission induced by neutrons in the few MeV range which are crucial to calculations of fast induced fission correlation.

We became aware though of extensive unpublished work by Frehaut and collaborators[3] in which neutron multiplicities were derived for the fast neutron induced fission of ^{235}U, ^{238}U, and ^{239}Pu, three most interesting nuclides from the nuclear material assay standpoint. The reason this P_ν data, despite its uniqueness and importance, was unpublished is that it is obviously flawed, due mainly it is presumed, to poor counting statistics and the way in which counting statistical uncertainties propagate in the transformations connecting experimentally observed neutron multiplicities with the P_ν distributions (see below). Certain P_ν at some energies are negative, physically and mathematically impossible for a probability, and, considered as a function of energy, the P_ν often exhibit fluctuations obviously related to poor counting statistics rather than to any real physical process.

Scope and Methodology

Our task then has been to salvage from this data kindly furnished by Frehaut (and to our knowledge the only such existing), the multiplicity P_ν as a function of incident neutron energy E_n. The overall procedure may be described as data smoothing guided by general physical and mathematical principles. These will be described immediately below and illustrated with particular cases cited later on.

Though the data furnished covers up to $E_n \approx 25$ MeV, results will be quoted only over the range 0-10 MeV, as in the region 10-15 MeV and certainly for the higher energies the data become prohibitively unreliable even though smoothing processes are employed. Another problem is that the furnished data only begin at E_n = 1.36 MeV. Fortunately comparatively accurate thermal neutron induced fission cross sections do exist for (^{235}U + n) and (^{239}Pu + n); more on this point later.

Any of the P_ν as a function of En has the appearance of a bell-shaped curve or a portion of one. For the smaller ν values only the decreasing tail of the bell appears, starting at E_n = 0. For intermediate values of ν the whole bell shaped curve appears except the ascending part cut off at E_n = 0; the function rises to a maximum and then declines, presumably approaching zero for high enough E_n. The P_ν, being probabilities, must sum to unity at any energy. Therefore, the only way for $\langle\nu\rangle$ to increase monotonically, as it is well known to do from experimental results, is for an increase in the P_ν for higher values of ν with a corresponding decrease in the relative importance of those P_ν for smaller values of ν. There is no evidence from the data of Frehaut et al. for more complicated behaviour of any of the P_ν as a function of E_n than this.

Therefore a basic premise of the fitting procedure was that each P_ν for any ν was considered to be a smooth function of neutron energy describable by a low order least squares fit polynomial in E_n. In fact, in the region 0-10 MeV it will be seen below there there was no need for polynomials higher than the fourth order. In some cases these polynomial fits are good representations of the data for energies beyond 10 MeV, though in other cases the fit rapidly becomes unusable after 10 MeV.

In the fitting procedure the attempt was made, however, to use data beyond E_n = 10 MeV to help establish the trend in the data up to that point. Similarly, thermal neutron data for ^{235}U and ^{239}Pu was used to anchor the start of the various P_ν

curves for these nuclides, since the Frehaut data starts only at E_n = 1.36 MeV. Not surprisingly, there is no thermal neutron data for (^{238}U+n) as the cross sections are too small below the effective threshold for induced fission. Instead, the fit obtained to the data between E_n = 1.36 MeV and E_n = 10 MeV was extrapolated down to E_n = 0 to yield an estimate for the multiplicity distribution between E_n = 0 and 1.36 MeV for the system (^{238}U + n).

After least squares fitting the P_ν as a polynomial in E_n, the P_ν set at a given energy will depart slightly from the normalization condition $\sum P_\nu = 1$, and so they were renormalized.

The average value for ν, $\langle\nu\rangle = \sum \nu P_\nu$, has been determined as a function of E_n by independent experiment to a greater accuracy than can be determined from the P_ν. These experiments basically determined a gross count rate G in terms of a detector efficiency ε, and a source strength q:

$$G = \varepsilon\langle\nu\rangle q \quad . \qquad (1)$$

The renormalized P_ν at any given energy predict a $\langle\nu\rangle = \sum \nu P_\nu$ close to but not precisely equal to the best available values for $\langle\nu\rangle$ determined as a function of E_n from Eq. (1). The differences were reconciled by considering them formally as though they had arisen due to error or uncertainties in the efficiency of a (hypothetical) detector in an experiment determining the P_ν. In such an experiment, the observed neutron multiplicity, Q_n, that n neutrons are observed from a given fission event is related to the P_ν by

$$Q_n = \sum P_\nu \binom{\nu}{n} \varepsilon^n (1-\varepsilon)^{\nu-n}, \qquad (2)$$

where ε is the detector efficiency. The P_ν are obtained from this expression by inverting the relation:

$$P_\nu = \sum Q_n \binom{n}{\nu} \varepsilon^{-n} (\varepsilon-1)^{n-\nu}. \qquad (3)$$

As applied in this instance the normalized P_ν derived from the least squares fitting the Frehaut et al. data were used to obtain a hypothetical set Q_n based on an assumed value for ε. The normalized set P_ν defines a value $\langle\nu\rangle = \sum \nu P_\nu$. A set P'_ν such that it would yield a value $\langle\nu\rangle' = \sum \nu P'_\nu$ considered more correct can be thought of as being related to a detector efficiency ε' such that $\langle\nu\rangle'\varepsilon' = \langle\nu\rangle\varepsilon$, since experimentally $\langle\nu\rangle$ and ε are inversely related. Then the hypothetical Q_n can be used to obtain the set P_ν' corresponding to ε' from

$$P_\nu' = \sum Q_n \binom{n}{\nu} (\varepsilon')^{-n} (\varepsilon'-1)^{n-\nu}. \qquad (4)$$

This procedure then produces a normalized set of P_ν' as a function of E_n which yields the proper value of $\langle\nu\rangle' = \sum \nu P_\nu'$ at any given E_n and can be considered to be related to the same set of observables Q_n as the original set P_ν that was smoothed and renormalized.

The P_ν' are expected to be close to the least squares fits to the P_ν, and indeed are. In principle a new least squares fit could be made to the P_ν', the results renormalized, and again reconciled to the best values available for $\langle\nu\rangle$. This process is rapidly convergent, so much so that the resultant changes in P_ν would be a marginal improvement considering the precision of the original data.

Data Smoothing (Polynomial Fitting) Procedure

The statistical uncertainties in the P_ν are naturally greater the fewer the observations of a given multiplicity are. The larger P_ν therefore have better statistical precision. Since all observed O_n with $n<\nu$ can be considered as stemming from a multiplicity ν with probability P_ν because the efficiency ε is less than 1 (see Eqs. 2 and 3), the error propagation from the observed O_n to the derived P_ν is not simple. Nevertheless there is a qualitative correspondence between the magnitude of the P_ν and the "smoothness" of the data points. Thus P_3, P_4, P_5, which are comparatively large in the region 0–10 MeV were relatively easy to fit while P_0, P_7, and P_8 were the most difficult. P_0 is small to begin with (≈.01) at thermal energy and decreases rapidly to zero at roughly 10 MeV. P_7 and P_8 start out even smaller at thermal energies; while they increase rapidly, another difficulty that manifests itself is an increase in the general variability of the data at higher energies which is not explicable on the basis of the number of fissions observed at a given energy, being inferior for high energies (≈10 MeV) compared to low energies (≈2 MeV). In fact, the higher energy data has significantly more fissions analyzed, for example, for (^{235}U + n) there were 4,532 fissions analyzed for E_n = 1.87 MeV, but 11,374 fissions analyzed for E_n = 9.74, and these are typical of their respective neighboring values. (Incidentally, a typical modern P_ν experiment for spontaneous fissions or thermal neutron energies would involve 10^5 or 10^6 fissions. This points out the basic problem of the Frehaut data, lack of sufficient statistics.)

It is not practical to present every case treated, but the problems encountered and solutions adopted in reducing the data to useable form can best be illustrated with actual examples, using (^{239}Pu + n) for ν = 0 to 8 and (^{235}U + n) for ν = 7,8 as fairly representative.

These cases are illustrated in Fig. 1-10. The general format of these is to show the raw data (i.e. thermal plus Frehaut data points) above and the data plus the fitted polynomial in powers of E_n below.

The polynomial used was the lowest order which would give a good fit. Although the (Hewlett-Packard 9845) software for polynomial fitting routine also produced mathematical goodness of fit parameters, the criteria that proved practical in deciding the goodness of fit were those based in the general knowledge of how the P_ν functions of energy behaved, and that the fitted curve should intercept the P_ν axis close to the thermal value, since these are assumed to be one of two orders of magnitude more accurate than the typical data for $E_n>0$.

The curve fitting routine did not allow for weighting the data points according to statistical uncertainty, but a simple subterfuge, entering a data pair more than once, produced the same effect. The only points weighted this way were the thermal values. It was found that giving the thermal value a weight of 5 or 10 tended to improve the agreement of the fitted curve at $E_n = 0$, while not noticeably affecting the goodness of fit among the Frehaut data points which start at $E_n = 1.36$ MeV. In a few cases, choice of the order of the polynomial between otherwise equivalent fits was decided on the basis of which gave best agreement with the thermal value. However, in all cases where there was a thermal value available, there was no difficulty in fitting a low order polynomial (fourth or less) to the thermal value and the Frehaut data, a point which will be commented on later.

Though the resulting fits are meant to be considered representations of the respective P_ν only in the region 0 to 10 MeV, data points for E_n up to about 14 MeV were used in order to be able to make more certain the course of the function in the neighborhood of 10 MeV. In some of the Figs. (1-10), the fitted polynomial is shown extending beyond 10 MeV. In some cases, the particular P_ν continues to be represented by the fitted curve beyond 10 MeV, but in other cases the formal analytical extension of the polynomial is clearly non-physical, as the curve would depart from zero when it should remain there, or reverse direction when it should continue monotonically. This point will also be discussed later.

In a relatively few situations, data points were discarded when they seemed to lie off the tentatively fitted curves by what seemed to be significantly more than the typical variability in the data for that particular P_ν and seemed to interfere with the goodness of fit to the rest of the data.

Some particular remarks: The curve for P_0 (Fig. 1) illustrates the inadequacy of a finite order polynomial representation for more than a limited range, as the data indicates P_0 is statistically zero from about E_n = 10 MeV on. Similarly for P_1 (Fig. 2) where the best fit to the data over the whole region tends to change direction at E_n = 13 MeV, while (considering the data up to $E_n \approx$ 10 MeV) it should asymptotically approach zero.

Fig. 5 (P_4) illustrates that a higher order least squares fitted polynomial, while a better fit by the least squares criterion, is not necessarily better physically, so that the third order polynomial was chosen as the better representation.

P_8 (Fig. 9) had two problems, the extreme scatter of the data points (upper part of the figure) though a trend is clearly visible, and the absence of a thermal value to serve as an "anchor point." The effect of scatter was greatly reduced by averaging groups of points with respect to ordinate and abscissa, and fitting the curve to these points as plotted in the lower part of the figure.

The fitted curve for P_8 predicts a small nonzero value for P_0 at $E_n = 0$ which is not inconsistent with the zero value reported considering the assigned standard deviations for that data.

Fig. 10 illustrates P_7 and P_8 for ($^{235}U + n$). A nonzero thermal value is reported for P_7, but not for P_8, for this system. As can be seen, the Frehaut data for ($^{235}U + n$) is consistent with $P_8 = 0$ even beyond $E_n = 10$ MeV. However in agreement with the non-zero thermal value for P_7, the Frehaut data does show a small non-zero component for P_7 from E_n = 1.36 to about 6 MeV, followed by what seems to be an abrupt rise starting about E_n = 8.5 MeV, with however much scatter to the data. It was decided that the abrupt rise is an artifact due to chance occurence of three consecutive low values in the region ≈7 - ≈8 MeV. Therefore a curve was fit which agrees with the thermal values and extrapolates to an average of the Frehaut data in the region 10 to 11 MeV.

"Normalizing" the Smoothed Data

Here normalized will be used to mean that $\sum P_\nu = 1$ and $\sum \nu P_\nu = \langle\nu\rangle$, where $\langle\nu\rangle$ is a prescribed value.

After smoothing each P_ν treated as a function of E_n by least squares fitting a polynomial to the data, the P_ν at a given energy will no longer sum to unity although they are close, about a percent or so off.

The renormalized data are then subjected to the transformation according to Eqs. (2), (3), and (4), subject to the condition that they produce the value of $\langle\nu\rangle$ appropriate to that energy.

The values of $\langle\nu\rangle$ vs. E_n used in principle could have been those evaluated from all the literature. In the present situation it was thought better to ensure consistency and minimize uncertainties due to the as yet untried technique for evaluating the data, by using $\langle\nu\rangle$ for the three nuclides as determined by the same group that furnished the P_ν data.[4]

A second choice made was to smooth the $\langle\nu\rangle$ data rather than use the quoted experimental points. This is because as long as one stays away from an energy region roughly less than 1 MeV, there is no indication of any structure in $\langle\nu\rangle$ vs E_n; the relation seems quite linear, with the exception of what seems to be a definite jog between 4 and 7 MeV for (^{235}U + n) (Fig. 11). Thus, with the exceptions noted, deviations from linearity could be considered statistical (Figs. 12 and 13).

The process of normalizing in both senses the smoothed distributions is illustrated in Tables I and II, for (^{235}U + n) and (^{239}Pu + n) respectively, using the data at $E_n = 0$. Column (a) contains the smoothed data at $E_n = 0$ normalized only in the sense $\sum P_\nu = 1$. Column (b) contains the data normalized additionally so that $\sum \nu P_\nu = \langle\nu\rangle$, where the value of

$\langle\nu\rangle$ was evaluated independently. Column (c) shows P_ν at $E_n = 0$ from our earlier evaluation of thermal P_ν data,[2] together with the standard deviations assigned at that time.

Final Results and Evaluation

The final results of this process of data evaluation are presented in tabular form (Tables III, IV, V) and graphically (Fig. 14 and 15) to show the general truncated (on the abscisssca scale) bell-shape. The $E_n = 0$ P_ν for $(^{238}U + n)$ are obtained of course by extrapolation of the fitted functions. The $(^{238}U + n)$ curves are qualitatively similar to those for $(^{239}Pu + n)$ and so are not presented.

The important question at this point is how accurately this salvaged data represents reality. There is only P_ν data for $E_n = 0$ and then only for $(^{235}U + n)$ and $(^{239}Pu + n)$. As mentioned above, the smoothed data of Frehaut extrapolates very well to those $E_n = 0$ values in the two cases where such are available, even before weighting those points (although in recognition of their superior statistical precision and accuracy, they were accordingly weighted in the final fitting process). This argues that the lack of smoothness in the

Frehaut data is statistical and that there may not be any serious systematic errors. Then a smoothing procedure which tends to level statistical fluctuations would have validity.

Another indication of validity in the procedure is that in the two cases (Tables I and II) where the thermal values obtained by the data processing procedure could be compared with P_ν values obtained independently, the two sets are well within the uncertainties assigned to the independently obtained P_ν values, even without weighting the $E_n = 0$ values.*

Finally, each P_ν set was derived independently on at least two occasions widely enough separated in time so that some of the subjective details of the process, such as which points to drop from the fitting procedure, how far beyond 10 MeV should points be included, what was the lowest order best fit, etc., were forgotten. Nevertheless, aside from minor differences due to our acquiring improved skills at fitting, the respective derived P_ν sets were all essentially equivalent well within the attributed uncertainties.

* The data in Tables I and II do have weighting for the thermal values, which of course increases the apparent agreement.

It would be very desirable to compare the Frehaut data as processed above with P_ν values derived from new experiments. However, it was the lack of such experiments together with the low probability in the present state of reactor physics or nuclear data research that such experiments would be performed that was a major reason for the present work.

All things considered, we do feel the present set of data will prove adequate for safeguards work, and the kind of methodology used may even indicate how to design future P_ν experiments so as to most economically and efficiently utilize experimental time and facilities.

The lack of a thermal P_ν set to "anchor" the fitted curves does however reduce the reliability of ($^{238}U + n$) compared to the others.

Further Remarks

A reason for our interest in P_ν is the hope that some information regarding the systematics of neutron multiplicity in fission would result. This could be important both from the standpoint of improving the theory of the fission process and to compensate for the lack of experimental data in many instances.

The more we have delved into the details of P_ν the more it seems to be so that such systematics as are noticed (e.g. Terrell,[5] et al.) are true only in a fairly approximate sense and are not necessarily accurate enough relations to be useful in many technical applications, such as neutron correlation counting.

In the present paper it is seen that for two nuclides $\langle\nu\rangle$ is quite linear over the range $E_n > 1$ MeV, but must be represented by two lines joined by a smooth transition for the third nuclide. That same nuclide has no discernable P_8 component in the studied region, whereas the other nuclides, only 3 and 4 nucleons away have small but definitely non-zero P_8 components.

On the other hand, a plot of the second factorial moment $\langle\nu(\nu-1)\rangle$ for the three nuclides reveals a curious consistency which we are at a loss to explain as being some artifact of the data smoothing process (Fig. 16). The three plots of $\langle\nu(\nu-1)\rangle$ each seem to be made up of two straight line segments with a smooth transition between. The extrapolations of these straight line portions seem to all intersect between 4 and 5 MeV.

These and other speculations can ultimately only be satisfied with more experimental work.

References

1(a) K. Böhnel, Nuclear Science and Engineering 90, 75 (1985).

(b) W. Hage and D. M. Cifarelli, Nuclear Science and Engineering 89, 159 (1985).

(c) R. Dierckx and W. Hage, Nuclear Science and Engineering 85, 325 (1983).

(d) N. Ensslin, Proc. 7th Annual Symp. on Safeguards and Nuclear Material Management, Liege, Belgium, 23 May 1985.

(e) W. Hage and D. M. Cifarelli, Nucl. Inst. and Methods A236, 165 (1985).

2. We have to date evaluated the P_ν data for thermal or spontaneous fission for the following nuclides based on what data could be gleened from the literature or from private communications (all the evaluated data is available at least in the form of BNL reports): ^{257}Fm, ^{252}No, ^{242,244,246,248}Cm, ^{246,250,252,254}Cf, ^{236,238,239,240,241,242}Pu, ^{233,235,238}U.

(a) N. E. Holden and M. S. Zucker, Proc. ANS/INMM Topical Meeting on Safeguards and Technology, Trans. Am. Nucl. Soc. 45, Suppl. 1 23 (1983), BNL 33965.

(b) M. S. Zucker and N. E. Holden, Proc. 6th ESARDA Symp. on Safeguards and Nuclear Material Management, Venice, Italy, May 1984, p. 341, BNL 34804.

(c) N. E. Holden and M. S. Zucker, IAEA Advisory Group, Meeting on Nuclear Standard Reference Data, Geel, Belgium, Nov. 1984, IAEA-TECDOC-335, p. 248, BNL 35513.

(d) M. S. Zucker and N. E. Holden, Trans. Amer. Nuc. Soc. 49, 195, June 1985, BNL 35829.

(e) N. E. Holden and M. S. Zucker, Int. Conf. on Nucl. Data for Basic and Applied Science, Santa Fe, N.M., May 1985, to be published in Radiation Effects (1986), BNL 36379.

(f) M. S. Zucker and N. E. Holden, Trans. Am. Nuclear Society 52, 636, June 1986, BNL 37575.

(g) N. E. Holden and M. S. Zucker, ANS Topical Meeting on Advances in Reactor Physics and Safety, Saratoga Springs, NY, Sept. 1986, BNL 37362.

(h) N. E. Holden and M. S. Zucker, 7th Annual Symposium on Safeguards and Nuclear Material Management, Liege, Belgium, May 1985, p. 229, BNL 36467.

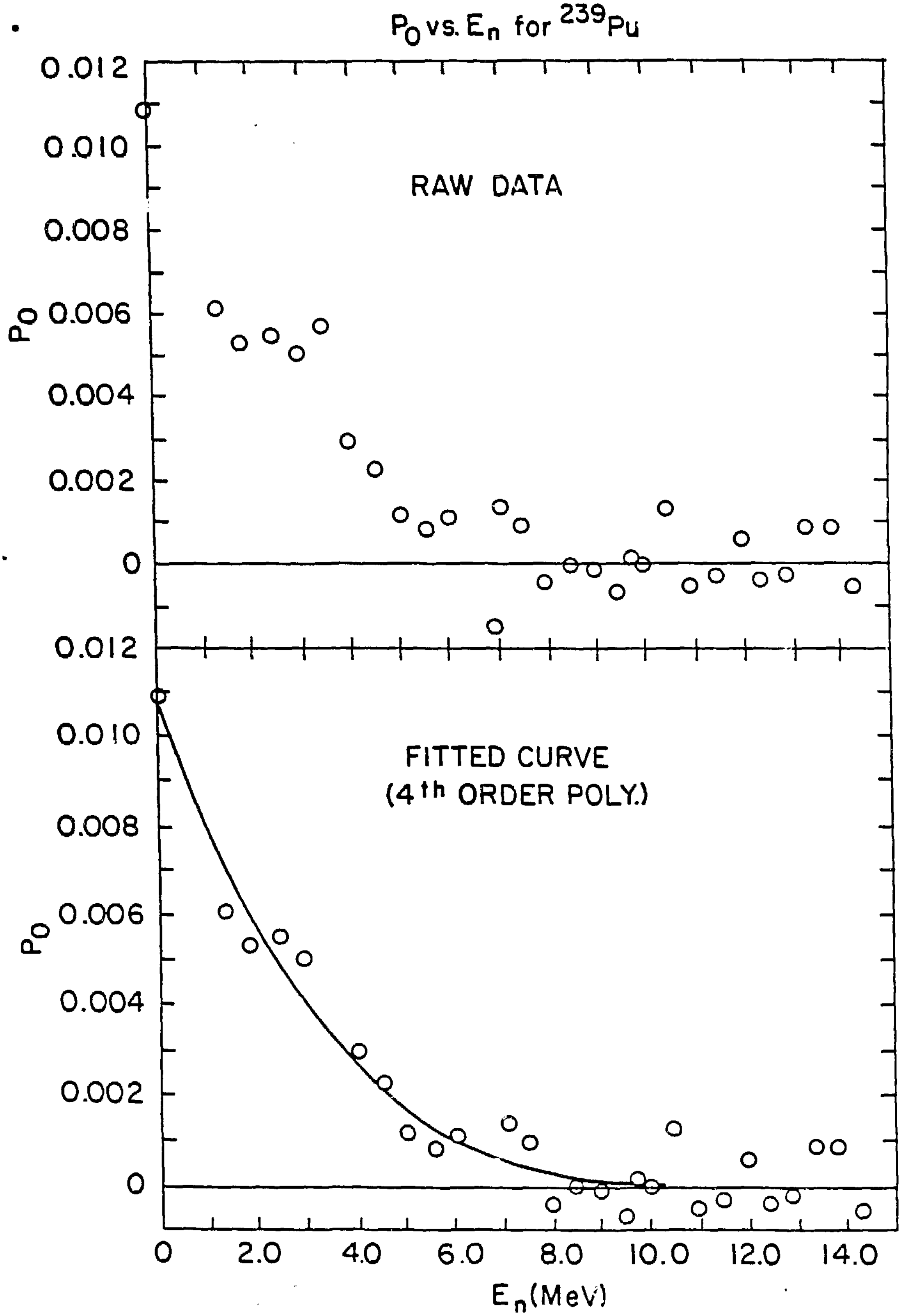
3. We are very much indebted to J. Frehaut and coworkers at the Commissariat a l'Energie Atomique, Centre d'Etudes de Bruyeres le Chatel, for so kindly furnishing their data.

4. M. Soleilhac, J. Frehaut, and J. Gauriau, Journal of Nuclear Energy, 23, 257 (1969).

5. J. Terrell, Phys. Rev. 108, 783 (1957).

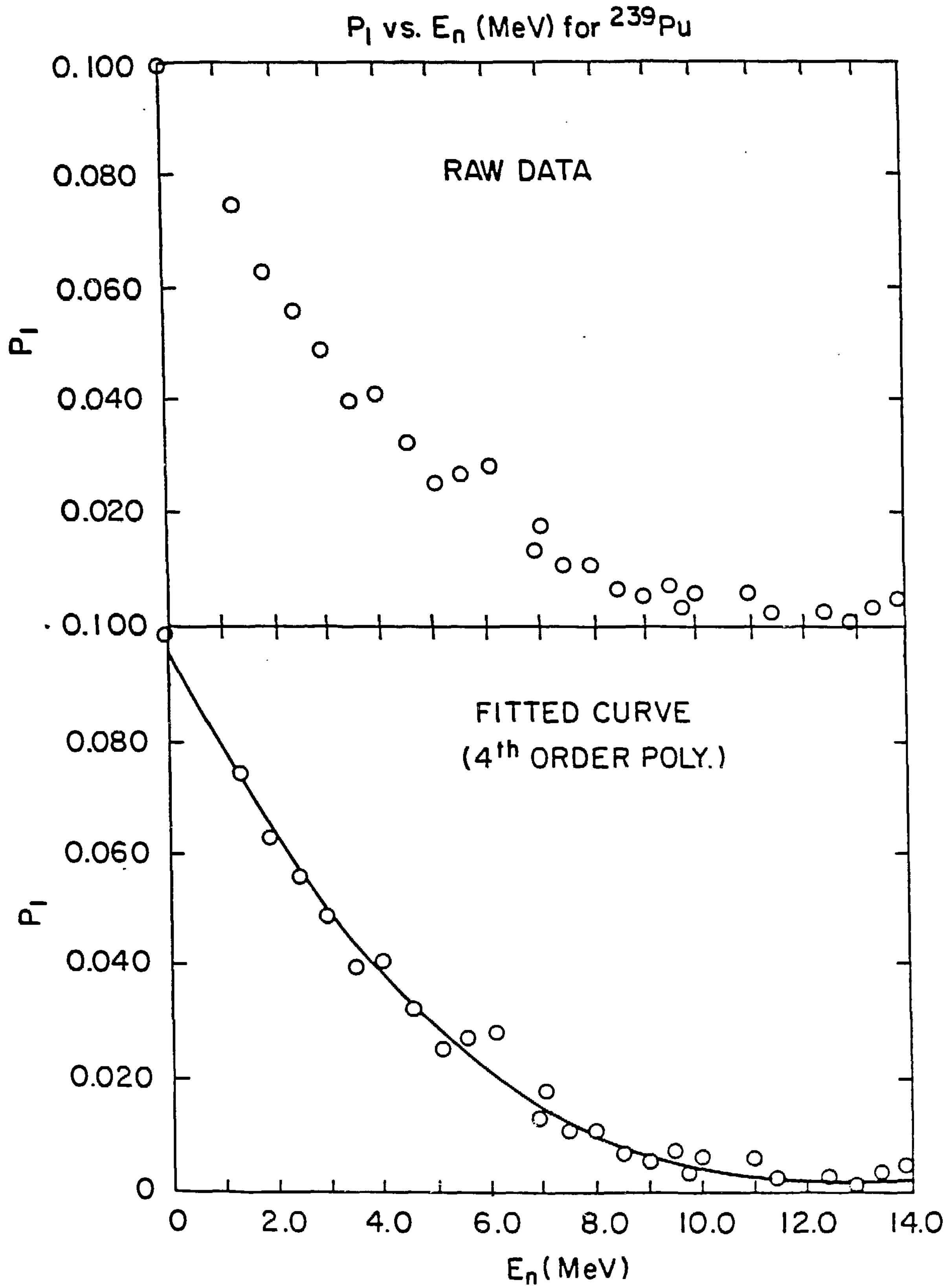
Figures and Tables

FIGURE 1



P_0 vs. E_n for ^{239}Pu
0.012
0.010
0.008
0.006
0.004
0.002
0
P_0
RAW DATA
FITTED CURVE
(4th ORDER POLY.)
0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
E_n(MeV)

FIGURE 2



P$_I$ vs. E$_n$ (MeV) for ^{239}Pu
0.100
0.080
0.060
0.040
0.020
P$_I$
RAW DATA
0.100
0.080
0.060
0.040
0.020
0
P$_I$
FITTED CURVE
(4th ORDER POLY.)
0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
E$_n$ (MeV)

FIGURE 3

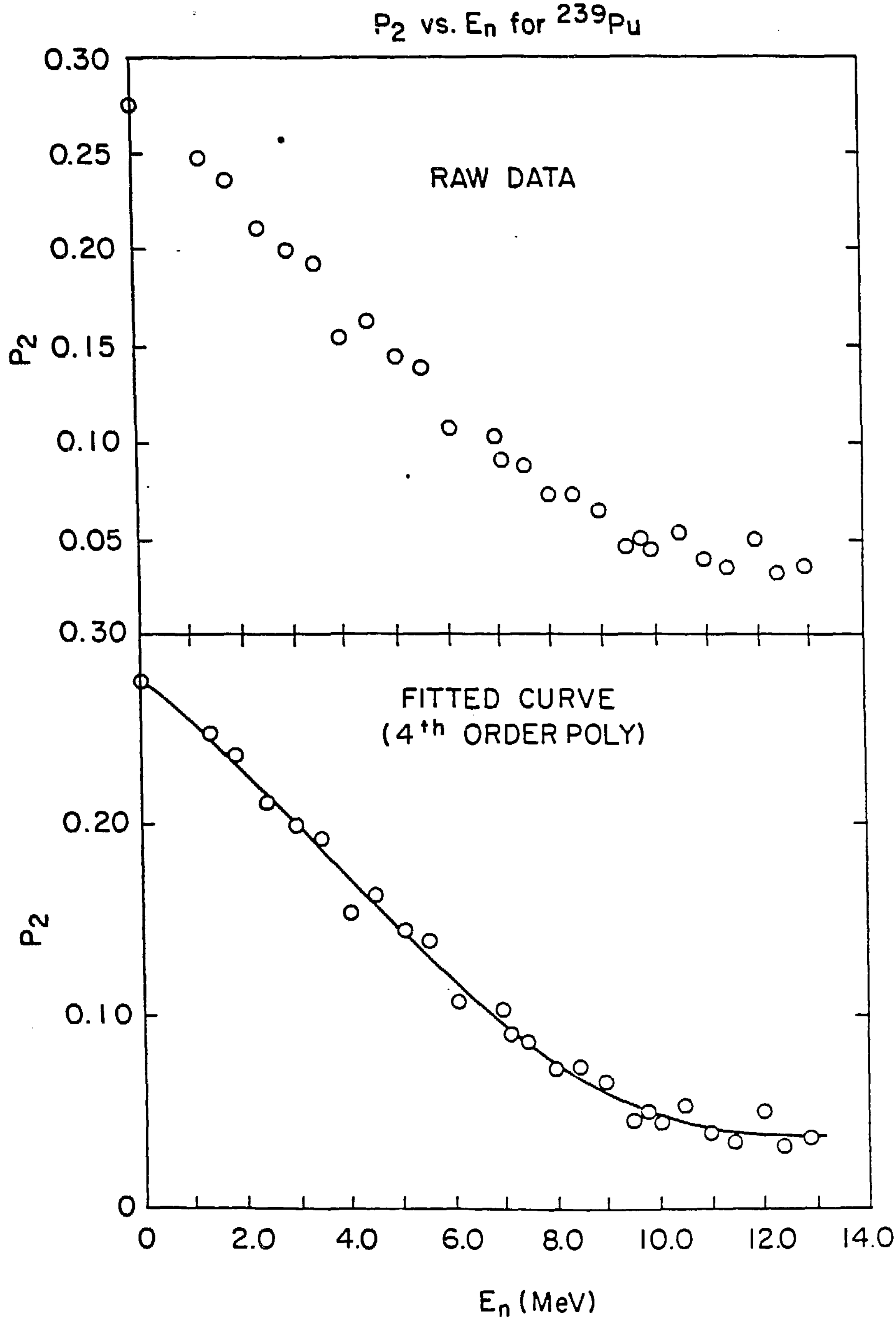
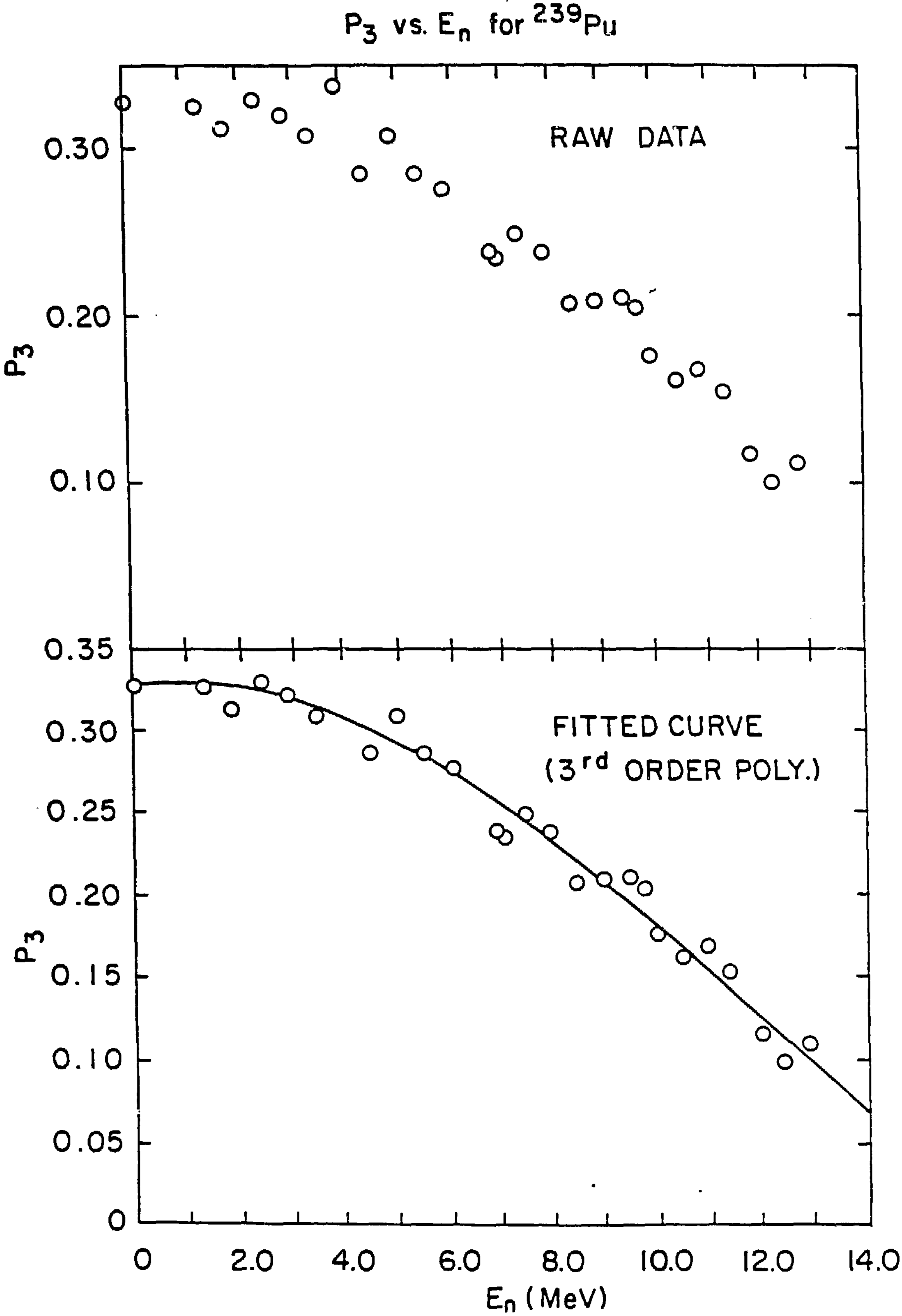
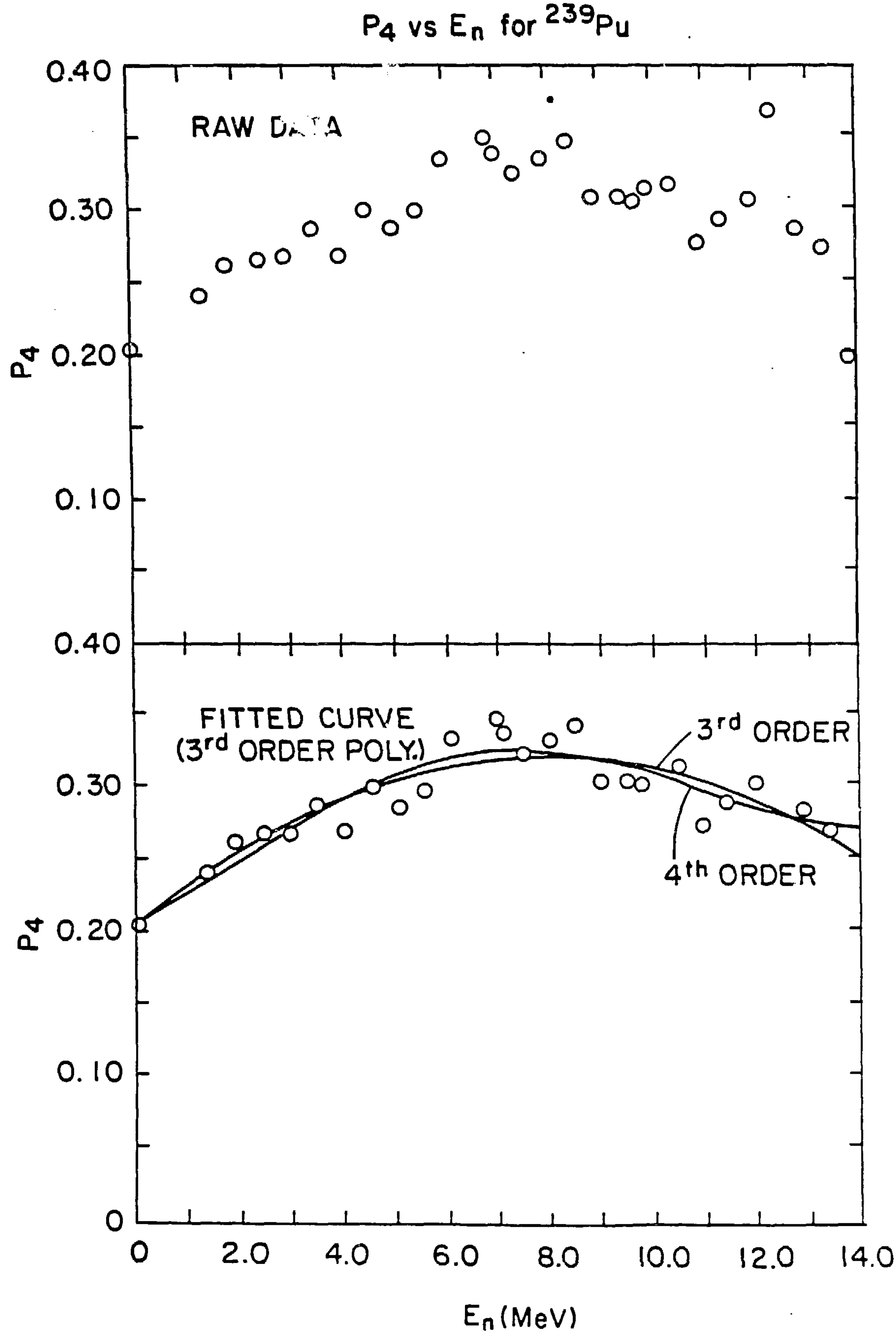

P2 vs. En for 239Pu
RAW DATA
FITTED CURVE
(4th ORDER POLY)
P2
0.30
0.25
0.20
0.15
0.10
0.05
0.30
0.20
0.10
0
0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
En (MeV)

FIGURE 4



P3 vs. En for 239Pu
RAW DATA
FITTED CURVE
(3rd ORDER POLY.)
0.30
0.20
0.10
0.35
0.25
0.15
0.05
0
P3
0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
En (MeV)

FIGURE 5



P4 vs En for 239Pu
RAW DATA
FITTED CURVE
(3rd ORDER POLY.)
3rd ORDER
4th ORDER
P4
0.40
0.30
0.20
0.10
0
0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
En (MeV)

FIGURE 6

P_5 vs. E_n for ^{239}Pu

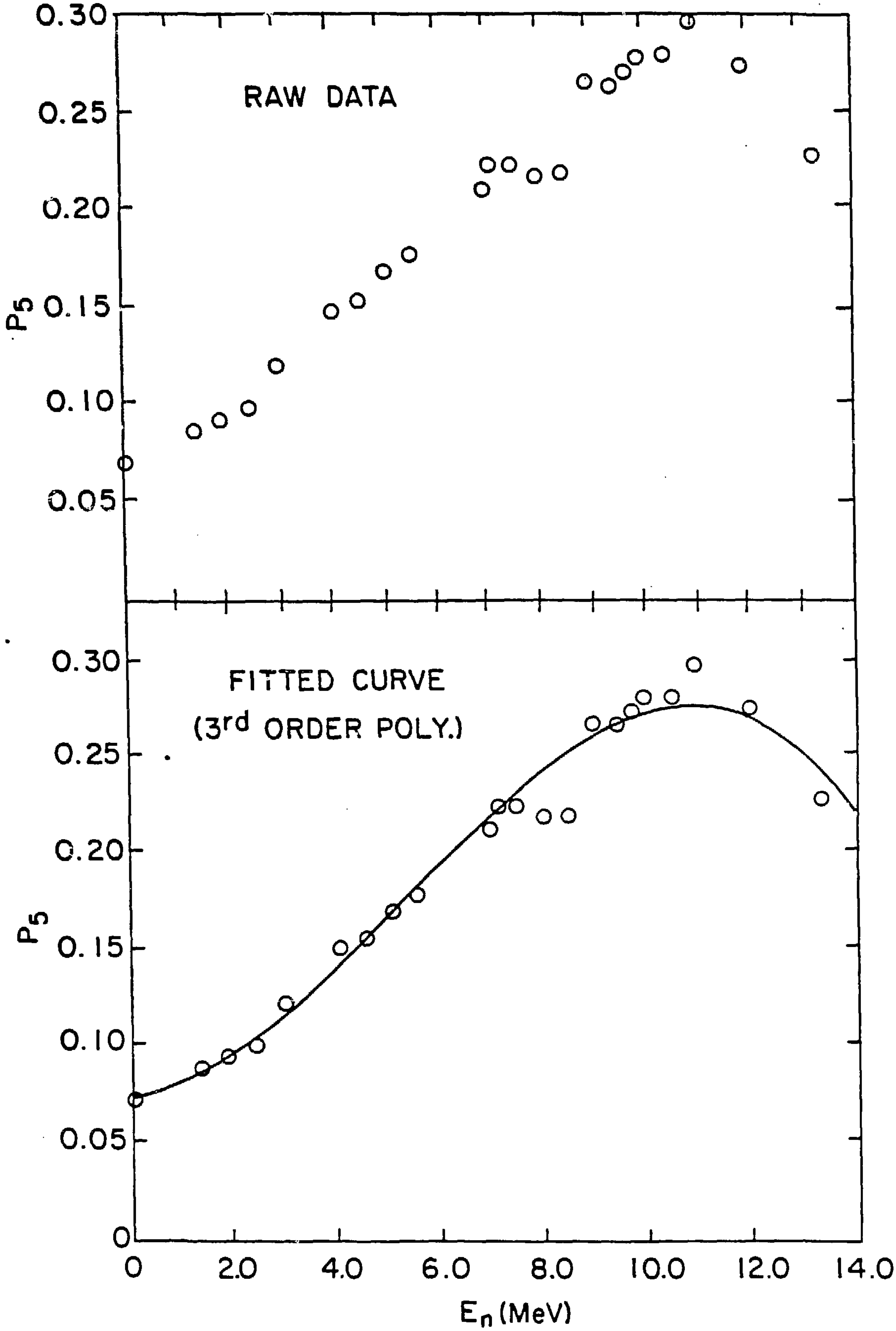
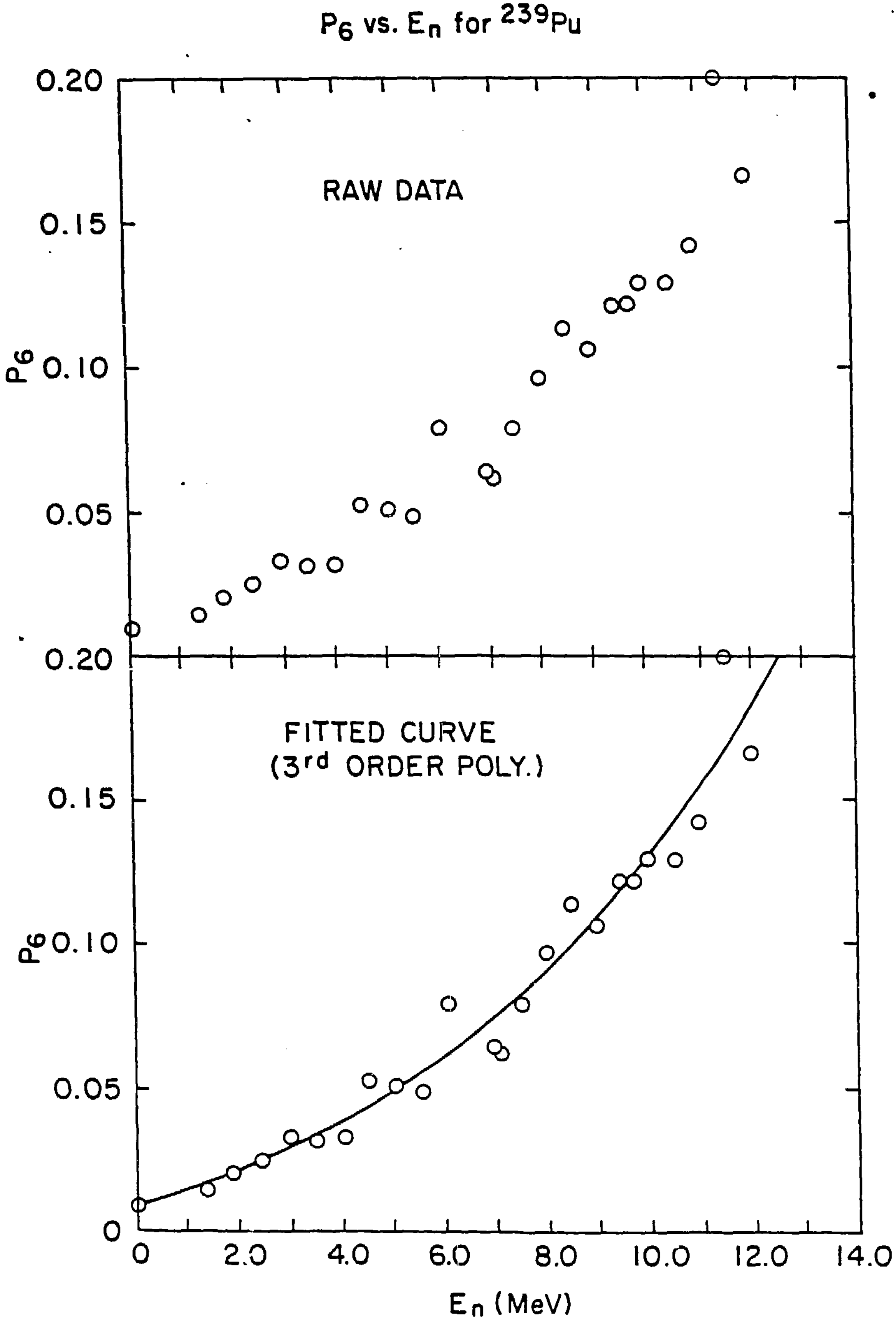
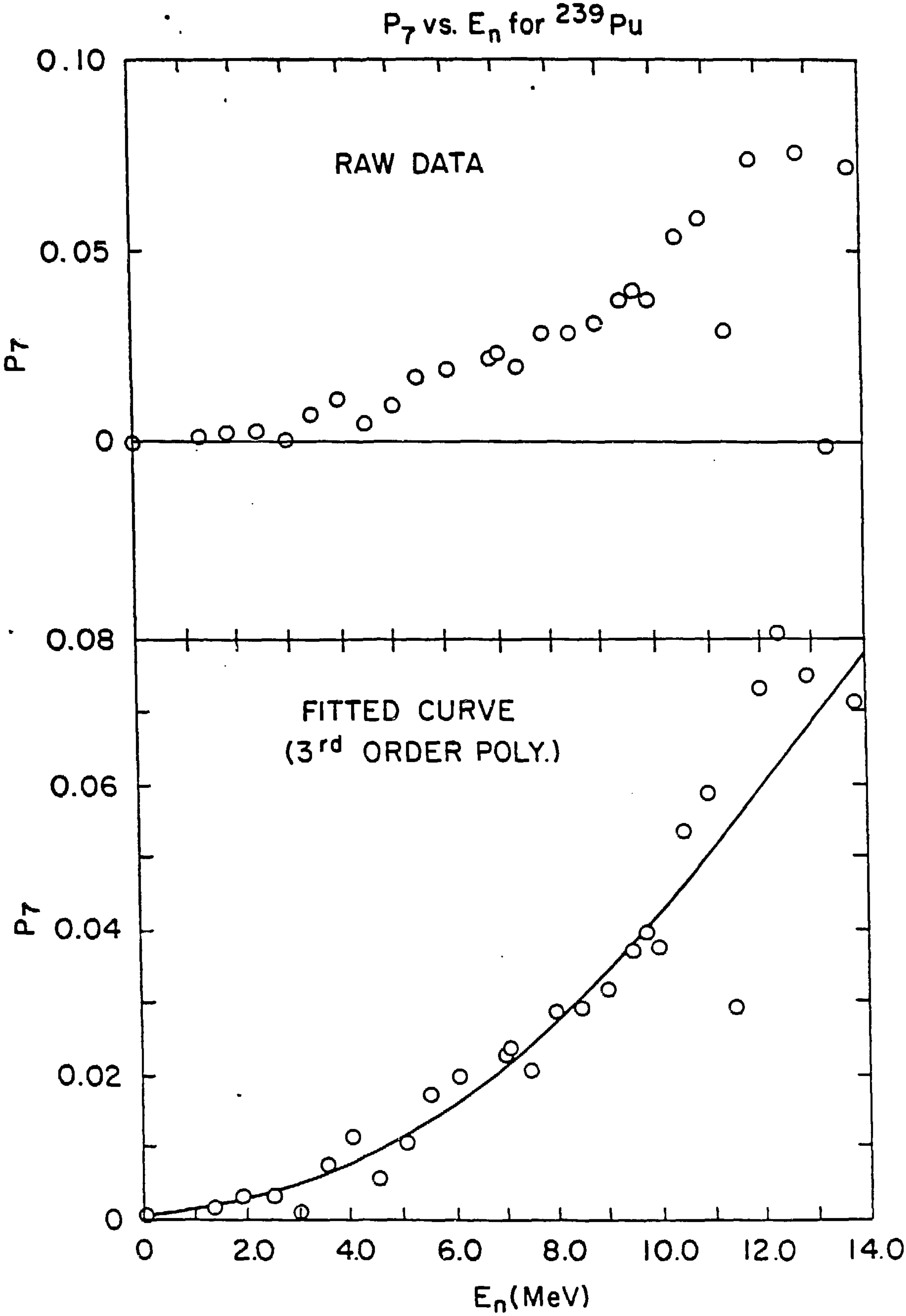


FIGURE 7



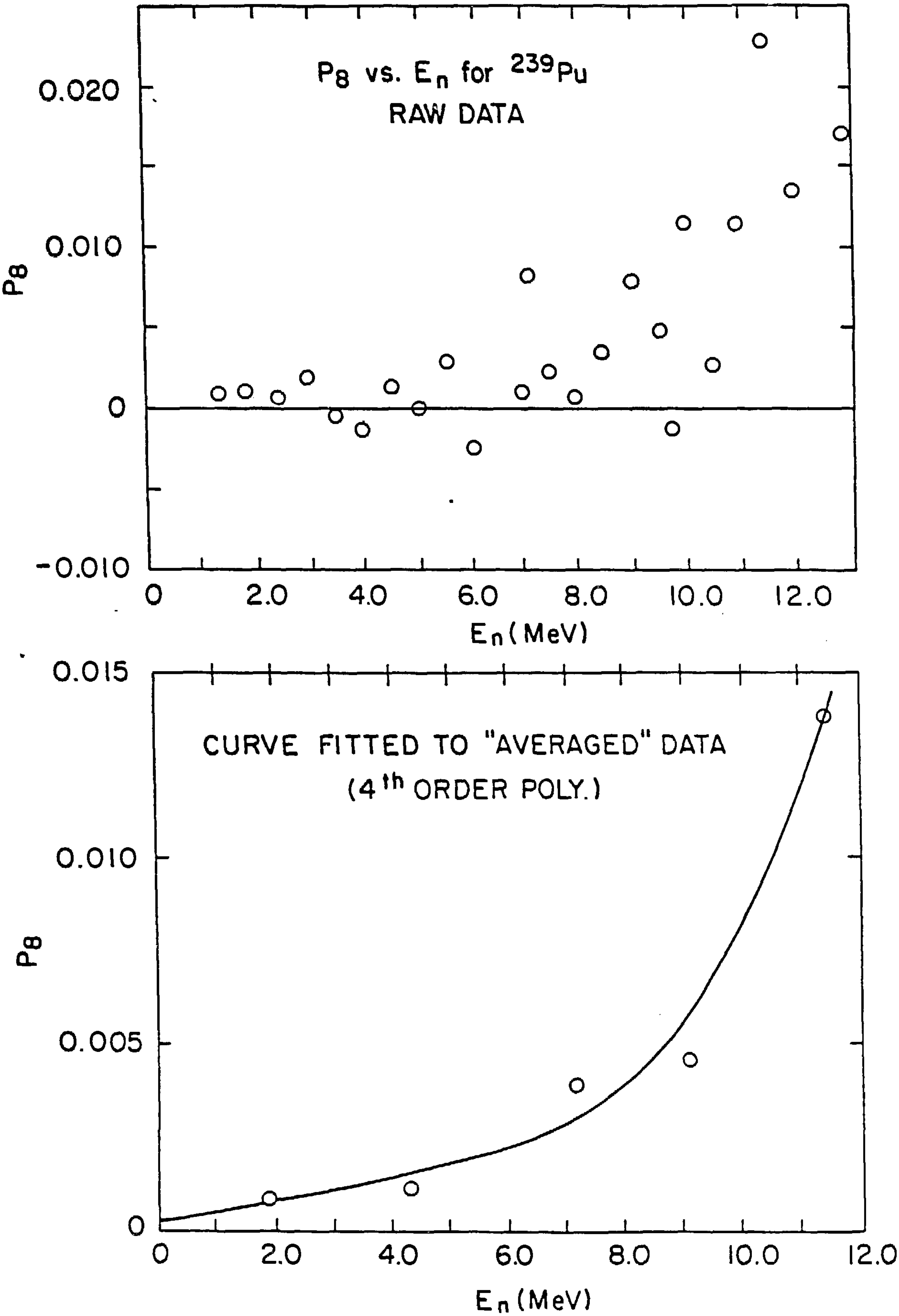
P6 vs. En for 239Pu
0.20
0.15
0.10
0.05
P6
RAW DATA
0.20
0.15
0.10
0.05
0
P6
FITTED CURVE
(3rd ORDER POLY.)
0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
En (MeV)

FIGURE 8



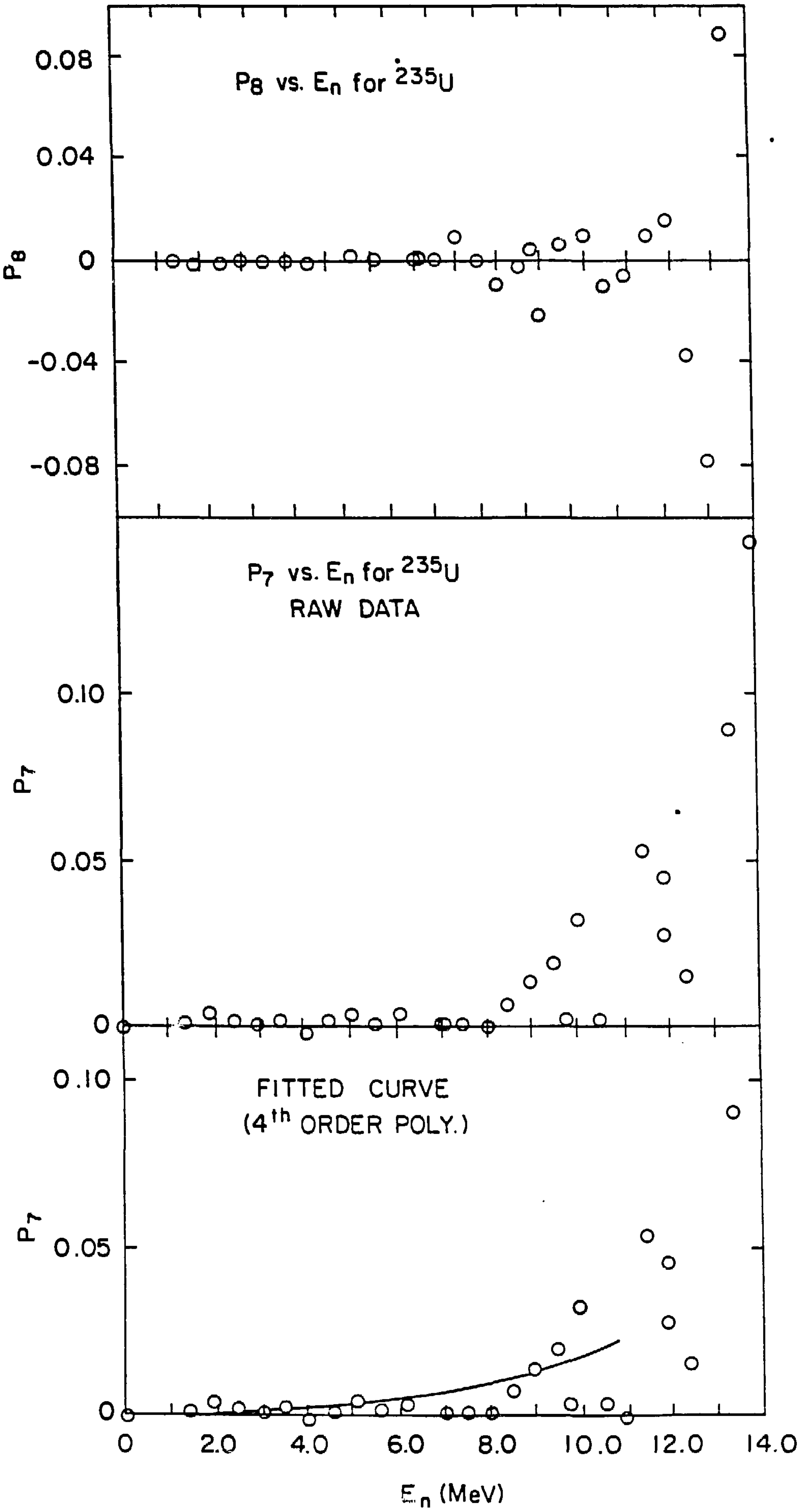
P_7 vs. E_n for ^{239}Pu
RAW DATA
FITTED CURVE
(3rd ORDER POLY.)
0.10
0.05
0
0.08
0.06
0.04
0.02
0
P_7
P_7
0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
E_n(MeV)

FIGURE 9



P_8 vs. E_n for ^{239}Pu
RAW DATA
0.020
0.010
0
-0.010
P_8
0
2.0
4.0
6.0
8.0
10.0
12.0
E_n (MeV)
0.015
CURVE FITTED TO "AVERAGED" DATA
(4th ORDER POLY.)
0.010
0.005
0
P_8
0
2.0
4.0
6.0
8.0
10.0
12.0
E_n (MeV)

FIGURE 10



P8 vs. En for 235U
0.08
0.04
0
-0.04
-0.08
P8
P7 vs. En for 235U
RAW DATA
0.10
0.05
0
P7
FITTED CURVE
(4th ORDER POLY.)
0.10
0.05
0
P7
0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
En (MeV)

FIGURE 11

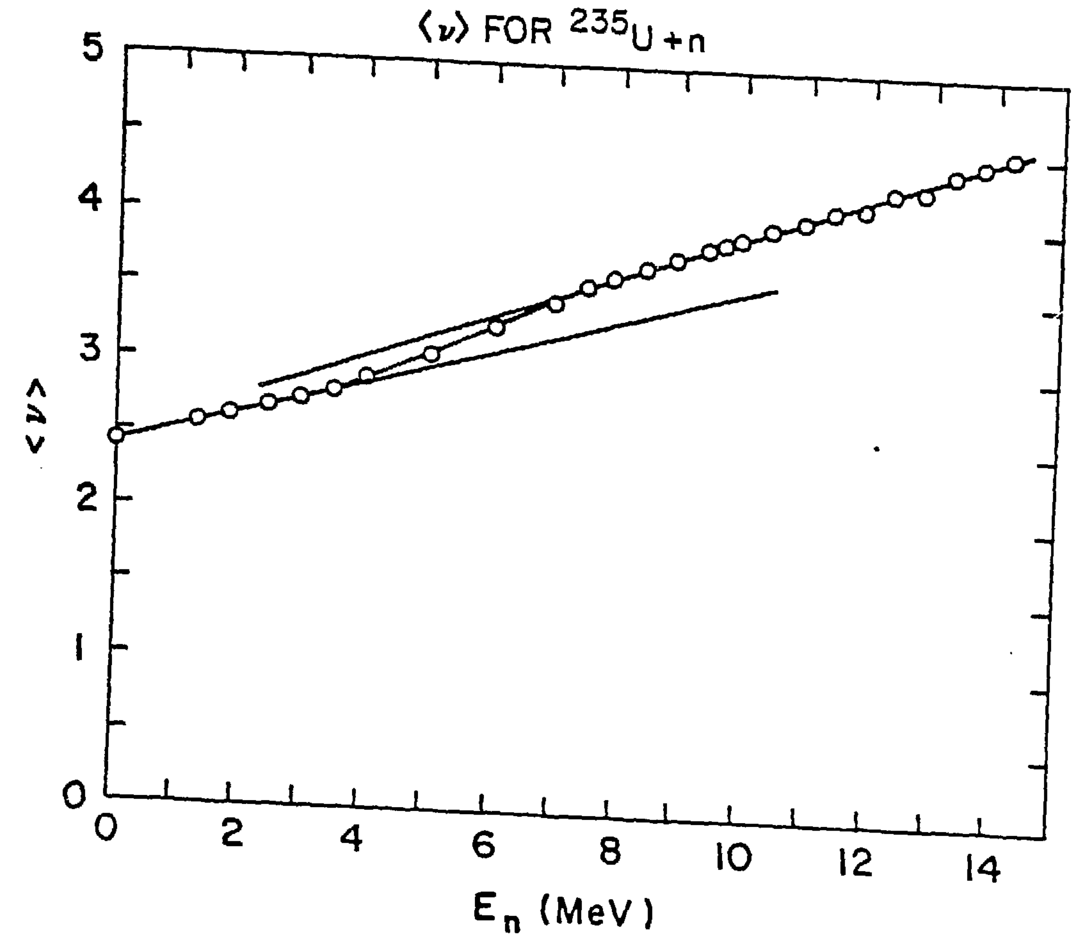


FIGURE 12

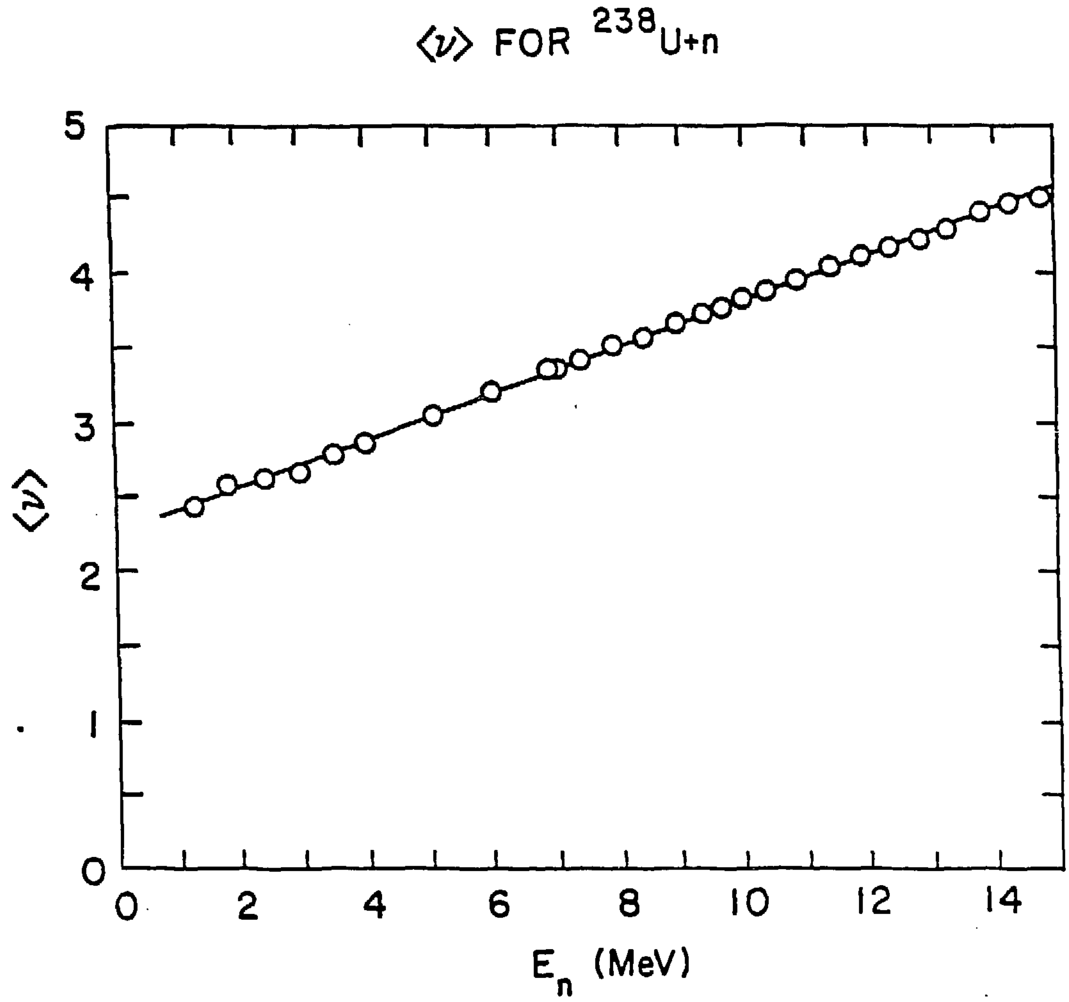


FIGURE 13

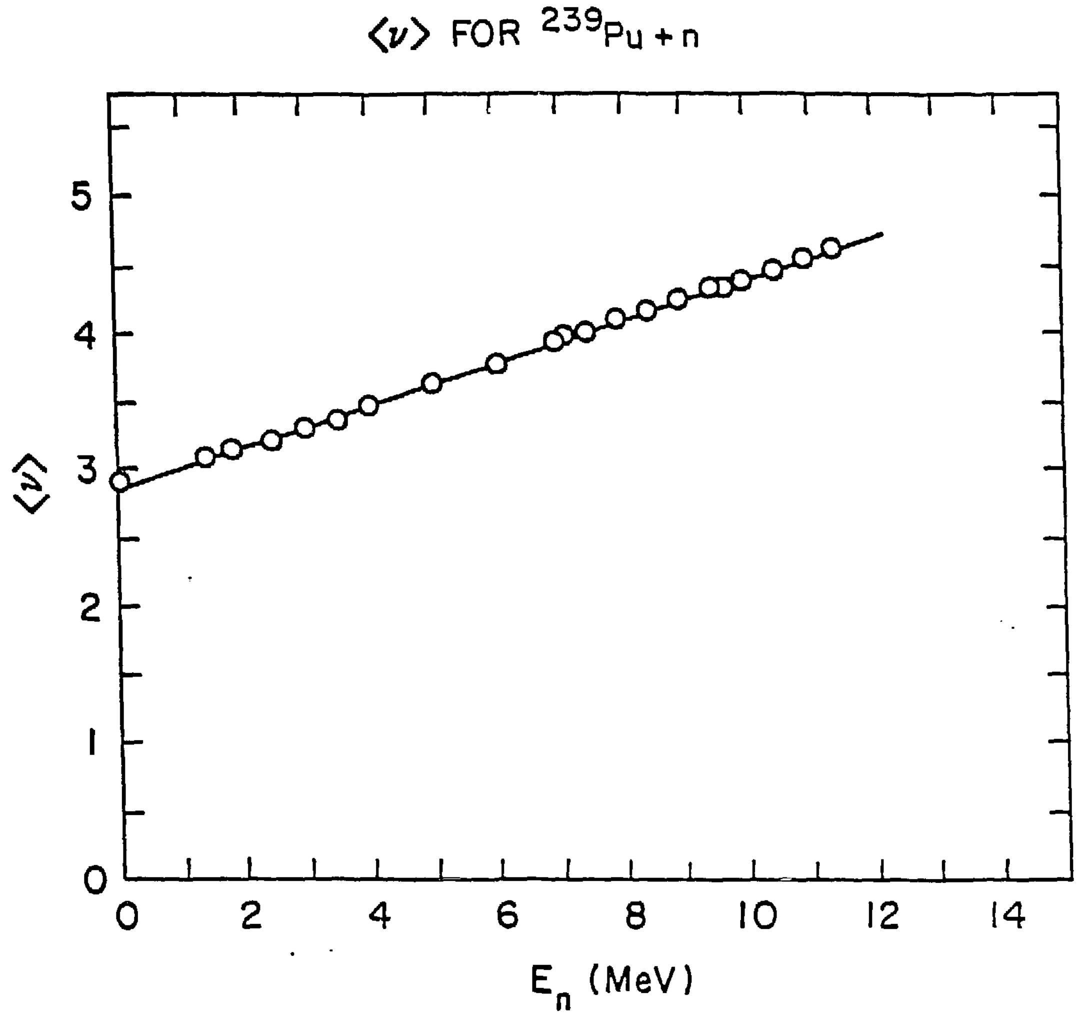


Table I

Comparison of $P_\nu(E_n = 0)$ for $(^{235}U + n)$, derived in different ways

a) curve fitted, $\langle\nu\rangle = 2.41052$.
b) curve fitted, normalized to $\langle\nu\rangle = 2.41400$.
c) consensus data, normalized to $\langle\nu\rangle = 2.41400$.

	(a)	(b)	(c)	
ν	P_ν	P_ν	P_ν	std. dev.
0	.0319004	.0316520	.0317223	.0015
1	.1725213	.1718003	.1717071	.0014
2	.3361397	.3357926	.3361991	.0031
3	.3038798	.3044640	.3039695	.0004
4	.1266155	.1271582	.1269459	.0036
5	.0261843	.0263510	.0266793	.0026
6	.0026170	.0026383	.0026322	.0009
7	.0001421	.0001436	.0001449	.00006
$\langle\nu\rangle$	2.4105200	2.4140000	2.4140000	.007
$\langle\nu(\nu-1)\rangle$	4.6231073	4.6364655	4.6382	.0297
$\langle\nu(\nu-1)(\nu-2)\rangle$	6.7769876	6.8063813	6.8176	.1683
$\langle\nu(\nu-1)\rangle/\langle\nu\rangle^2$	.7956325	.7956325	.79593	.00510
$\langle\nu^2\rangle-\langle\nu\rangle^2$	1.2230207	1.2230695	1.2248	.0297
$\langle\nu^2\rangle$	7.0336273	7.0504654	7.0522	.0297

Table II

Comparison of $P_\nu(E_n = 0)$ for ($^{239}Pu + n$), derived in different ways

a) curve fitted, $\langle\nu\rangle = 2.87737$.
b) curve fitted, normalized to $\langle\nu\rangle = 2.87600$.
c) consensus data, normalized to $\langle\nu\rangle = 2.87600$.

	(a)	(b)	(c)	
P_ν	P_ν	P_ν	P_ν	std. dev.
0	.0108819	.0108593	.0108601	.00003
1	.0993848	.0995994	.0993044	.0028
2	.2747747	.2749800	.2748737	.0003
3	.3269322	.3268549	.3270500	.0041
4	.2046922	.2044754	.2047660	.0087
5	.0727458	.0726004	.0727720	.0133
6	.0097395	.0097138	.0097430	.0027
7	.0006463	.0006452	.0006310	.0009
8	.0002726	.0002716	--	--
$\langle\nu\rangle$	2.8773708	2.8760000	2.8760000	.009
$\langle\nu(\nu-1)\rangle$	6.7569620	6.7505254	6.7435	.0184
$\langle\nu(\nu-1)(\nu-2)\rangle$	12.6350212	12.6169716	12.5447	.0539
$\langle\nu(\nu-1)\rangle/\langle\nu\rangle^2$	.8161309	.8161309	.81528	.00223
$\langle\nu^2\rangle-\langle\nu\rangle^2$	1.3550705	1.3551498	1.3481	.0184
$\langle\nu^2\rangle$	9.6343328	9.6265254	9.6195	.0184

TABLE III. P_ν vs. E_n (MeV) for ^{235}U

E_n / ν	0	1	2	3	4	5	6	7	8	9	10
0	.0317223	.0237898	.0183989	.0141460	.0115208	.0078498	.0046272	.0024659	.0012702	.0007288	.0004373
1	.1717071	.1555525	.1384891	.1194839	.1032624	.0802010	.0563321	.0360957	.0216090	.0134879	.0080115
2	.3361991	.3216515	.3062123	.2883075	.2716849	.2456595	.2132296	.1788634	.1472227	.1231200	.1002329
3	.3039695	.3150433	.3217566	.3266568	.3283426	.3308175	.3290407	.3210507	.3083032	.2949390	.2779283
4	.1269459	.1444732	.1628673	.1836014	.2021206	.2291646	.2599806	.2892537	.3123950	.3258251	.3342611
5	.0266793	.0356013	.0455972	.0569113	.0674456	.0836912	.1045974	.1282576	.1522540	.1731879	.1966100
6	.0026322	.0034339	.0055694	.0089426	.0128924	.0187016	.0265604	.0360887	.0462449	.0551737	.0650090
7	.0001449	.0004546	.0011093	.0019504	.0027307	.0039148	.0056322	.0079244	.0107009	.0135376	.0175099
$\langle\nu\rangle$	2.4140000	2.5236700	2.6368200	2.7623400	2.8738400	3.0386999	3.2316099	3.4272800	3.6041900	3.7395900	3.8749800
$\langle\nu(\nu-1)\rangle$	4.6382	5.1013758	5.6229883	6.2281944	6.7892476	7.6254916	8.6457809	9.7357124	10.7748700	11.6133228	12.4970553
$\langle\nu(\nu-1)(\nu-2)\rangle$	6.8176	8.0012201	9.4764699	11.2637536	12.9882286	15.5726214	18.8596276	22.5586171	26.2791158	29.4444477	32.9645971
$\langle\nu(\nu-1)\rangle/\langle\nu\rangle^2$	.79593	.8009810	.8087349	.8162212	.8220467	.8258330	.8278780	.8288359	.8294624	.8304405	.8322787
$\langle\nu^2\rangle-\langle\nu\rangle^2$	1.2248	1.2561356	1.3069885	1.3600120	1.4041316	1.4304940	1.4340880	1.4167443	1.3888742	1.3683791	1.3565652
$\langle\nu^2\rangle$	7.0522	7.6250458	8.2598084	8.9905344	9.6630876	10.6641915	11.8773909	13.1629923	14.3790600	15.3539128	16.3720353

TABLE IV. P_ν vs. E_n (MeV) for ^{238}U

E_n / ν	0	1	2	3	4	5	6	7	8	9	10
0	.0396484	.0299076	.0226651	.0170253	.0124932	.0088167	.0058736	.0035997	.0019495	.0008767	.0003271
1	.2529541	.2043215	.1624020	.1272992	.0984797	.0751744	.0565985	.0420460	.0309087	.0226587	.0168184
2	.2939544	.2995886	.2957263	.2840540	.2661875	.2436570	.2179252	.1904095	.1625055	.1356058	.1111114
3	.2644470	.2914889	.3119098	.3260192	.3344938	.3379711	.3368863	.3314575	.3217392	.3076919	.2892434
4	.1111758	.1301480	.1528786	.1779579	.2040116	.2297901	.2541575	.2760413	.2943792	.3080816	.3160166
5	.0312261	.0363119	.0434233	.0526575	.0640468	.0775971	.0933127	.1112075	.1313074	.1536446	.1782484
6	.0059347	.0073638	.0097473	.0130997	.0173837	.0225619	.0286200	.0355683	.0434347	.0522549	.0620617
7	.0005436	.0006947	.0009318	.0013467	.0020308	.0030689	.0045431	.0065387	.0091474	.0124682	.0166066
8	.0001158	.0001751	.0003159	.0005405	.0008730	.0013626	.0020831	.0031316	.0046284	.0067176	.0095665
$\langle\nu\rangle$	2.2753781	2.4305631	2.5857481	2.7409331	2.8961181	3.0513031	3.2064881	3.3616731	3.5168581	3.6720432	3.8272281
$\langle\nu(\nu-1)\rangle$	4.3405824	4.8960202	5.5151635	6.1926892	6.9241053	7.7066250	8.5393781	9.4232501	10.3605657	11.3547324	12.4099025
$\langle\nu(\nu-1)(\nu-2)\rangle$	6.9937088	8.1395680	9.6174313	11.4229409	13.5518908	16.0083662	18.8082439	21.9797085	25.5622336	29.6048171	34.1638967
$\langle\nu(\nu-1)\rangle/\langle\nu\rangle^2$	.8383799	.8287612	.8248709	.8242945	.8255267	.8277392	.8305523	.8338535	.8376716	.8420955	.8472268
$\langle\nu^2\rangle-\langle\nu\rangle^2$	1.4386152	1.4189462	1.4148184	1.4209081	1.4327235	1.4474776	1.4643005	1.4840772	1.5091327	1.5428747	1.5894556
$\langle\nu^2\rangle$	6.6159604	7.3265832	8.1009115	8.9336223	9.8202234	10.7579281	11.7458662	12.7849232	13.8774238	15.0267756	16.2371306

TABLE V. P_ν vs. E_n (MeV) for ^{239}Pu

E_n	0	1	2	3	4	5	6	7	8	9	10
ν											
0	.0108826	.0084842	.0062555	.0045860	.0032908	.0022750	.0014893	.0009061	.0004647	.0002800	.0002064
1	.0994916	.0790030	.0611921	.0477879	.0374390	.0291416	.0222369	.0163528	.0113283	.0071460	.0038856
2	.2748898	.2536175	.2265608	.1983002	.1704196	.1437645	.1190439	.0968110	.0775201	.0615577	.0492548
3	.3269196	.3289870	.3260637	.3184667	.3071862	.2928006	.2756297	.2558524	.2335926	.2089810	.1822078
4	.2046061	.2328111	.2588354	.2792811	.2948565	.3063902	.3144908	.3194566	.3213289	.3200121	.3154159
5	.0726834	.0800161	.0956070	.1158950	.1392594	.1641647	.1892897	.2134888	.2356614	.2545846	.2687282
6	.0097282	.0155581	.0224705	.0301128	.0386738	.0484343	.0597353	.0729739	.0886183	.1072344	.1295143
7	.0006301	.0011760	.0025946	.0048471	.0078701	.0116151	.0160828	.0213339	.0274895	.0347255	.0432654
8	.0001685	.0003469	.0005205	.0007233	.0010046	.0014140	.0020017	.0028246	.0039531	.0054786	.0075217
$\langle\nu\rangle$	2.8760000	3.0088800	3.1628300	3.3167800	3.4707300	3.6246800	3.7786300	3.9325800	4.0865300	4.2404900	4.3944400
$\langle\nu(\nu-1)\rangle$	6.7479888	7.4107740	8.2397008	9.1241373	10.0544377	11.0243551	12.0311784	13.0754119	14.1602527	15.2911453	16.4750997
$\langle\nu(\nu-1)(\nu-2)\rangle$	12.5894114	14.5928444	17.3210445	20.4416846	23.9063613	27.6864348	31.7771206	36.1974684	40.9883672	46.2105526	51.9416469
$\nu(\nu-1)\rangle/\langle\nu\rangle^2$	.8158242	.8185663	.8236823	.8293894	.8346726	.8391002	.8426349	.8454740	.8479331	.8503700	.8531413
$\langle\nu^2\rangle-\langle\nu\rangle^2$	1.3526129	1.3662954	1.3990374	1.4398876	1.4792012	1.5107299	1.5317640	1.5428066	1.5470550	1.5498799	1.5584365
$\langle\nu^2\rangle$	9.6239888	10.4196539	11.4025308	12.4409173	13.5251677	14.6490351	15.8098084	17.0079919	18.2467828	19.5316353	20.8695397

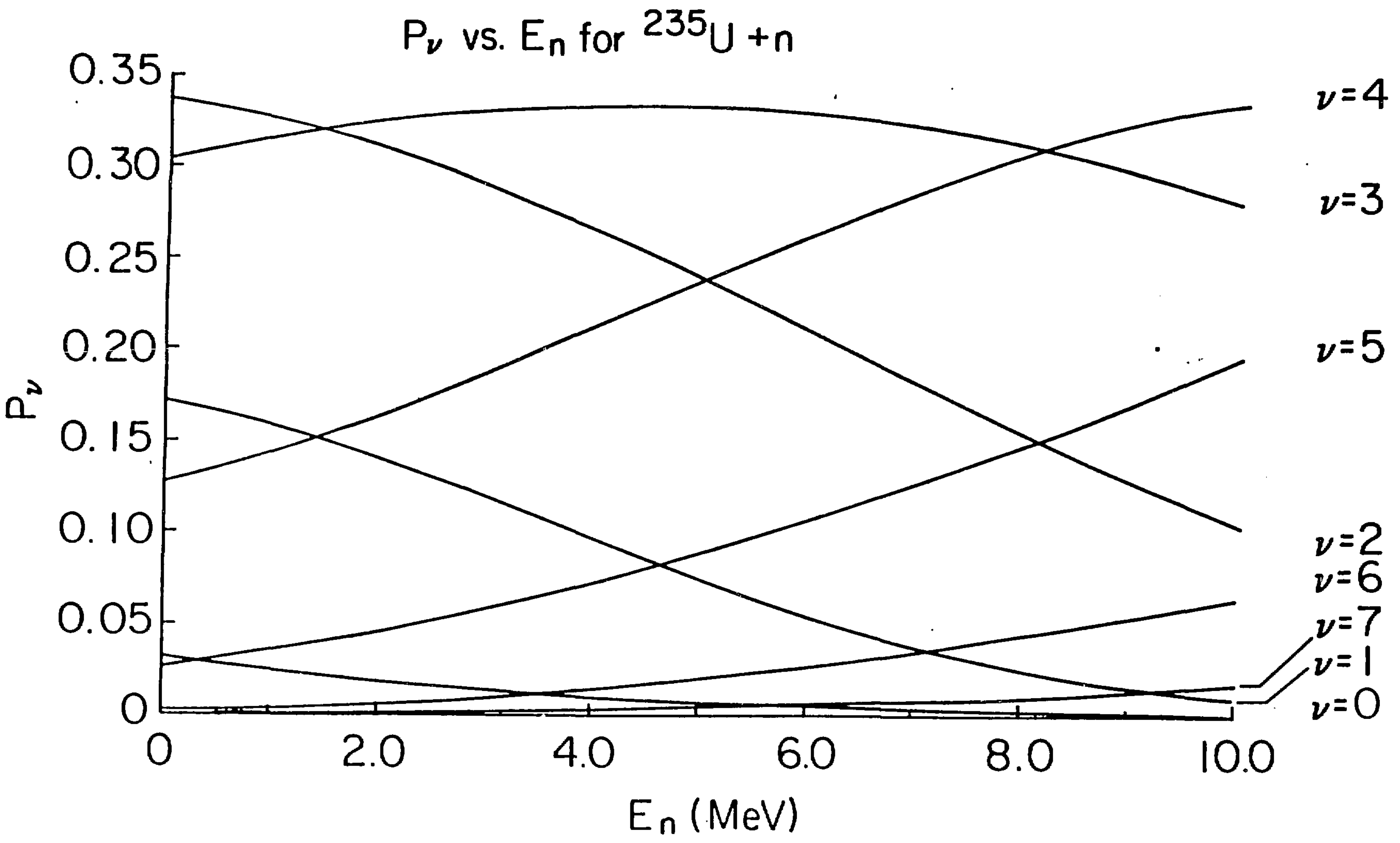

Pν vs. En for 235U +n
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
Pν
ν=4
ν=3
ν=5
ν=2
ν=6
ν=7
ν=1
ν=0
0
2.0
4.0
6.0
8.0
10.0
En (MeV)

FIGURE 14

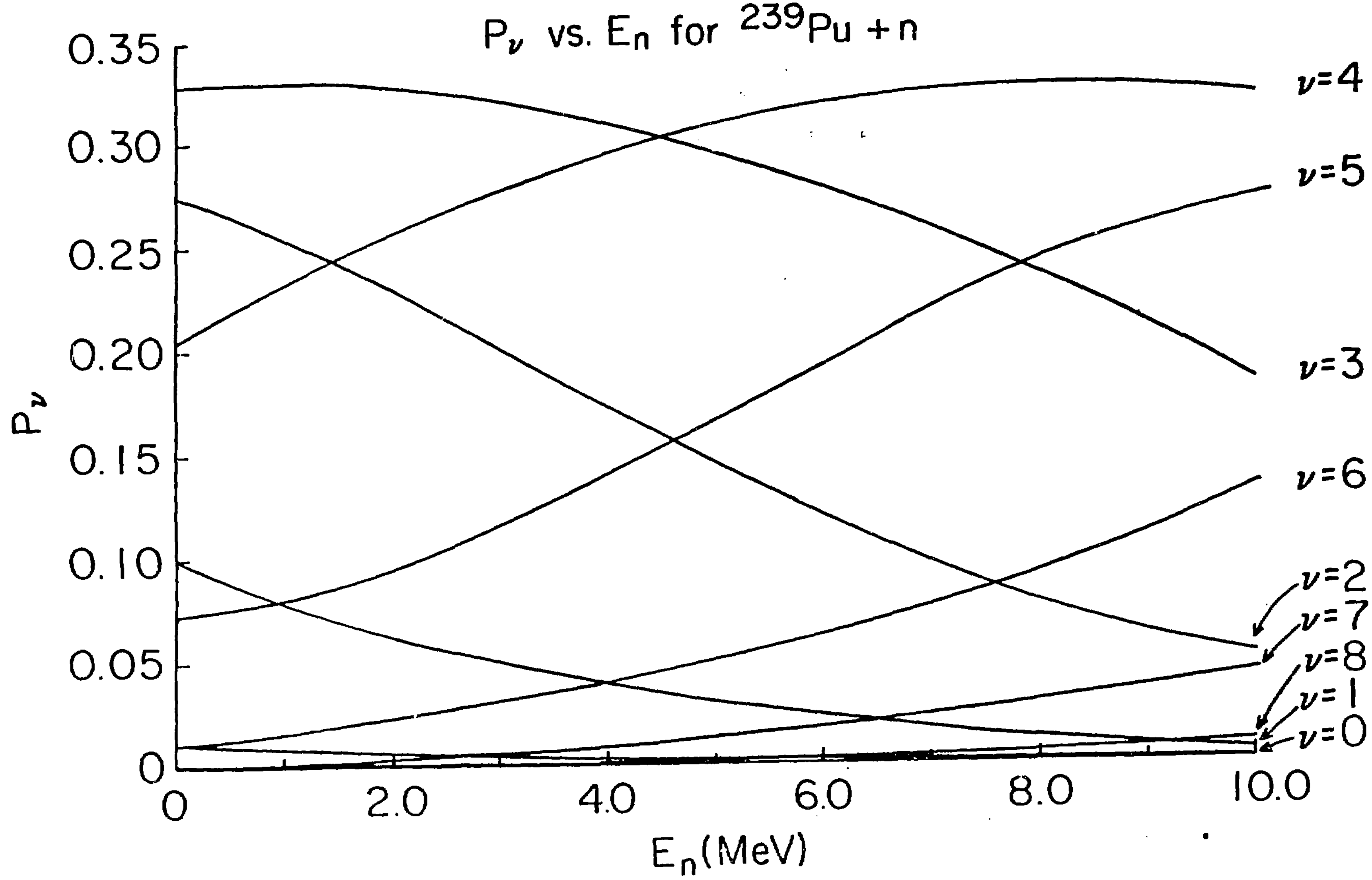
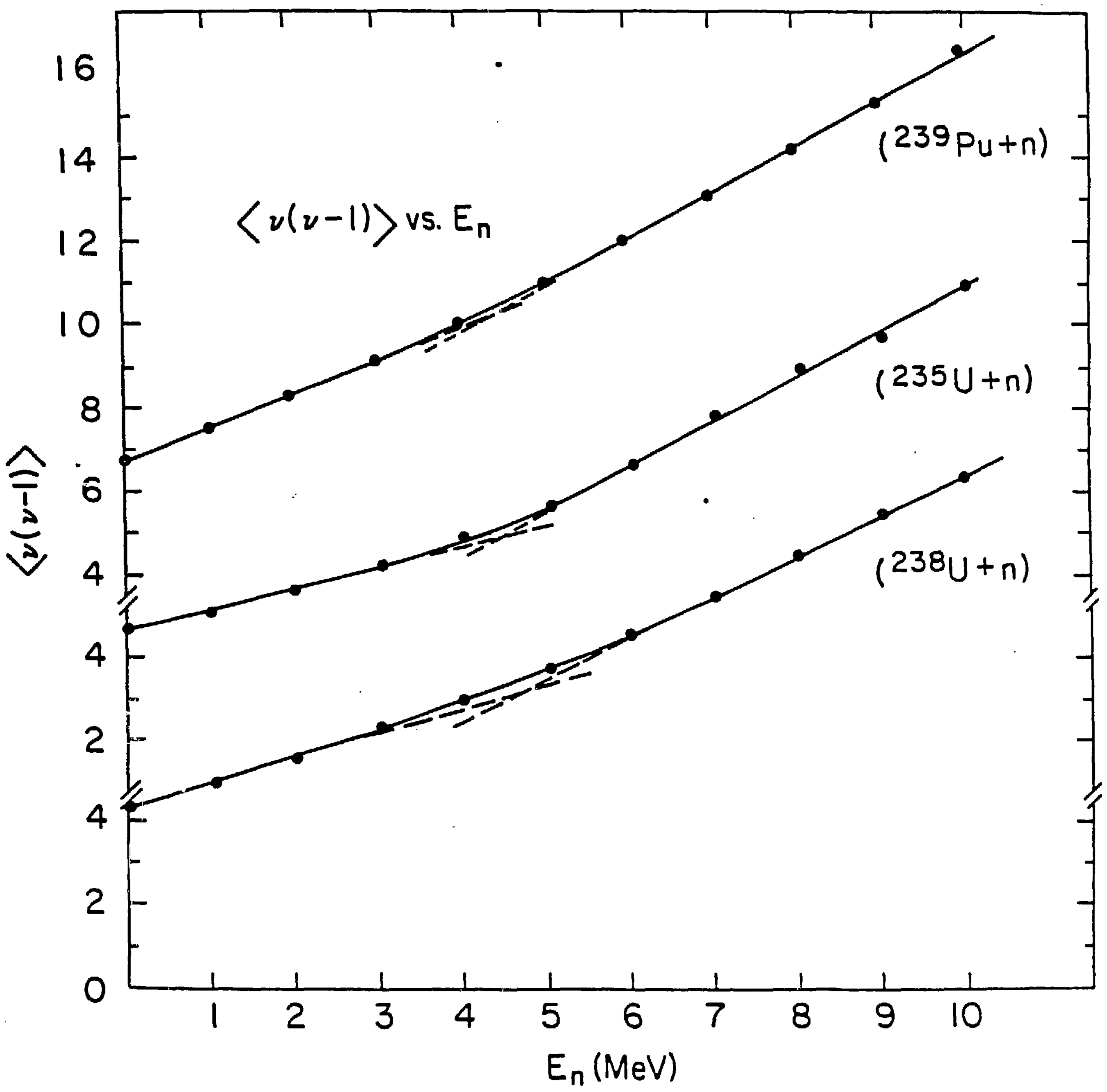

P_ν vs. E_n for 239Pu + n
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
P_ν
0
2.0
4.0
6.0
8.0
10.0
E_n(MeV)
ν=4
ν=5
ν=3
ν=6
ν=2
ν=7
ν=8
ν=1
ν=0

FIGURE 15

FIGURE 16



⟨ν(ν−1)⟩ vs. E_n
(239Pu+n)
(235U+n)
(238U+n)
⟨ν(ν−1)⟩
E_n (MeV)
16
14
12
10
8
6
4
4
2
4
2
0
1
2
3
4
5
6
7
8
9
10