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DEVELOPING CONVECTION ABOVE A FINITE HORIZONTAL SURFACE

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The problem described -- the initiation of convection above a suddenly heated horizontal surface of finite extent -- is of interest to both the engineering and geophysical fields. The apparatus used in the described experiment consisted of an enclosure with a square planform. A heated strip, whose width was equal to one-fourth of the length of a side of the enclosure, was centered on the lower inside surface of the enclosure. The top of the fluid layer was maintained at a constant temperature, and the depth of the layer was equal to the width of the heated strip. The flow field, heater surface temperature, and heat flux distribution were studied. Flow was observed to initiate near the edges of the heated strip as two convection cells. In time, the two cell boundaries merged into a central plume rising above the heated strip. The velocity field was found to be the most sensitive indicator of convection; surface temperature, the least. Despite the finite width of the heated strip and the finite depth of the layer, the initiation of convection appeared to readily conform to Howard's model of conduction layer instability developed for infinite layers. The average Rayleigh number for the critical conduction layer was 1120.

1. INTRODUCTION

This paper reports the results of an experimental study of the initiation of convection above a suddenly heated horizontal surface of finite extent. The phenomenon of flow initiation in fluid layers is of interest to both the engineering and geophysical fields (Mollendorf et al. 1984; Olson et al. 1988). While a number of previous studies -- both theoretical (Currie 1967; Kim and Kim 1986) and experimental (Nielsen and Sabersky 1973; Mollendorf et al. 1984; Kukalka and Mollendorf 1988) -- exist, the present study is the first that deals specifically with surfaces of finite extent.

A fairly extensive review of the literature for horizontal layers heated suddenly from below exists in the papers of Mollendorf and colleagues. However, only a few studies of specific interest to the present experiment are reviewed here. Currie (1967) first analyzed the stability of an infinite layer heated suddenly from below using a quasi-steady linear stability analysis. In this analysis, the onset of convective instability was defined as the point at which the fastest growing wave component (of a disturbance) is neutrally stable. Kim and Kim (1986) analyzed the same problem using a time-dependent calculation with the inclusion of random fluctuations. The onset of convection was defined there as the time at which the Nusselt number (heat-transfer coefficient) started to grow, as a result of convection, for the first time. A more intuitive approach to the problem is based on Howard's (1966) hypothesis of the instability of a conduction boundary layer next to the heated surface; this hypothesis was first advanced to model the periodic release of thermals in turbulent thermal convection. According to Howard's theory, the onset of convection is characterized by a critical Rayleigh number based on the thickness of the conduction boundary layer. A demonstration of conduction boundary layer instability can be found in Goldstein et al. (1977).

Nielsen and Sabersky (1973) carried out an extensive study of transient heat transfer in Benard convection using silicone oils. They found good general agreement with the trend of Currie's result. As pointed out in the later work by Kim and Kim (1986), the Nielsen and Sabersky data are in much closer quantitative agreement with the Kim and Kim result. Mollendorf et al. (1984) studied the transient transport above a suddenly heated water layer and found agreement with Howard's theory of conduction boundary layer instability. Recently, Kukulka and Mollendorf (1986) extended the study to include downward-facing surfaces. They also made interesting observations on the effects of side walls next to the edge of the heated surface.

Although the primary motivation of the present study is to seek an understanding of the physics of convection initiation, a secondary motivation is to assist in developing a more comprehensive view of the problem. We have observed that while a number of studies exist for the "infinite layers," the studies do not seem to be regarded as a whole. By making complementary observations, perhaps a more unified view can be achieved.

2. EXPERIMENT

The experimental apparatus, Figure 1, consists of a transparent Lexan (polycarbonate) enclosure with a square planform measuring 55.9 cm by 55.9 cm. An electrically heated strip, with a width equal to one-fourth of the length of a side of the enclosure, is centered on the lower inside surface of the enclosure. The heated strip consists of a 3-mm-thick copper plate with a thin, flexible, etched foil heater glued onto the underside. The heater fits into a centered recess, which was machined into the Lexan sheet that forms the bottom of the enclosure. The depth of the fluid layer is the same as the width of the heated strip. The top of the fluid layer is bounded by a constant temperature plate. The working fluid is a commercial corn sweetener, 42/43 corn syrup. The viscosity of the working fluid is highly temperature dependent. Typical properties of the syrup at 25°C are as follows: density, 1.423 gm/cm^3; specific heat, 2.3 J/gm K; thermal expansion coefficient, 3.96×10^{-4} K^{-1}; thermal conductivity, 0.380 W/mK; and viscosity, 748 P. The corresponding Prandtl number is 4.5×10^5. Details of the experimental apparatus and the working fluid can be found in Chu and Hickox (1988).

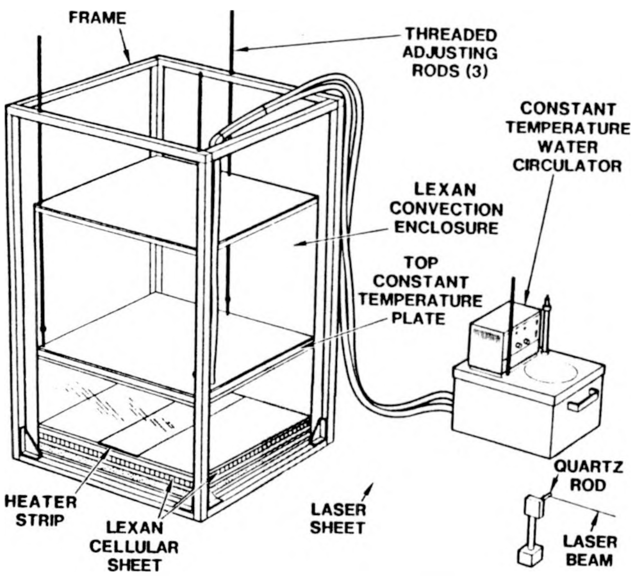

Figure 1. Schematic of Experimental Apparatus.

Initially, the fluid layer is at a uniform temperature. At time zero, a constant energy input is applied to the heated strip, while maintaining the top surface at the initial temperature. The temperature response of the heated strip is modelled using a one-dimensional transient solution of a heater with finite heat capacity at the interface of two semi-infinite bodies (syrup and Lexan); see Mollendorf et al. (1984) for details. Measurements are made of the transient flow field, heater surface temperature, and heat flux distribution.

The results of three experiments with three different heat fluxes are reported here and are designated as Cases A, B, and C in Figure 2. Except for early times, the response of the heated strip closely follows that of the surface of a semi-infinite solid with an imposed constant heat flux. The value of this effective surface flux is a constant fraction of the applied flux to the heated strip. For the present heater assembly and combination of materials, the fraction of energy into the test section is 0.56 of the applied energy to the heater. The heat flux cited in Figure 2 is this effective flux. Initially, the temperature of the heated surface closely follows the conduction solution. With the initiation of convection, the response curve departs smoothly from the conduction solution. A temperature overshoot, followed by a gradual approach to steady state, was observed. The same data plotted in log-log scales in Figure 3 show that the transient responses are essentially similar.

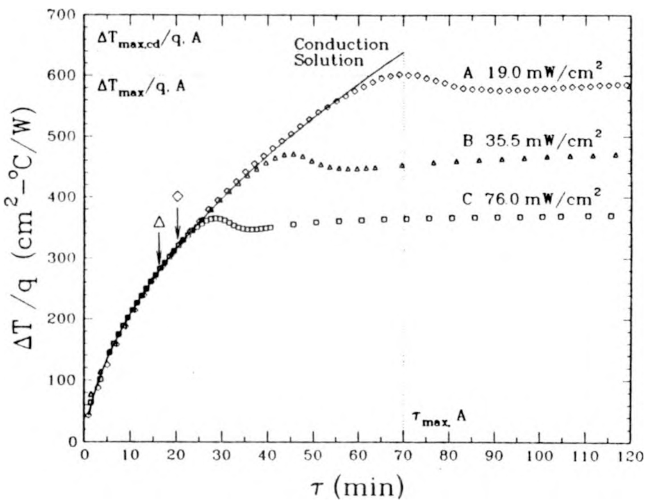

Figure 2. Heated Strip Surface Temperature vs. Time.

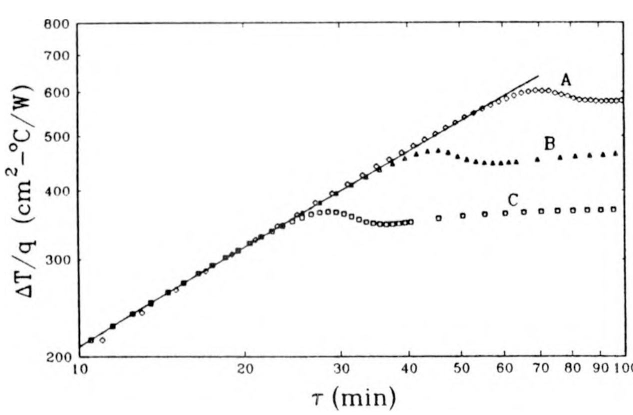

Figure 3. Heated Strip Surface Temperature vs. Time Log-Log Scales.

The developing flow field was examined by photographing the paths of seeded particles in the working fluid. The particles were illuminated by a sheet of laser light cutting through the central plane of the test section; see Figure 1. Figure 4 is a series of time-lapse photographs illustrating the development of the flow field for Case A, q = 19.0 mW/cm^2. In this figure, the photographs are 4.5-min time exposures taken 10 min apart. Incipient fluid motion (streaks) can be observed in the horizontal direction near the edge of the heated strip at 40 min into the experiment, long before the temperature response shows any significant departure from the conduction solution. (Actually, incipient motion can be observed as early as 20 min into the experiment.) With time, the development of two counter-rotating cells, driven by a central plume rising from the heated strip, can be clearly observed. At incipient motion, the centers of the cells are located directly above the edges of the heated strip; as the central plume develops, they move progressively closer. When the distance between the cell centers reaches a minimum, the cells begin to rise from the heated strip and move apart horizontally. The upward motion is stopped by the presence of the top surface; the cells eventually settle downward to steady state positions. The loci of the center of the convection cells are shown in Figure 5. The distance between the two cells as a function of time and the temperature of the heated surface are plotted in Figure 6; there is a close correspondence between the movement of the cells and the temperature of the heated surface.

By construction, the heated strip provides a constant heat flux only on the average; it still allows local variations of heat flux over the surface. This variation of heat flux over the heated strip is observed using a slit deflection shadowgraph. The method is based on the deflection of a slit of parallel light by the thermal boundary layer. The amount of deflection is a measure of the local heat flux. The results for Case B are shown in Figure 7. The heat flux is uniform during the conduction phase. At incipient motion, a slight depression is observed near the edge of the heated strip. The movements of the two depressions closely replicate the movements of the cells. Eventually, the two depressions merge into one at the closest approach of the two cells; the cell boundaries merge into a single plume, and the heater surface temperature reaches a maximum.

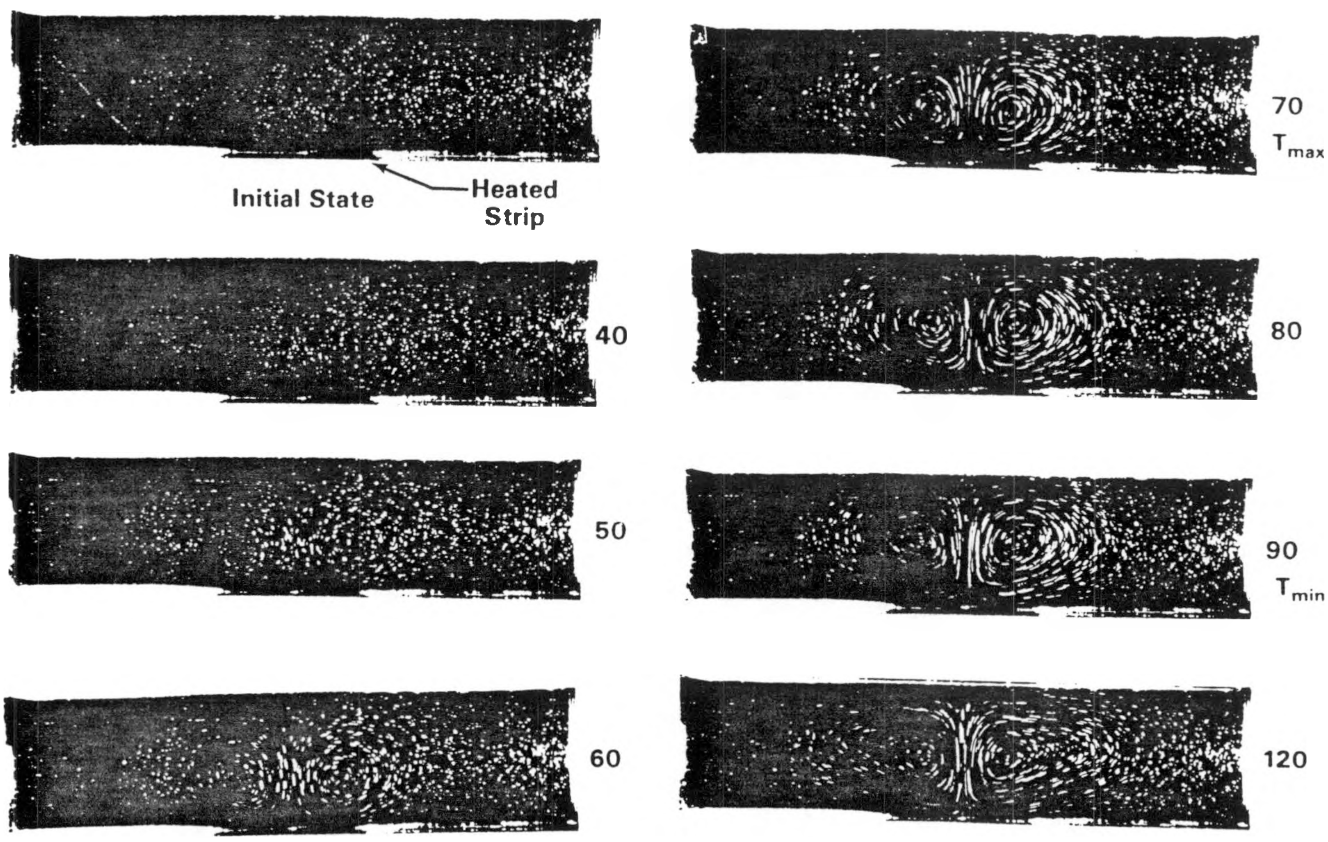

Figure 4. Flow Development for Case A, q = 19.0 mW/cm^2. Numbers shown next to each photo indicate the minutes passed since the initial state.

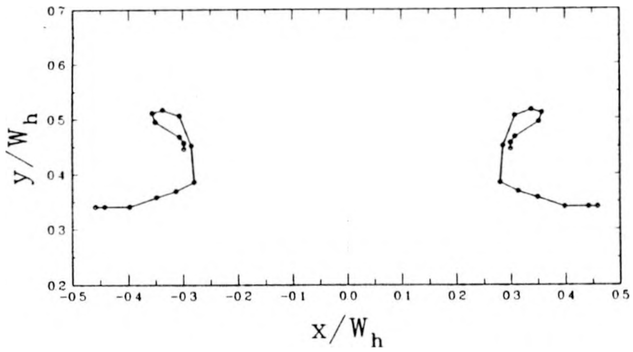

Figure 5. Loci at Cell Centers for Case A.

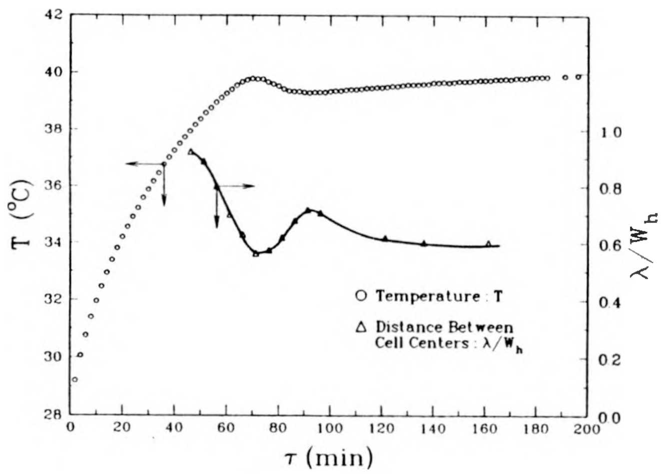

Figure 6. Heated Strip Surface Temperature vs. Time, and Cell Distance vs. Time, for Case A.

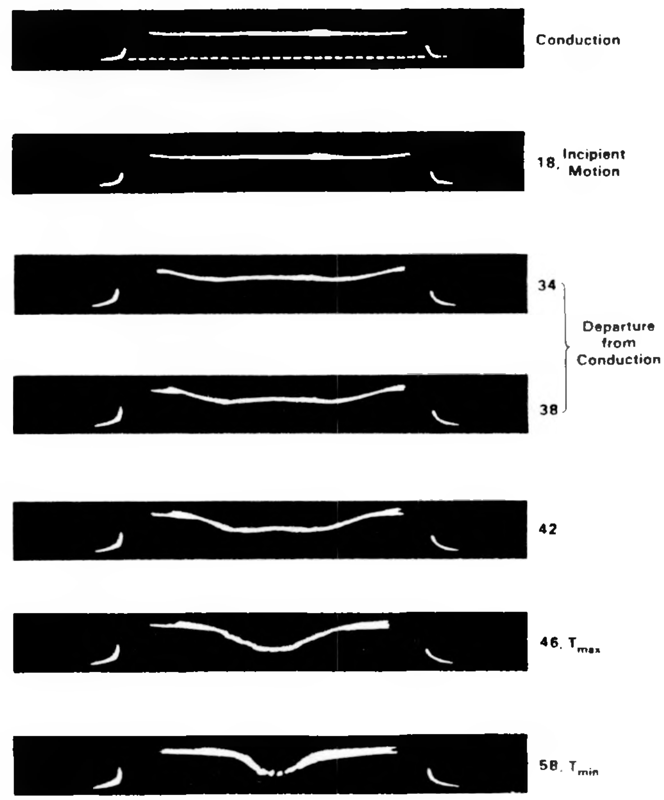

Figure 7. Surface Flux Distribution for Case B, $q = 35.5\ mW/cm^2$.

3. DISCUSSION

While the width of the heated surface is finite, the developing convection has many features in common with the "infinite layer" studies. The most obvious feature shared by these studies is the overshoot of the surface temperature predicted by Kim and Kim (1986) and observed experimentally by Mollendorf et al. (1984).

The conventional method of representing the results of linear stability analyses for the onset of convection uses the Rayleigh number, Ra, and a dimensionless heat flux, H, based on the layer depth, D:

$$Ra = \frac{g\beta}{\nu\alpha}\Delta T D^3;\quad H = \frac{g\beta}{\nu\alpha}\frac{q}{k}D^4 \qquad (1)$$

where q is the heat flux at the lower surface. In Figure 8, the onset of convection data cast in the Ra versus H form are compared with the analyses of Kim and Kim (1986) and Currie (1967). The two analyses show the same trend for large H; the difference is mainly due to the different criteria used to define the onset of convection. The fact that the two critical Rayleigh numbers are off only by a constant factor is in agreement with the experimental observation, shown in Figure 3, that the onset transients are similar. To compare the two analyses, two criteria are also used to examine the onset of convection in the present experiment. One is based on the "incipent motion"; the other is based on the "minimum Nusselt number" or "maximum surface temperature" criterion (Kim and Kim 1986). The first criterion might in principle be more appropriate; however, because time-lapse photographs are required to resolve the minute motion near the onset of convection, it impossible to pinpoint the exact moment of the initiation of convection. Thus, only cases A and B are examined using the incipient motion criterion since time-lapse photographs were not available for Case C. The times for incipent motion are marked by arrows in Figure 2.

As first observed by Nielsen and Sabersky (1973), the onset of convection for high heating rates or deep layers takes place before the thermal wave penetrates the fluid layer. Therefore, the onset of convection should be independent of the layer depth and Ra_C should be proportional to $H^{3/4}$:

$$Ra_C = CH^{3/4} \qquad (2a)$$

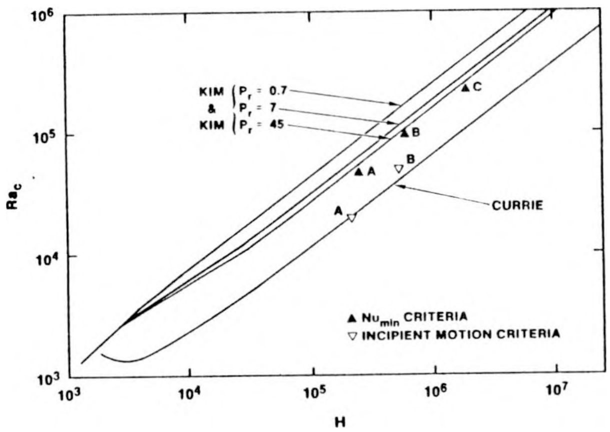

Figure 8. Comparison of the Stability Analyses: Critical Rayleigh Number vs. Dimensionless Heat Flux H.

Indeed the Currie analysis, the Kim and Kim analysis, and the present experiment all follow this correlation. As one might expect, the data corresponding to the "incipent motion" criterion are in general agreement with the Currie analysis, and the data using the "minimum Nusselt number" criterion are closer to the Kim and Kim result. The "minimum Nusselt number" data are found to be well represented by an $H^{3/4}$ correlation, as seen in Figure 9:

$$Ra_C = 3.82\ H^{3/4} \qquad (2b)$$

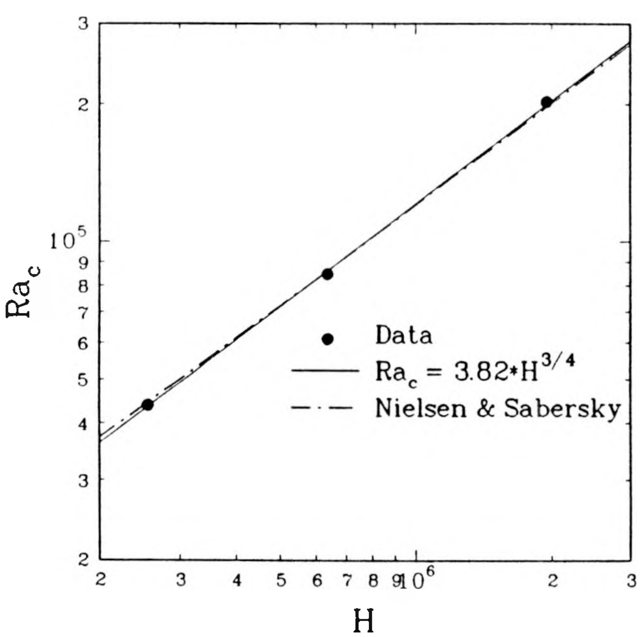

Figure 9. Comparison with Nielsen and Sabersky Results: Critical Rayleigh Number vs. Dimensionless Heat Flux H.

Properties used in reducing the data are evaluated at a bulk temperature calculated from the semi-infinite conduction temperature profile in the layer. The correlation is in excellent agreement with the Nielsen and Sabersky data for "infinite (in width) layers." The nearly exact agreement between the present experiment and the Nielsen and Sabersky data is somewhat fortuitous since the two sets of data are based on different onset criteria. Nielsen and Sabersky defined the onset of instability as the time in which motion was first observed on a shadowgraph that was set up to observe the planform of convection. This criterion actually corresponds to the condition of observable change in the horizontal temperature gradient. Because the onset transients are similar, further discussion is restricted to the data using the "minimum Nusselt number" criterion.

By assuming that heat transfer occurs by conduction alone before the onset of convection is observed, the correlation between Ra_C and H, equation (2a), and the semi-infinite conduction solution can be combined to yield:

$$\left[\frac{g\beta}{\nu\alpha}\ \Delta T_{max}\sqrt{\frac{4\alpha\tau_{max}}{\pi}}^{\,3}\right]^{1/4} = C \qquad (3a)$$

where τ_{max} is the time corresponding to the maximum temperature of the heater surface, T_{max}. Introducing the definition of conduction thermal boundary layer thickness, $\delta = \sqrt{\pi\alpha\tau_{max}}$, results in the expression:

$$Ra_{c,\delta} = \frac{g\beta}{\nu\alpha}\ \Delta T_{max}\delta^3 = C^4\left(\frac{\pi}{2}\right)^3 = \text{constant} \qquad (3b)$$

This is exactly the statement of Howard's theory of conduction boundary layer instability. Therefore, the stability analyses and Howard's theory for deep layers are completely equivalent. For the present experiment, the above expression has to be modified to account for the fact that the temperature of the heated surface deviates from the conduction solution as the surface temperature approaches T_{max}:

$$Ra_{c,\delta} = C^4\left(\frac{\pi}{2}\right)^3\left(\Delta T_{max,cd}/\Delta T_{max}\right)^3 \qquad (3c)$$

where $T_{max,cd}$ is the temperature that the heated strip would have reached had it followed the conduction solution to T_{max} (see Figure 2). For the present experiment, the average value of this correction term is 1.066. The resulting critical Rayleigh number, which is based on the thermal boundary layer thickness, is then calculated to be 1000. Alternately, the critical Rayleigh number can be determined by calculating the thermal boundary layer thickness, δ. The critical Rayleigh numbers thus calculated are 1040, 1050, and 1260 for Cases A, B, and C, respectively. The average value of 1120 is about 12% higher than the estimate based on the deep layer correlation. The main source of this discrepancy is due to the fact that the heated strip does not follow the semi-infinite solution at small times. The critical Rayleigh number obtained in the present experiment is somewhat higher than the average value of 948 of Mollendorf et al. (1984). The difference mainly reflects the fact that the two studies use different criteria for the onset of convection.

Because the onset of convection is independent of the layer depth, the critical Nusselt number based on the layer thickness should then follow a $Ra^{1/3}$ correlation. This is indeed the case:

$$Nu_c = 0.169 Ra_c^{1/3} \qquad (4)$$

This correlation is approximately 13% higher than the correlation for an upward facing finite surface recommended by Lloyd and Moran (1974) for the turbulent regime. Because the onset of convection is controlled by the conduction boundary layer thickness, the critical Nusselt number based on the conduction boundary layer thickness was found to be essentially constant: 1.70, 1.75, and 1.79 for Cases A, B, and C respectively. Furthermore, similar to infinite layers, the ratio of the cell distance to the thermal boundary layer thickness at the onset of convection was also found to be essentially constant, having a value of 1.95 for Case A and 1.99 for Case B.

For the convenience of data comparison, all the data for the present experiment are summarized in Table 1.

4. *CONCLUDING REMARKS*

By using complementary methods of observation, we were able to illustrate in detail the sequence of events that occur during the initiation of convection above a finite horizontal surface. The results have many features in common with the results of the infinite layer studies. Perhaps it could be argued that the present experiment simply isolates a single "unit" from an infinite number of repeating "units" for an "infinite layer." Convective instability can therefore be thought of as a local phenomenon. Conversely, all experiments are finite. It is entirely likely that the first instability could have occurred near the edge of an enclosure out of the "normal" field of observation in other experiments also. The "edge instability" observed by Kukalka and Mollendorf (1988) is a good example.

It is useful to note that while the critical Rayleigh number depends on the criteria used for the onset of convection, the similarity of the onset transients means that the general trend of the data should still be consistent among different experiments.

We have shown that the deep layer results of the stability calculations are completely equivalent to Howard's theory of conduction boundary layer instability.

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Table 1. Summary of Experimental Data

Case	q (W/m^2)	T_{ref} (°C)	ΔT (°C)	T_{max} (°C)	τ_{max} (min)	Ra_c (x10^{-4})	H (x10^{-5})	Nu_c	δ (10^{-2}m)	$Ra_{c,\delta}$	$Nu_{c,\delta}$	λ_{min}/δ
A	190	31.3	11.4	39.8	70.0	4.37	2.56	5.90	3.91	1040	1.70	1.95
B	355	33.5	16.6	45.8	45.5	8.46	6.37	7.57	3.15	1050	1.75	1.99
C	760	36.4	27.7	56.9	28.5	20.2	19.5	9.71	2.50	1260	1.79	