

ASPECTS OF INCOMPRESSIBILITY [†]

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[†]Prepared for the "Gross Properties of Nuclei and Nuclear Excitations International Workshop XVIII", Hirschegg, Kleinwalsertal, Austria January 15-20, 1990

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INTRODUCTION

The nuclear matter incompressibility K_{∞} (usually called the "compressibility" for some reason lost in antiquity) has been receiving an increasing amount of attention lately. Quite a spirited discussion is underway between groups who favor rather low values for use in simulations of supernova explosions, those who favor much higher values for the explanation of certain measurements in high-energy nuclear collisions, and many others who favor various intermediate values¹⁾. The purpose of this paper is to present some preliminary results concerning the value of K_{∞} arising from a statistical model of macroscopic nuclear properties that is currently under development by Wladek Swiatecki and myself.

This model, which is described in the next section, is meant to serve as a replacement for the traditional Liquid Drop Model and Droplet Model and their various extensions. It is itself an extension of the Thomas-Fermi approach of Seyler and Blanchard. However, it is important to note that we do not regard this approach as a poor approximation to Hartree-Fock, but rather as a vast improvement over the traditional LDM type approaches with their obvious limitations for light nuclei, at the drip lines, for large deformations, and other extreme situations such as large amounts of angular momentum or electric charge.

THE THOMAS-FERMI METHOD AND THE SEYLER-BLANCHARD FORCE

The phenomenological, momentum-dependent, two-body force of Seyler and Blanchard²⁾ has been employed in general studies of saturating two component systems³⁾, for predicting nuclear masses and sizes⁴⁾, for studying nuclei at finite temperatures in equilibrium with their associated vapor⁵⁾, and for a detailed study of the behavior of the surface energy of a two-component system⁶⁾. The nuclear properties are obtained by minimizing the energy of a system of particles whose kinetic energy distribution is obtained from the density by the Thomas-Fermi assumption and whose potential energy is calculated with the phenomenological Seyler-Blanchard force. The Euler equation that results is solved by computer iteration.

[†]This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Nuclear Physics Division of the US Department of Energy under Contract DE - AC03 - 76SF00098.

We found that it was necessary to generalize the original Seyler-Blanchard force slightly in order to obtain the best possible agreement with the measured charge distributions while retaining agreement with measured values of the nuclear masses.[†]

The interaction that was used for two like (*l*), or unlike (*u*), nucleons with separation *r* and relative momentum of magnitude *p* (where *p* is in units of the Fermi momentum of standard nuclear matter) was

$$V(r, p) = -\frac{C}{4\pi a^3} \frac{e^{-r/a}}{r/a} [\alpha_{l,u} + \beta_{l,u} p^2 + \gamma_{l,u}/p], \quad (1)$$

The parameters of the model were determined by a fit to nuclear masses⁷⁾ and constrained by comparing our calculated charge distributions with those obtained from electron scattering experiments⁸⁾. This led to the following nuclear properties:

| | | | |
|-------------------------|---------|-------------|-----|
| nuclear radius constant | $r_0 =$ | 1.13 fm, | |
| volume energy | $a_1 =$ | 16.527 MeV, | |
| symmetry energy | $J =$ | 31.375 MeV, | (2) |
| surface energy | $a_2 =$ | 20.268 MeV, | |
| compressibility | $K =$ | 301.27 MeV. | |

If in addition the properties of pure neutron matter are adjusted to agree with Friedman & Pandharipande⁹⁾ the following parameter values result:

| | | | |
|-------------------------------|---|--------------------------|-----|
| $C = 455.46 \text{ MeV fm}^3$ | , | $a = 0.59542 \text{ fm}$ | |
| $\alpha_l = 0.74597$ | , | $\alpha_u = 2.86331$ | |
| $\beta_l = 0.25255$ | , | $\beta_u = 1.23740$ | (3) |
| $\gamma_l = 0.21329$ | , | $\gamma_u = 0.0$ | |

THE SURFACE ENERGY AND THE COMPRESSIBILITY

The surface energy and the compressibility are closely related since, after all, the surface energy arises in part from the fact that there is a loss of binding associated with reduced density, and the compressibility coefficient K_∞ is the quantity that governs this effect for small density deviations. In fact, we find in our work that the value of K_∞ is determined by the requirement that the surface diffuseness correspond to the one measured in electron scattering and the surface energy is the one that corresponds to a fit of the model to nuclear masses. The effect on the surface energy of varying the diffuseness *b* or the compressibility K_∞ can be seen in fig. 1.

Even though K_∞ has been determined, the effective value of the compressibility K_{eff} for a finite nucleus can be quite a bit smaller because the resistance of the nucleus to changes in scale consists not only of a bulk effect but depends also on surface, curvature and higher order effects. See fig. 2.

[†]We are currently engaged in an extension of the Seyler-Blanchard, Thomas-Fermi approach to the calculation of fission barriers as a function of angular momentum. One of the consequences of this project will be a more precisely determined set of force parameters.

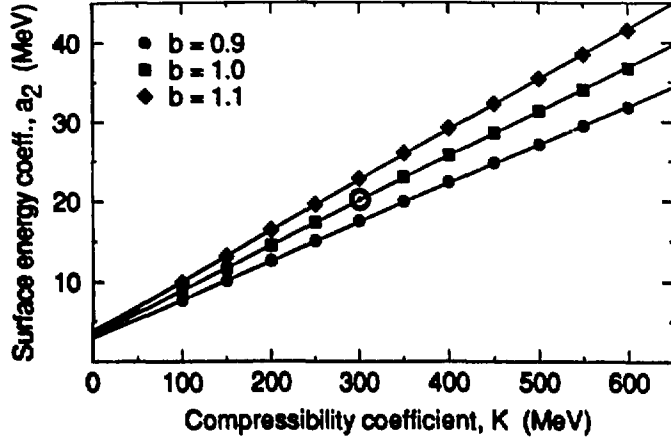


Fig. 1 The value of the surface energy coefficient a_2 is plotted against K_∞ for three different values of the nuclear diffuseness b . The point corresponding to our choice of parameters (given in eq. (3)) is in the circle in the center of the figure.

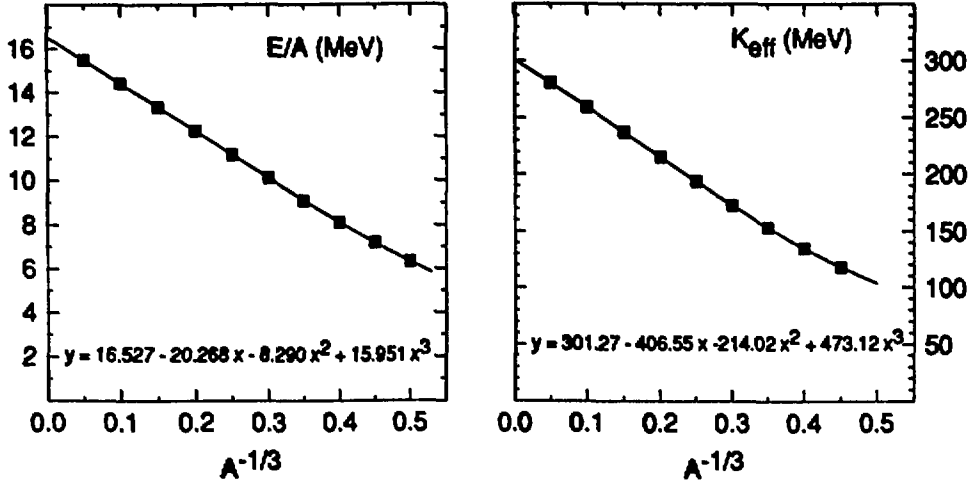


Fig. 2 A plot of the energy per particle E/A versus $A^{-1/3}$ for finite $N = Z$ nuclei (without Coulomb energy) is compared with a similar plot for the quantity K_{eff} .

PREDICTED GMR ENERGIES

The effective compressibility depends not only on the size of the nucleus but also on its composition. In fig. 3 the effect of the neutron excess and the Coulomb repulsion can be clearly seen. In addition, in fig. 4 we show our prediction for the energy of the Giant Monopole Resonance based on these values of K_{eff} and the simple hydrodynamical expression¹⁰⁾ $E_{GMR} = \hbar \frac{\pi}{\sqrt{15}} \sqrt{K_{eff}/B}$, where $B = m\langle r^2 \rangle$, $\langle r^2 \rangle = \frac{3}{5}R^2 + 3b^2$, $R = 1.13 A^{1/3}$ fm and $b = 1$ fm¹¹⁾.

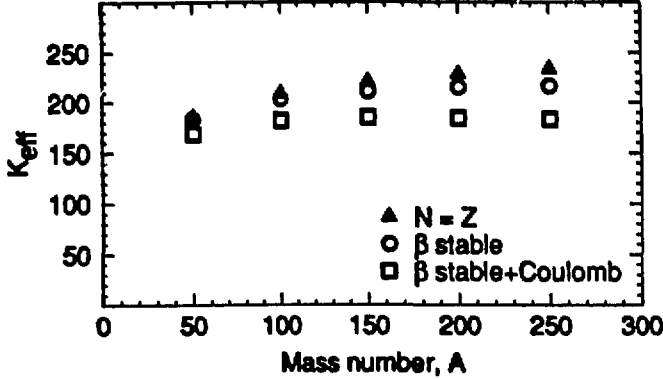


Fig. 3 The effective value of the compressibility K_{eff} for a number of nuclei is plotted versus their mass number A . The triangles correspond to $N = Z$ and have the same values as in fig. 2. The circles show the reduction that occurs when the N, Z ratio is changed to correspond to β stability. The effect of adding the Coulomb repulsion is indicated by the square symbols.

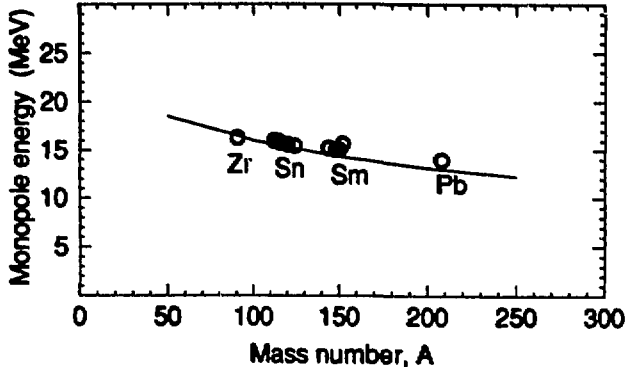


Fig. 4 The solid line corresponds to our estimate of the energy of the Giant Monopole Resonance using the hydrodynamical expression above. The circles correspond to measured values¹⁾ whose errors are claimed to be smaller than the size of the symbols.

CONCLUSION

A Thomas-Fermi nuclear model has been used to display the relationship between the compressibility and the surface energy. In addition it has been used to display the effect of finite size, the effect of neutron excess and the effect of Coulomb repulsion on the effective value of the compressibility. A comparison is also made between measured values of the Giant Monopole Resonance and the results of a simple scaling model.

The author wishes to acknowledge discussions with W.J. Swiatecki who was responsible for a number of the ideas presented here. He also wants to acknowledge the important contribution made by P. Möller.

References

- 1) Sharma, M.M., Proc. NATO Adv. Study Institute on Nuclear Equation of State, Peñiscola, Spain, May 22 - June 2 (1989).
- 2) Seyler, R.G. and Blanchard, C.H., Phys. Rev. **124**, 227 (1961); **131**, 355 (1963).
- 3) Myers, W.D. and Swiatecki, W.J., Ann. of Phys. **55**, 395 (1969).
- 4) von Groote, H., Proc. 3rd Int. Conf. on nuclei far from stability, ed. R. Klapisch, CERN 76-13 p. 595 (1976).
- 5) Küpper, W.A., Wegmann, G. and Hilf, E.R., Ann. of Phys. **88**, 454 (1974); Küpper, W.A., Ph.D. Thesis, Univ. Munich (1978).
- 6) Myers, W.D., Swiatecki, W.J. and Wang, C.S., Nucl. Phys. **A436**, 185 (1985).
- 7) Möller, Peter, private communication.
- 8) de Vries, H., de Jager, C.W. and de Vries, C., At. Dat. and Nucl. Dat. Tables **36** 495(1987).
- 9) Friedman, B. and Pandharipande, V.R., Nucl. Phys. **A361**, 502 (1981).
- 10) This expression was derived from eq. (6A-50) in (A. Bohr and B.R. Mottelson, "Nuclear Structure" Vol. 2, W.A. Benjamin, Inc., 1975), which is $\omega = \pi u_c / R_o$, where $u_c = \sqrt{\frac{K}{9m}}$. To arrive at our expression, which includes a diffuseness correction, we replaced R_o by $\sqrt{\frac{2}{3}} \langle r^2 \rangle$.
- 11) Myers, W.D. and Schmidt, K.-H., Nucl. Phys. **A410**, 61 (1983).