SCATTERING AND LOCALIZABILITY OF ECH POWER IN CIT*

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ABSTRACT

The theory of scattering by drift-wave density fluctuations is applied to electron-cyclotron heating (ECH) in the Compact Ignition Tokamak (CIT). It is found for CIT that the scattering angles are small and have a Gaussian distribution. An analytic result is given for the average number of scattering events suffered by a ray during propagation through the turbulence layer; this average number is 1.3 for the turbulence level expected in CIT. Localizability of ECH power in CIT is also studied for two choices of steering mirror. Better access to outer flux surfaces and better localization is achieved if the power is steered within a poloidal plane.

PLASMA AND MICROWAVE-BEAM PARAMETERS

We study the propagation of electron-cyclotron power in a plasma representative of the beginning of the flat portion of the current evolution. The equilibrium magnetic field $\boldsymbol{B}_0$ is a solution [1] of the Grad-Shafranov equation. This equilibrium is characterized by a field strength of 11.2 T and a safety factor $q = 1.16$ at the magnetic axis. The $q = 2$ surface is at $\psi = 0.76$. (In this paper, flux surfaces are specified by values of the poloidal flux ψ, normalized to have values of zero at the magnetic axis and unity at the separatrix.) The electron-temperature profile is $T_e(\psi) = \hat{T}_e(1 - \psi)$ with $\hat{T}_e = 20\,\text{keV}$. The density profile is $n_e(\psi) = \hat{n}_e(1 - \psi)^{\alpha}$ with $\hat{n}_e = 5 \times 10^{20}\,\text{m}^{-3}$ and $\alpha = 0.5 - 1.5\psi^{20}$. Note that $\alpha \approx 0.5$ except very close to $\psi = 1$, where increasing α yields a smaller $n_e(\psi)$ and a bounded $dn_e/d\psi$.

The ray-tracing calculations in the next section use a ray bundle that represents a Gaussian beam with a Rayleigh length of 1.5 m and a waist of radius $w_0 = 2.2\,\text{cm}$ located in the plasma core. Such a beam could be focussed and directed towards the plasma from a mirror of diameter 0.4 m located 6.7 m from the magnetic axis.

SCATTERING BY DRIFT-WAVE DENSITY FLUCTUATIONS

The scattering theory was formulated by Ott et al. [2], and a numerical method described by Hui et al. [3]. That work is directly applicable to the present situation of ordinary-mode propagation nearly perpendicular to $\boldsymbol{B}_0$.

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We assume that the density fluctuations are located on the outer flux surfaces and have a radial profile given by

$$\langle(\delta n)^2\rangle(\boldsymbol{x}) = \langle(\delta n)^2\rangle(\psi_t)\left[1 - \frac{(\psi - \psi_t)^2}{(\delta\psi_t)^2}\right] \quad ,$$

with $\psi_t = 0.8$ and $\delta\psi_t = 0.2$. At each point in space, the density fluctuations are assumed to have negligible parallel wavenumbers and a perpendicular-wavenumber spectrum

$$S(k') = \langle(\delta n)^2\rangle \frac{1}{\pi\zeta_0^2} \exp\left(-k'^2/\zeta_0^2\right)$$

that is Gaussian and isotropic in the two directions $\perp \boldsymbol{B_0}$.

Our estimate of the turbulence level is taken from Liewer[4]:

$$\langle(\delta n)^2\rangle^{1/2} = 10 n_e(\psi_t) r_s / L_n \quad ,$$

where $r_s \equiv c_s/\Omega_i$ is the sound speed over the ion gyrofrequency and L_n is the density-gradient scale length. Values for these parameters are calculated using the assumed density and temperature profiles and the solution $\boldsymbol{B_0}$ of the Grad-Shafranov equation: $\psi = \psi_t$ at $R \approx 2.73\,\mathrm{m}$, where $B_0 \approx 9\,\mathrm{T}$, $T_e(\psi_t) \approx 4\,\mathrm{keV}$, $r_s \approx 1\,\mathrm{mm}$, $L_n \approx 0.23\,\mathrm{m}$, $n_e(\psi_t) \approx 2.2 \times 10^{20}\,\mathrm{m}^{-3}$, $\langle(\delta n)^2\rangle^{1/2}/n_e(\psi_t) \approx 5\%$, and

$$\left[\langle(\delta n)^2\rangle(\psi_t)\right]^{1/2} = 0.1 \times 10^{20}\,\mathrm{m}^{-3} \quad .$$

We assume that the typical perpendicular wavenumber ζ_0 of the turbulence scales as $1/r_s$. The numerical coefficient in $\zeta_0 = 0.4/r_s$ is consistent with the results of Ref. [5]. Our estimate of this wavenumber for CIT is $\zeta_0 = 0.4\,\mathrm{mm}^{-1}$.

The scattering angles are predominantly small in CIT because the dimensionless parameter $\gamma = (2\omega/c\zeta_0)^2 \approx 1 \times 10^3$ is large. Then the distribution of scattering angles β is essentially Gaussian, $G(\beta) = \exp(-\frac{1}{4}\gamma\beta^2)$. The largeness of γ allows us to write a simple equation for the rate of increase, during propagation of a ray through the turbulence layer, of the probability of occurrence of a scattering event:

$$\frac{dp}{ds} = \sqrt{\pi}\,\frac{(4\pi e^2/m)^2}{2c^2\omega^2\zeta_0}\,\langle(\delta n)^2\rangle(\boldsymbol{x}) \quad .$$

To make further analytic progress, we relate distance s along a ray to poloidal flux ψ by $\psi(s) \approx 1 - s|\nabla\psi|$, which is a good approximation for rays propagating nearly perpendicular to outer flux surfaces and close to the equatorial plane $Z = 0$. Performing the s-integration, we find $p = \int(dp/ds)\,ds$, the average number of scattering events for a ray during passage through the entire turbulent layer. The result is

$$p \approx 3.4 \times 10^{-30}\mathrm{m}^4\,\frac{\delta\psi_t\,\langle(\delta n)^2\rangle(\psi_t)}{|\nabla\psi|\,\zeta_0\,f^2} \quad , \qquad (1)$$

where $\langle(\delta n)^2\rangle(\psi_t)$, $|\nabla\psi|$, and ζ_0 are in MKS units, and the microwave frequency f is in GHz.

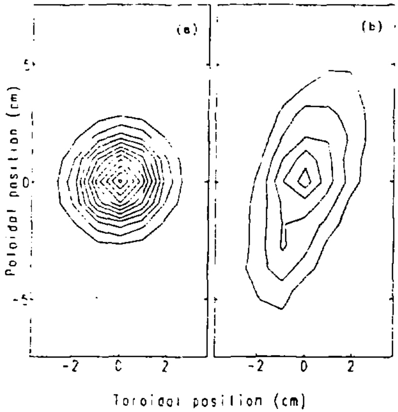

Fig. 1. Profiles of power density impinging on a surface $R = R_{res}$ near the magnetic axis for CIT plasmas (a) without turbulence and (b) with turbulence. Contour levels are multiples of 0.01 W/cm^2 for each Watt of incident power.

We evaluate Eq. (1) for CIT by using $|\nabla\upsilon| \approx 1.35\,\mathrm{m}^{-1}$, $f = 308\,\mathrm{GHz}$, and the parameters given previously. The result is $p \approx 1.3$, which indicates that most rays are scattered at least once. However, most of the scattering angles satisfy $\beta < c\zeta_0/\omega \approx 4^\circ$. An angular deflection of 4° yields a 3.5 cm displacement along the 0.5 m ray path from the turbulence layer to the plasma center. For the turbulence parameters assumed here we thus find that, due to scattering, the beam broadens by only a few centimeters.

The beam broadening is illustrated in Fig. 1, in which we compare beam profiles with and without turbulence. The two cases each have an initially Gaussian beam represented by 500 rays, which are traced through the plasma to a cylindrical surface $R = R_{res}$, where the cyclotron resonance occurs. The profiles are computed from the locations of ray hits on this surface. The broadened beam in Fig. 1(b) is primarily wider in the direction perpendicular to magnetic field lines in the turbulence layer, where the field-line direction is noticeably different from the toroidal direction.

LOCALIZABILITY OF POWER DEPOSITION

For purposes of controlling sawteeth and disruptions, we wish to deposit electron-cyclotron power on a narrow range of flux surfaces near the $q = 1$ and $q = 2$ surfaces. Study of our ability to deposit power locally begins with the constraint of fixed $f = 308\,\mathrm{GHz}$, chosen to satisfy cyclotron resonance at the nominal 11 T field. We obtain underestimates of the width of the deposition layer by ignoring both scattering and the finite width and divergence of the beam. Cyclotron absorption is computed in the weakly relativistic approximation [6].

Injection of power along the tokamak equatorial plane ($Z = 0$) can result in power deposition away from the magnetic axis if a Doppler shift is produced by a mirror that steers the beam into a direction oblique to the magnetic field lines [7]. This method has limited ability to deposit power on outer surfaces (e.g., near $q = 2$) and leads to poor localization (see Fig. 2).

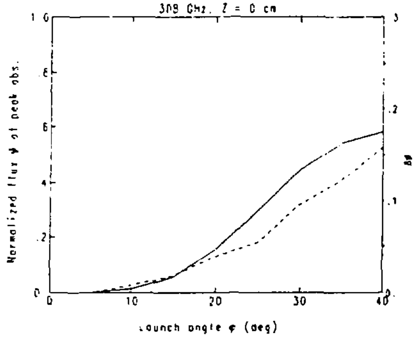

Fig. 2. Localizability of power deposition with steering in the equatorial plane. The solid curve (and the left axis) shows the location in ψ of the peak of the absorption profile. The dashed curve (and the right axis) shows the half-width in ψ of the profile at the $1/e$ point. The launch angle ϕ is measured between the ray and the line from the launch point $(3, 0, 0)$ m to the machine center.

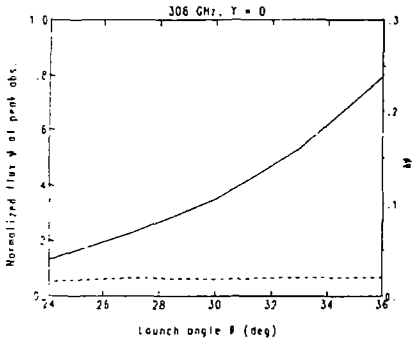

Fig. 3. Localizability of power deposition with steering in the poloidal plane. The launch angle θ is measured between the ray and a horizontal plane. Note that the range of angles θ in this figure is smaller than the range of ϕ in Fig. 2.

A more successful method involves using a mirror to steer the beam in the poloidal plane. For CIT we locate the mirror at $(X, Y, Z) = (3.5, 0, -0.3)$ m and aim the beam to intersect the resonance surface $R = R_{res}$ at the desired value of ψ. As shown in Fig. 3, this method can access flux surfaces as far out as $q = 2$ and can deposit power very locally.

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